CE3203PC: STRUCTURAL ANALYSIS-II

Topic Name: Two Hinged Arches

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ANALYSIS OF TWO-HINGED ARCHES

ANALYSIS OF TWO-HINGED ARCHES

A typical two-hinged arch is shown in Figure. In the case of two-hinged arch, we have four unknown reactions, but there are only three equations of equilibrium available. Hence, the degree of statical indeterminacy is one for two-hinged arch.

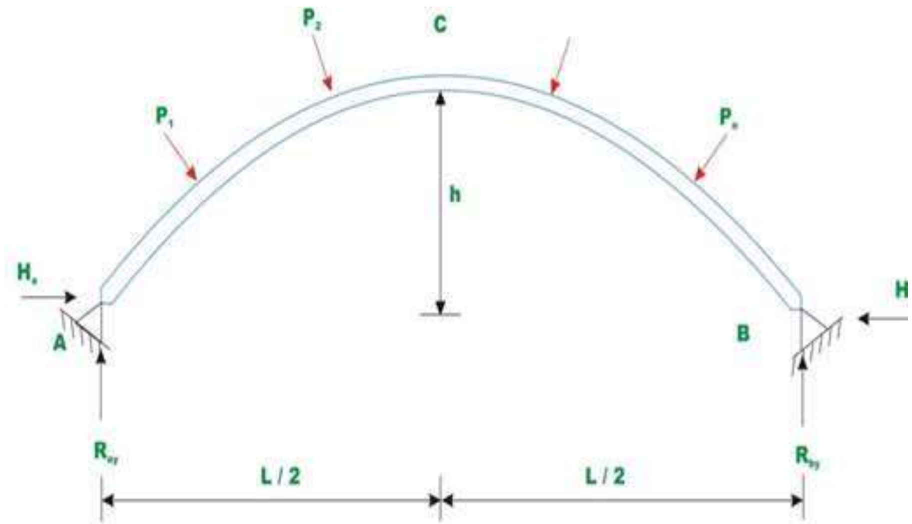


Fig. a Two-Hinged Arch

Symmetrical Two Hinge

Figure shows a two-hinged arch hinged only at the abutments A and B. The vertical reactions R_a and R_b ,of the course, may be determined by taking moments about either hinge.

The horizontal thrust at each support may be determined from the condition that the horizontal displacement of either hinge with respect to the other is zero.

Let M be the beam moment at any section X.

Actual bending moment at the section is given by

$$M_x = (M - H_y)$$

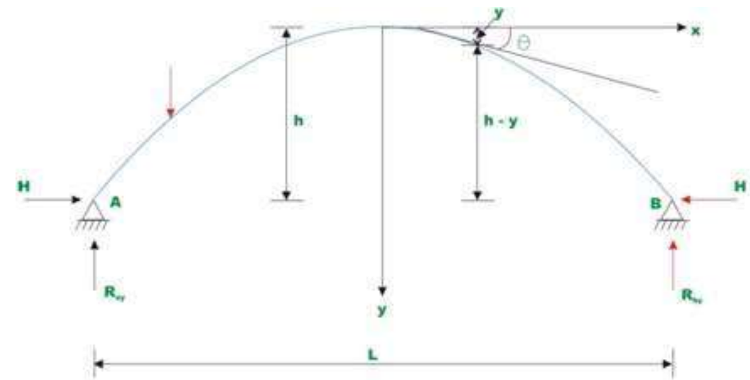


Fig.a

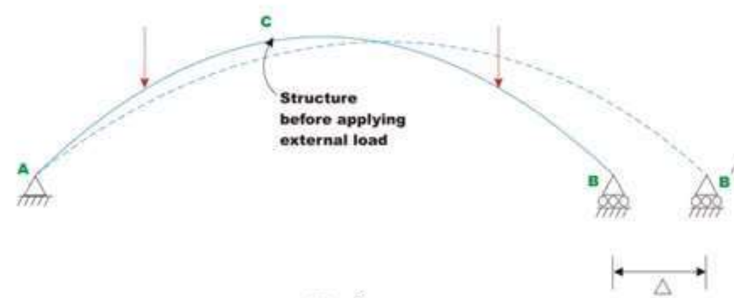


Fig. b

Therefore, total strain energy stored by the whole arch is

$$W_i = \int M_x^2\, ds / 2EI$$

$$= \int (M - Hy)^2\, ds / 2EI$$

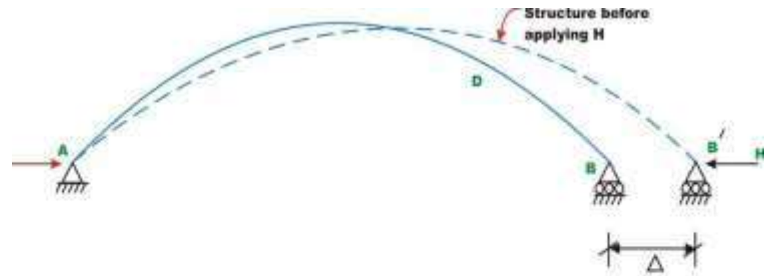


Fig. c

By the first theorem of castigliano the horizontal end relative to other is given by $\partial W_i / \partial H$.

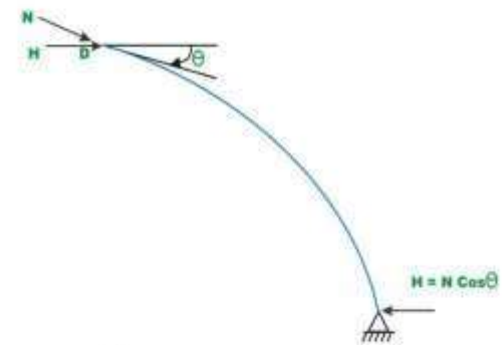


Fig. d

Since such a relative horizontal displacement of one end with respect to the other end is not possible in two hinged arch.

$$\partial W_i / \partial H = 0$$

$$\partial W_i/\partial H = \int 2(M-Hy)(-y)ds/2EI$$

$$= \int Myds/2EI - H\int y^2 ds/2EI = 0$$

$$\mathbf{H = \int Myds/2EI / \int y^2 ds/2EI}$$

If the arch is of uniform flexural rigidity EI,

$$\mathbf{H = \frac{\int Myds}{\int y^2 ds}}$$

TEMPERATURE EFFECT

Consider an unloaded two-hinged arch of span L. When the arch undergoes a uniform temperature change of T ° C , then its span would increase by α LT if it were allowed to expand freely (Fig a). α is the co-efficient of thermal expansion of the arch material. Since the arch is restrained from the horizontal movement, a horizontal force is induced at the support as the temperature is increased.

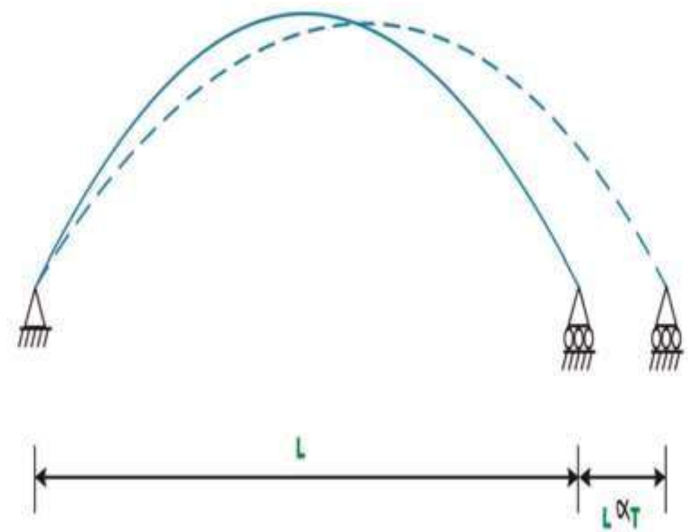


Fig. a

Considering the end B as a roller end with an external horizontal force H applied at B, the bending moment at any section is given by

$$M_x = -Hy$$

Strain energy stored by the arch $= W_i = \int M_x^2 ds/2EI$

$$= \int H^2 y^2 ds/2EI$$

By the first theorem of castigliano,

Inward horizontal movement of B $= \delta$

$$\delta = \partial W_i/\partial H$$

$$= \int 2H(y^2 ds/2EI)$$

$$\delta = H \int y^2 ds/EI$$

The condition that H may represent the horizontal thrust for the two hinged arch subjected to the rise of temperature is,

$$\delta = \alpha T l$$

$$H \int y^2 ds/EI = \alpha T l$$

$$H = \alpha T l / \int y^2 ds/EI$$

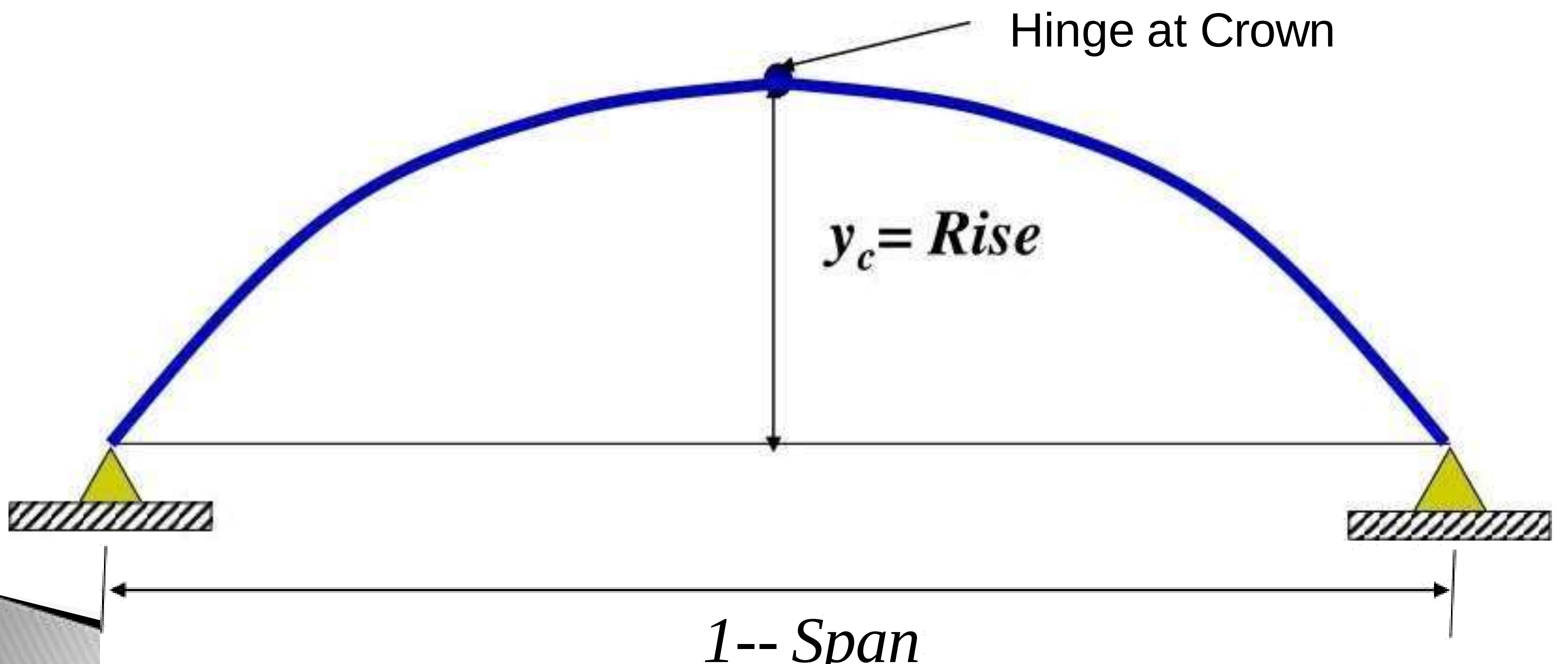
the arch section is of uniform flexural rigidity,

$$\mathbf{H = EI\, \alpha T l / \int y^2 ds}$$

- Arch is a curved structure or humped beam, primarily bears the applied loads by compression.
- The hinge introduced anywhere in the arch makes the structure determinate as
  - Both supports are assumed to be hinged
  - Hinge introduced in the arch provides a further equation to analyze the arch i.e., moment of all forces about hinge is equal to zero.

Introduction

- Normally the third hinge is introduced at the top most point on the arch curve known as crown.



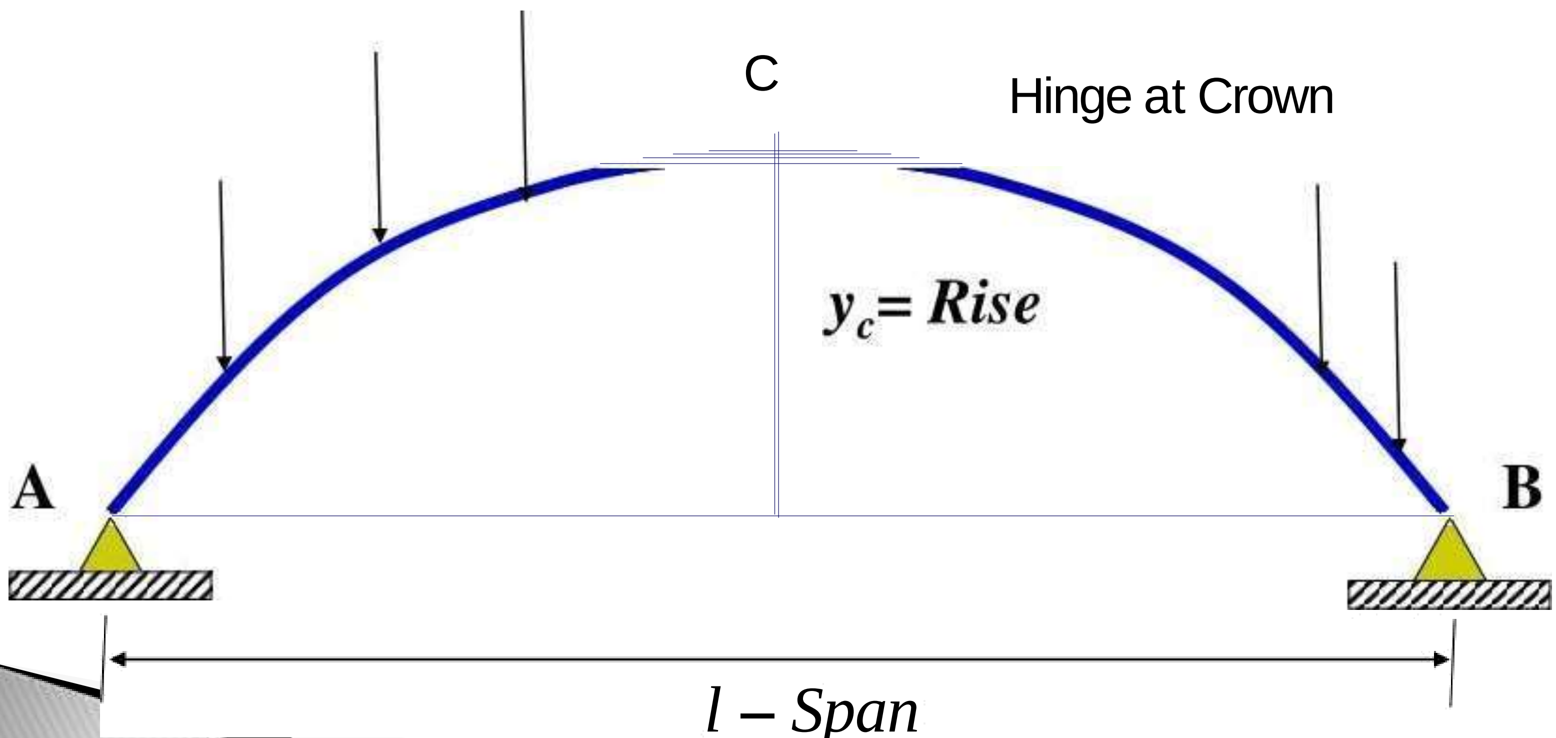
Introduction

- One can solve the arch as beam if we know the horizontal reactions at various supports.

- With the help of third hinge, we can easily determine the horizontal reactions and hence the arch can be analyzed.

Arch Formula’s

- Consider an arch ACB, hinged at A, B and C.
- / is the horizontal span and y_c is central rise.



Arch Formula's

- Obtain the vertical reactions Vy and V_B at the ends as usual. To find the horizontal thrust, MC the moment at central hinge must be zero.

General Derivation

- To find *H* the horizontal thrust, M the moment at central hinge C must be known.

$$3f, = y, + H.y, — 0$$

$$H \quad —— \quad ——— \quad (1)$$

$$X.-$$

- We can find moment at any cross-section X of the arch whose coordinates are (x, y).

$$M, \; — y \; + Hy \quad —— \quad — (2)$$

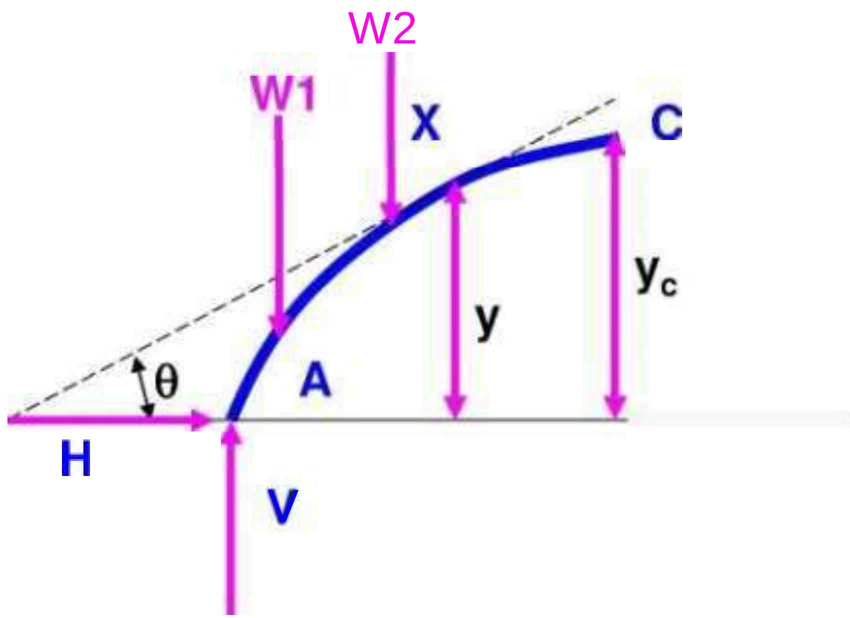
General Derivation

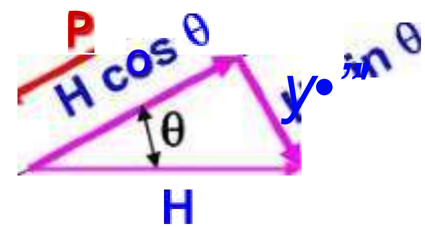
- The vertical and horizontal actions on the section, considering the portion AX arc,
- Vertical reaction can be find out by vertical shear force at the section as for a straight horizontal beam.
- Horizontal thrust at both ends is same.

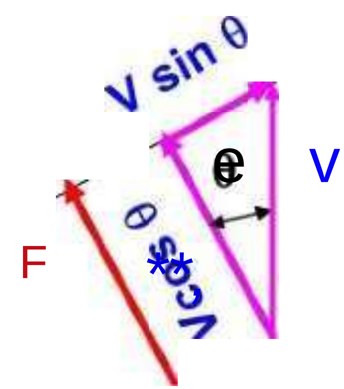
$$= \quad -W1 - \mathrm{JV2}$$

$$H = H$$

General Derivation







P —Tł cost + V sin6————(3)

F —H sink— V cost————(4)

Æ0

General Derivation

- Draw the tangent at X to the centre-line of the arch and let its inclination to the horizontal must be 6.
- Resolving V and H normally to the section and tangentially, i.e., along the tangent at X)
- If the resultant T is required, use eq.5

$P \quad C \cos d + V \sin k$ ——— (3)

F —/ $\sin 6$ — $V \cos 6$ ——— (4)

$T \quad \sqrt{V^2 + H^2}..or..\sqrt{P^2 + F'}$ —— (S)

Parabolic Arch

- If the three hinged arch is parabolic in shape and if it carries a uniformly distributed load over the entire span,
  - every action of the arch will be purely in compression,
  - » resisting only a normal thrust;
  - there will be no shear force nor B. M. at the section.

Parabolic Arch

- The linear arch for a given load system on an arch represents the y-diagram.

x With a uniformly distributed load over the entire span, the y-diagram is a parabola.

- The linear arch which is parabolic will, then, have three points (at the hinges) in common with the centre-line of the actual arch, which is also parabolic.

Parabolic Arch

- The linear arch will therefore be identical with the actual arch.
- x For any other loading on a parabolic arch, there will be three straining actions, P , F and M at any section.
- To obtain the bending moment, it will be necessary to calculate the rise at any section of the arch.

x ——, y ——
y,

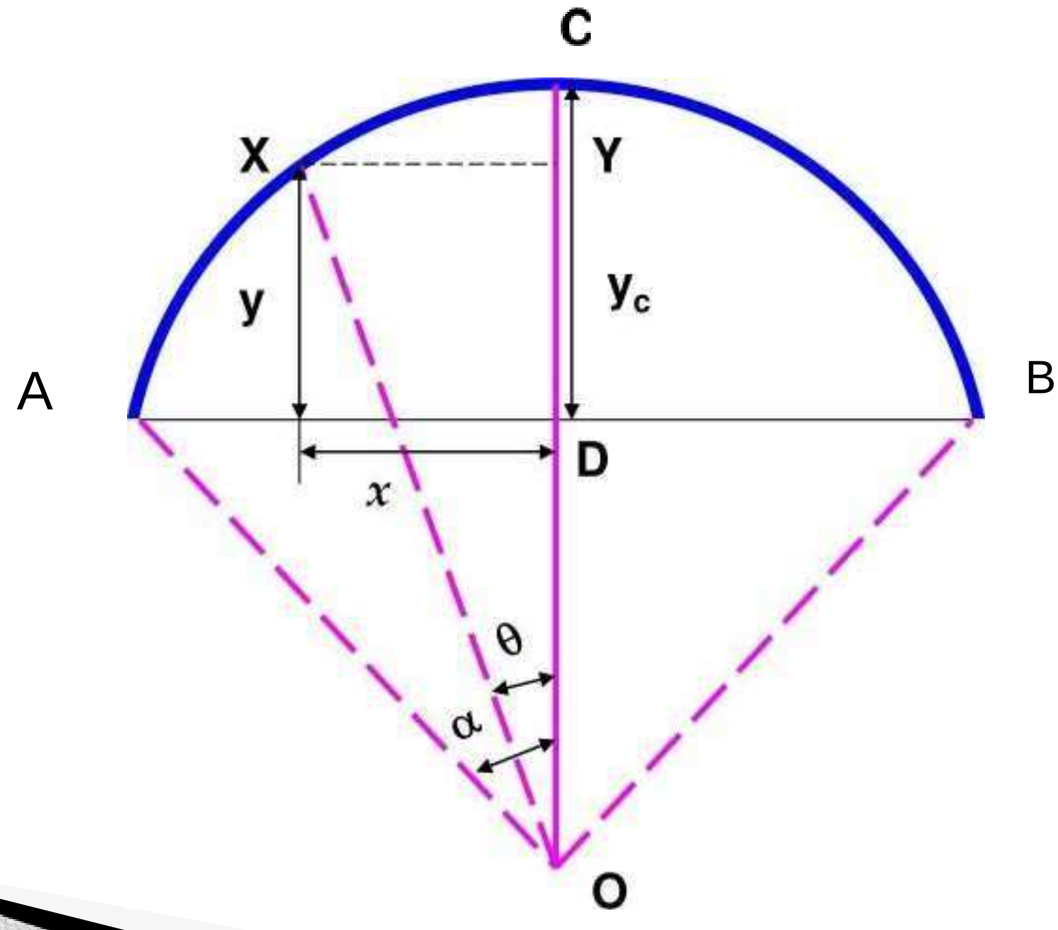
2 · 2
4y

4y · x{l —x)
J 2

d› - tan8= 4/. (/ —2x)
dx

It may also be noted that at quarter points, where z=//'4, the *rize* ^/4 y,

Circular Arch



Circular Arch

- If the centre-line of the arch is a segment of a circle of radius R, it is more convenient to have the origin at D, the middle of the span. Let (x, y) be the coordinates of a section.

Circular Arch

$AO \quad OB \quad R$

$AD — DB \quad \frac{'}{2}$

$DC \quad — y$

$OY \quad — OD \quad + DY \quad — R \quad — \quad) + \quad y$

•y =•

07€ ^{2} = $OY^{2} + XY$

$$R^2 = x^2 + \{(R - y_c) + y\}^2$$

$y \ (2A — y,) = \frac{l^2}{4}$

$\sin o \quad \frac{AD}{AO} \quad \frac{1}{2A}$

$\cos T \quad \frac{OD}{} \quad \frac{(A — y)}{}$

$x = OX \sin 8 \ — / t \sin Ø$

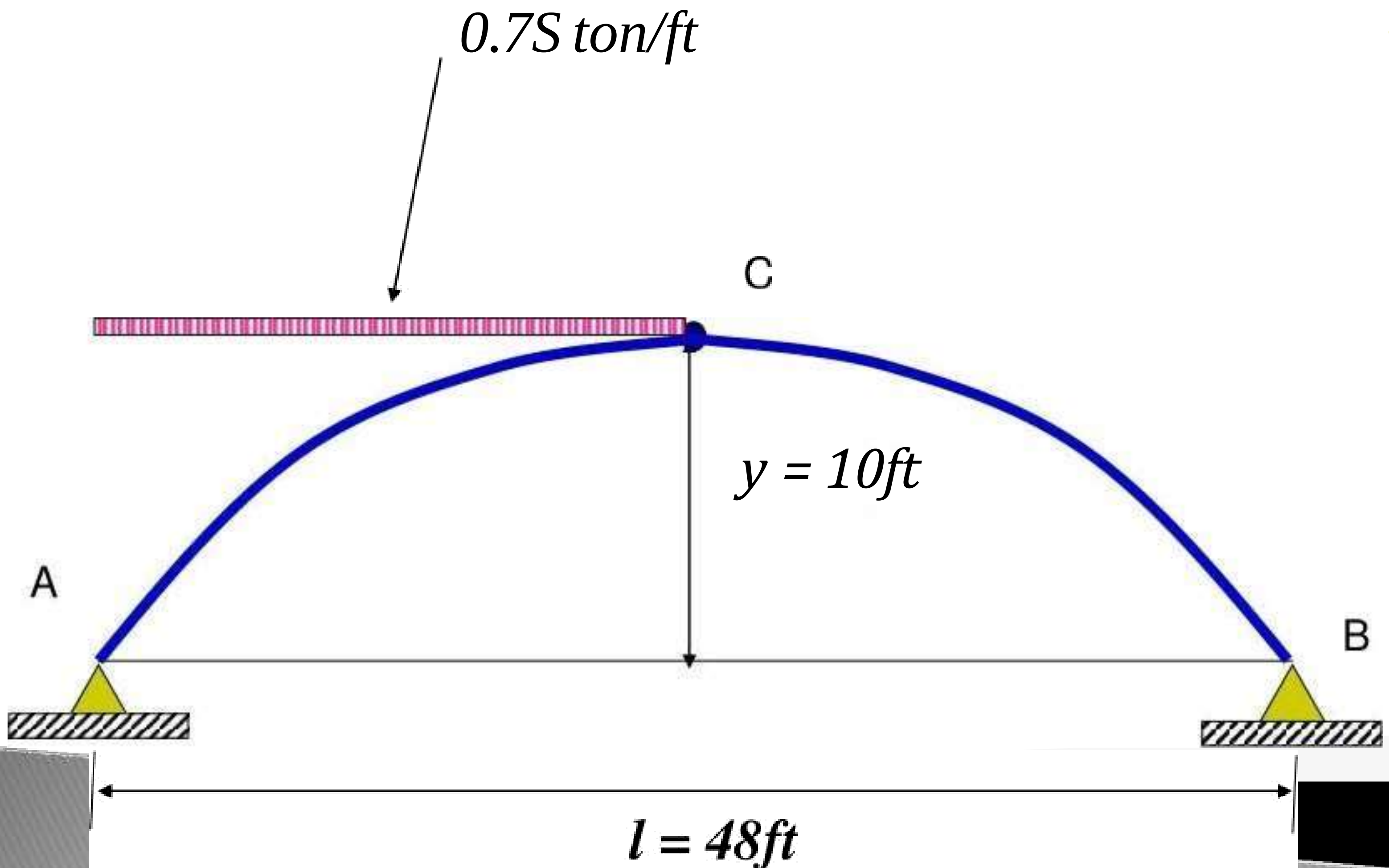
$y = OY \ — OD \ — R \cos Ø \ — A c o s o$

y' = /t(«o»B —€oso)

Æ8

Example 01

- A three hinged Parabolic arch, at the crown and springings has a horizontal span of 48ft. It carries UDL of .75 Ton/ft run over the left hand half of the span. Rise = 10’. Calculate the
  - Reactions
  - Normal Thrust
  - Shear force and BM at 6, 12, 30ft from left hinge.


0.7S ton/ft
C
y = 10ft
A
B
l = 48ft

Solution

$V_A = 13.5$ T; $V_B = 4.5$ T; $H = 10.8$ T

y = 5/288*x(48-x), 0 = 5/288*(48-2x)

x	y	0	cos0	sin8	M	V	P	F
6	4.375	32º	0.848	0.529	-20.25	9	13.93	1.91
12	7.5	22◦37'	0.923	0.348	-27	4.5	11.7	0
30	9.66	11°46’	0.979	0.203	20.25	4.5	11.49	2.2

Example 02

- Circular arch of span 80 ft with central rise 16 ft is hinged at crown and supports. Carries a point load of 10 tons 20ft from left support.
  - Reactions
  - Normal Thrust
  - Maximum and Minimum BM

Solution

$V_A = 7\cdot5$

T; $V_B =$

2.5 T; $H =$

6.25 T

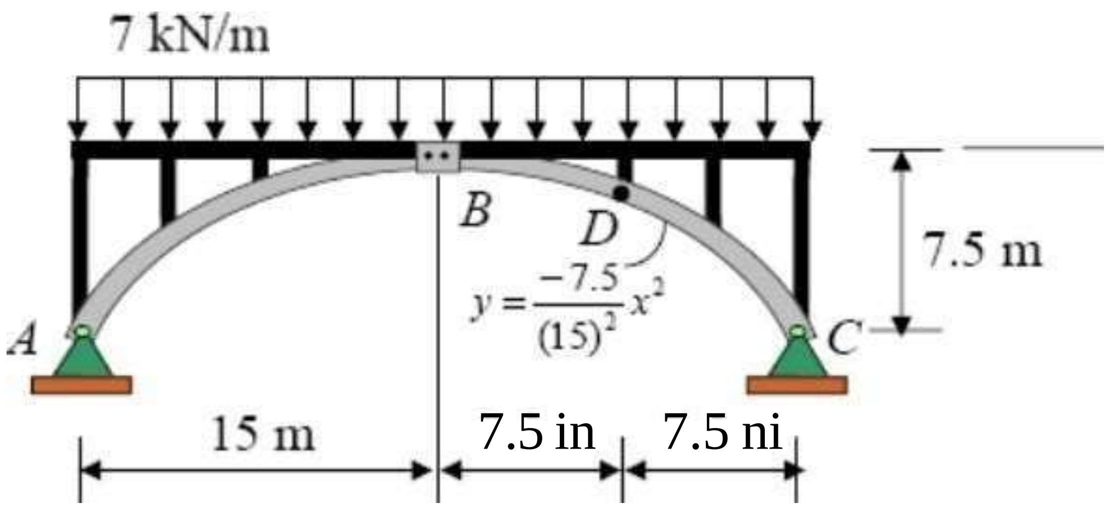
y,(2R−Yc)− /4t

$R^2=x^2+\{y1(R$ c)t^2

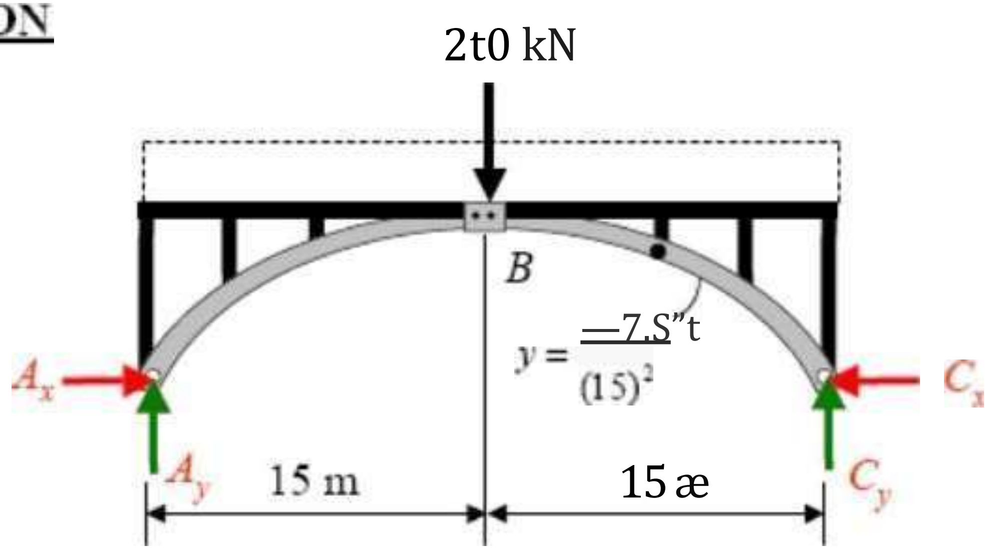
Example •
6

The three-hinRcd open-spaudrcl arch bridpc shown iii the figure bclou has a parabolic shape and supports the unifomi load . Show that the parabolic arch is subjected only to axial compression at an intermediate point D along its axis.

Assume the load is iuiifoivily tmnsmitted to the arch ribs.



SOLUTION



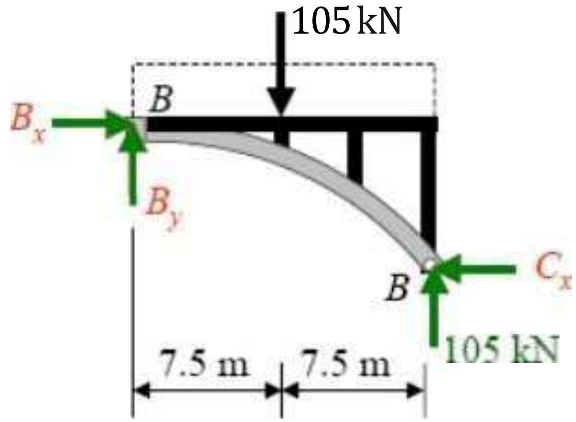
Entire arch :

$+\circlearrowleft \Sigma M_A = 0$: C,,(30)—210(15) 0

C ,—105 IN

+Î £P = 0: d ,—210+105= 0

At,—t05 kN



Arch segment *BC* :

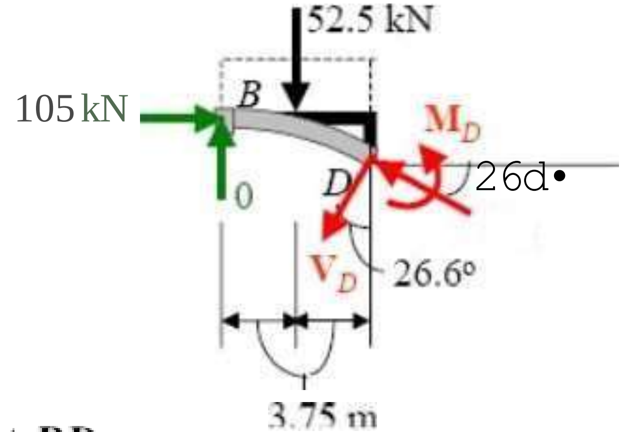
$$+\circlearrowright\ \Sigma M_B = 0: \quad -105(7.5)+105(15)-C_x(7.5)=0$$

$$C_x = 105\ \text{kN}$$

ZF,−0 : B_E —105 k.Y

+Î ZF› = 0: B› —105+ 105 —0

B —0



Arch segment *BD*

:

Asection of the arch taken through point *D*. *x* —7.5 m y = -7.5(7.5*)/(t5)* = -1.875 m: is shown in the figure. The slope of the segment at *D* is

$$\tan\theta = \frac{dy}{dx} = \frac{-15}{(15)^2} x\Big|_{x=7.5} = -0.5 ,$$ θ= 26.6s

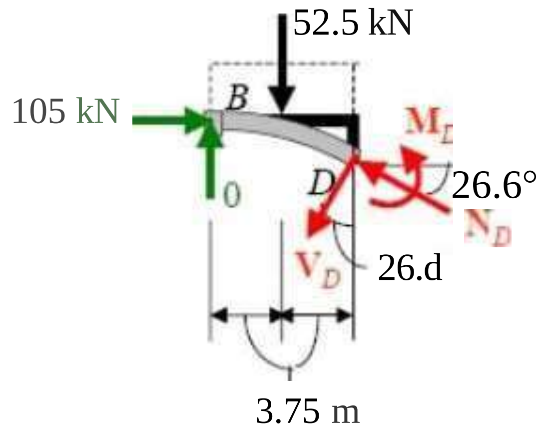
$\xrightarrow{+} \Sigma F_x = 0$: 105 - *Np* cos 26.6° - Kpsin 26.6e = 0

<f . —0: -52.5 + *Np* sin 26.6° - Kpcos 26,go 0

$\circlearrowleft + \Sigma M_D = 0$: *MQ* + 52.5(3.75) - 105(1.875) = 0

N —117.40 kN. *V* —0, I —0 kN

Alternate Method



y y y y

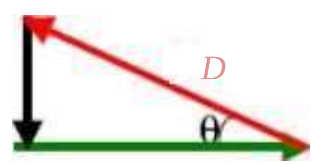
6 e

Arch segment *BD :*

A section of the arch taken throiigb point *D*. *x* - 7.5 m, y - -7.5(7.5*)/(15)2= -1.875 m, iS shoWn in tlle figure. T6e Slope Of the Segment at *D E*

$$\tan\theta = \frac{dy}{dx} = \frac{-15}{(15)^2}\, x\Big|_{x=7.5} = -0.5 \qquad 6 = 26.6^\circ$$

7.5 iv = (7.5)(7)= 52.5 kN



Nq=25 kN

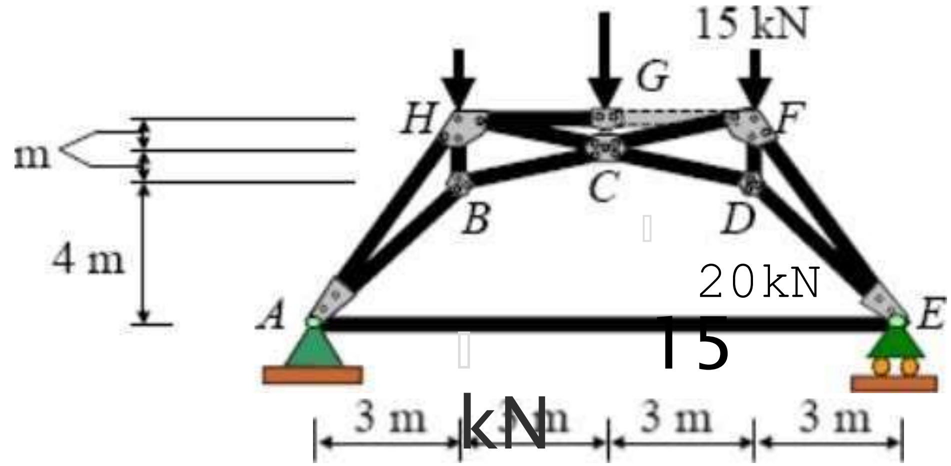
$T \quad = Tg = \quad (1\ \ 5) + (\ 2\ 5)$

$T_{max} = 117.4$ kN

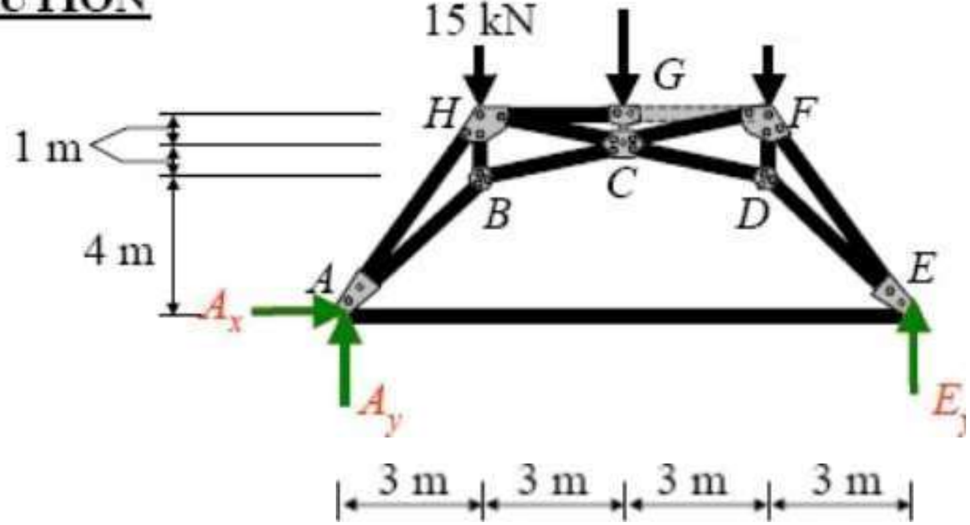
Notes : Since the arch is a parabola. their arc no shear acid bending moment, ooly I_D *E* present

Example S-7

- The three-hinged tied arch is subjected to the loading shown in the figure below. Detemiine the force in members CA rind *CB*. Tb*c* dashed member GF of the trust is intended to carry no force.



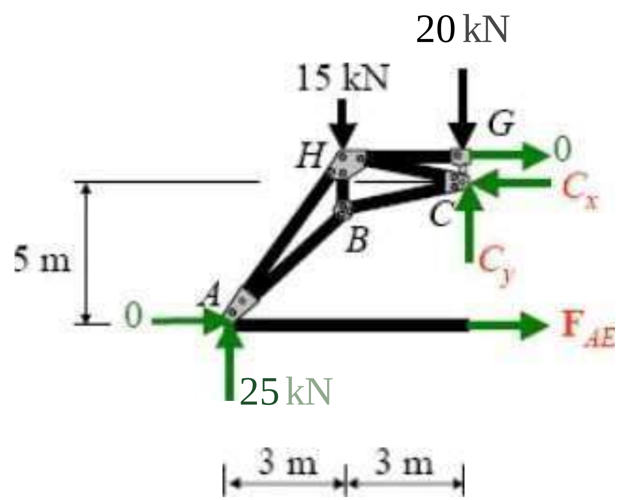
SOLUTION



$$\overset{+}{\circlearrowright}\ \Sigma M_A = 0: \qquad E_y(12) - 15(3) - 20(6) - 15(9) = 0$$

$$E_y = 25\ \text{kN}$$

$$A_y = 25\ \text{kN}$$



-+) - 0: N (5)—25(6)+15(3)= 0

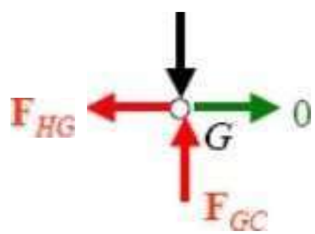
F -- 21.0 kN

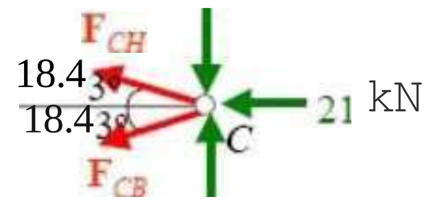
ZF,=0: -Cg + 21= 0

Cq - 21.0 kN

$+\uparrow \Sigma F_y = 0$: 25—15—20+ C' = 0

Cl —10 kN





Joint G :

$\xrightarrow{+} \Sigma F_x = 0: \quad F_{HG} = 0$

$+\uparrow \Sigma F_y = 0: \quad F_{GC} - 20 = 0$

F -- 20 kN (C)

Joint C :

−*ZF,=0 :

-N cos18.43 - N cos18.43 - 21= 0

+Î ZF = 0:

N sinl8.43 - Ny cosl8.43 - 20 10 = 0

Thus,

F -- 4.75 kN (T).

F - -26.88 kN (C)

1.2 Moment Distribution method

Outline of the presentation

- Introduction to moment distribution method.
- Important terms.
- Sign conventions.
- Fixed end moments (FEM)
- Examples;
- (A) example of simply supported beam
- (B) example of fixed supported beam with sinking of support.

- The moment distribution method was first introduced by Prof. Hardy Cross of Illinois University in 1930.

- This method provides a convinient means of analysing statically indeterminate beams and rigid frames.

- It is used when number of reduntants are large and when other method becomes very tedious.

Important terms

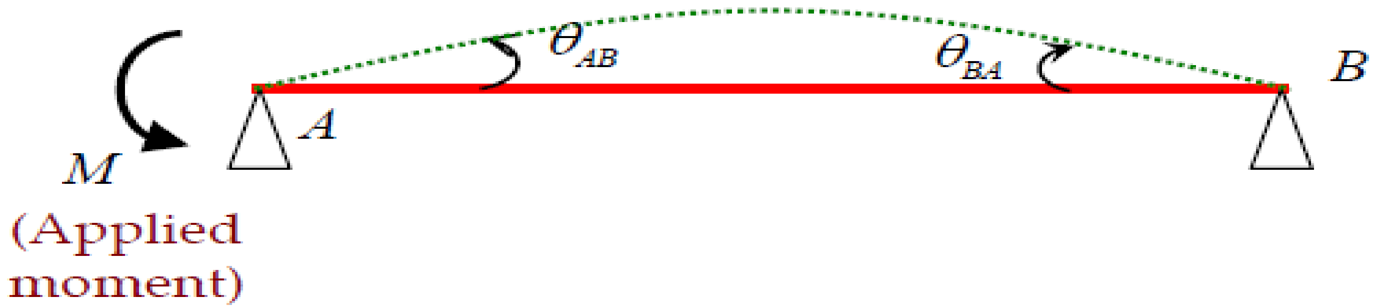
1. Stiffness

The moment required to produce a unit rotation (slope) at a simply supported end of a member is called Stiffness. It is denoted by 'K'.

A) Stiffness when both ends are hinged.

B) Stiffness when both ends are fixed.

A) Beam hinged at both ends:



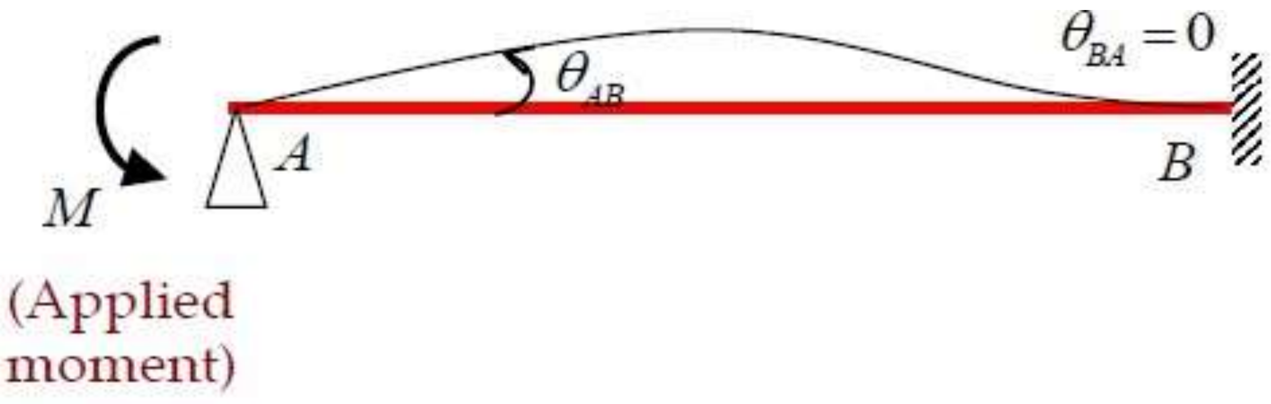
$$\frac{M_{AB}}{\theta_{AB}} = \frac{3EI}{L}$$

i.e., the moment required at A to induce a unit rotation at A is $\frac{3EI}{L}$

(when the far end B is free to rotate)

This moment, i.e., moment required to induce a unit rotation, is called stiffness (denoted by k).

B) Beam hinged at near end and fixed at far end:



$$M_{AB} = \frac{2EI}{L}(2\theta_{AB} + 0) \qquad \Rightarrow \frac{M_{AB}}{\theta_{AB}} = \frac{4EI}{L}$$

i.e., the moment required at A to induce a unit rotation at A is $\frac{4EI}{L}$

(when the far end B is fixed against rotation)

A moment applied at the near end induces at a fixed far end a moment equal to half its magnitude, in the same direction.

Half of moment applied at the near end is carried over to the fixed far end.

Carry over factor is 1/2.

Cont..

Distribution factor (D.F.)

- The factor by which the applied moment is distributed to the member is known as the distribution factor.
- Figure:

  - far-end pined (DF = 1)
  - far-end fixed (DF = 0)

-

Cont..

Several members meeting at a joint

$$M_1 = \frac{3E_1I_1}{L_1}\theta = k_1\theta$$

$$M_2 = \frac{4E_2I_2}{L_2}\theta = k_2\theta$$

$$M_3 = \frac{3E_3I_3}{L_3}\theta = k_3\theta$$

$$M_4 = \frac{4E_4I_4}{L_4}\theta = k_4\theta$$

$$M_1 : M_2 : M_3 : M_4 :: k_1 : k_2 : k_3 : k_4$$

Cont….

$$M_1 = \frac{k_1}{k_1 + k_2 + k_3 + k_4} M = \frac{k_1}{\sum k} M$$

$$M_i = \frac{k_i}{\sum k} M$$

A moment applied at a joint, where several members meet, will be distributed amongst the members in proportion to their stiffness.

$$M_i = \frac{k_i}{\sum k} M$$

distribution factor

Sign Conventions

A) Support moments :

clockwise moment = +ve

anticlockwise moment = -ve

B) Rotation (slope):

clockwise moment = +ve

anticlockwise moment = -ve

Cont...

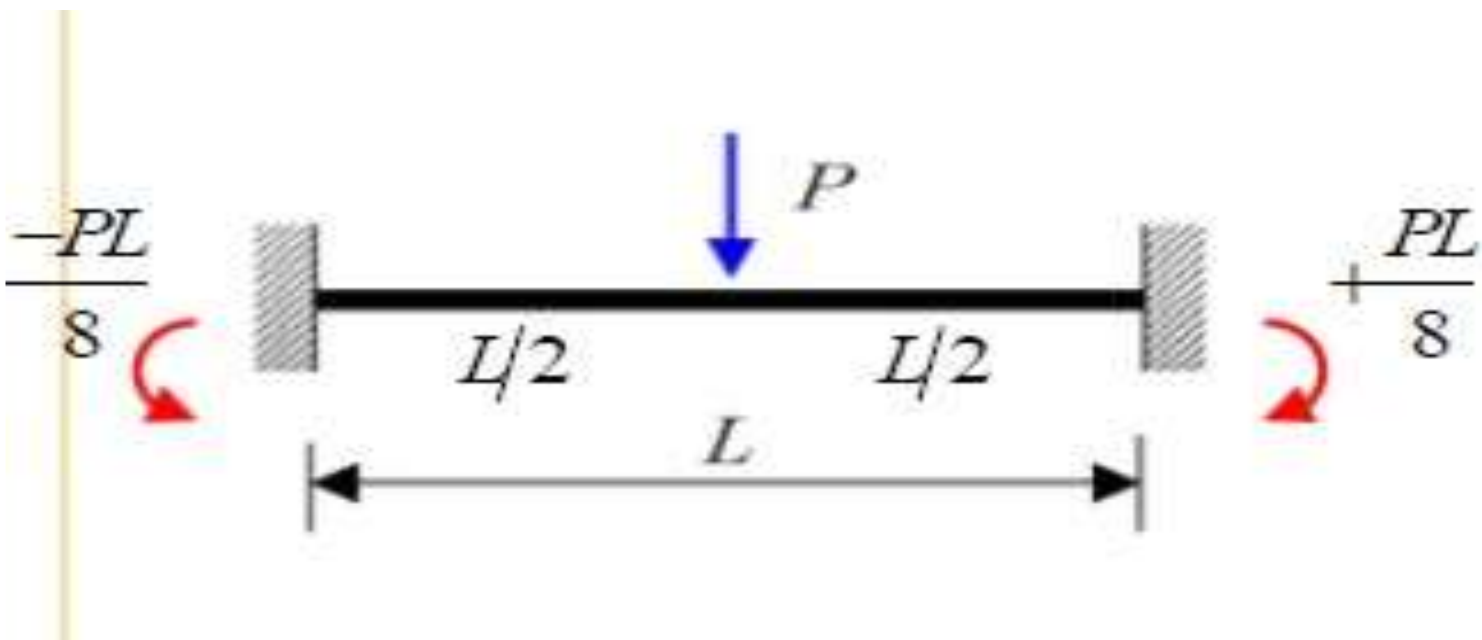
C) Sinking (settlement)

- The settlement will be taken as +ve, if it rotates the beam as a whole in clockwise direction.

- The settlement will be taken as -ve, if it rotates the beam as a whole in anti-clockwise direction.

Fixed End Moments

- The fixed end moments for the various load cases is as shown in figure;
- a) for centric loading;



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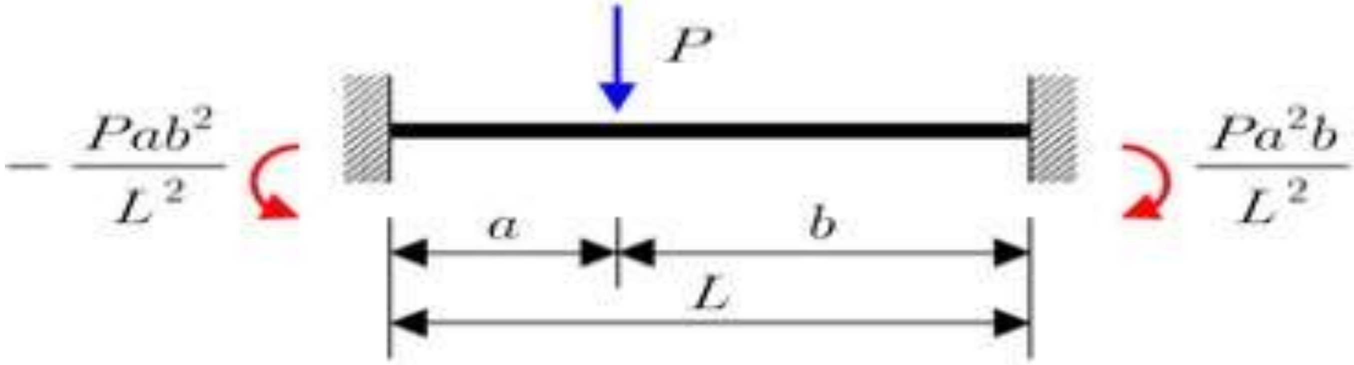
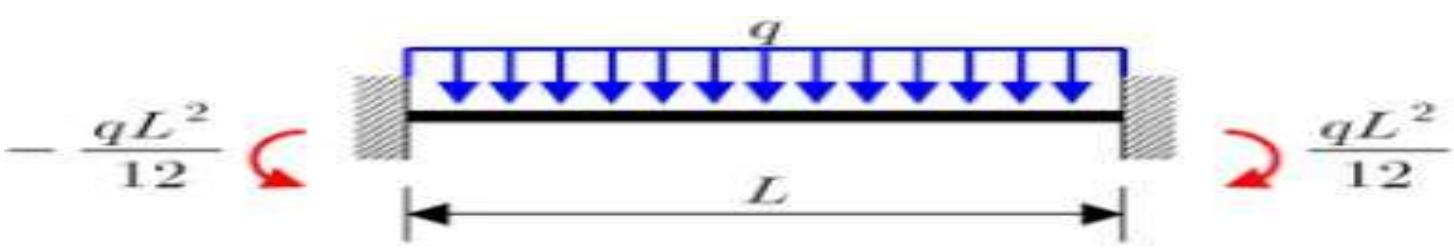
b) for eccentric loading, udl,rotation,sinking of supports & uvl





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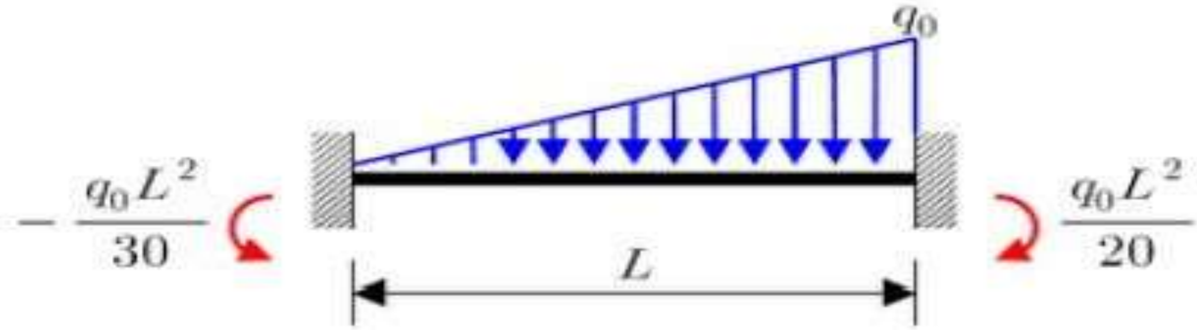
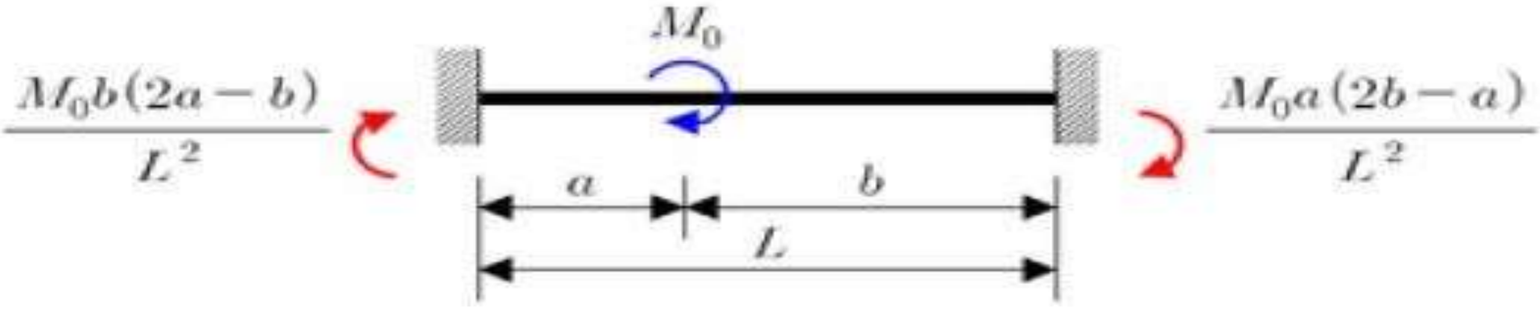
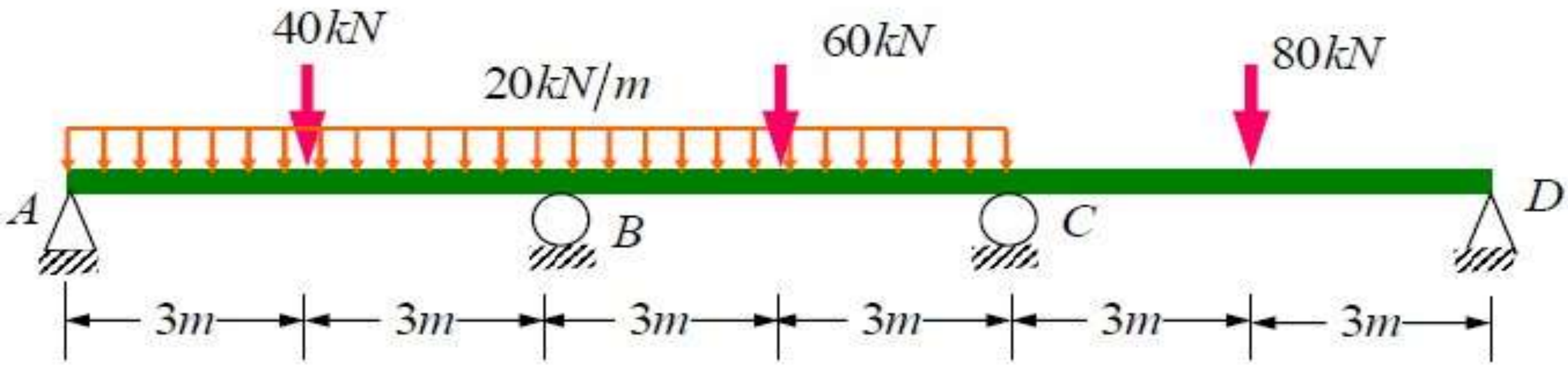

q_0
$-\frac{q_0 L^2}{30}$
$\frac{q_0 L^2}{20}$
L


M_0
$\frac{M_0 b(2a-b)}{L^2}$
$\frac{M_0 a(2b-a)}{L^2}$
a
b
L

Cont...

- Fixed end moment for sinking of supports : $-\frac{6EI\delta}{L^2}$

Example 1



Fixed end moments

$$-FEM_{AB} = FEM_{BA} = \frac{wl^2}{12} + \frac{Pl}{8} = \frac{20\times 6^2}{12} + \frac{40\times 6}{8} = 60 + 30 = 90\,kNm$$

$$-FEM_{BC} = FEM_{CB} = \frac{wl^2}{12} + \frac{Pl}{8} = \frac{20\times 6^2}{12} + \frac{60\times 6}{8} = 60 + 45 = 105\,kNm$$

$$-FEM_{CD} = FEM_{DC} = \frac{Pl}{8} = \frac{80\times 6}{8} = 60\,kNm$$

Joint	Member	K (Stifness)	ΣK (Total stiffness)	D.F
A	—	—	—	1
B	BA	$\frac{3EI}{6} = 0.5EI$	1.16EI	0.43
	BC	$4EI/6 = 0.66EI$		0.57
C	CB	$\frac{4EI}{6} = 0.66EI$	1.16EI	0.57
	CD	$3EI/6 = 0.5EI$		0.43
D	—	—	—	1

Cont..

A	B		C		D	
	0.429	0.571	0.571	0.429		Distribution factors
-90	+90	-105	+105	-60	+60	Fixed End Moments
+90	+45			-30	-60	Release A& D, and carry over
0	**+135**	**-105**	**+105**	**-90**	**0**	Initial moments
	-12.87	-17.13	-8.565	-6.435		Distribution
		-4.283	-8.565			Carry over
	+1.837	+2.445	4.89	3.674		Distribution
		+2.445	1.223			Carry over
	-1.049	-1.396	-0.698	-0.524		Distribution
0	**+122.92**	**-122.92**	**+93.29**	**- 93.29**	**0**	Final Moments

Example-5 : Analyse the beam shown in figure by moment distribution method and draw SFD and BMD. **(G.U., Dec. 2006)**

Solution :

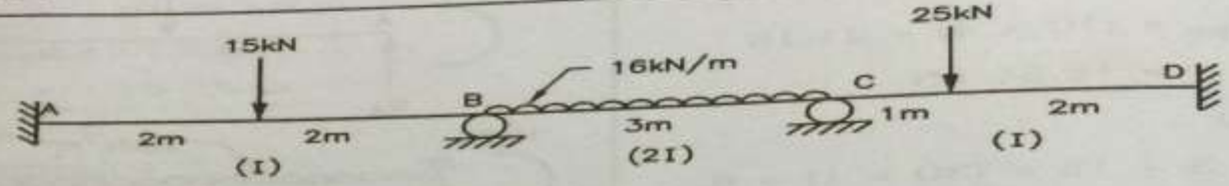


FIG. 3.21

(a) Fixed End Moments (FEM) :

$$M_f\,AB = -\frac{Wl}{8} = -\frac{15 \times 4}{8} = -7.5 \text{ kN.m}$$

$$M_f\,BA = +\frac{Wl}{8} = 7.5 \text{ kN.m}$$

$$M_f\,BC = -\frac{wl^2}{12} = -\frac{16 \times 3^2}{12} = -12 \text{ kN.m}$$

$$M_f\,CB = +\frac{wl^2}{12} = 12 \text{ kN.m}$$

$$M_f\,CD = -\frac{W\,ab^2}{l^2} = -\frac{25 \times 1 \times 2^2}{3^2} = -11.11 \text{ kN.m}$$

$$M_f\,DC = +\frac{W\,ba^2}{l^2} = \frac{25 \times 2 \times 1^2}{3^2} = 5.55 \text{ kN.m}$$

(b) Distribution Factors (D.F.) :

Sr.No.	Joint	Member	k	Σk	D.F. = $\frac{k}{\Sigma k}$
1.	B	BA	$\frac{4\,EI}{4} = 1.0\,EI$	3.67 EI	0.27
		BC	$\frac{4\,E\,(2I)}{3} = 2.67\,EI$		0.73
2.	C	CB	$\frac{4\,E\,(2I)}{3} = 2.67\,EI$	4.0 EI	0.67
		CD	$\frac{4\,EI}{3} = 1.33\,EI$		0.33
3.	A	–	–	–	0
4.	D	–	–	–	0

(c) Moment Distribution (M.D.) :

	A	B		C		D	
	0	0.27	0.73	0.67	0.33	0	D.F.
Sum ↓	−7.5	7.5	−12	12	−11.11	5.55	F.E.M.
	0	1.21	3.28	−0.59	−0.30	0	Balance
	0.60	0	−0.30	1.64	0	−0.15	C.O.
	0	0.08	0.22	−1.10	−0.54	0	Balance
	0.04	0	−0.55	0.11	0	−0.27	C.O.
	0	0.15	0.40	−0.07	−0.04	0	Balance
	0.075	0	−0.035	0.20	0	−0.02	C.O.
	0	0.009	0.026	−0.13	−0.07	0	Balance
	−6.79	8.95	−8.95	12.06	−12.06	5.11	Final moments

↳ balanci

$M_{AB} = -6.79$ kN.m.

$M_{BA} = 8.95$ kN.m. $M_{BC} = -8.95$ kN.m

$M_{CB} = 12.06$ kN.m. $M_{CD} = -12.06$ kN.m

$M_{DC} = 5.11$ kN.m

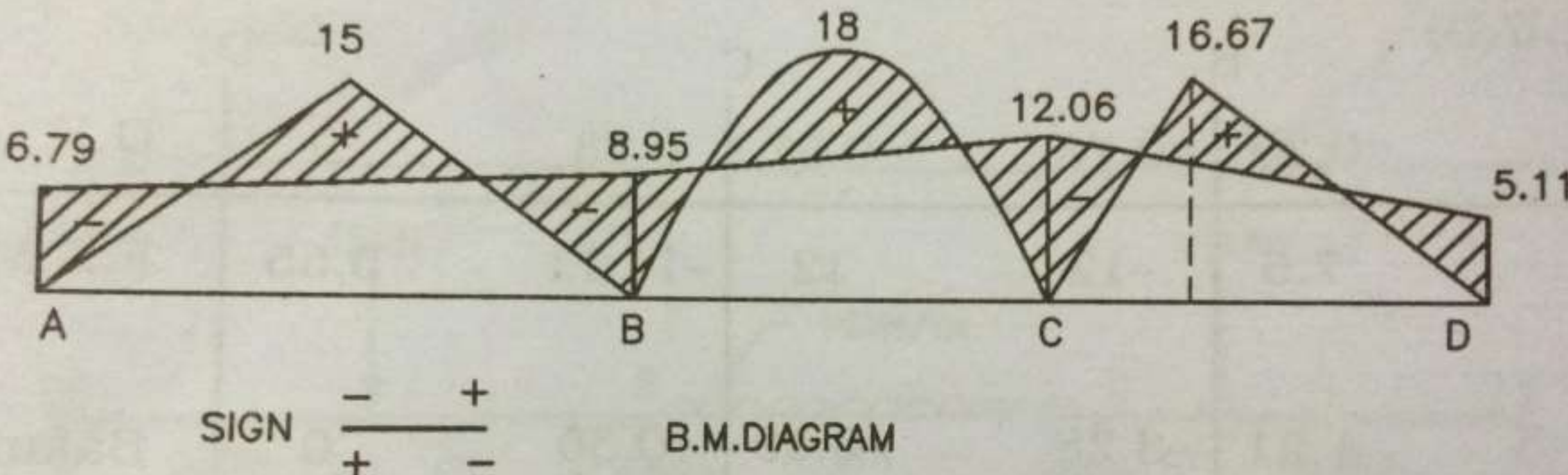
(d) B.M. diagram :

Simply supported moments.

Span AB. $M = \dfrac{Wl}{4} = \dfrac{15 \times 4}{4} = 15$ kN.m

Span BC. $M = \dfrac{wl^2}{8} = \dfrac{16 \times 3^2}{8} = 18$ kN.m

Span CD. $M = \dfrac{W\,ab}{l} = \dfrac{25 \times 1 \times 2}{3} = 16.67$ kN.m


15
18
16.67
6.79
8.95
12.06
5.11
A
B
C
D
SIGN
B.M.DIAGRAM

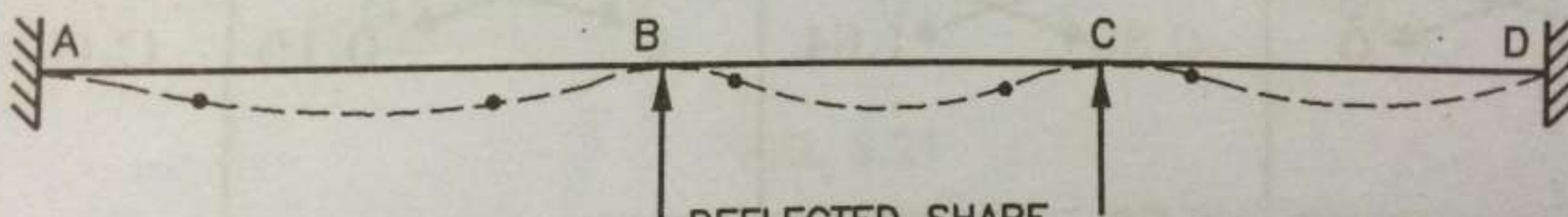
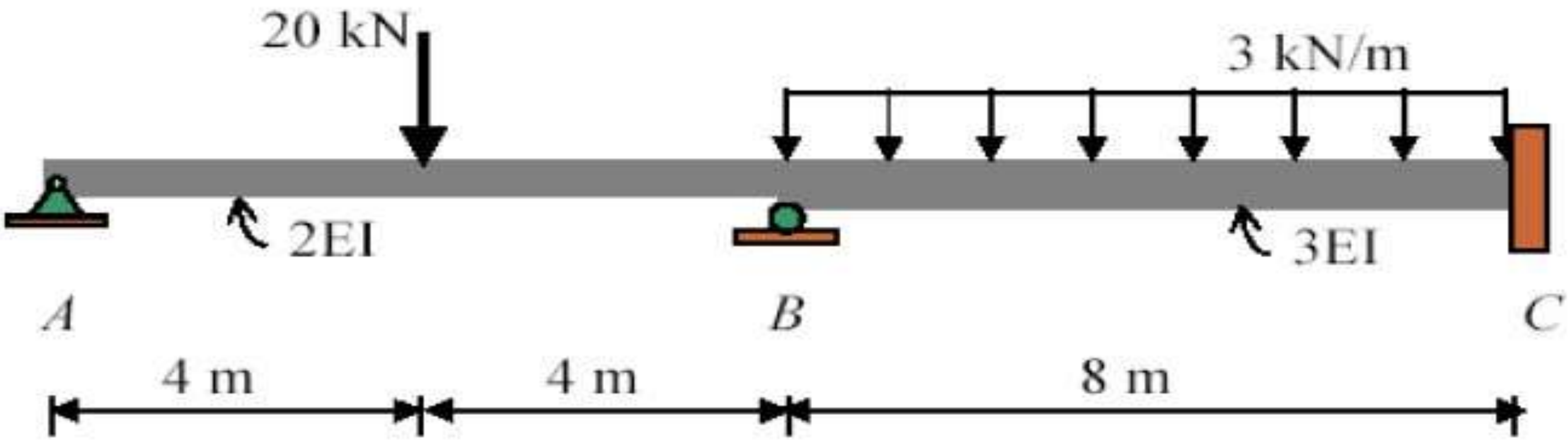

A
B
C
D
DEFLECTED SHAPE

FIG. 3.22

DRCFBY
HDCRFY

Support B settles by 10 mm. $E = 200GPa, \quad I = 50\times10^6 mm^4$



$$DF_{BA} = \frac{3(2EI/8)}{3(2EI/8)+4(3EI/8)} = 0.333 \qquad DF_{BC} = \frac{4(3EI/8)}{3(2EI/8)+4(3EI/8)} = 0.6$$

$$\frac{R/}{8} \quad \frac{20XÚ}{8} = 20$$

$$\frac{›‹/^2}{12} \text{ -- } \frac{3 \quad 8^2}{12} \text{ -- } \quad k$$

$$FEM_{AB} = \frac{\cdot \ \cdot}{8} - \frac{EI\delta}{L^2}.$$

$$2ft \quad \frac{6 \times 2 \times 200 \times 10'' \times 30 \quad 10' \quad 10^{*'''} X 10 X 10^{*'}}{2}$$

- —20-18.73 —38.75

$$= 20 - \frac{6 \times 2 \times 200 \times 10' \times 50 \times 10^6 \times 10^{*12} \quad 10 \times 10^{*'}}{8^2}$$

$$- 20 - 18.75 - 1.25 \ kNm$$

$$_{BC} - \frac{}{12} \cdot \frac{}{L^2}$$

$$= -16 + \frac{6 \times 3 \times 200 \times 10^6 \times 50 \times 10^6 \times 10^{-12} \times 10 \times 10^{-3}}{}$$

g2

=—1ö+28125=ł2ł25ÆVœ

$$\frac{wl^2}{12} \quad \frac{6EI\delta}{L}$$

$$\$ \quad \frac{6 \text{ x } 3 \text{ x } 200 \text{ x } 0^6 \text{ x } 50 \text{ x } 0^6 \text{ x } 0^{*12} \text{ x } 10 \text{ x } 10^{*'}}{}$$

= 1ò + 28.125 = 44.123

A	B	B	C	
1	0.333	0.667	0	
-38.75	+1.25	12.125	44.125	Fixed End Moments
38.75	19.375			Release A, and carry over
0.0	20.625	12.125	44.125	M∘ *
	-10.906	-21 844	0	„stribution
			-10.922	∘*∧
0.0	+ 9.719	-9.719	+33.203	Final Moments


Thank you!

UNIT-II

Cables and Suspension Bridge

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Introduction

- Cables are used as temporarily guys during the erection and as permanent guys for supporting masts and towers.
- Cables are used in the suspension bridges. A suspension bridge consists of two cables with the number of suspenders (hangers) which support the roadway.
- Figure 1 shows a typical suspension bridges in which the cable is supported over towers.

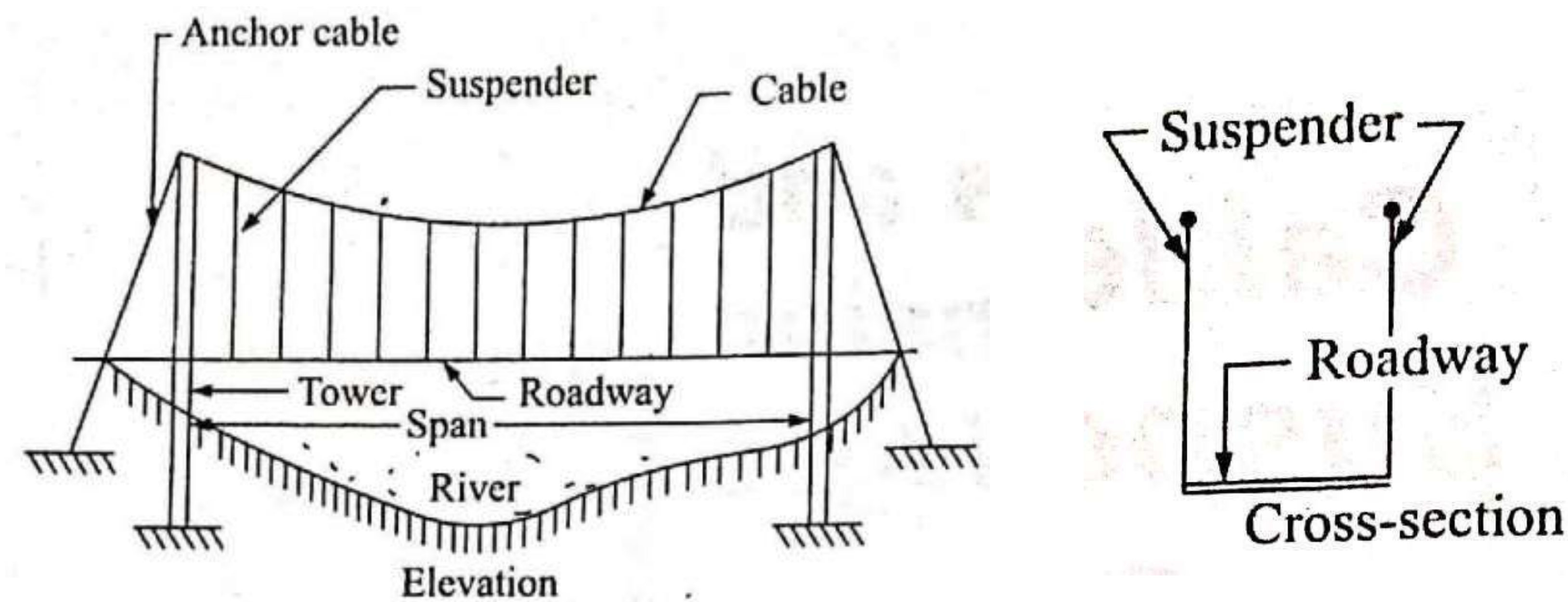
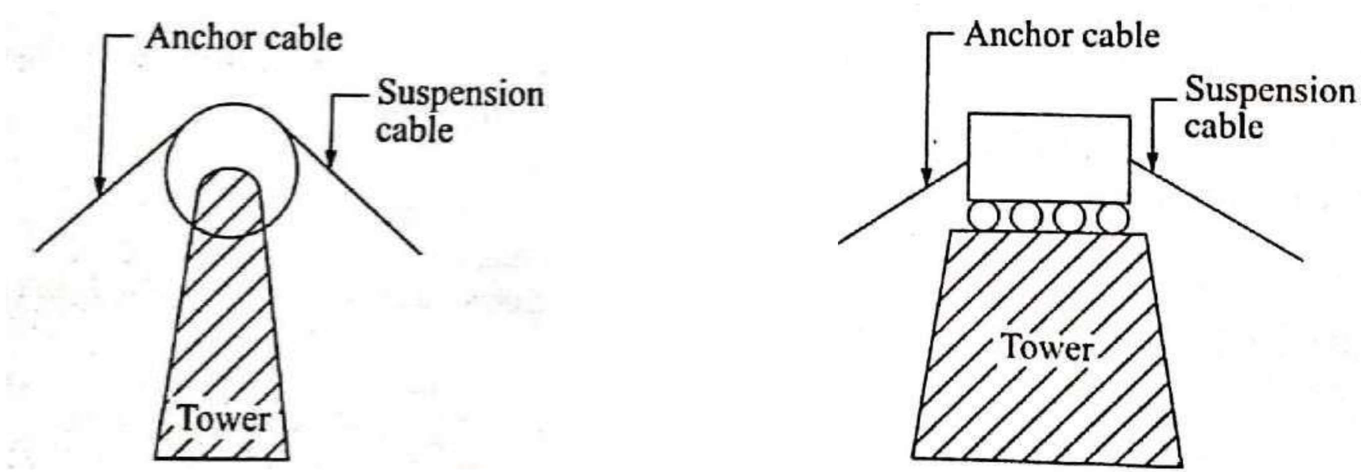


Figure 1: A typical suspension bridge

- To reduce the bending moment in the towers anchor cables are provided.
- The central sag or dip of the cable varies from $\left(\frac{1}{10}\right)$ th to $\left(\frac{1}{15}\right)$ th of span.
- The cables will be having either guided pulley support or roller pulley support as shown in Figure 2.



a) Guided pulley support

b) Roller pulley support

Figure 2: Support system

- In case of pedestrian suspension bridges, suspenders support the roadway directly.

- For heavy traffic, large spans stiffening girders are provided to support the roadway as shown in Figure 3.
- Laksman Jhula (Rishikesh) and Howrah bridge (Kalkata) are popular example of suspension bridges.
- Since, the number of suspenders are very large, the load on the cable may be taken as uniformly distributed load.

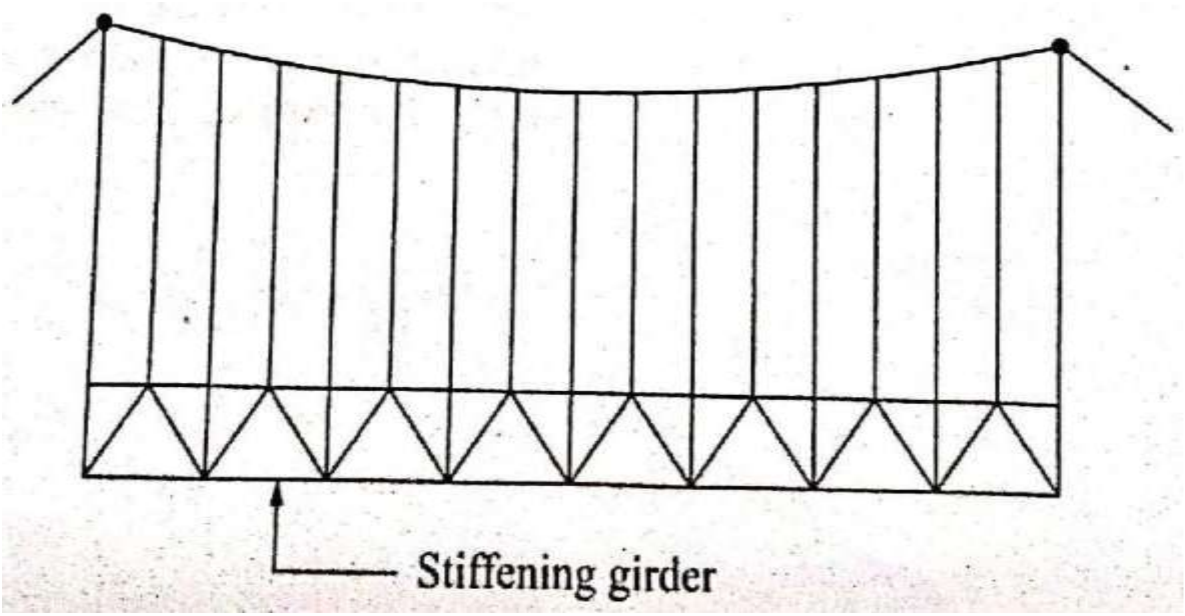


Figure 3: Equilibrium of Cable

Equilibrium of Cable

- A cable is a flexible structure which can not resist Bending Moment.
- In deflected shape of cable, the bending moment at any point of cable is zero which is achieved by developing horizontal thrust at the support.

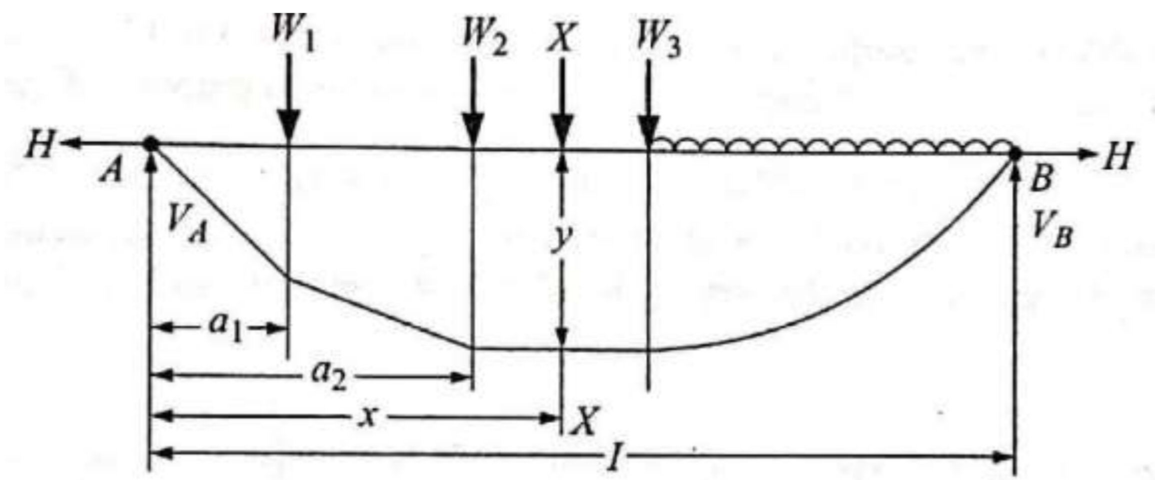


Figure 4: Equilibrium of Cable

- Consider the cable shown in Figure 4, which is subjected to various loads.
- Let the horizontal force developed at support is *H*
- Let the vertical reactions at support A and B is V_A and V_B respectively.

At section X-X, let the deflection be '*y*'

- Moment at section x-x = $M_x = V_A x - W_1 (x-a_1) - W_2 (x-a_2) - H y$
- Since the cable is flexible, $M_x = 0$
- Therefore, $H y = V_A x - W_1 (x-a_1) - W_2 (x-a_2)$
- $H y$ = Beam Moment
- The loaded cable can be analyzed by using above equation at any segment of cable.

Cable Subjected to Concentrated Loads

- Consider the cable of length L spanning over a horizontal gap l subjected to the concentrated loads as shown in Figure 5.
- Let V_A and V_B be the vertical reactions and H be the horizontal reactions at supports.

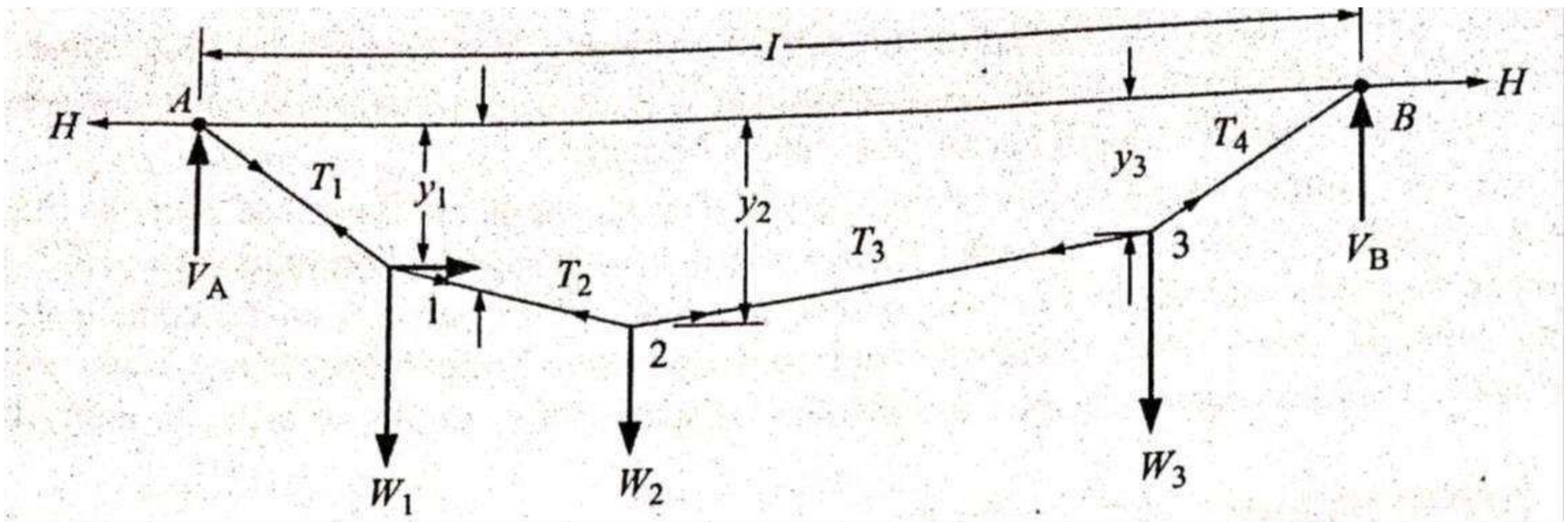


Figure 5: Cable subjected to concentrated loads

- The equilibrium condition is $H\,y = M_{\text{beam}}$

$$\text{or} \quad \boldsymbol{y = \frac{M_{\text{beam}}}{H}}$$

- Hence, the deflected shape is similar to the beam moment diagram.
- If M_1, M_2 and M_3 are the beam moments at load points 1, 2 and 3 respectively.
- y_1, y_2 and y_3 are the deflections at 1, 2 and 3 respectively
- y_1, y_2 and y_3 can be found using above equation i.e. $y_1 = \frac{M_1}{H}$, $y_2 = \frac{M_2}{H}$ and $y_3 = \frac{M_3}{H}$
- If the horizontal thrust is known or position of cable at any one point is known, the deflections at all points can be calculated.
- The actual length of the cable is the sum of lengths of each segments.
- After finding the deflections, slope of the various segments can be found.
- Using equilibrium equations of load points 1, 2 and 3, forces in the various segment of cable can be found.

Cable Subjected to a Uniformly Distributed Load

- Let a cable of length L be supported at points A and B which are at a horizontal distance l and are at the same level as shown in Figure 6.
- The cable is subjected to a uniformly distributed load w/unit horizontal length.

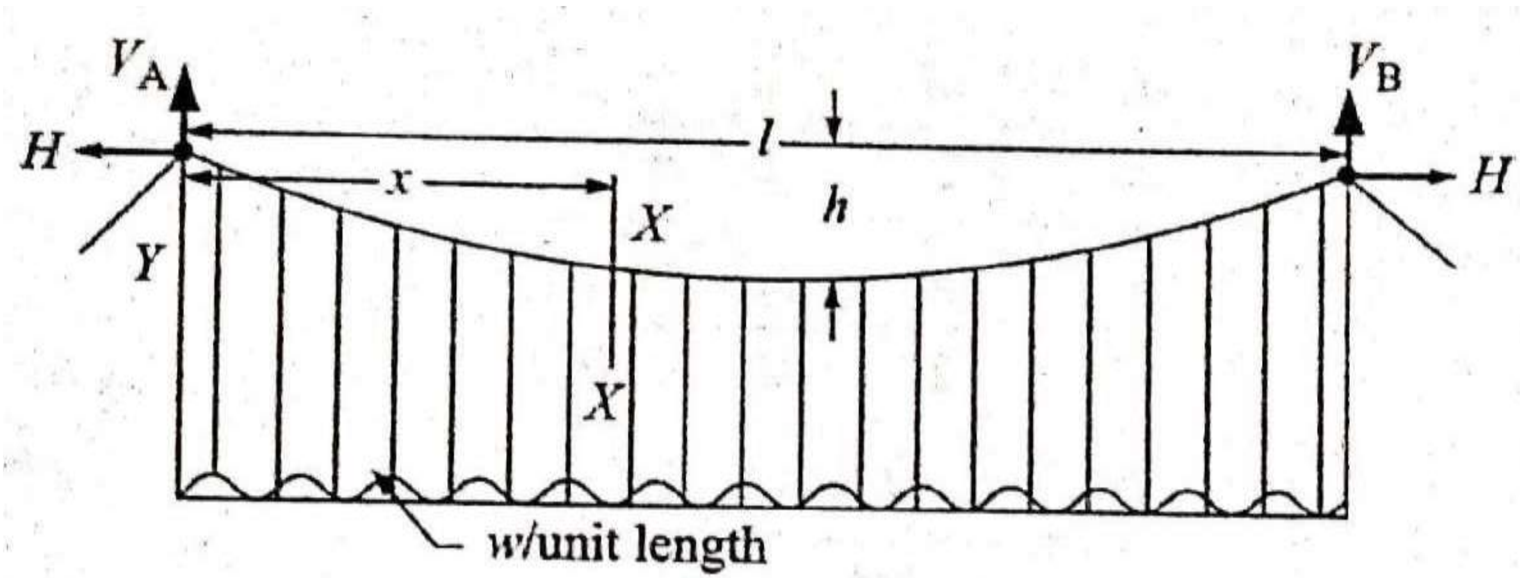


Figure 6: A typical cable subjected to *udl*

- The vertical reactions at A and B is V_A and $V_{B.}$

$$V_A = V_B = \frac{wl}{2}$$

- Taking moment at central point and equate to zero (Though the bending moment is zero at all points in the cable)

$$H\,h - \frac{wl}{2} \times \frac{l}{2} + \frac{wl}{2} \times \frac{l}{4} = 0$$

or $$H = \frac{wl^2}{8h}$$

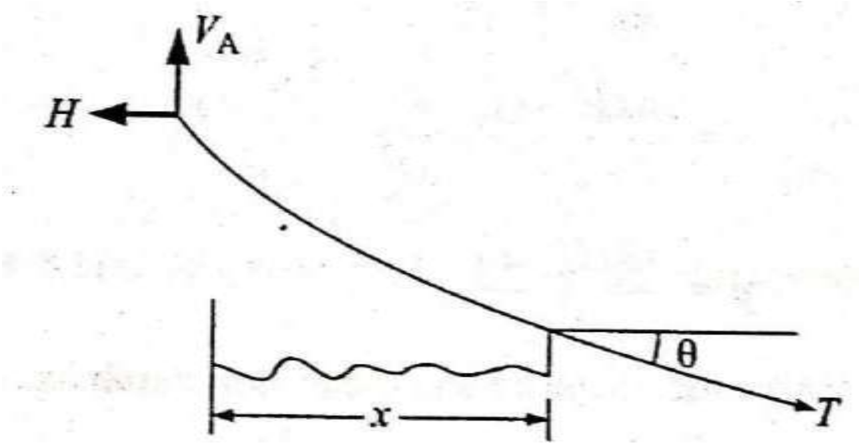


Figure 7: Free body diagram of cable

- If V is shear force at any section X-X distance x from A as shown in Figure 7.

Then, $T = \sqrt{V^2 + H^2}$

$V_{max} = \frac{wl}{2}$ at support

Therefore, $T_{max} = \sqrt{\left(\frac{w}{2}\right)^2 + \left(\frac{wl^2}{8h}\right)^2} = \frac{wl}{2}\sqrt{\left(1 + \frac{l^2}{16h^2}\right)}$

$V_{min} = 0$ at centre

$T_{min} = \sqrt{0 + H^2} = H$

- At any point, since cable can not resist shear

$V = T \sin\theta$

- Now to find the shape of the cable, consider the portion on left side of section X-X. Let θ be the slope. Then,

$\sum H = 0,\ \ T\cos\theta = H$

$\sum V = 0,\ \ T\sin\theta = V_A - w\,x$

or $T \sin \theta = \frac{wl}{2} - w\,x$

Therefore, $\tan \theta = \left[\frac{wl}{2} - wx\right] \times \frac{1}{H}$

i.e. $\frac{dy}{d} = \left[\frac{wl}{2} - wx\right] \times \frac{1}{H}$

Therefore, $y = \left[\frac{wl}{2}x - \frac{wx^2}{2}\right] \times \frac{1}{H}$

or $y = \frac{wx(l-x)}{2H}$

Substituting the value of H = $\frac{wl^2}{8h}$, we get

$$y = \frac{wx(l-x)}{2} \times \frac{8h}{wl^2} = \frac{4hx(l-x)}{l^2}$$

- Which is a parabola. Thus the shape of the cable is a parabola.
- To find the length of the cable in any curve (L)

$$\frac{ds}{d} = \sqrt{1 + \left(\frac{d}{dx}\right)^2} = 1 + \frac{1}{2}\left[\frac{4h(l-2x)}{l^2}\right]^2$$

Therefore, Length of the cable = $L = \int_0^l ds = l + \frac{8h^2}{3l}$

Forces on Anchor Cables and Towers

- The forces on anchor cable and towers depends upon the type of support given to cables.

There are two types of support:

- Guided Pulley Support
- Roller Support

Guided Pulley Support:

- Let the inclination of main cable to horizontal be **‘θ’.**
- Inclination of anchor cable to horizontal be ‘**α**’
- Assuming the pulley as friction less.
- Tension in anchor cable = tension in main cable.
- Let the tension be *T*

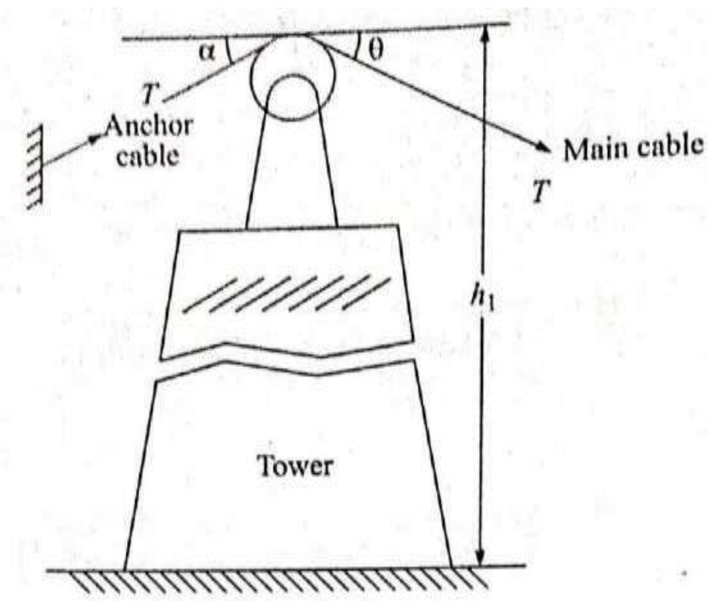


Figure 8: Guided Pulley Support

Vertical load transmitted to tower = $T \sin\theta + T \sin\alpha$

Vertical load transmitted to tower = $T(\sin\theta + \sin\alpha)$

Horizontal load transmitted to tower = $T \cos\theta - T \cos\alpha$

Horizontal load transmitted to tower = $T(\cos\theta - \cos\alpha)$

Bending moment on the tower = Horizontal force on tower × Height of tower

Bending moment on the tower = $T(\cos\theta - \cos\alpha) \times h_1$

Roller Support:

- In this case, the suspensible cable and the anchor cables are connected to a saddle resting on a tower.

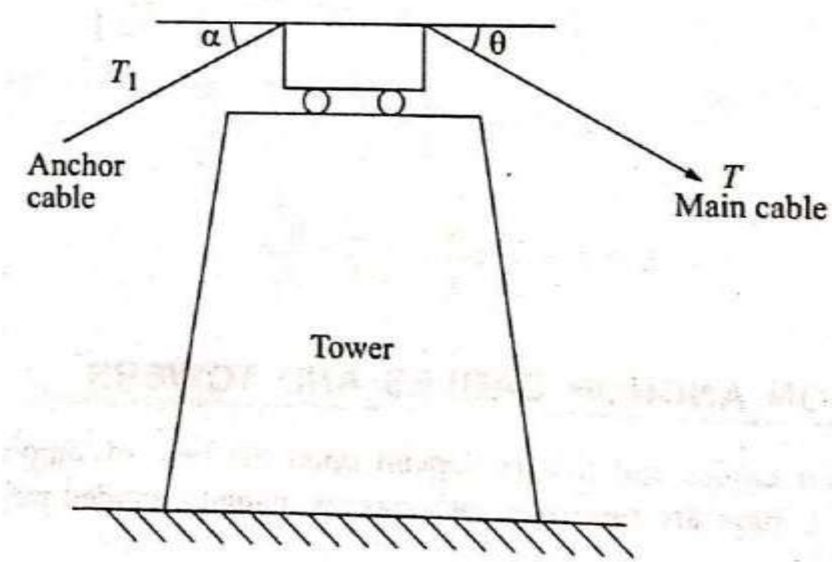


Figure 9: Roller Support

- In this arrangement, the two cables need not have the same tension.
- Let T be the tension in main cable and T_1 in the anchor cable.
- Assume saddle have frictionless rollers

$$T_1 \cos \alpha = T \cos \theta$$

$$T_1 = T \left(\frac{\cos \theta}{\cos \alpha}\right)$$

- Since, saddle is having frictionless rollers, there is no horizontal force and hence, no bending moment on tower,

Vertical force on the tower = $T_1 \sin \alpha + T \sin \theta$

Thanks

Cables and Suspension Bridge (Solved Problems)

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Q1. A bridge cable is suspended from towers 80 m apart and carries a load of 30 kN/m on the entire span. If the maximum sag is 8 m, calculate the maximum tension in the cable. If the cable is supported by saddles which are stayed by wires inclined at 30° to the horizontal, determine the forces acting on the towers. If the same inclination of back stay passes over pulley, determine the forces on the towers.

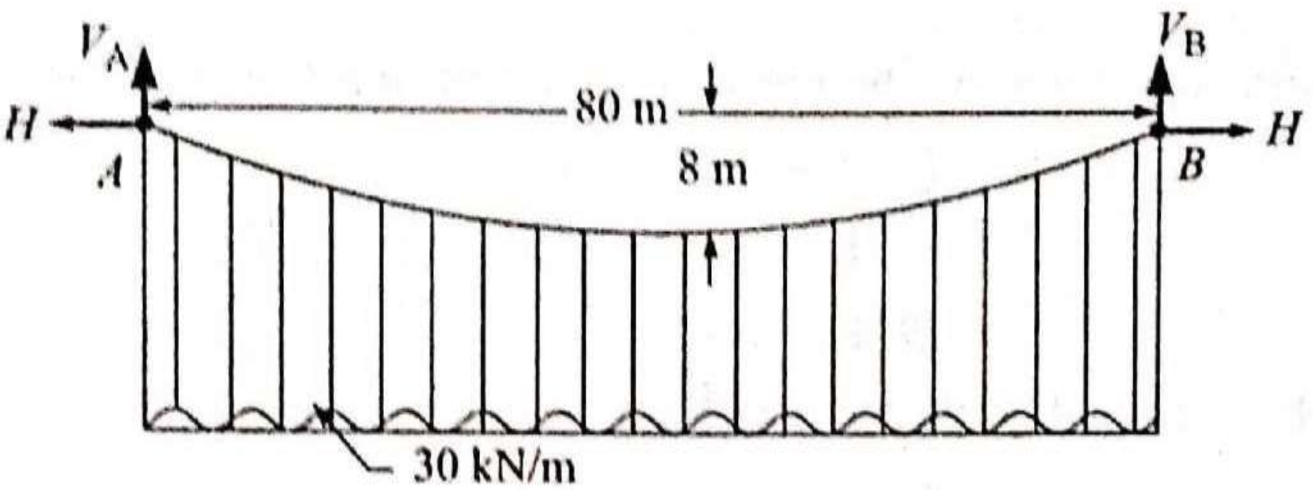

Figure 1(a): Example 1

- Reaction at both ends = $V_A = V_B = \frac{wl}{2} = \frac{30 \times 80}{2} = 1200$ kN
- For horizontal reaction, taking moment about central point '*C*'

$$H \times 8 - \frac{wl}{2} \times \frac{l}{2} + \frac{wl}{2} \times \frac{l}{4} = 0$$

or $$H = \frac{wl^2}{64} = \frac{30 \times 80^2}{64} = 3000 \text{ kN}$$

- Maximum tension occurs at support

$$T_{\max} = \sqrt{V^2 + H^2} = \sqrt{1200^2 + 3000^2}$$

$$T_{\max} = 3231.1 \text{ kN}$$

$$H = T_{\max} \cos \theta$$

$$\theta = \cos^{-1}\left(\frac{H}{T}\right) = \cos^{-1}\left(\frac{3000}{3231.1}\right) = 21.80^{\circ}$$

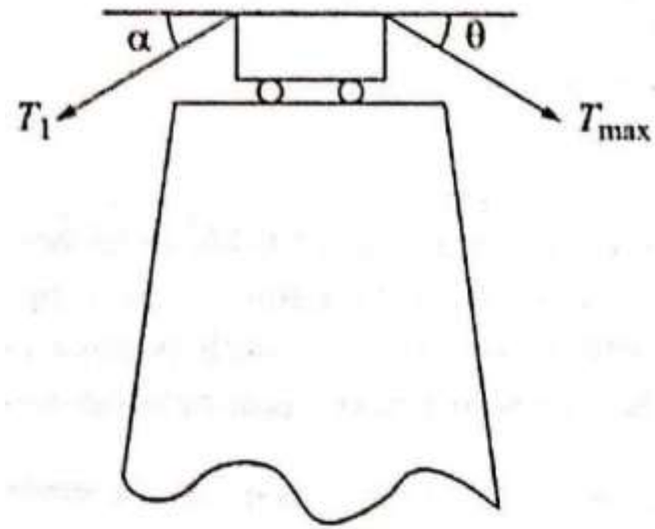

Figure 1(b): saddle support

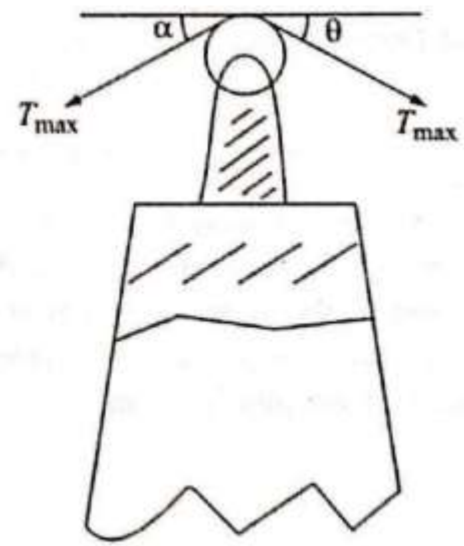

Figure 1(c) : Pulley support

If the cable is supported by saddle (Figure 1b)

The anchor cable tension T_1 can be found by equating horizontal tension

$$T_1 \cos\alpha = T_{max} \cos\theta$$

$$T_1 \times \cos 30° = 3231.80 \times \cos 21.80°$$

$$T_1 = 3464.1 \text{ kN}$$

- There is no horizontal force on the tower.
- The vertical force on the tower = $T_1 \sin\alpha + T_{max} \sin\theta$

Vertical force = 3464.1 sin 30° + 3231.1 sin 21.80° = 2931.98 kN

If the cable is supported over pulley (Figure 1c)

- The vertical force on tower = $T_{max} (\sin\alpha + \sin\theta)$

Vertical force = 3231.1 (sin 30° + sin 21.80°) = 2815.48 kN

- Horizontal force on the tower = Tmax (cos θ – cos α)

Horizontal force = 3231.1 (cos 21.8° - cos 30°) = 201.82 kN

Q2. A cable of span 120 m and dip 10 m carries a load of 6 kN/m of horizontal span. Find the maximum tension in the cable and the inclination of the cable at the support. Find the forces transmitted to the supporting pier if the cable passes over smooth pulleys on top of the pier. The anchor cable is at 30° to the horizontal. Determine the maximum bending moment for the pier if the height of the pier is 15 m.

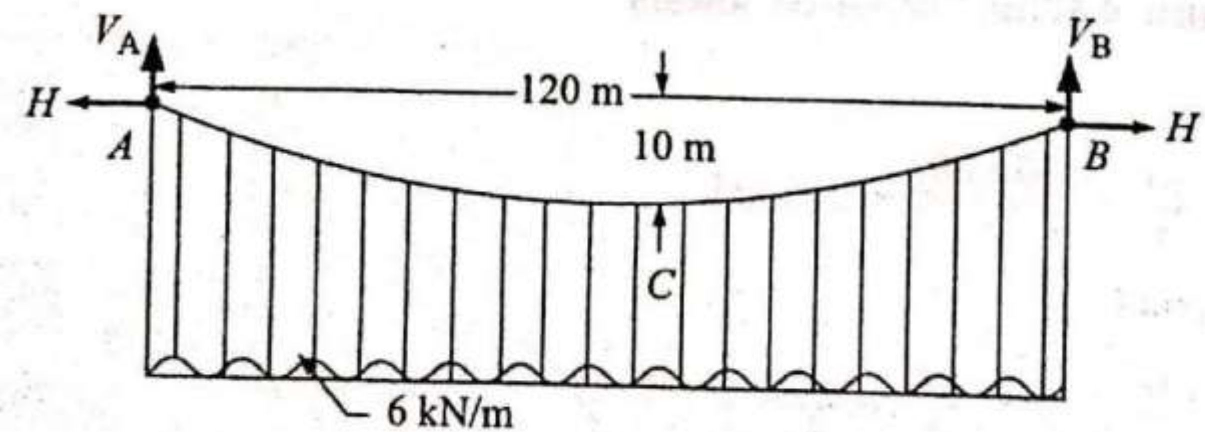

Figure 2 (a): Example 2

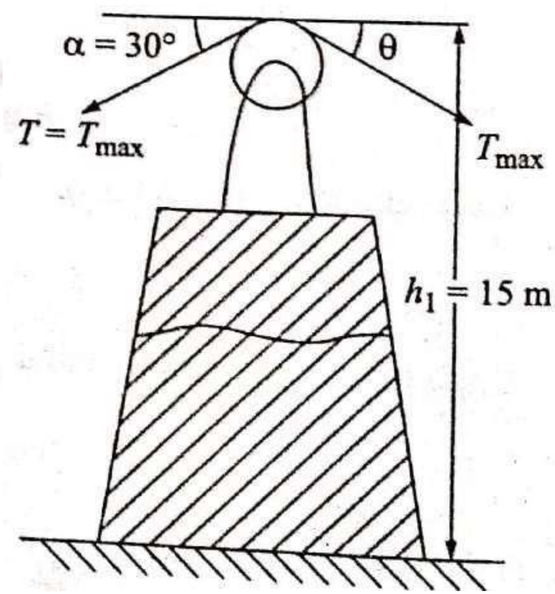

Figure 2 (b): Forces on pier

- Due to symmetry, Reaction at A and B is

$$V_A = V_B = \frac{wl}{2} = \frac{6\times120}{2} = 360 \text{ kN}$$

- Taking moment about central point C,

$$H \times h - \frac{wl}{2} \times \frac{l}{2} + \frac{wl}{2} \times \frac{l}{4} = 0$$

$$H = \frac{wl^2}{8h} = \frac{6 \times 120 \times 120}{8 \times 10} = 1080 \text{ kN}$$

$$T_{\max} = \sqrt{V^2 + H^2} = \sqrt{360^2 + 1080^2} = 1138.42 \text{ kN}$$

$$\cos\theta = \frac{H}{T_{max}} = \frac{1080}{1138.42}$$

$$\theta = 18.435$$

- Horizontal force transferred to pier = $T_{\max}$ (cos 18.435° - cos 30°)
- Horizontal force transferred to pier = 1138.42 (cos 18.435° - cos 30°) = 94.099 kN
- Maximum bending moment in the pier = $H\,h_1$ = 94.099 × 15 = 1411.49 kNm
- Vertical force on the pier = T (sin θ + sin α) = 1138.42 (sin 18.435° + sin 30°)

= 929.21 kN

Q3. A light flexible cable 18 m long is supported at two ends at the same level. The supports are 16 m apart. The cable is subjected to uniformly distributed load of 1 kN/m of horizontal length over its entire span. Determine the reactions developed at the support.

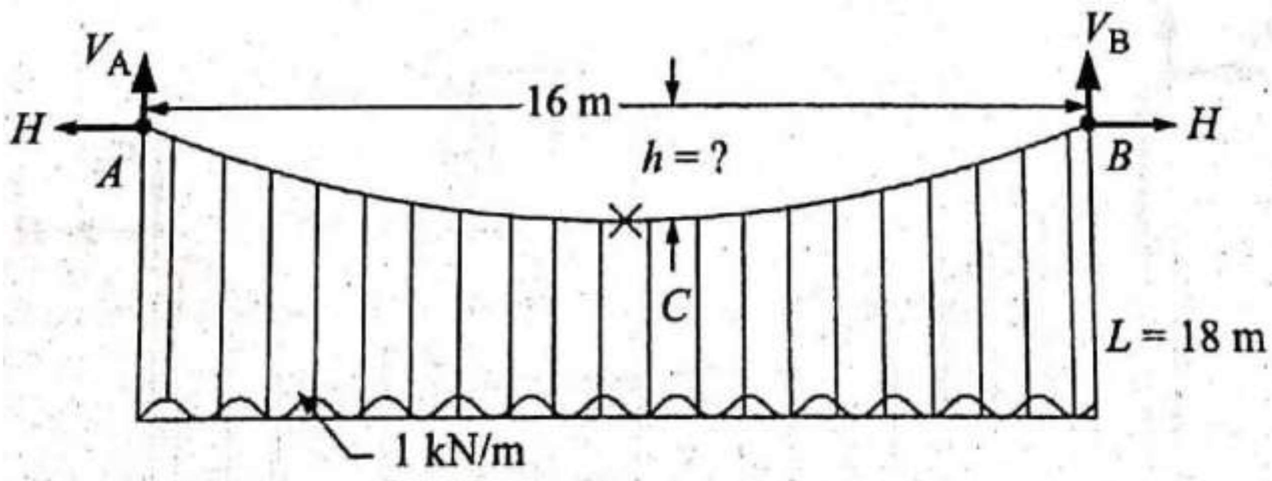

Figure 3: Example 3

- The length of the cable = $L = l + \frac{8}{3} \times \frac{h^2}{}$

  where, l = span, h = central dip

- Applying this we get, $18 = 16 + \frac{8}{3} \times \frac{h^2}{16}$

  or $h = 3.464$ m

- Let H = horizontal force, and V_A = vertical reaction at A

$$V_A = \frac{wl}{2} = \frac{1 \times 16}{2} = 8 \text{ kN}$$

$$H \times 3.464 = \frac{wl^2}{8} = \frac{1 \times 162}{8}$$

$$H = 9.237 \text{ kN}$$

$$T_{max} = \sqrt{V^2 + H^2} = = \sqrt{8^2 + 9.237^2} = 12.220 \text{ kN}$$

- Inclination 'θ' with horizontal

$$T_{max} \cos \theta = H$$

$$\theta = \cos^{-1}\left(\frac{H}{T_{max}}\right) = \cos^{-1}\left(\frac{9.237}{1.220}\right) = 40.898^\circ$$

Thanks

Suspension bridge with three-hinged stiffening Girder

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Introduction

- Consider the suspension cable stiffened with a three-hinged girder as shown in Figure 1.
- The girder can be a heavy beam or a truss which has three hinges two at the ends and one at the centre.
- The cable and the girder are connected by a number of hangers/suspenders.
- Since the number of suspenders are very large, the load on cable or girder, due to the forces in the suspenders, may be taken as uniformly distributed load.

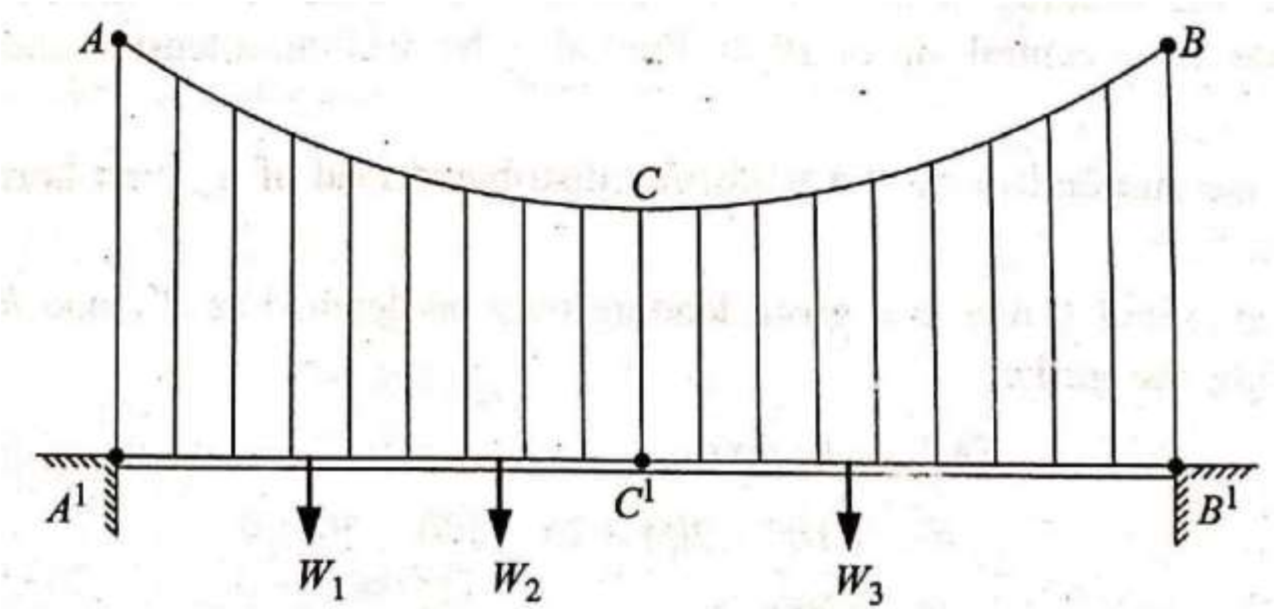

Figure 1: Typical suspension bridge with three-hinged stiffening girder

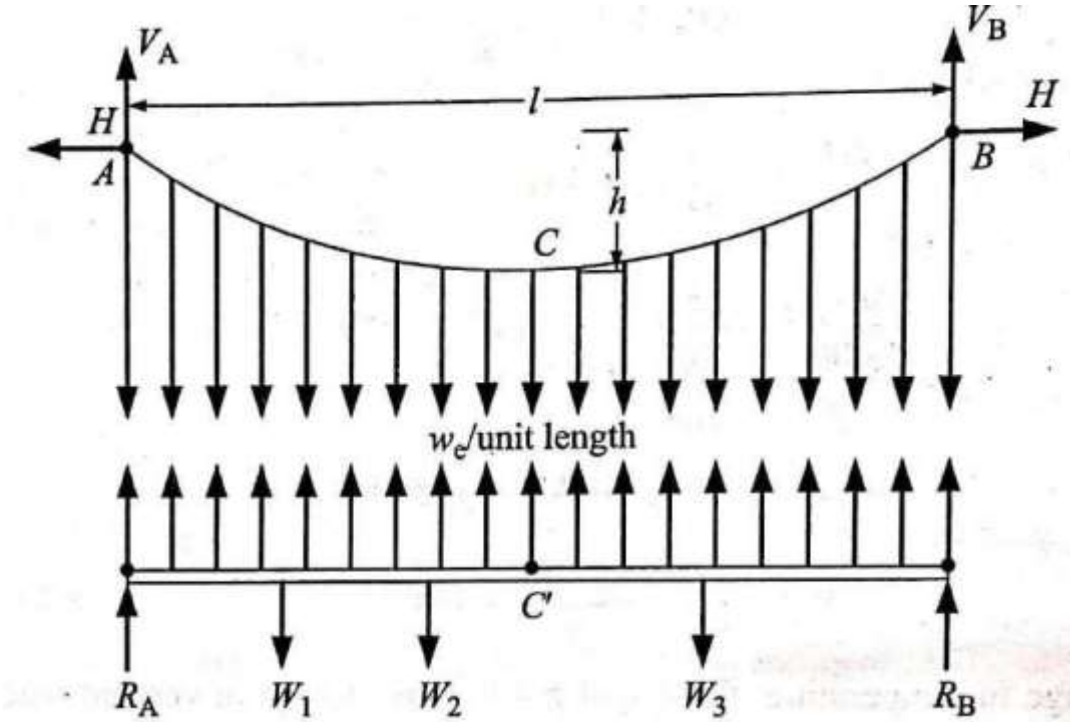

Figure 2: Free Body Diagram of cable and girder

- Let the uniformly distributed load = w_e per unit horizontal length
- Let C' is the central hinge of girder
- Let the uniformly distributed load w_e exerted by suspender on the girder
- **The beam may be analyzed for the given load along with w_e**

Due to w_e alone,

- The bending moment at section x-x = $\frac{w_e x}{8}(l - x)$
- Maximum bending moment at C = $\frac{w_e l^2}{8}$ (Hogging moment)
- The shear force at section x-x = $-w_e\left(\frac{}{2} - x\right)$
- The cable can be analyzed for the uniformly distributed load w_e

Q. A three-hinged stiffening girder of a suspension bridge of span 100 m is subjected to two point loads of 200 kN and 300 kN at the distance of 25 m and 50 m from the left end. Find the shear force and bending moment for the girder at a distance 30 m from the left end. The supporting cable has a central dip of 10 m. Find also the maximum tension and its slope in the cable.

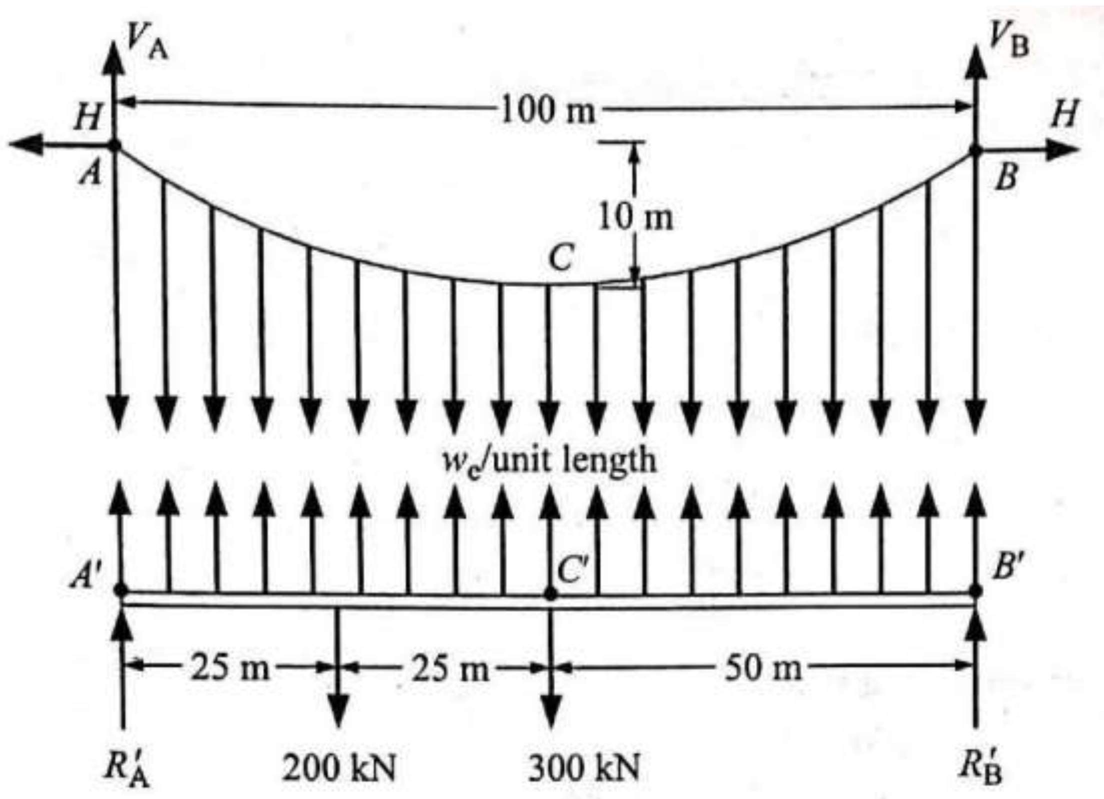

Figure 3

- Let the suspenders exert a uniformly distributed load of w_e per unit horizontal length as shown in Figure 3.
- Reactions at A and B due to a given loading only be denoted as R'_A and R'_B, respectively.
- To find out the reactions in the girder, take $\sum M'_A = 0$

$$R'_B \times 100 - 200 \times 25 - 300 \times 50 = 0$$

$$R'_B = 200 \text{ kN}$$

- Using $\sum V = 0$, Find R'_A

$$R'_A + R'_B = 200 + 300$$

$$R'_A = 300 \text{ kN}$$

- Moment at the central hinge of girder $(C') = 0$ $(\sum M_{c'} = 0)$
- Bending moment due to given loading + Bending moment due to $w_e = 0$

$$R'_B \frac{l}{2} - \frac{w_e l^2}{8} = 0$$

$$200 \times \frac{100}{2} = \frac{w_e \times 1002}{8} \text{ (After simplification)}$$

$$w_e = 8 \text{ kN/m}$$

- Shear Force and Bending moment at a distance 30 m from the left end,
- SF = SF due to given loading + SF due to w_e

$$SF = R'_A - 200 - w_e\left(\frac{100}{2} - 30\right)$$

$$= 300 - 200 - 8(50\text{-}30)$$

$$= -60 \text{ kN} = 60 \text{ kN } (\uparrow)$$

- BM = Moment due to given loading + Moment due to w_e

$$BM = 300 \times 30 - 200 \times 5 - \frac{w_e x}{8}(l - x)$$

$$BM = 300 \times 30 - 200 \times 5 - \frac{8 \times 30}{8}(100 - 30) = -400 \text{ kNm}$$

- BM = 400 kN (Hogging)
- For the analysis of cable
- First finding vertical reaction at A and B i.e. V_A and V_B, Take $\sum V = 0$

$$V_A = V_B = w_e \times \frac{l}{2} = 8 \times \frac{100}{2} = 400 \text{ kN}$$

- For getting horizontal reaction, taking moment about C, we get

$$H \times h = \frac{w_e l}{2} \times \frac{l}{2} - w_e \times \frac{l}{2} \times \frac{l}{4} = \frac{w_e l^2}{8}$$

$$H \times 10 = \frac{w_e l^2}{8} = \frac{8 \times 100^2}{8} = 10000$$

or $\quad H = 1000$ kN

- Maximum Tension in the cable = $T_{max} = \sqrt{V^2{}_A + H^2}$

$$T_{max} = \sqrt{400^2 + 1000^2} = 1077.033 \text{ kN}$$

- Its slope to horizontal is $T_{max} \cos\theta = H$

$$\theta = \cos^{-1}\left(\frac{1000}{1077.033}\right) = 21.80^\circ$$

Thanks

UNIT-III

Flexibility matrix method

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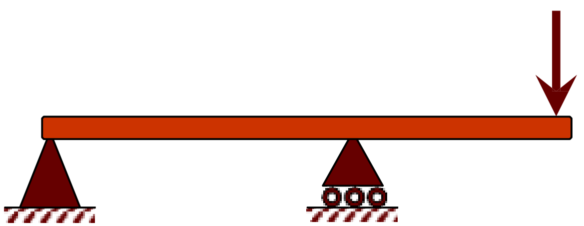
Introduction

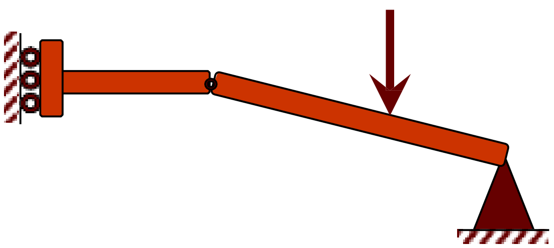
- What is statically ***DETERMINATE*** structure?
  - When all the forces (reactions) in a structure can be determined from the equilibrium equations its called statically determinate structure
  - Structure having unknown forces equal to the available equilibrium equations



No. of unknown = 3

No. of equilibrium equations = 3

3 = 3 thus statically determinate



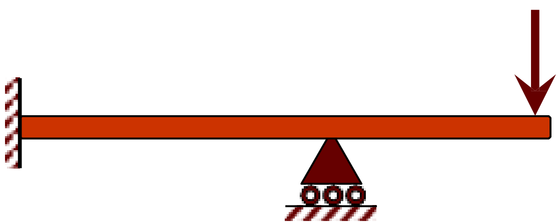
No. of unknown = 6

No. of equilibrium equations = 6

6 = 6 thus statically determinate

Introduction

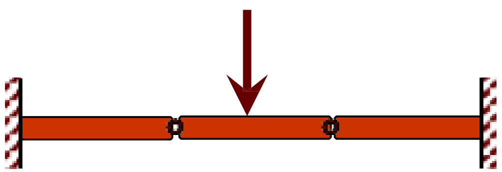
- What is statically ***INETERMINATED*** structure
  - Structure having more unknown forces than available equilibrium equations
  - Additional equations needed to solve the unknown reactions



No. of unknown = 4

No. of equilibrium equations = 3

4 > 3 thus statically Indeterminate



No. of unknown = 10

No. of equilibrium equations = 9

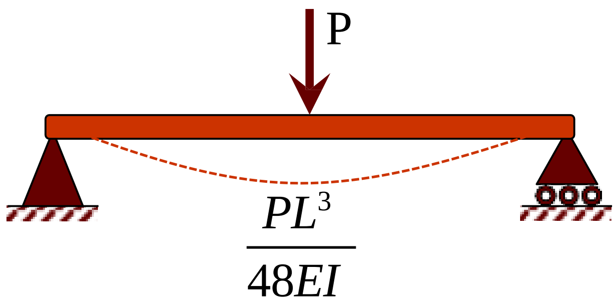
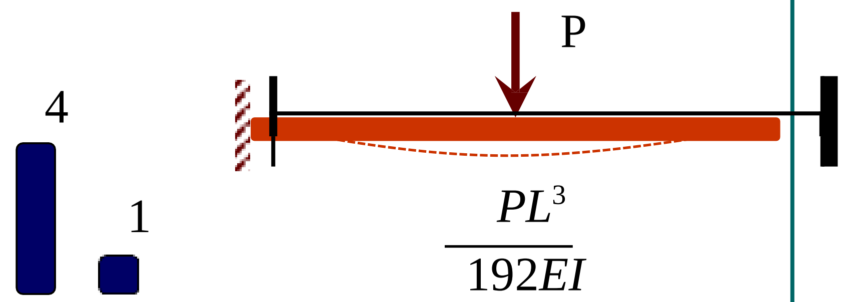
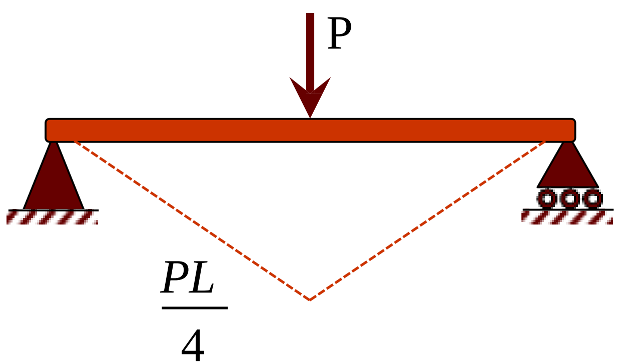
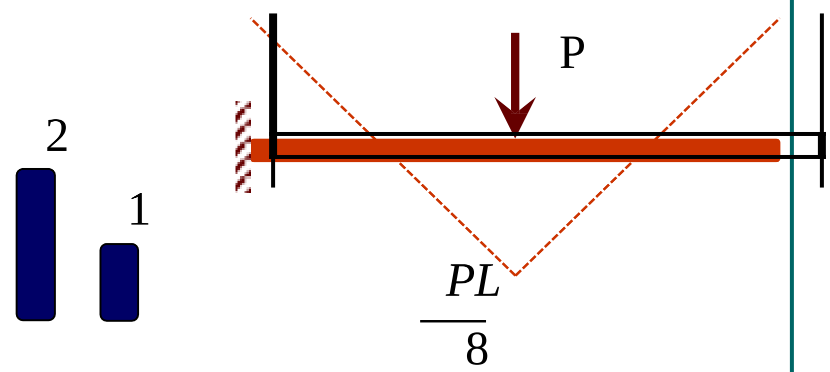
10 > 9 thus statically Indeterminate

Indeterminate Structure

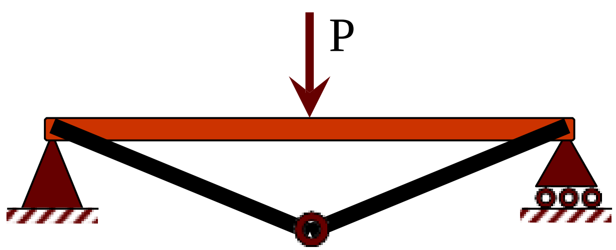
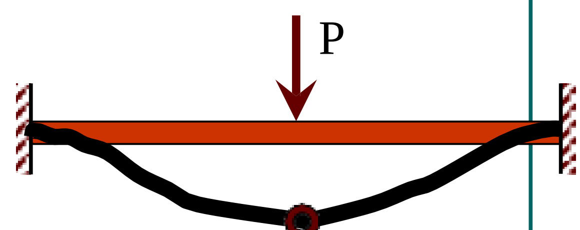
Why we study indeterminate structure

- Most of the structures designed today are statically indeterminate
- Reinforced concrete buildings are considered in most cases as a statically indeterminate structures since the columns & beams are poured as continuous member through the joints & over the supports
- More stable compare to determinate structure or in another word safer.
- In many cases more economical than determinate.
- The comparison in the next page will enlighten more

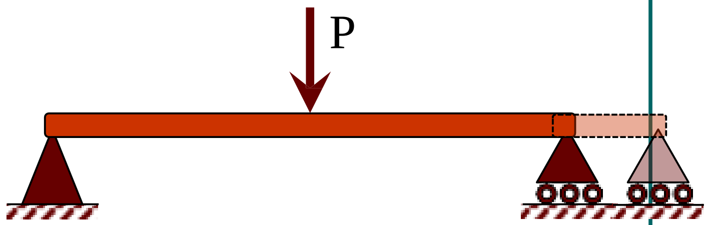
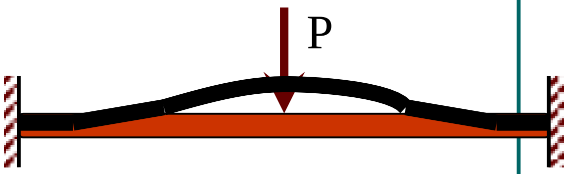
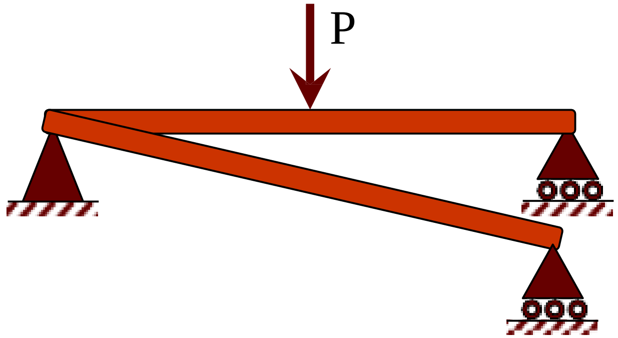
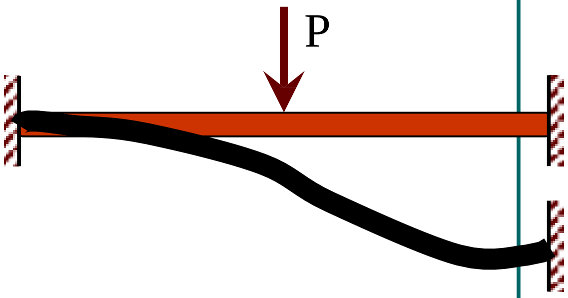
Contrast

	Indeterminate Structure	Determinate Structure
Deflection	Considerable compared to indeterminate structure 	Generally smaller than determinate structure 
Stress	High moment caused thicker member & more material needed 	Less moment, smaller cross section & less material needed 

Contrast

	Indeterminate Structure	Determinate Structure
Stability in case of over load	✶Support will not develop the horizontal force & moments that necessary to prevent total collapse ✶No load redistribution ✶When the plastic hinge formed certain collapse for the system	✶Will develop horizontal force & moment reactions that will hold the beam ✶Has the tendency to redistribute its load to its redundant supports ✶When the plastic hinge formed the system would be a determinate structure
	 Plastic Hinge	 Plastic Hinge

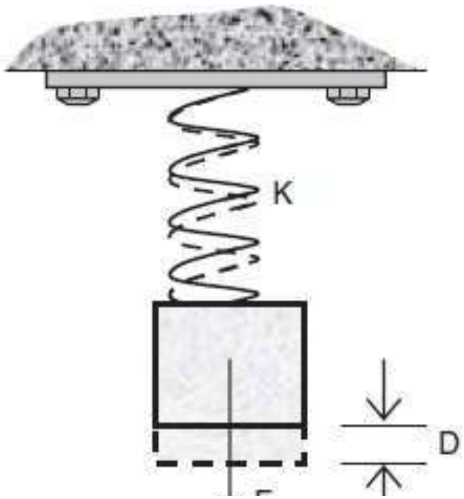
Contrast

	Indeterminate Structure	Determinate Structure
Temperature	No effect & no stress would be developed in the beam  	Serious effect and stress would be developed in the beam 
Differential Displacement	No effect & no stress would be developed  	Serious effect and stress would be developed 

ANALYSIS OF STATICALLY INDETERMINATE STRUCTURES

MATRIX STIFFNESS METHOD OF ANALYSIS

MATRIX STIFFNESS METHOD OF ANALYSIS



$$\{F\}_{n\times 1} = [K]_{n\times n}\{D\}_{n\times 1}$$

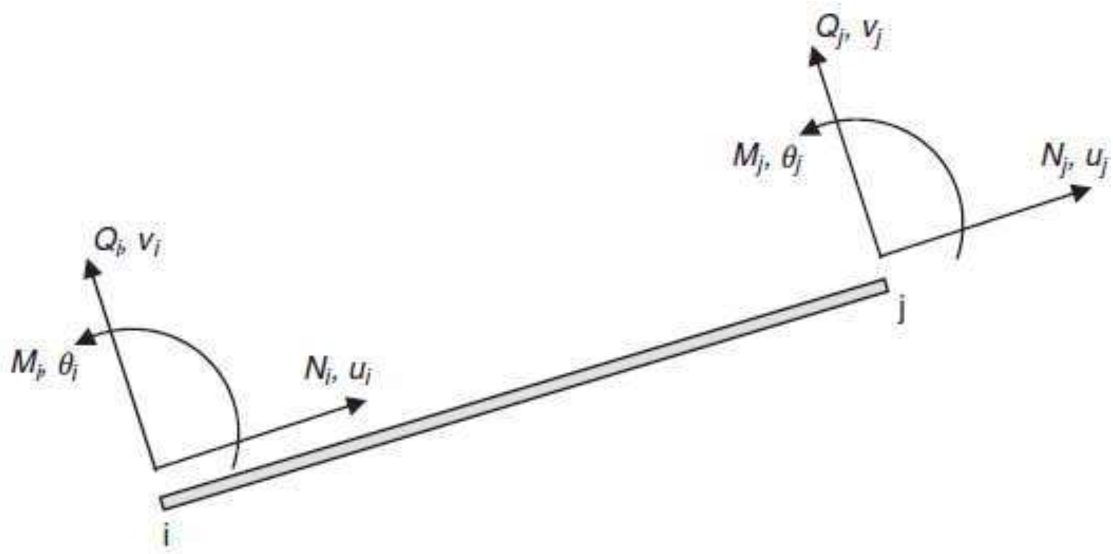
$$F = KD$$

$$\{F\}_{n\times 1} = [K]_{n\times n}\{D\}_{n\times 1}$$

$$D = F/K$$

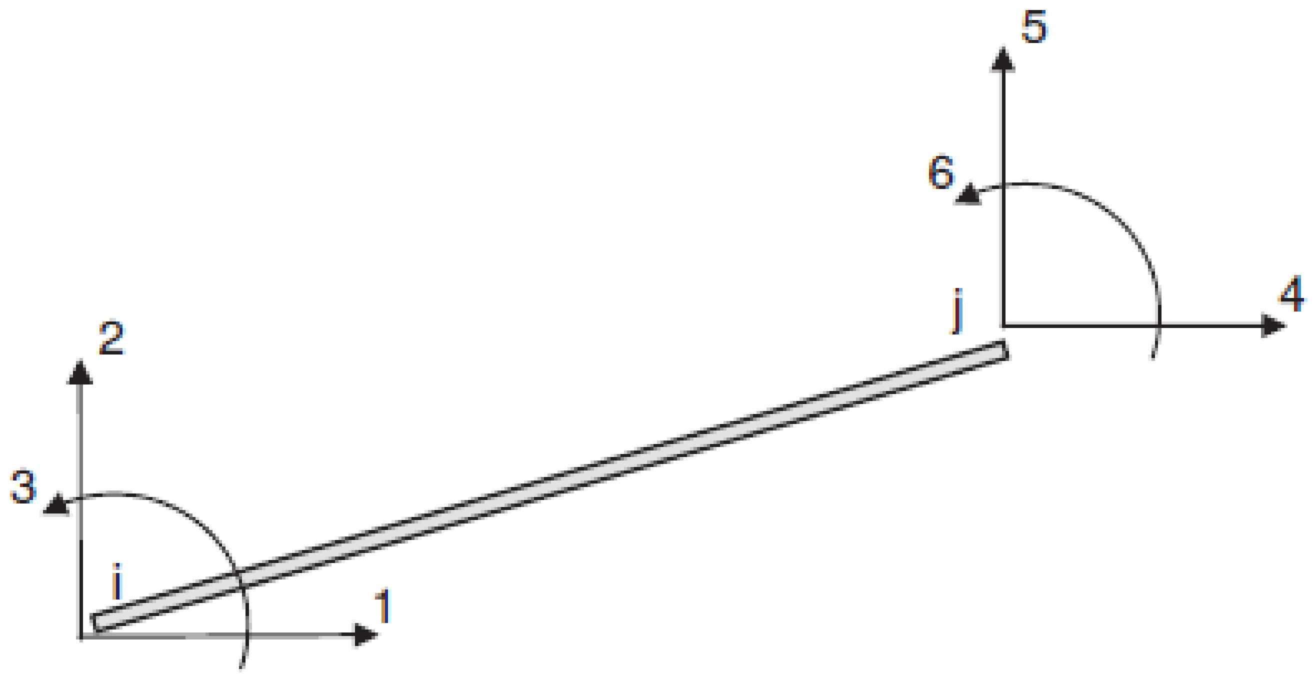
$$\{D\} = [K]^{-1}\{F\}$$

LOCAL COORDINATE SYSTEM



$M_{i,j}$, $\theta_{i,j}$ = bending moments and corresponding rotations at ends i, j, respectively; $N_{i,j}$, $u_{i,j}$ are axial forces and corresponding axial deformations at ends i, j, respectively; and $Q_{i,j}$, $v_{i,j}$ are shear forces and corresponding transverse displacements at ends i, j, respectively. The directions of the actions and movements are positive when using the stiffness method.

DEGREES OF FREEDOM



MEMBER STIFFNESS MATRIX

The structure stiffness matrix $[K]$ is assembled on the basis of the equilibrium and compatibility conditions between the members. For a general frame, the equilibrium matrix equation of a member is

$$\{P\} = [K_e]\{d\} \tag{1.9}$$

where $\{P\}$ is the member force vector, $[K_e]$ is the member stiffness matrix, and $\{d\}$ is the member displacement vector, all in the member's local coordinate system. The elements of the matrices in Equation (1.9) are given as

$$\{P\} = \begin{Bmatrix} N_i \\ Q_i \\ M_i \\ N_j \\ Q_j \\ M_j \end{Bmatrix}; [K_e] = \begin{bmatrix} K_{11} & 0 & 0 & K_{14} & 0 & 0 \\ 0 & K_{22} & K_{23} & 0 & K_{25} & K_{26} \\ 0 & K_{32} & K_{33} & 0 & K_{35} & K_{36} \\ K_{41} & 0 & 0 & K_{44} & 0 & 0 \\ 0 & K_{52} & K_{53} & 0 & K_{55} & K_{56} \\ 0 & K_{62} & K_{63} & 0 & K_{65} & K_{66} \end{bmatrix}; \{d\} = \begin{Bmatrix} u_i \\ v_i \\ \theta_i \\ u_j \\ v_j \\ \theta_j \end{Bmatrix}$$

ELEMENS OF MEMBER STIFFNESS MATRIX

AXIAL LOADING

A member under axial forces N_i and N_j acting at its ends produces axial displacements u_i and u_j as shown in Figure 1.10. From the stress-strain relation, it can be shown that

$$N_i = \frac{EA}{L}(u_i - u_j) \qquad (1.10a)$$

$$N_j = \frac{EA}{L}(u_j - u_i) \qquad (1.10b)$$

where E is Young's modulus, A is cross-sectional area, and L is length of the member. Hence, $K_{11} = -K_{14} = -K_{41} = K_{44} = \frac{EA}{L}$.

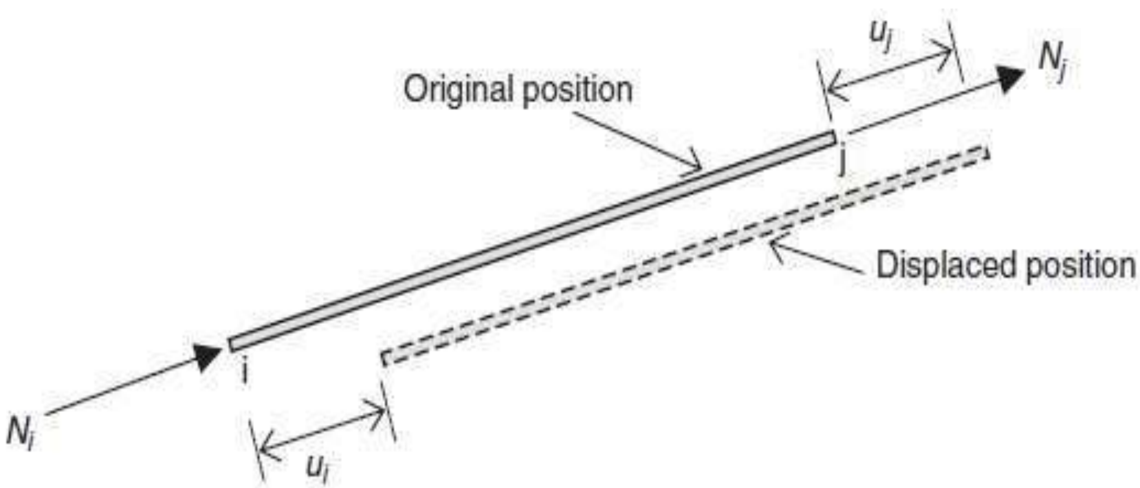
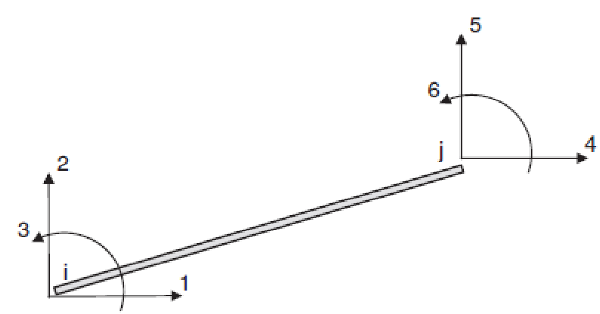


FIGURE 1.10. Member under axial forces.



BENDING MOMENTS AND SHEAR

For a member with shear forces Q_i, Q_j and bending moments M_i, M_j acting at its ends as shown in Figure 1.11, the end displacements and rotations are related to the bending moments by the slope-deflection equations as

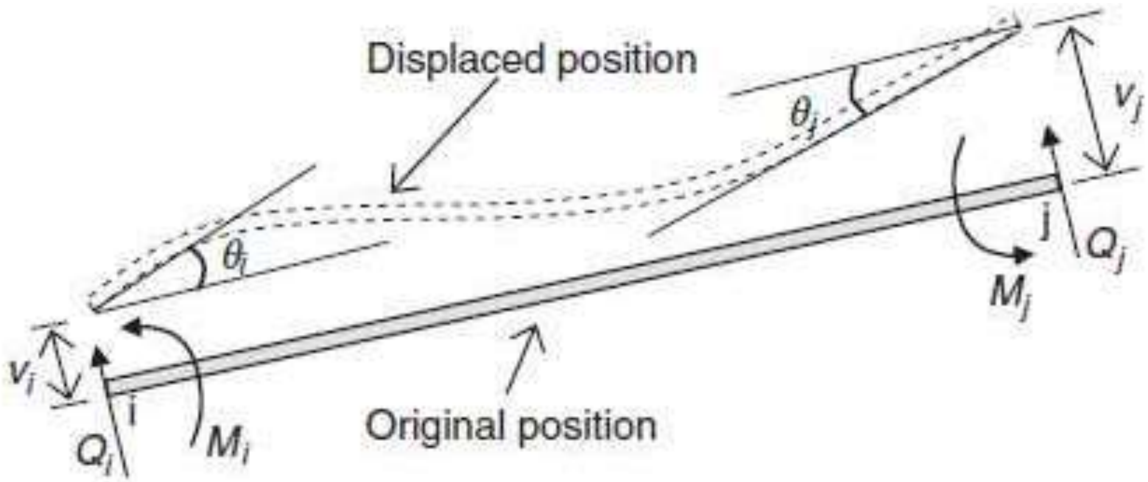


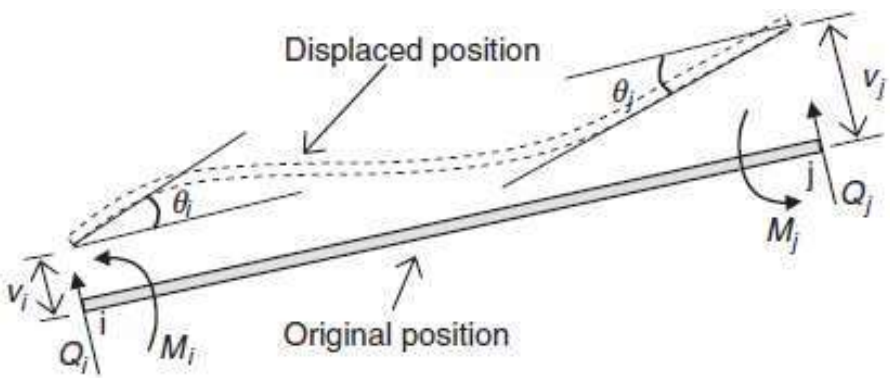
FIGURE 1.11. Member under shear forces and bending moments.

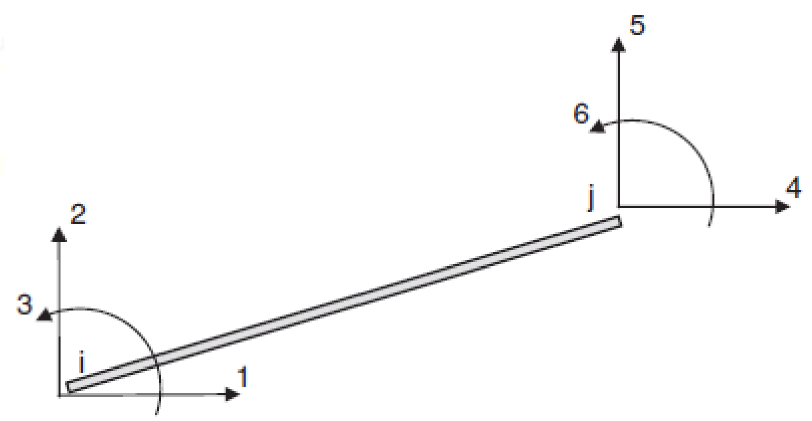
BENDING MOMENTS AND SHEAR

$$M_i = \frac{2EI}{L}\left[2\theta_i + \theta_j - \frac{3(v_j - v_i)}{L}\right]$$

$$M_j = \frac{2EI}{L}\left[2\theta_j + \theta_i - \frac{3(v_j - v_i)}{L}\right]$$

Hence, $K_{62} = -K_{65} = \dfrac{6EI}{L^2}$, $K_{63} = \dfrac{2EI}{L}$, and $K_{66} = \dfrac{4EI}{L}$.

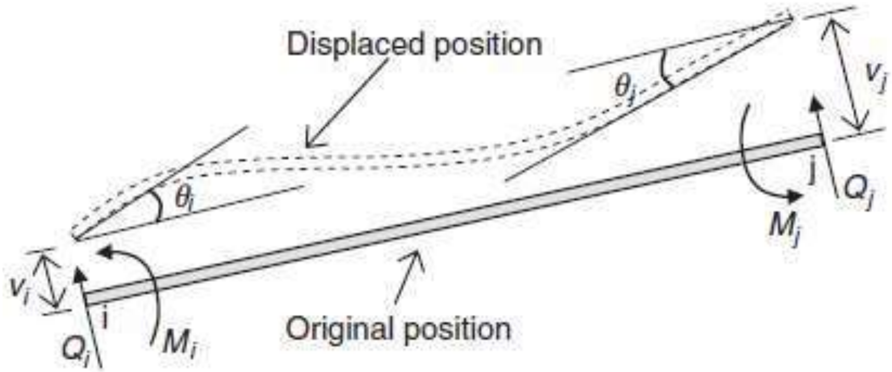




BENDING MOMENTS AND SHEAR

By taking the moment about end j of the member in Figure 1.11, we obtain

$$Q_i = \frac{M_i + M_j}{L} = \frac{2EI}{L^2}\left[3\theta_i + 3\theta_j - \frac{6(v_j - v_i)}{L}\right] \tag{1.12a}$$



Also, by taking the moment about end i of the member, we obtain

$$Q_j = -\left(\frac{M_i + M_j}{L}\right) = -Q_i \tag{1.12b}$$

$$K_{22} = K_{55} = -K_{25} = -K_{52} = \frac{12EI}{L^3} \text{ and } K_{23} = K_{26} = -K_{53} = -K_{66} = \frac{6EI}{L^2}.$$

STIFFNESS MATRIX

In summary, the resulting member stiffness matrix is symmetric about the diagonal:

$$[K_e] = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \tag{1.13}$$

COORDINATES TRANSFORMATION

In order to establish the equilibrium conditions between the member forces in the local coordinate system and the externally applied loads in the global coordinate system, the member forces are transformed into the global coordinate system by force resolution. Figure 1.12 shows a member inclined at an angle α to the horizontal.

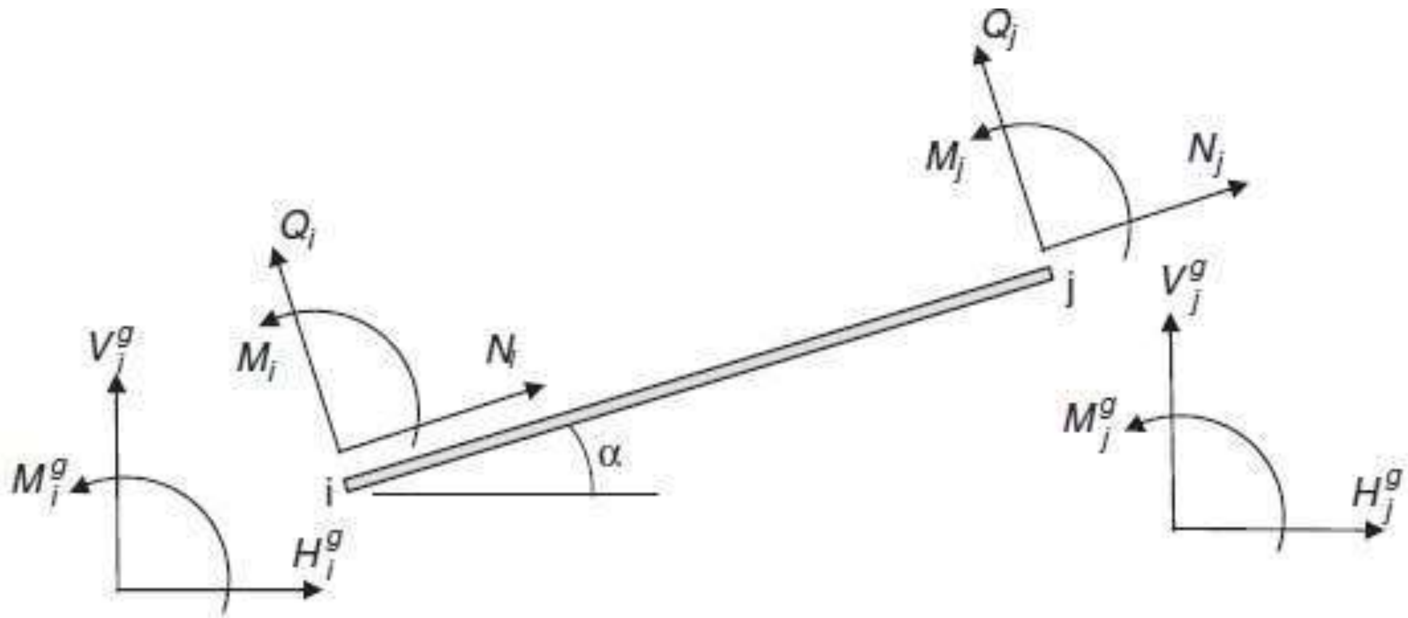
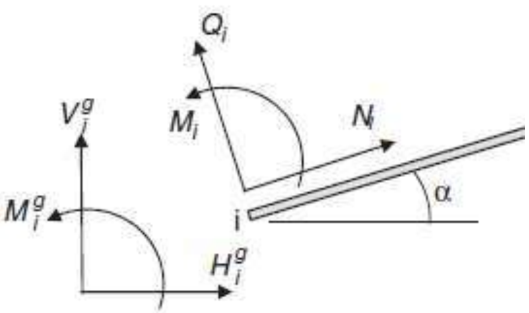


FIGURE 1.12. Forces in the local and global coordinate systems.

LOAD TRANSFORMATION

The forces in the global coordinate system shown with superscript "g" in Figure 1.12 are related to those in the local coordinate system by

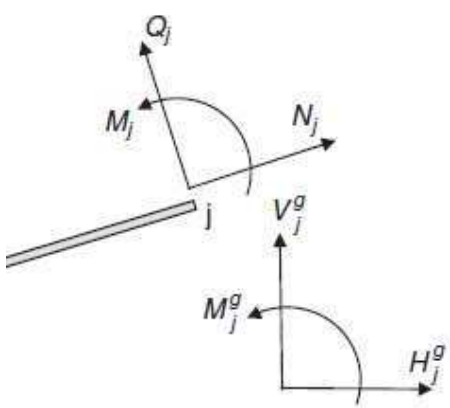


$$H_i^g = N_i \cos\alpha - Q_i \sin\alpha \quad (1.14a)$$

$$V_i^g = N_i \sin\alpha + Q_i \cos\alpha \quad (1.14b)$$

$$M_i^g = M_i \quad (1.14c)$$

Similarly,



$$H_j^g = N_j \cos\alpha - Q_j \sin\alpha \quad (1.14d)$$

$$V_j^g = N_j \sin\alpha + Q_j \cos\alpha \quad (1.14e)$$

$$M_j^g = M_j \quad (1.14f)$$

In matrix form, Equations (1.14a) to (1.14f) can be expressed as

$$\{F_e^g\} = [T]\{P\} \quad (1.15)$$

LOAD TRANSFORMATION

In matrix form, Equations (1.14a) to (1.14f) can be expressed as

$$\{F_e^g\} = [T]\{P\} \tag{1.15}$$

where $\{F_e^g\}$ is the member force vector in the global coordinate system and $[T]$ is the transformation matrix, both given as

$$\{F_e^g\} = \begin{Bmatrix} H_i^g \\ V_i^g \\ M_i^g \\ H_j^g \\ V_j^g \\ M_j^g \end{Bmatrix} \text{ and } [T] = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & 0 & 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

DISPLACEMENT TRANSFORMATION

The displacements in the global coordinate system can be related to those in the local coordinate system by following the procedure similar to the force transformation. The displacements in both coordinate systems are shown in Figure 1.13.

From Figure 1.13,

$$u_i = u_i^g \cos\alpha + v_i^g \sin\alpha \tag{1.16a}$$

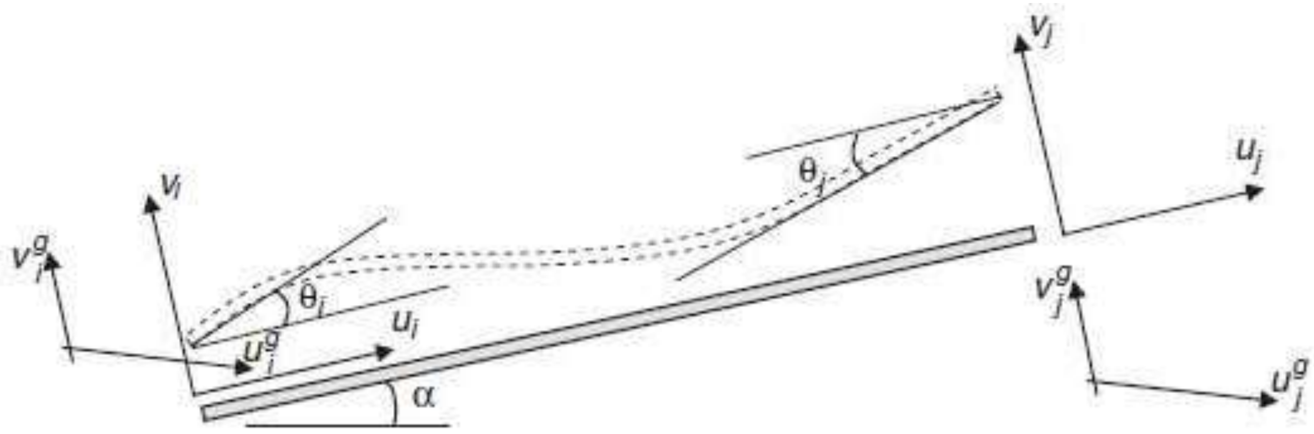


FIGURE 1.13. Displacements in the local and global coordinate systems.

DISPLACEMENT TRANSFORMATION

$$v_i = -u_i^g \sin\alpha + v_i^g \cos\alpha \tag{1.16b}$$

$$\theta_i = \theta_i^g \tag{1.16c}$$

$$u_j = u_j^g \cos\alpha + v_j^g \sin\alpha \tag{1.16d}$$

$$v_j = -u_j^g \sin\alpha + v_j^g \cos\alpha \tag{1.16e}$$

$$\theta_j = \theta_j^g \tag{1.16f}$$

DISPLACEMENT TRANSFORMATION

In matrix form, Equations (1.16a) to (1.16f) can be expressed as

$$\{d\} = [T]^t \{D_e^g\} \tag{1.17}$$

where $\{D_e^g\}$ is the member displacement vector in the global coordinate system corresponding to the directions in which the freedom codes are specified and is given as

$$\{D_e^g\} = \begin{Bmatrix} u_i^g \\ v_i^g \\ \theta_i^g \\ u_j^g \\ v_j^g \\ \theta_j^g \end{Bmatrix}$$

and $[T]^t$ is the transpose of $[T]$.

Member Stiffness in Global Coordinate System

From Equation (1.15),

$$\begin{aligned}\{F_e^g\} &= [T]\{P\} \\ &= [T][K_e]\{d\} \qquad \text{from Equation (1.9)} \\ &= [T][K_e][T]^t\{D_e^g\} \quad \text{from Equation (1.17)} \\ &= [K_e^g]\{D_e^g\}\end{aligned}$$

where $[K_e^g] = [T][K_e][T]^t$ = member stiffness matrix in the global coordinate system.

Member Stiffness in Global Coordinate System

An explicit expression for $[K_e^g]$ is

$$[K_e^g] = \begin{bmatrix} C^2\frac{EA}{L} + S^2\frac{12EI}{L^3} & SC\left(\frac{EA}{L} - \frac{12EI}{L^3}\right) & -S\frac{6EI}{L^2} & -\left(C^2\frac{EA}{L} + S^2\frac{12EI}{L^3}\right) & -SC\left(\frac{EA}{L} - \frac{12EI}{L^3}\right) & -S\frac{6EI}{L^2} \\ & S^2\frac{EA}{L} + C^2\frac{12EI}{L^3} & C\frac{6EI}{L^2} & -SC\left(\frac{EA}{L} - \frac{12EI}{L^3}\right) & -\left(S^2\frac{EA}{L} + C^2\frac{12EI}{L^3}\right) & C\frac{6EI}{L^2} \\ & & \frac{4EI}{L} & S\frac{6EI}{L^2} & -C\frac{6EI}{L^2} & \frac{2EI}{L} \\ & & & C^2\frac{EA}{L} + S^2\frac{12EI}{L^3} & SC\left(\frac{EA}{L} - \frac{12EI}{L^3}\right) & S\frac{6EI}{L^2} \\ & \text{Symmetric} & & & S^2\frac{EA}{L} + C^2\frac{12EI}{L^3} & -C\frac{6EI}{L^2} \\ & & & & & \frac{4EI}{L} \end{bmatrix} \quad (1.19)$$

where C = cos α; S = sin α.

Assembly of Structure Stiffness Matrix

Consider part of a structure with four externally applied forces, F1, F2, F4, and F5, and two applied moments, M3 and M6, acting at the two joints p and q connecting three members A, B, and C as shown in Figure 1.14. The freedom codes at joint p are {1, 2, 3} and at joint q are {4, 5, 6}. The structure stiffness matrix [K] is assembled on the basis of two conditions: compatibility and equilibrium conditions at the joints.

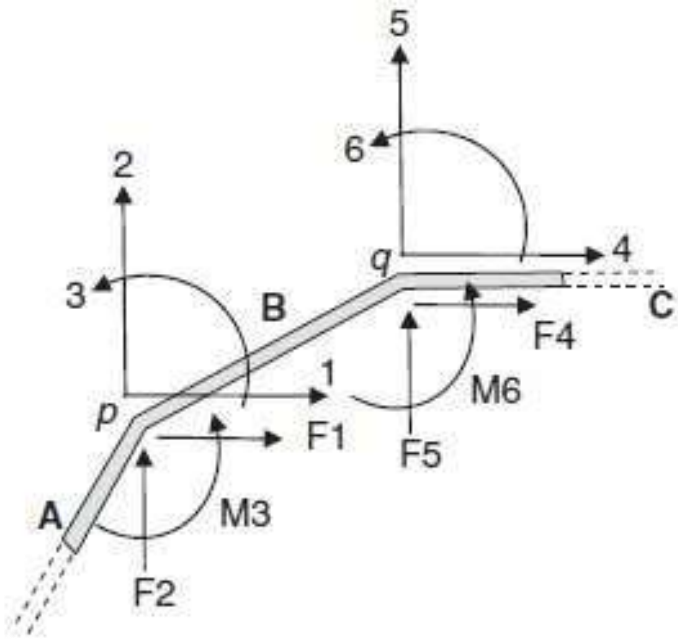


FIGURE 1.14. Assembly of structure stiffness matrix [K].

Compatibility Condition

At joint p, the global displacements are $D1$ (horizontal), $D2$ (vertical), and $D3$ (rotational). Similarly, at joint q, the global displacements are $D4$ (horizontal), $D5$ (vertical), and $D6$ (rotational). The compatibility condition is that the displacements ($D1$, $D2$, and $D3$) at end p of member A are the same as those at end p of member B. Thus, $(u_j^g)_A = (u_i^g)_B = D1$, $(v_j^g)_A = (v_i^g)_B = D2$, and $(\theta_j^g)_A = (\theta_i^g)_B = D3$. The same condition applies to displacements ($D4$, $D5$, and $D6$) at end q of both members B and C.

The member stiffness matrix in the global coordinate system given in Equation (1.19) can be written as

$$[K_e^g] = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} \end{bmatrix} \tag{1.20}$$

where $k_{11} = C^2 \frac{EA}{L} + S^2 \frac{12EI}{L^3}$, etc.

Compatibility Condition

For member A, from Equation (1.18),

$$\left(H_j^g\right)_A = \ldots.. + \ldots.. + \ldots.. + (k_{44})_A D_1 + (k_{45})_A D_2 + (k_{46})_A D_3 \quad (1.21a)$$

$$\left(V_j^g\right)_A = \ldots.. + \ldots.. + \ldots.. + (k_{54})_A D_1 + (k_{55})_A D_2 + (k_{56})_A D_3 \quad (1.21b)$$

$$\left(M_j^g\right)_A = \ldots.. + \ldots.. + \ldots.. + (k_{64})_A D_1 + (k_{65})_A D_2 + (k_{66})_A D_3 \quad (1.21c)$$

Similarly, for member B,

$$(H_i^g)_B = (k_{11})_B D_1 + (k_{12})_B D_2 + (k_{13})_B D_3 + (k_{14})_B D_4 + (k_{15})_B D_5 + (k_{16})_B D_6 \quad (1.21d)$$

$$(V_i^g)_B = (k_{21})_B D_1 + (k_{22})_B D_2 + (k_{23})_B D_3 + (k_{24})_B D_4 + (k_{25})_B D_5 + (k_{26})_B D_6 \quad (1.21e)$$

$$(M_i^g)_B = (k_{31})_B D_1 + (k_{32})_B D_2 + (k_{33})_B D_3 + (k_{34})_B D_4 + (k_{35})_B D_5 + (k_{36})_B D_6 \quad (1.21f)$$

Compatibility Condition

$$\left(H_j^g\right)_B = (k_{41})_B D_1 + (k_{42})_B D_2 + (k_{43})_B D_3 + (k_{44})_B D_4 + (k_{45})_B D_5 + (k_{46})_B D_6 \tag{1.21g}$$

$$\left(V_j^g\right)_B = (k_{51})_B D_1 + (k_{52})_B D_2 + (k_{53})_B D_3 + (k_{54})_B D_4 + (k_{55})_B D_5 + (k_{56})_B D_6 \tag{1.21h}$$

$$\left(M_j^g\right)_B = (k_{61})_B D_1 + (k_{62})_B D_2 + (k_{63})_B D_3 + (k_{64})_B D_4 + (k_{65})_B D_5 + (k_{66})_B D_6 \tag{1.21i}$$

Similarly, for member C,

$$(H_i^g)_C = (k_{11})_C D_1 + (k_{12})_C D_2 + (k_{13})_C D_3 + \ldots\ldots + \ldots\ldots + \ldots\ldots \tag{1.21j}$$

$$(V_i^g)_C = (k_{21})_C D_1 + (k_{22})_C D_2 + (k_{23})_C D_3 + \ldots\ldots + \ldots\ldots + \ldots\ldots \tag{1.21k}$$

$$(M_i^g)_C = (k_{31})_C D_1 + (k_{32})_C D_2 + (k_{33})_C D_3 + \ldots\ldots + \ldots\ldots + \ldots\ldots \tag{1.21l}$$

Equilibrium Condition

Any of the externally applied forces or moments applied in a certain direction at a joint of a structure is equal to the sum of the member forces acting in the same direction for members connected at that joint in the global coordinate system. Therefore, at joint *p*,

$$F1 = \left(H_j^g\right)_A + \left(H_i^g\right)_B \tag{1.22a}$$

$$F2 = \left(V_j^g\right)_A + \left(V_i^g\right)_B \tag{1.22b}$$

$$M3 = \left(M_j^g\right)_A + \left(M_i^g\right)_B \tag{1.22c}$$

Also, at joint q,

$$F4 = \left(H_j^g\right)_B + \left(H_i^g\right)_C \tag{1.22d}$$

$$F5 = \left(V_j^g\right)_B + \left(V_i^g\right)_C \tag{1.22e}$$

$$M6 = \left(M_j^g\right)_B + \left(M_i^g\right)_C \tag{1.22f}$$

Equilibrium Condition

$$\begin{Bmatrix} \bullet \\ F1 \\ F2 \\ M3 \\ F4 \\ F5 \\ M6 \\ \bullet \end{Bmatrix} = \begin{bmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & (k_{44})_A + (k_{11})_B & (k_{45})_A + (k_{12})_B & (k_{46})_A + (k_{13})_B & (k_{14})_B & (k_{15})_B & (k_{16})_B & \bullet \\ \bullet & (k_{54})_A + (k_{21})_B & (k_{55})_A + (k_{22})_B & (k_{56})_A + (k_{23})_B & (k_{24})_B & (k_{25})_B & (k_{26})_B & \bullet \\ \bullet & (k_{64})_A + (k_{31})_B & (k_{65})_A + (k_{32})_B & (k_{66})_A + (k_{33})_B & (k_{34})_B & (k_{35})_B & (k_{36})_B & \bullet \\ \bullet & (k_{41})_B & (k_{42})_B & (k_{43})_B & (k_{44})_B + (k_{11})_C & (k_{45})_B + (k_{12})_C & (k_{46})_B + (k_{13})_C & \bullet \\ \bullet & (k_{51})_B & (k_{52})_B & (k_{53})_B & (k_{54})_B + (k_{21})_C & (k_{55})_B + (k_{22})_C & (k_{56})_B + (k_{23})_C & \bullet \\ \bullet & (k_{61})_B & (k_{62})_B & (k_{63})_B & (k_{64})_B + (k_{31})_C & (k_{65})_B + (k_{32})_C & (k_{66})_B + (k_{33})_C & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{bmatrix} \begin{Bmatrix} \bullet \\ \bullet \\ D1 \\ D2 \\ D3 \\ D4 \\ D5 \\ D4 \\ \bullet \\ \bullet \end{Bmatrix} \tag{1.23}$$

where the "•" stands for matrix coefficients contributed from the other parts of the structure. In simple form, Equation (1.23) can be written as

$$\{F\} = [K]\{D\}$$

which is identical to Equation (1.7). Equation (1.23) shows how the structure equilibrium equation is set up in terms of the load vector $\{F\}$, structure stiffness matrix $[K]$, and the displacement vector $\{D\}$.

Assembly of Structure Stiffness Matrix

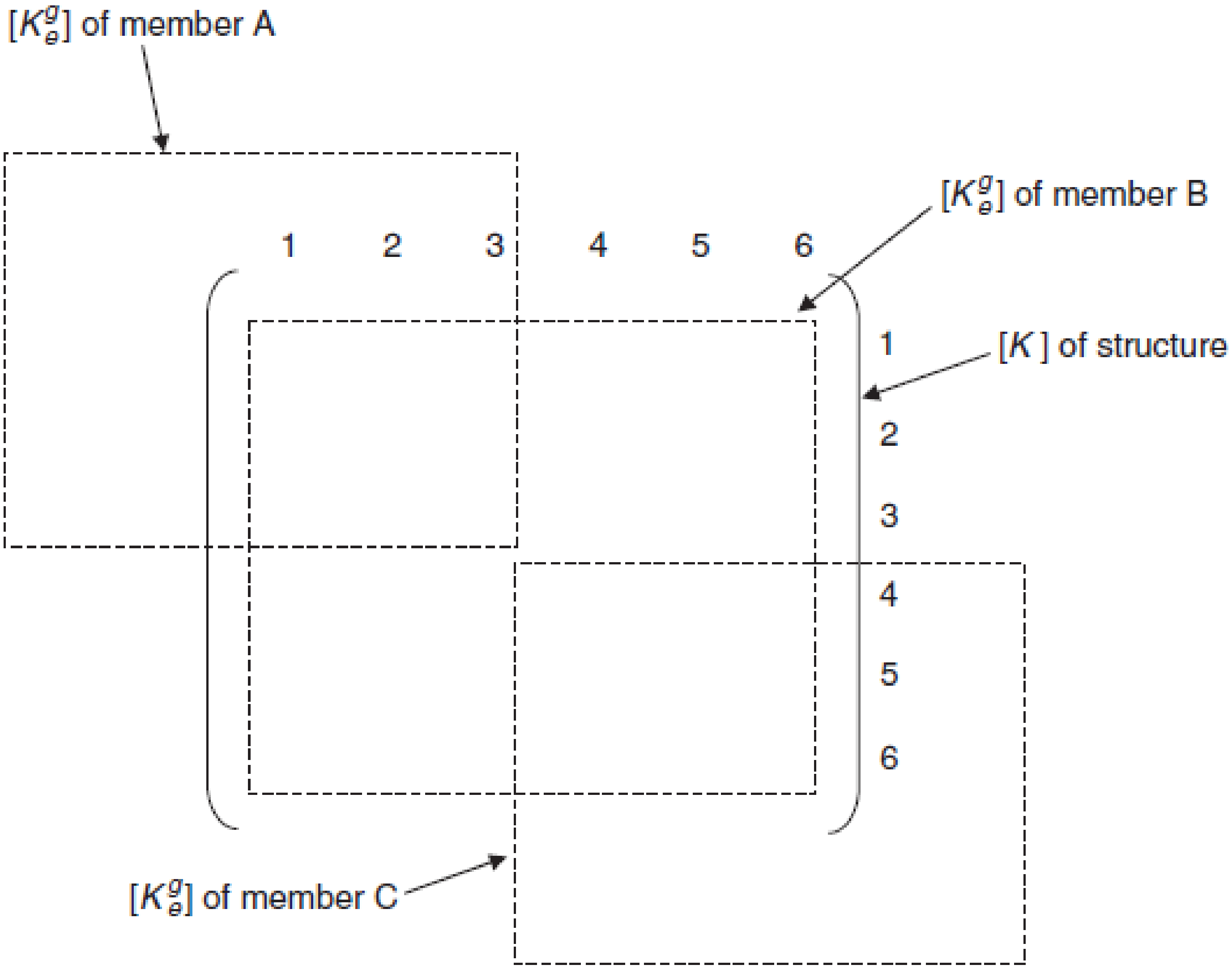


FIGURE 1.16. Assembly of structure stiffness matrix.

ASSEMBLY OF LOAD VECTOR

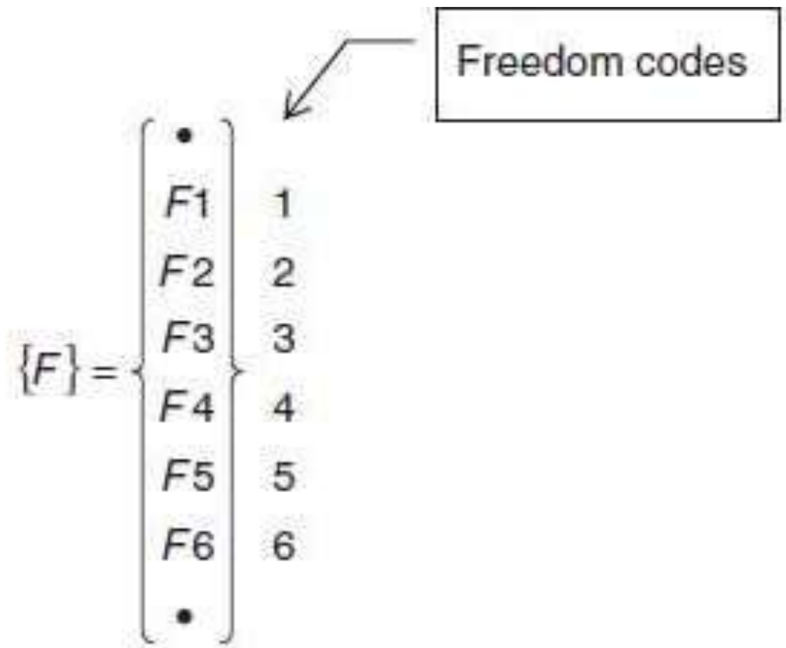


FIGURE 1.17. Assembly of load vector.

METHODS OF SOLUTION

The displacements of the structure can be found by solving Equation (1.23). Because of the huge size of the matrix equation usually encountered in practice, Equation (1.23) is solved routinely by numerical methods such as the Gaussian elimination method and the iterative Gauss–Seidel method. It should be noted that in using these

numerical methods, the procedure is analogous to inverting the structure stiffness matrix, which is subsequently multiplied by the load vector as in Equation (1.8):

$$\{D\} = [K]^{-1}\{F\} \tag{1.8}$$

METHODS OF SOLUTION

The numerical procedure fails only if an inverted [K] cannot be found. This situation occurs when the determinant of [K] is zero, implying an unstable structure. Unstable structures with a degree of statically indeterminacy, f_r, greater than zero (see Section 1.2) will have a zero determinant of [K]. In numerical manipulation by computers, an exact zero is sometimes difficult to obtain. In such cases, a good indication of an unstable structure is to examine the displacement vector {D}, which would include some exceptionally large values.

CALCULATION OF MEMBER FORCES

Member forces are calculated according to Equation (1.9). Hence,

$$\begin{aligned}\{P\} &= [K_e]\{d\} \\ &= [K_e]\,[T]^t\{D_e^g\}\end{aligned} \tag{1.24}$$

where $\{D_e^g\}$ is extracted from $\{D\}$ for each member according to its freedom codes and

$$[K_e][T]^t = \begin{bmatrix} C\dfrac{EA}{L} & S\dfrac{EA}{L} & 0 & -C\dfrac{EA}{L} & -S\dfrac{EA}{L} & 0 \\ -S\dfrac{12EI}{L^3} & C\dfrac{12EI}{L^3} & \dfrac{6EI}{L^2} & S\dfrac{12EI}{L^3} & -C\dfrac{12EI}{L^3} & \dfrac{6EI}{L^2} \\ -S\dfrac{6EI}{L^2} & C\dfrac{6EI}{L^2} & \dfrac{4EI}{L} & S\dfrac{6EI}{L^2} & -C\dfrac{6EI}{L^2} & \dfrac{2EI}{L} \\ -C\dfrac{EA}{L} & -S\dfrac{EA}{L} & 0 & C\dfrac{EA}{L} & S\dfrac{EA}{L} & 0 \\ S\dfrac{12EI}{L^3} & -C\dfrac{12EI}{L^3} & -\dfrac{6EI}{L^2} & -S\dfrac{12EI}{L^3} & C\dfrac{12EI}{L^3} & -\dfrac{6EI}{L^2} \\ -S\dfrac{6EI}{L^2} & C\dfrac{6EI}{L^2} & \dfrac{2EI}{L} & S\dfrac{6EI}{L^2} & -C\dfrac{6EI}{L^2} & \dfrac{4EI}{L} \end{bmatrix}$$

MEMBER FORCES

$$\{P\} = \begin{Bmatrix} N_i \\ Q_i \\ M_i \\ N_j \\ Q_j \\ M_j \end{Bmatrix} = \begin{bmatrix} C\frac{EA}{L} & S\frac{EA}{L} & 0 & -C\frac{EA}{L} & -S\frac{EA}{L} & 0 \\ -S\frac{12EI}{L^3} & C\frac{12EI}{L^3} & \frac{6EI}{L^2} & S\frac{12EI}{L^3} & -C\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ -S\frac{6EI}{L^2} & C\frac{6EI}{L^2} & \frac{4EI}{L} & S\frac{6EI}{L^2} & -C\frac{6EI}{L^2} & \frac{2EI}{L} \\ -C\frac{EA}{L} & -S\frac{EA}{L} & 0 & C\frac{EA}{L} & S\frac{EA}{L} & 0 \\ S\frac{12EI}{L^3} & -C\frac{12EI}{L^3} & -\frac{6EI}{L^2} & -S\frac{12EI}{L^3} & C\frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ -S\frac{6EI}{L^2} & C\frac{6EI}{L^2} & \frac{2EI}{L} & S\frac{6EI}{L^2} & -C\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{Bmatrix}$$

SUMMARY

1. Assign freedom codes to each joint indicating the displacement freedom at the ends of the members connected at that joint. Assign a freedom code of "zero" to any restrained displacement.

2. Assign an arrow to each member so that ends i and j are defined. Also, the angle of orientation α for the member is defined in Figure 1.18 as:

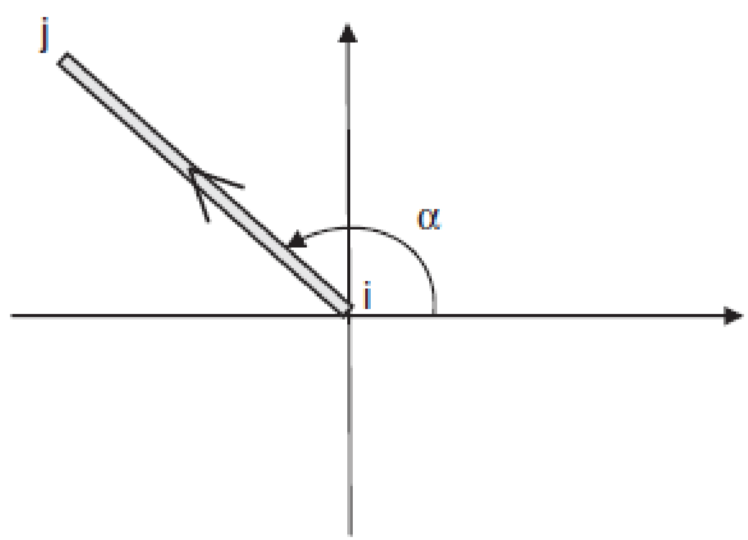


FIGURE 1.18. Definition of angle of orientation for member.

SUMMARY

3. Assemble the structure stiffness matrix $[K]$ from each of the member stiffness matrices.

4. Form the load vector $\{F\}$ of the structure.

5. Calculate the displacement vector $\{D\}$ by solving for $\{D\} = [K]^{-1}\{F\}$.

6. Extract the local displacement vector $\{D_e^g\}$ from $\{D\}$ and calculate the member force vector $\{P\}$ using $\{P\} = [K_e][T]^t\{D_e^g\}$.

Sign Convention for Member Force

Positive member forces and displacements obtained from the stiffness method of analysis are shown in Figure 1.19. To plot the forces in conventional axial force, shear force, and bending moment diagrams, it is necessary to translate them into a system commonly adopted for plotting.

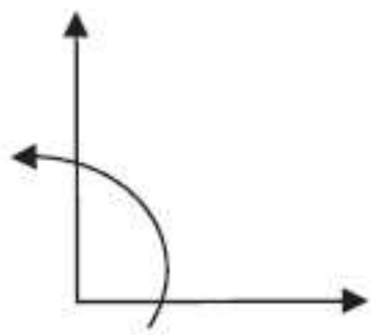


FIGURE 1.19. Direction of positive forces and displacements using stiffness method.

Axial Force

For a member under compression, the axial force at end *i* is positive (from analysis) and at end *j* is negative (from analysis), as shown in Figure 1.20.

FIGURE 1.20. Member under compression.

Shear Force

A shear force plotted positive in diagram is acting upward (positive from analysis) at end *i* and downward (negative from analysis) at end *j* as shown in Figure 1.21. Positive shear force is usually plotted in the space above the member.

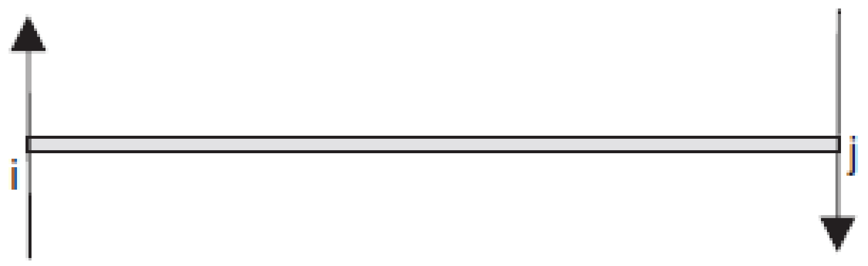


FIGURE 1.21. Positive shear forces.

Bending Moment

A member under sagging moment is positive in diagram (clockwise and negative from analysis) at end *i* and positive (anticlockwise and positive from analysis) at end *j* as shown in Figure 1.22. Positive bending moment is usually plotted in the space beneath the member. In doing so, a bending moment is plotted on the tension face of the member.

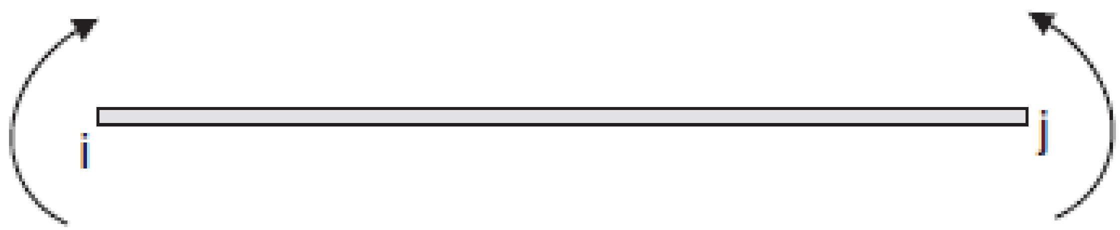


FIGURE 1.22. Sagging moment of a member.

THANK YOU

UNIT- V
INFLUENCE LINE DIAGRAMS

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Topics

- Introduction to Influence lines
- Influence lines for Beams
- Qualitative Influence lines
- Maximum Influence at a point due to Series of concentrated Loads
- Absolute maximum shear and moment

Introduction

(a)

(b)

Influence lines offer a quick and easy way of performing multiple analyses for a single structure. Response parameters such as *shear force* or *bending moment at a point* or *reaction at a support* for several load sets can be easily computed using influence lines

Influence Lines

- If Structure is subjected to moving load, the variation of the shear and bending moment in the member is best described using the Influence line.
- Once the IL constructed, one can tell at glance where the moving load should be placed on the structure so that it creates the greatest influence.

- Further more, magnitude of the associated reaction, shear, moment or deflection at the pt can be calculate from the ordinates of the ILD.

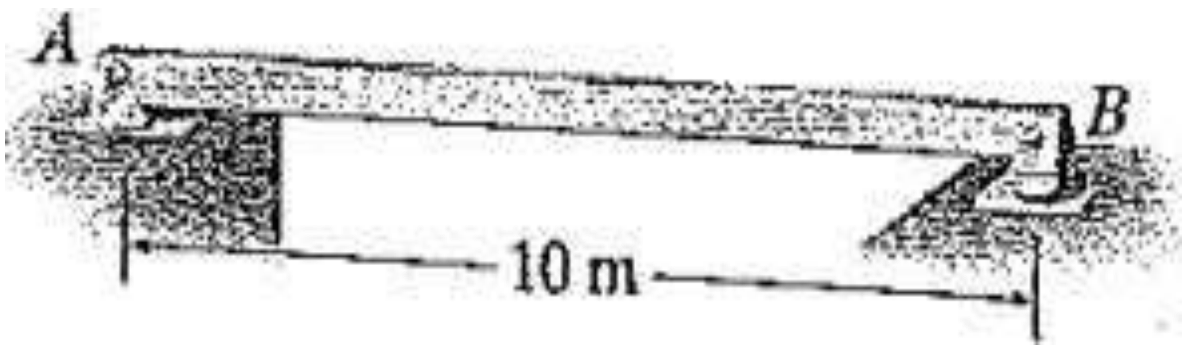
- So, that its play an important part in the design of bridges, industrial crane rails, conveyors and other structure where loads moves across the span.

Construction IL using Equilibrium Methods

- The most basic method of obtaining influence line is described below.
  - For a particular location of the unit load, solve for the equilibrium of the whole system and if required, as in the case of an internal force, also for a part of the member to obtain the response parameter for that location of the unit load. This gives the ordinate of the influence line at that particular location of the load.
  - Repeat this process for as many locations of the unit load as required to determine the shape of the influence line for the whole length of the member. It is often helpful if we can consider a generic location (or several locations) x of the unit load.
  - Joining ordinates for different locations of the unit load throughout the length of the member, we get the influence line for that particular response parameter.

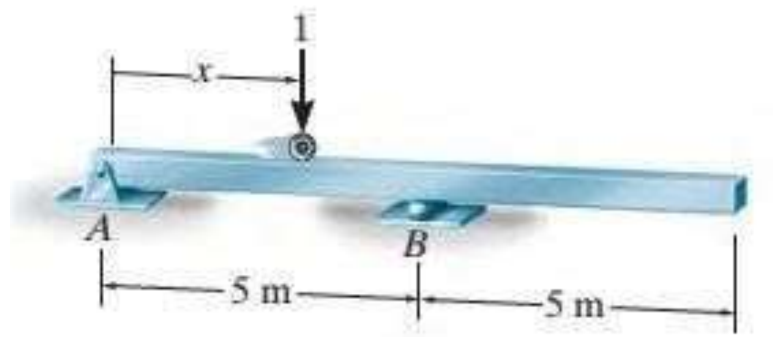
Example 8.1

Construct the influence line for the vertical reaction at A of the beam in Fig.



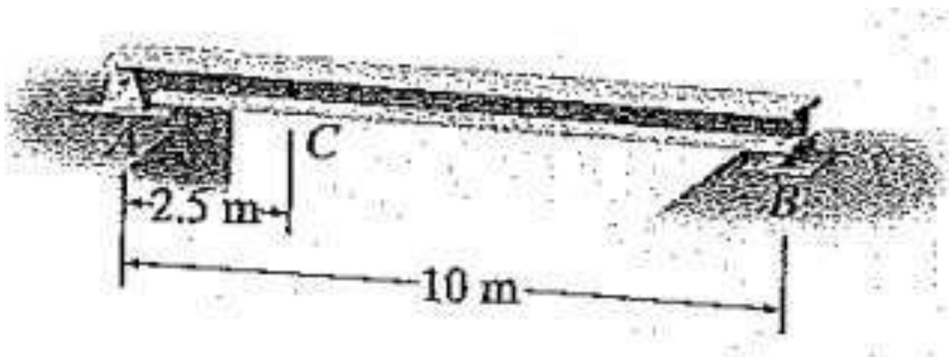
Example 8.2

Construct the influence line for the vertical reaction at B of the beam in Fig.



Example 8.3

Construct the influence line for the shear and moment at C of the beam in Fig.

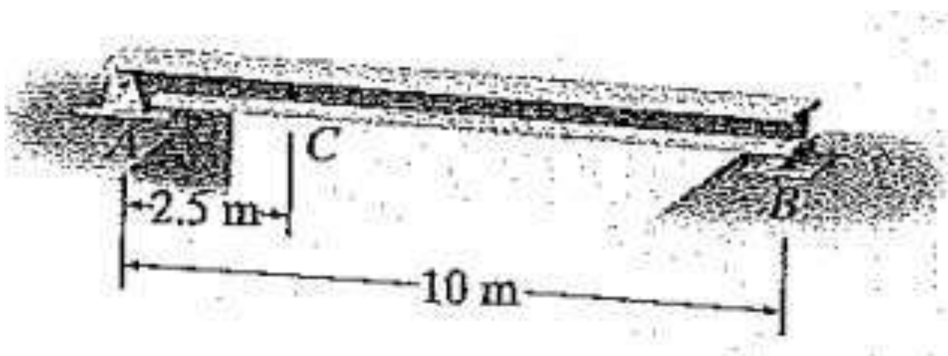


Influence Lines for Beams

- **Loading:** Once the Influence line for a function (reaction, shear or moment) has been constructed, then it will be possible to position the live load on the beam which will produce the maximum value of the function. Two types of loadings will be considered.
  - **concentrated force**
  - **uniform load**

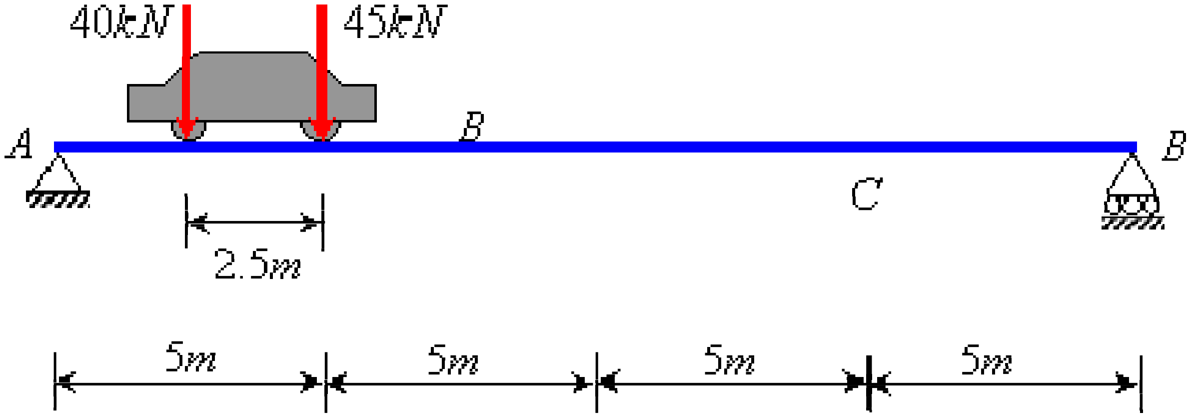
Example 8.4

Determine the maximum positive live shear that can be developed at point C in the beam shown in fig. due to a concentrated moving load of 4kN and a uniform moving load of 2 kN/m.



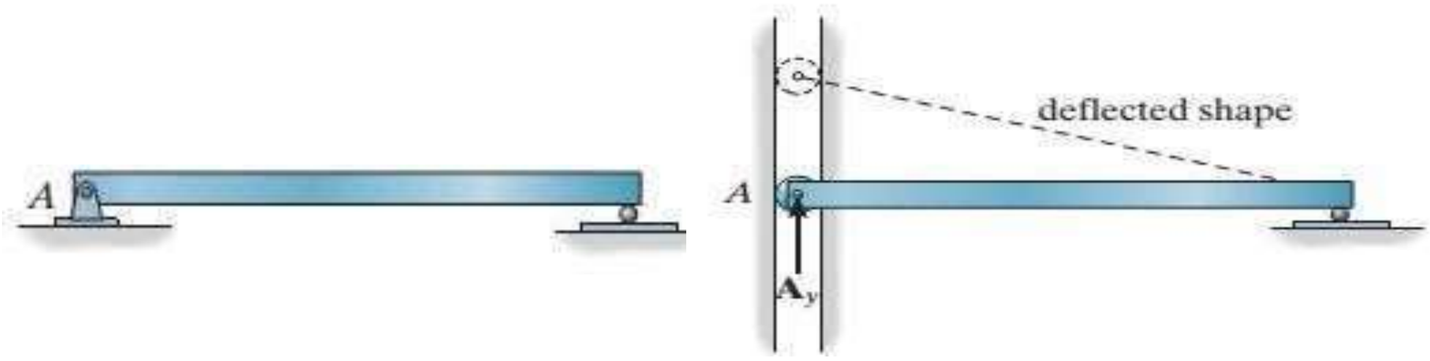
Homework 8.1

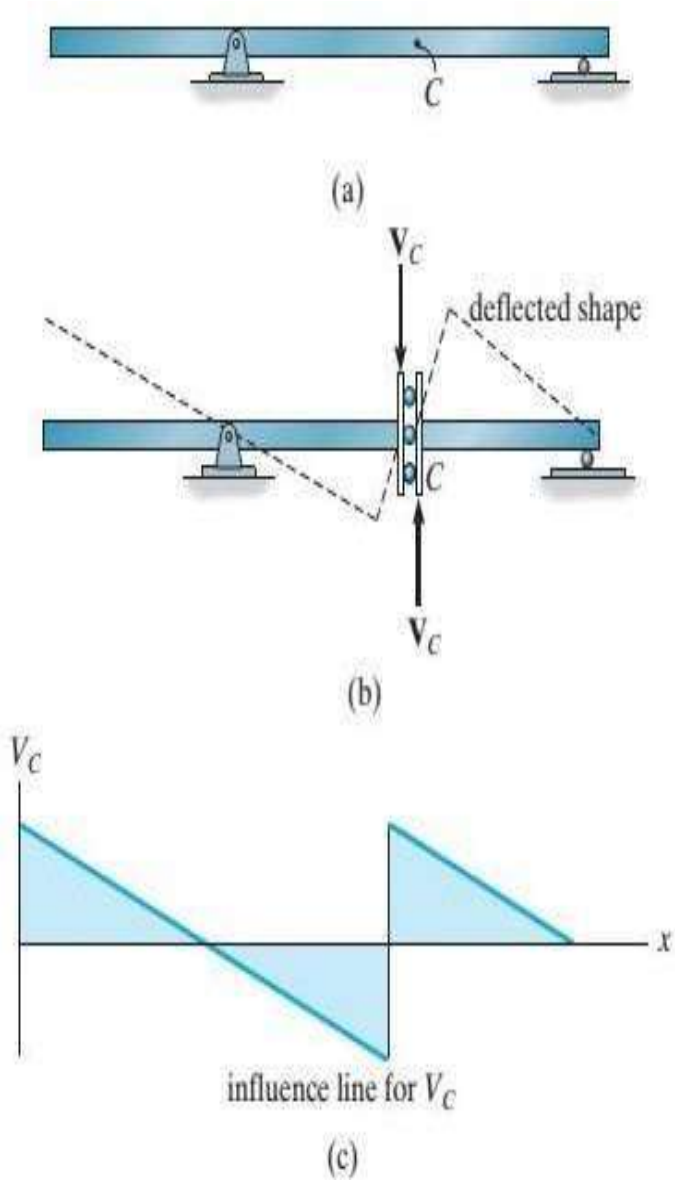
Find the maximum shear force at C for the moving load combination in fig. (**Ans: 58.75 kN**)

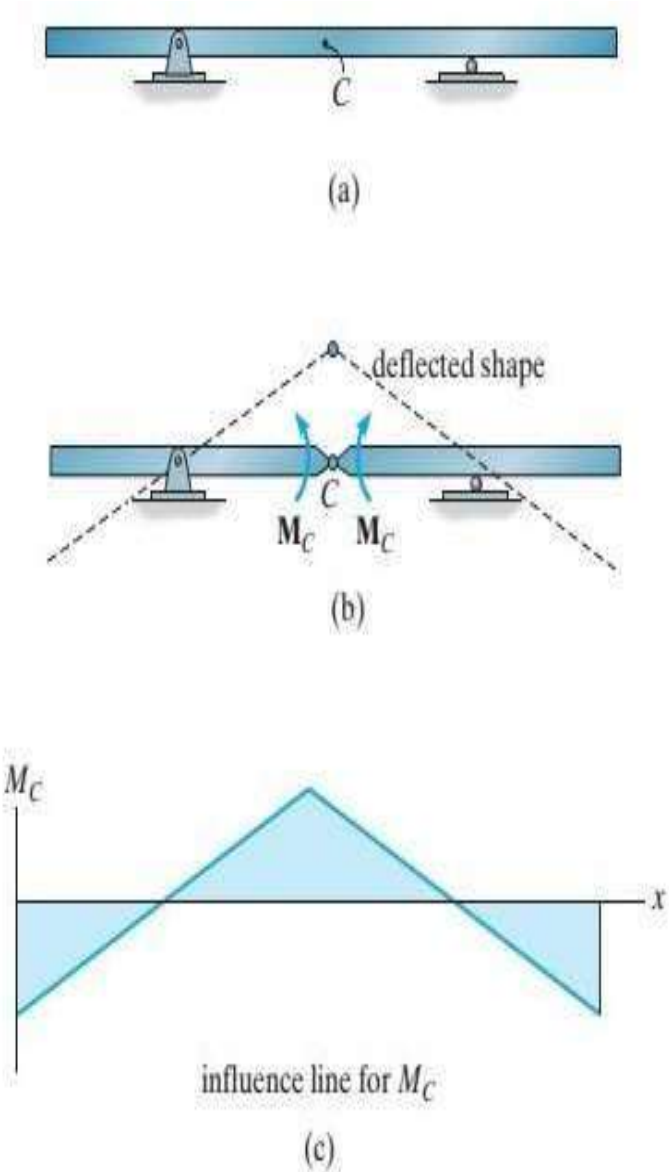


Qualitative Influence Lines

- In 1886, Muller Breslau developed a technique for rapidly constructing the shape of an IL.
- Muller Breslau Principle: *The IL for a function is to the same scale as the deflected shape of the beam when the beam is acted upon by the function.*

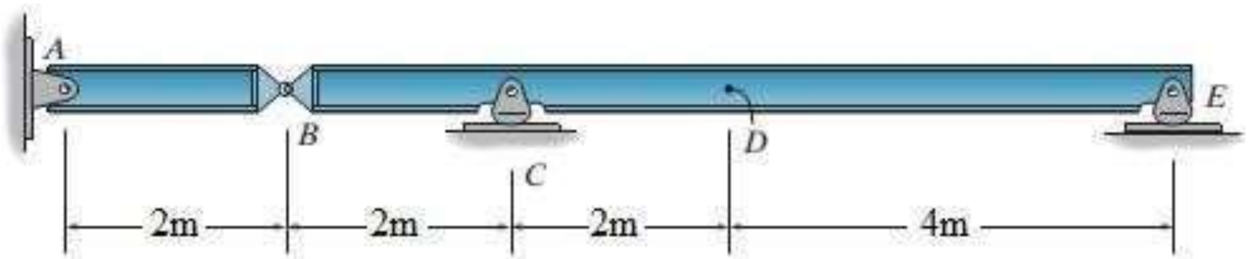



C
(a)
V_C
deflected shape
C
V_C
(b)
V_C
x
influence line for V_C
(c)

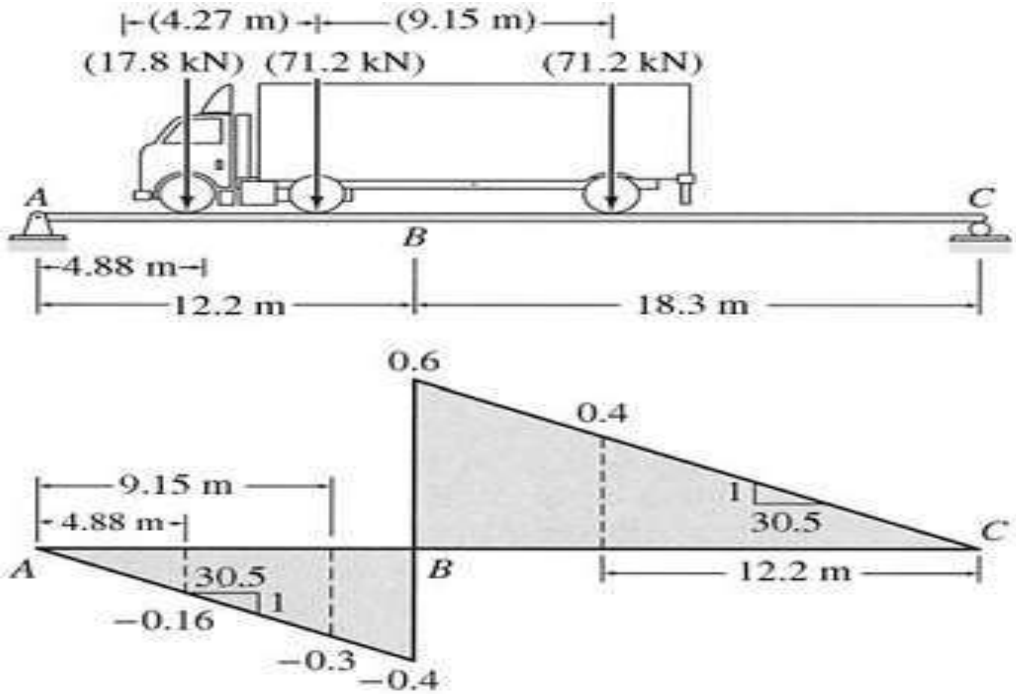

C
(a)
deflected shape
C
M_C
M_C
(b)
M_C
x
influence line for M_C
(c)

Example 8.5

Determine the maximum positive moment that can be developed at point D in the beam shown in fig. due to concentrated moving load of 16 kN, a uniform moving load of 3 kN/m and beam weight of 2 kN/m.



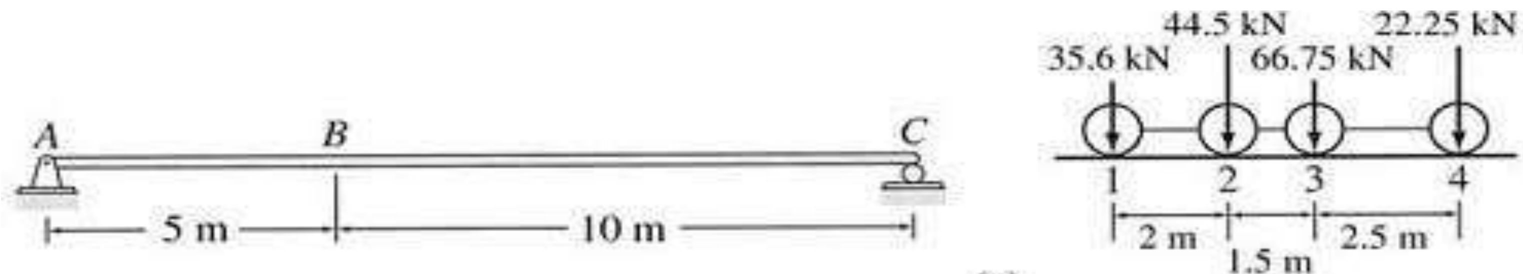
Maximum Influence at a point due to Series of concentrated Loads



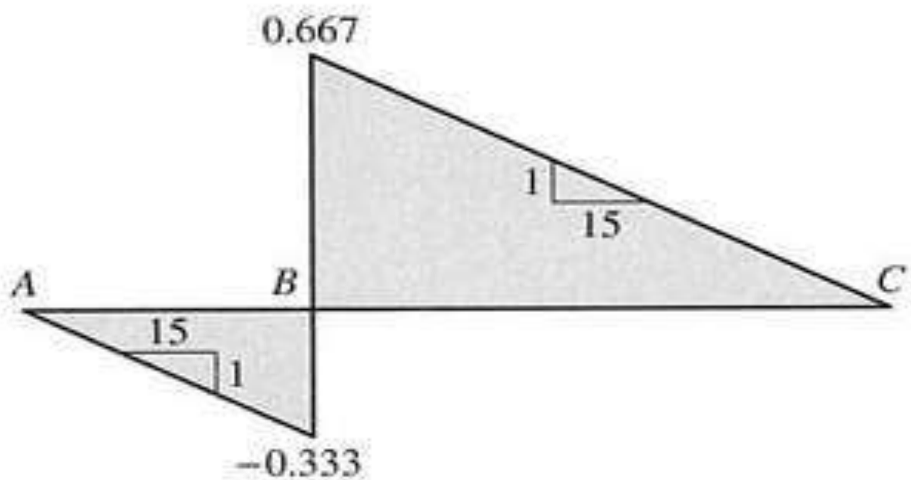
Influence Line for S_B kN/kN

$$S_B = -17.8(0.16) - 71.2(0.3) + 71.2(0.3) = 4.272 \text{ kN}$$

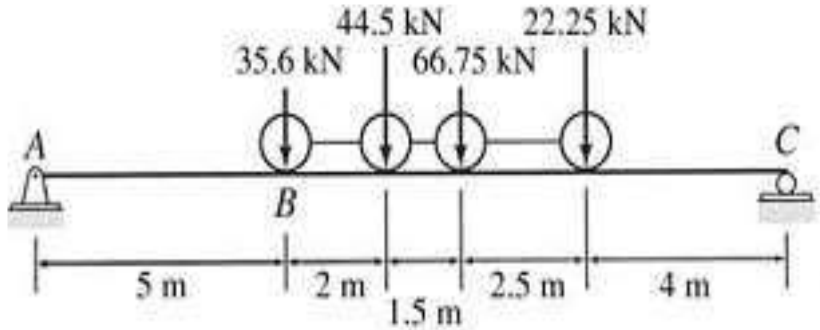
Example 8.6



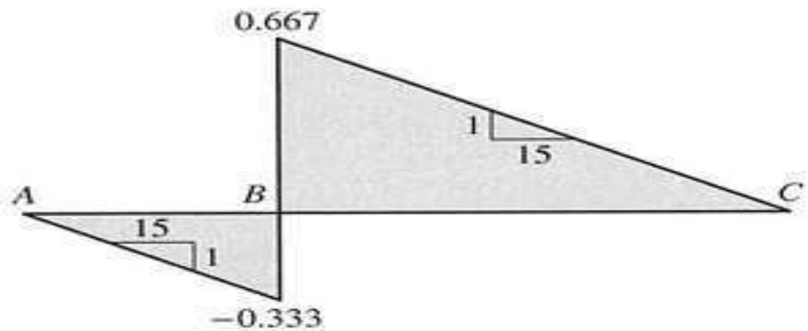
(a)



(b) Influence Line for S_B (kN/kN)

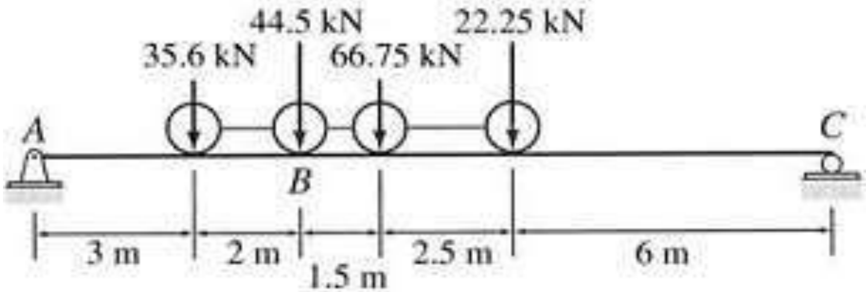


(c) Loading Position 1

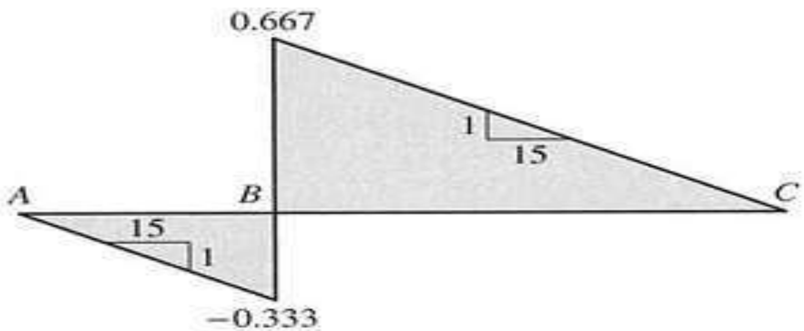


$$S_B = 35.6(10)\left(\frac{1}{15}\right) + 44.5(8)\left(\frac{1}{15}\right) + 66.75(6.5)\left(\frac{1}{15}\right) + 22.25(4)\left(\frac{1}{15}\right)$$

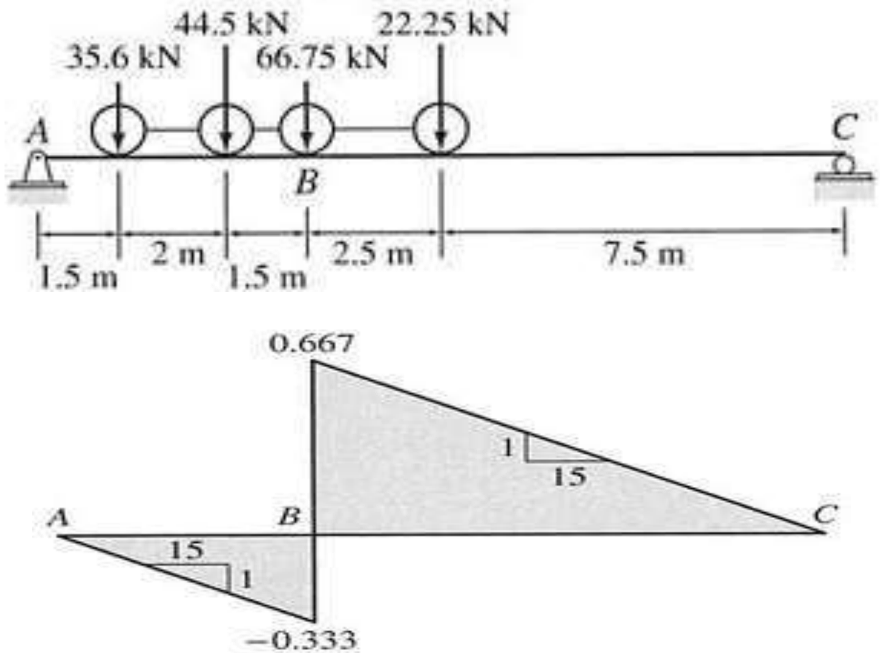
$$= 82.32 \text{ kN}$$



(d) Loading Position 2



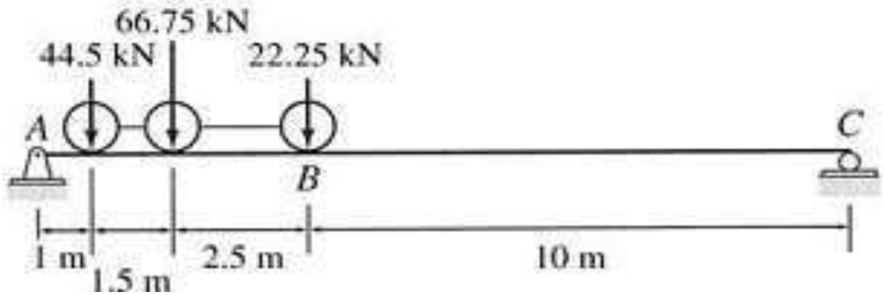
$$S_B = -35.6(3)\left(\frac{1}{15}\right) + 44.5(10)\left(\frac{1}{15}\right) + 66.75(8.5)\left(\frac{1}{15}\right) + 22.25(6)\left(\frac{1}{15}\right)$$
$$= 69.28 \text{ kN}$$



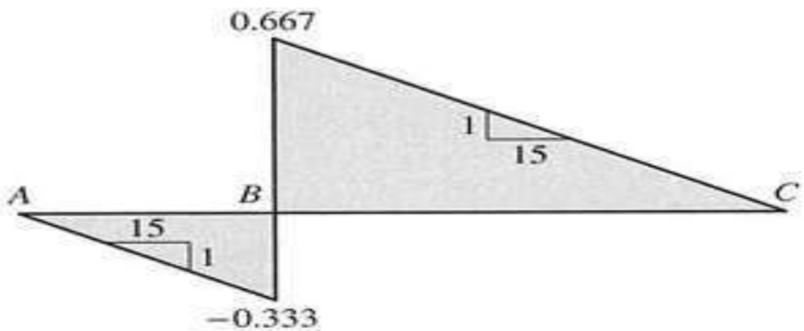
(e) Loading Position 3

$$S_B = -35.6(1.5)\left(\frac{1}{15}\right) - 44.5(3.5)\left(\frac{1}{15}\right) + 66.75(10)\left(\frac{1}{15}\right) + 22.25(7.5)\left(\frac{1}{15}\right)$$

$$= 41.68 \text{ kN}$$



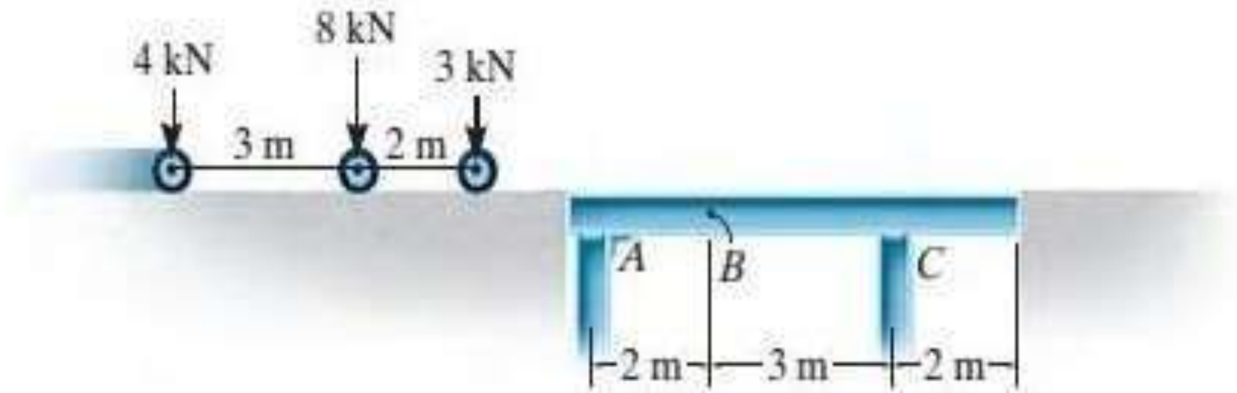
(f) Loading Position 4



$$S_B = -44.5(1)\left(\frac{1}{15}\right) - 66.75(2.5)\left(\frac{1}{15}\right) + 22.25(10)\left(\frac{1}{15}\right) = 0.742\text{ kN}$$

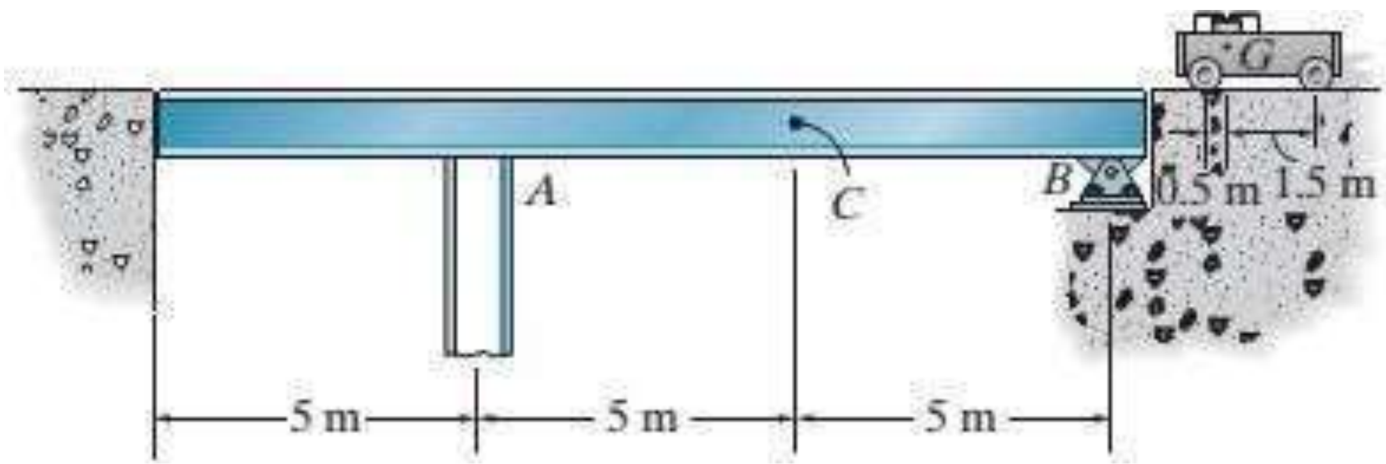
Homework 8.2

- Determine the maximum positive moment created at point B in the beam shown in Fig. due to wheel loads of the crane.

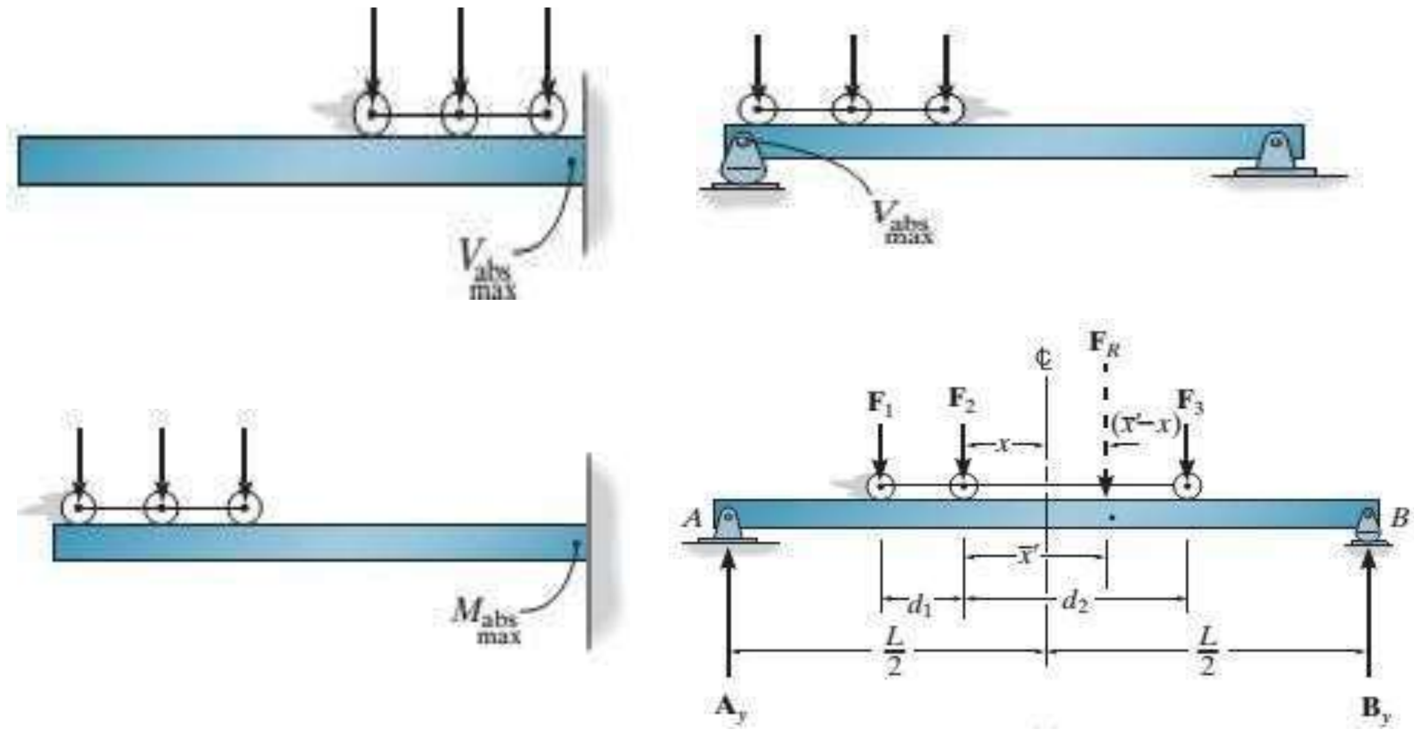


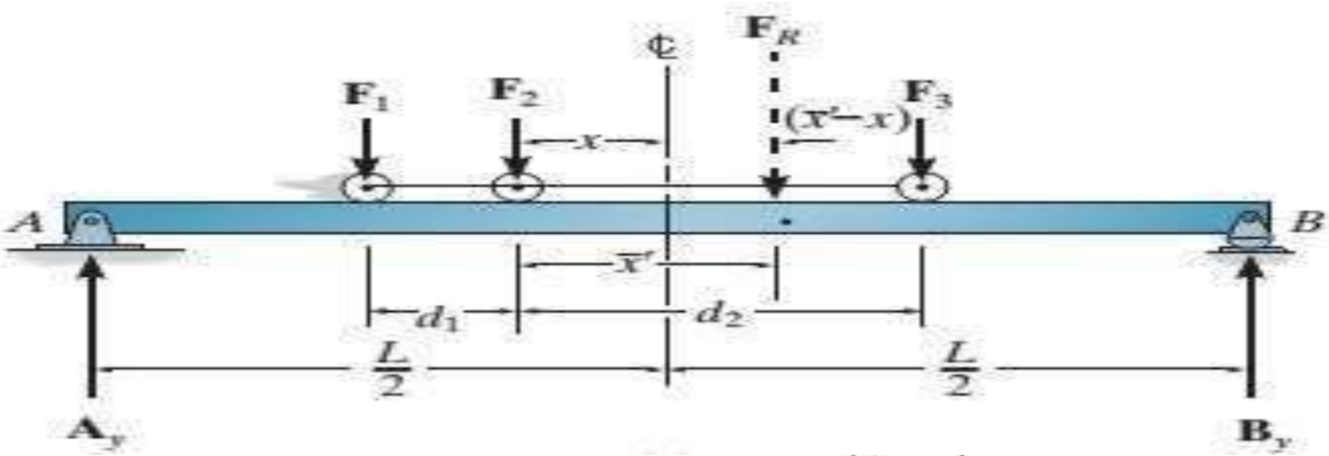
Homework 8.3

- Determine the maximum moment at C on the single girder caused by the moving dolly that has a mass of 2 Mg and a mass center at G. Assume A is a roller.

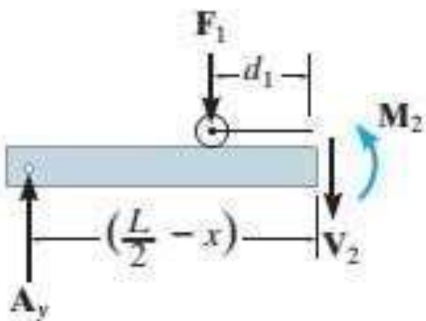


Absolute maximum shear and moment





$\Sigma M_B = 0;$ $\quad A_y = \frac{1}{L}(F_R)\left[\frac{L}{2} - (\overline{x}' - x)\right]$



$\Sigma M = 0;$ $\quad M_2 = A_y\left(\frac{L}{2} - x\right) - F_1 d_1$

$$= \frac{1}{L}(F_R)\left[\frac{L}{2} - (\overline{x}' - x)\right]\left(\frac{L}{2} - x\right) - F_1 d_1$$

$$= \frac{F_R L}{4} - \frac{F_R \overline{x}'}{2} - \frac{F_R x^2}{L} + \frac{F_R x \overline{x}'}{L} - F_1 d_1$$

For maximum M_2 we require

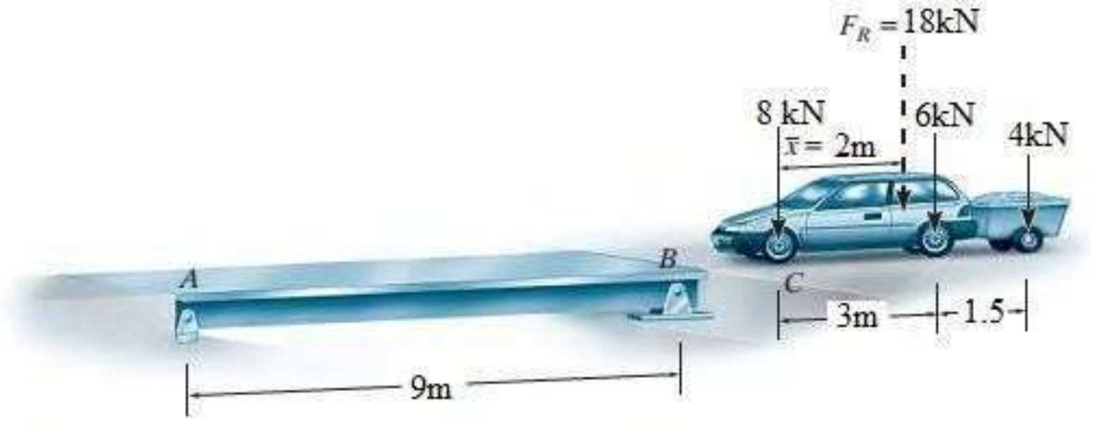
$$\frac{dM_2}{dx} = \frac{-2F_R x}{L} + \frac{F_R \overline{x}'}{L} = 0$$

or

$$x = \frac{\overline{x}'}{2}$$

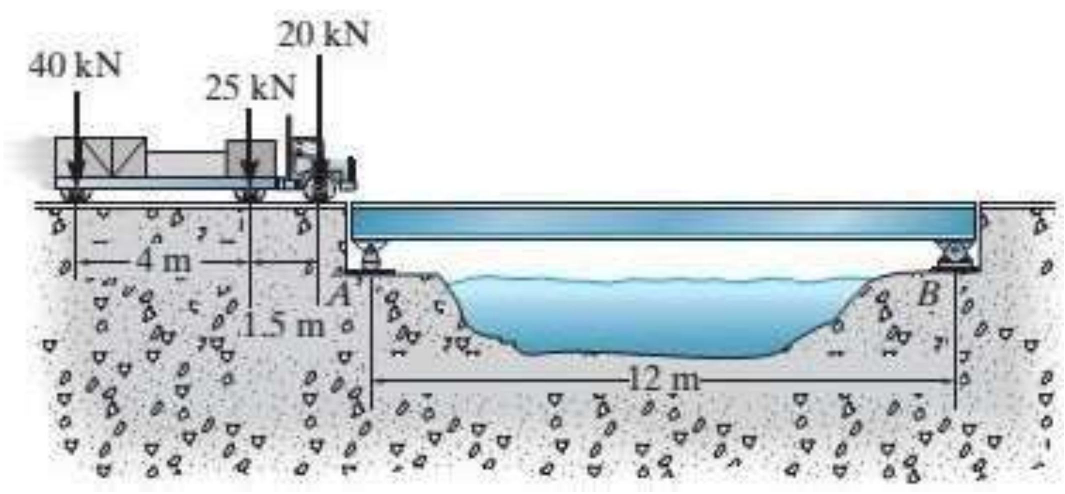
Example 8.7

- Determine the absolute maximum moment in the simply supported bridge deck shown in fig.



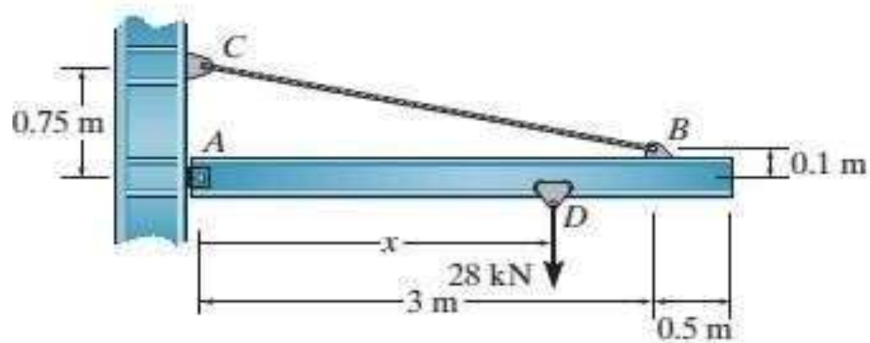
Homework 8.4

- Determine the absolute maximum shear and moment in the simply supported beam shown in fig.



Project Problem

- The chain hoist on the wall crane can be placed anywhere along the boom (0.1m <x< 3.4m) and has a rated capacity of 28 kN. Use an impact factor of 0.3 and determine the absolute moment in the boom and the maximum force developed in the tie rod BC.



- Influence line diagrams are drawn for various stress resultants like reaction, shear force, bending moment at specified points.
- Influence line diagram for a stress resultant is the one in which ordinate represent the value of the stress resultant for the position of unit load at the corresponding abscissa.
- For example If Figure 1 represents ILD for moment at section '*C*' in the beam *AB*, then the ordinate '*O*' represents the value of bending moment at '*C*' when a unit load is acting at section 1-1.

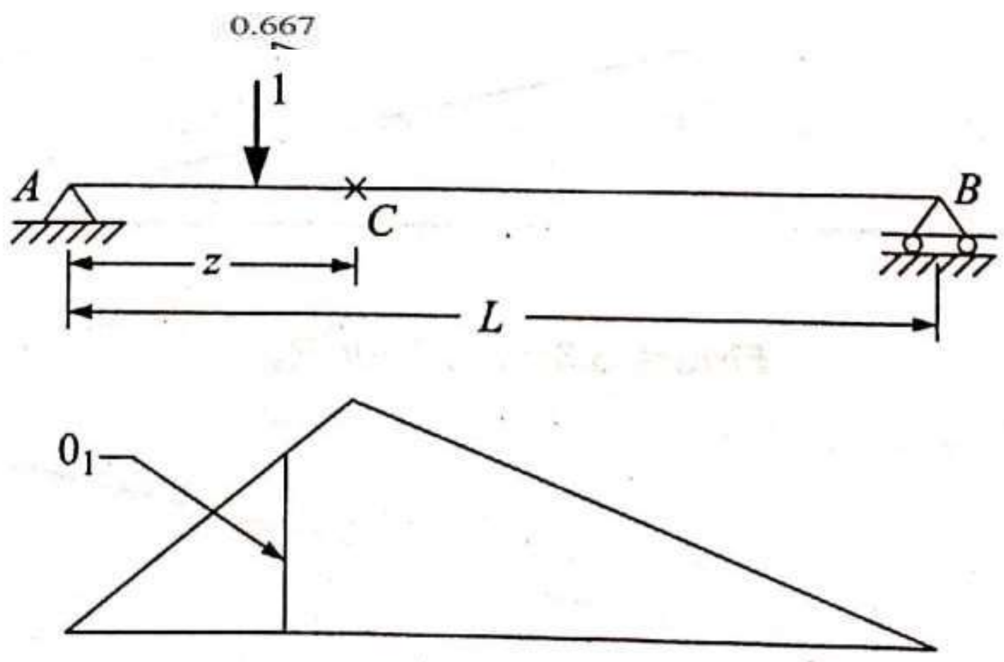


Figure 1: ILD for moment at '*C*'

Sign Convention

- Sign convention followed for shear force and bending moment

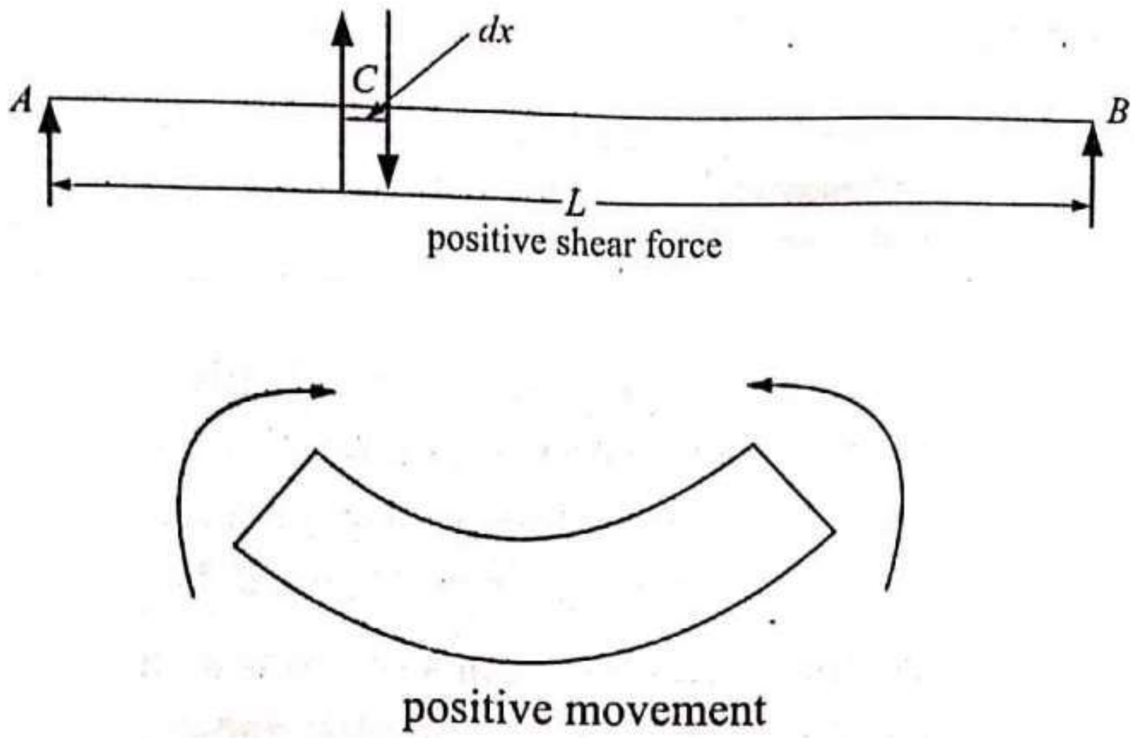


Figure 2: positive sense of SF and BM

Construction of Influence Lines

- The construction of influence lines can be done by using any one of the two approaches, one can construct the influence line at a specific point '*P*' in a member for any parameter (Reaction, Shear or Moment). In the present approaches it is assumed that the moving load is having dimensionless magnitude of unity.
- Classification of the approaches for construction of influence lines is given in Figure 3.

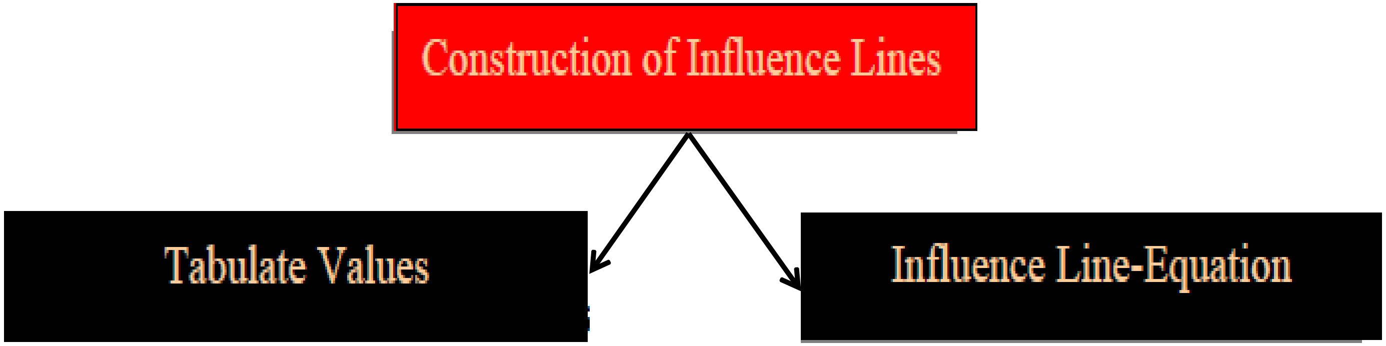


Figure 3: Classification of approach

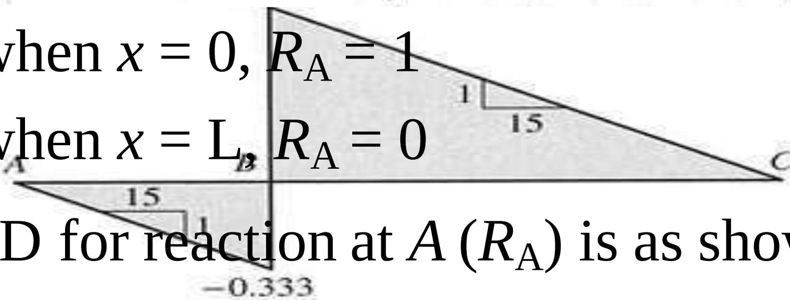
- Let the unit load be at a distance x from support A as shown in Figure 4.
- By taking moment about B, find out the reaction

$$R_A \times L = 1 \times (L\text{-}x)$$

$$R_A = \frac{1(L-x)}{L} = \left(1 - \frac{x}{L}\right), \text{ linear variation with } x$$

when $x = 0$, $R_A = 1$

when $x = L$, $R_A = 0$



- Hence, ILD for reaction at A (R_A) is as shown in Figure 4.

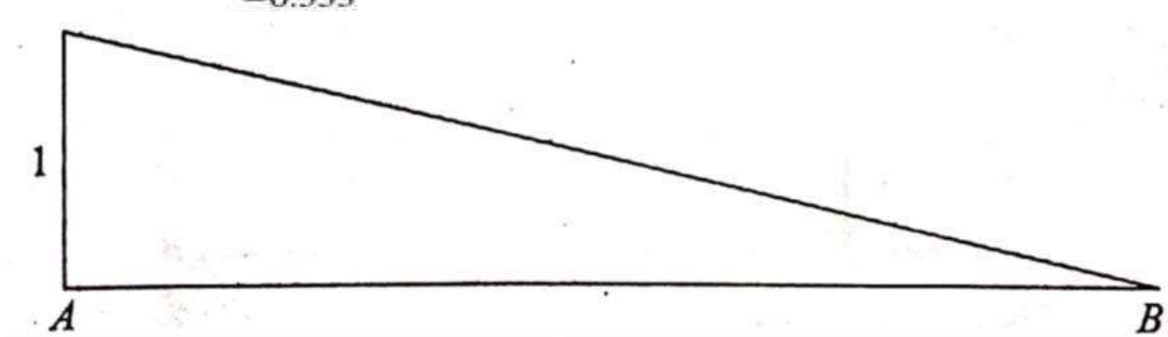


Figure 5: ILD for R_A

ILD for reaction at B (R_B):

- By taking moment about A, find out the reaction

$$R_B \times L = 1 \times x$$

$$R_B = \frac{x}{L}, \text{ linear variation}$$

when x = 0, $R_B = 0$

when x = L, $R_B = 1$

- Hence, ILD for reaction at B (R_B) is as shown in Figure 6.

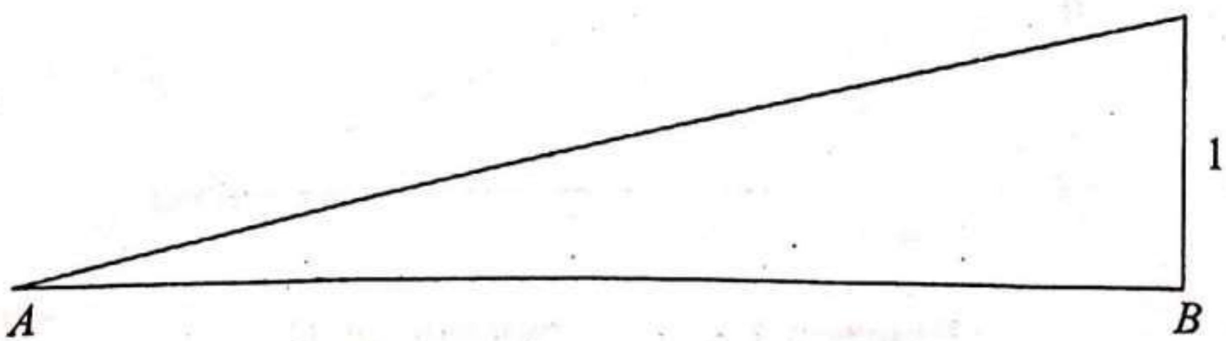


Figure 6: ILD for R_B

ILD for Shear Force at C (F_C):

- Let C be the section at a distance z from A as shown in Figure 4.
- **When $x < z$**

  Shear force at C $F_C = -R_B = -\frac{x}{L}$, linear variation,

  when $x = 0$, $F = 0$

  when $x = z$, $F = -\frac{z}{L}$
- **When $x > z$**

  Shear force at C = $F_C = R_A = \frac{L-x}{L}$, linear variation

  when $x = z$, $F_c = \frac{L-z}{L}$

  when $x = L$, $F_C = 0$

  Hence, ILD for shear force at C (F_C) is as shown in Figure 7.

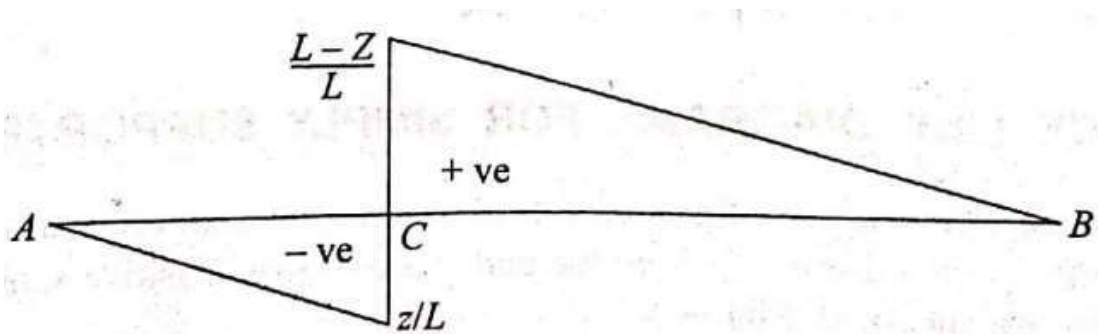


Figure 7: ILD for F_c

ILD for Moment at C (M_C):

- Let C be the section at a distance z from A as shown in Figure 4.

- **When $x < z$**

$M_C = R_B (L-z) = \frac{X}{L}(L-z)$ linear variation with x,

when $x = 0$, $M_C = 0$

when $x = z$, $M_C = \left(\frac{z(L-z)}{L}\right)$

- **When $x > z$**

$M_C = R_A z = \left(\frac{L-x}{L}\right)z$; linear variation with x

when $x = z$, $M_C = \frac{z(L-z)}{L}$

When $x = L$, $M_C = 0$

Hence, ILDfor moment at C (M_C) is as shown in Figure 8.

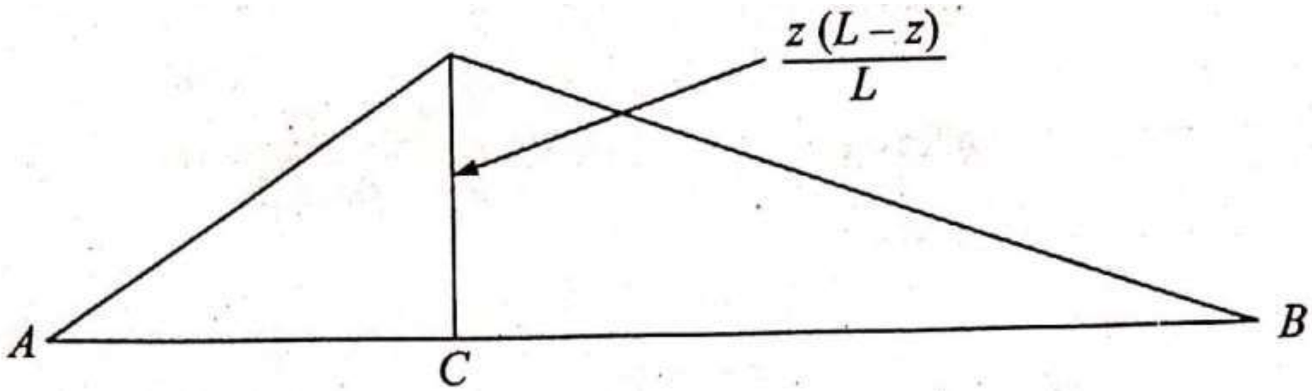


Figure 8: ILD for Moment at M_C

Influence Line Diagrams for Cantilever Beams

- Consider a cantilever beam of span L as shown in Figure 9.
- Influence line diagram for shear force and bending moment at fixed end A and at C are to be determined.
- Let a unit load act at a distance x from the free end B.

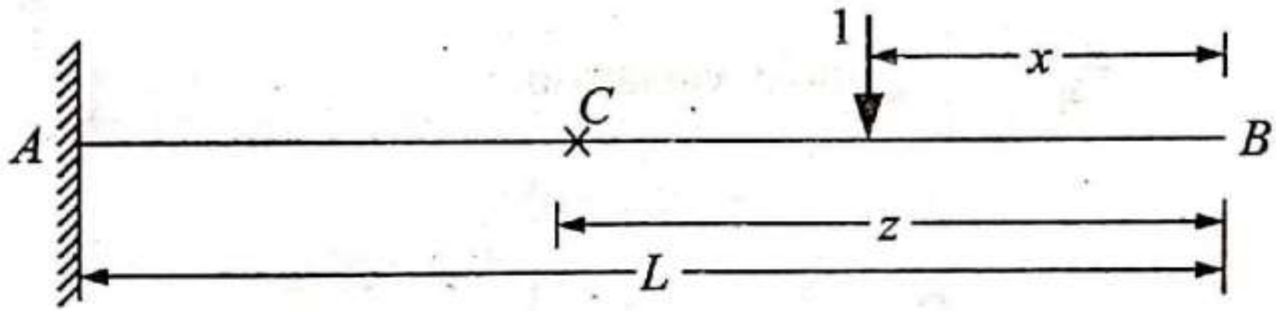
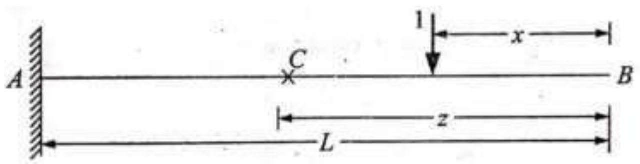


Figure 9: Cantilever with unit load

ILD for Shear Force at A (F_A):

Shear force at $A = F_A = 1$, Constant



Hence, ILD for F_A is as shown in Figure 10.

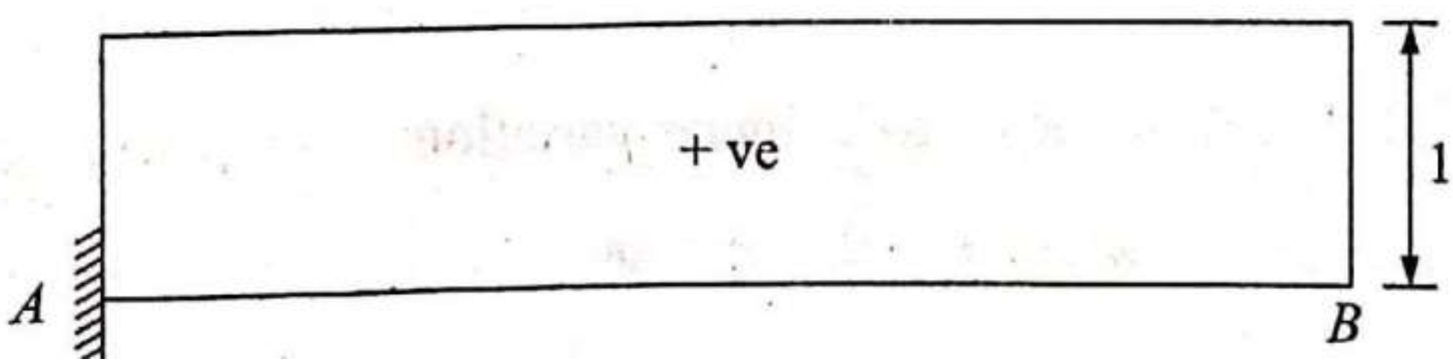
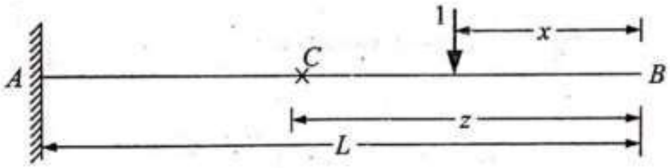


Figure 10: ILD for F_A

ILD for moment at A (M_A):

Moment at $A = M_A = -(L-x)$, Linear variation



when $x = 0$, $M_A = -L$

when $x = L$, $M_A = 0$

Hence, ILD for M_A is shown in Figure 11.

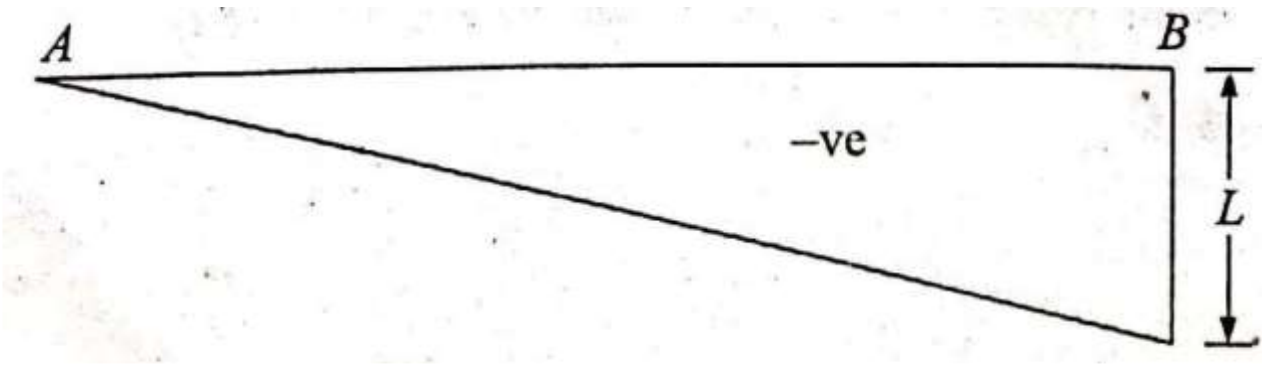
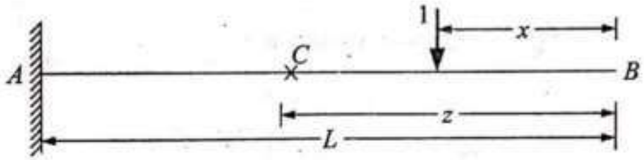


Figure 11: ILD for moment M_A

ILD for Shear Force at C (F_C):



- **When $x < z$**

  Shear force at C $= F_C = 1$ (Constant)

  when $x = 0$, $F_C = 1$ and when $x = z$, $F_C = 1$

- **When x > z**

  Shear force at $C = F_C = 0$

  when $x = z$, $F_C = 0$, when $x = L$, $F_C = 0$

Hence, ILD for shear force (F_C) is shown in Figure 12.

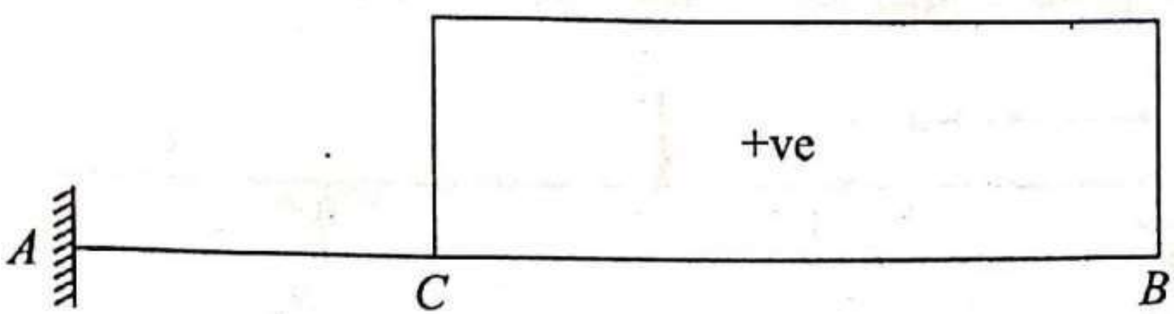
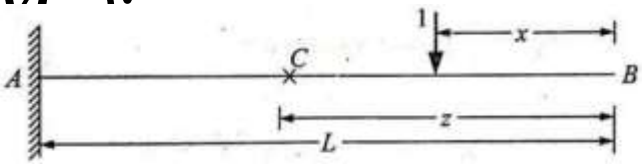


Figure 12: ILD for shear force at C (F_C)

ILD for Moment at *C* (M_C)



- **When $x \le z$**

  Moment at $C = M_C = -1\,(z-x)$, Linear variation

  when $x = 0$, $M_C = -z$

  when $x = z$, $M_C = 0$

- **When $x > z$**

  Moment at $C = M_C = 0$, Constant

  Hence, ILD for moment at C (M_C) is shown in Figure 13.

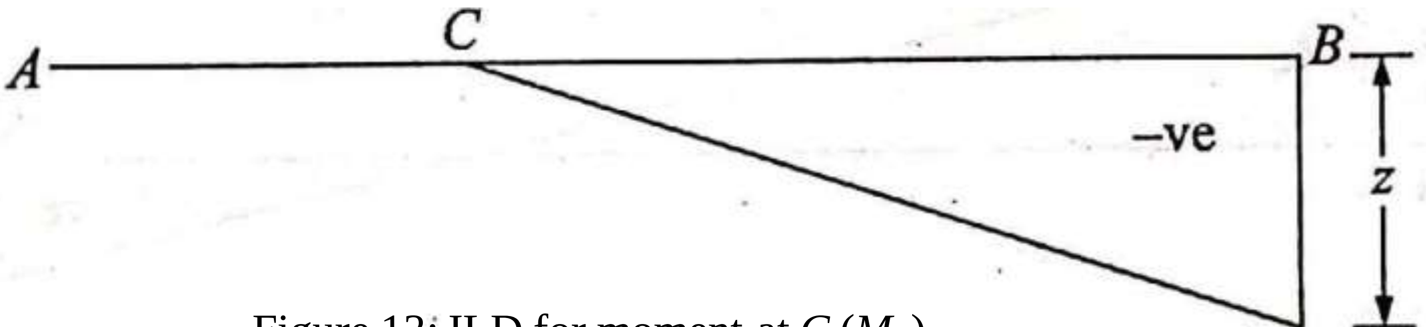


Figure 13: ILD for moment at C (M_C)

Thanks