STRCUTURAL ANALYSIS- II

UNIT - I

PART A : TWO HINGED ARCHES

INTRODUCTION

Mainly three types of arches are used in practice: three-hinged, two-hinged and hingeless arches. In the early part of the nineteenth century, three-hinged arches were commonly used for the long span structures as the analysis of such arches could be done with

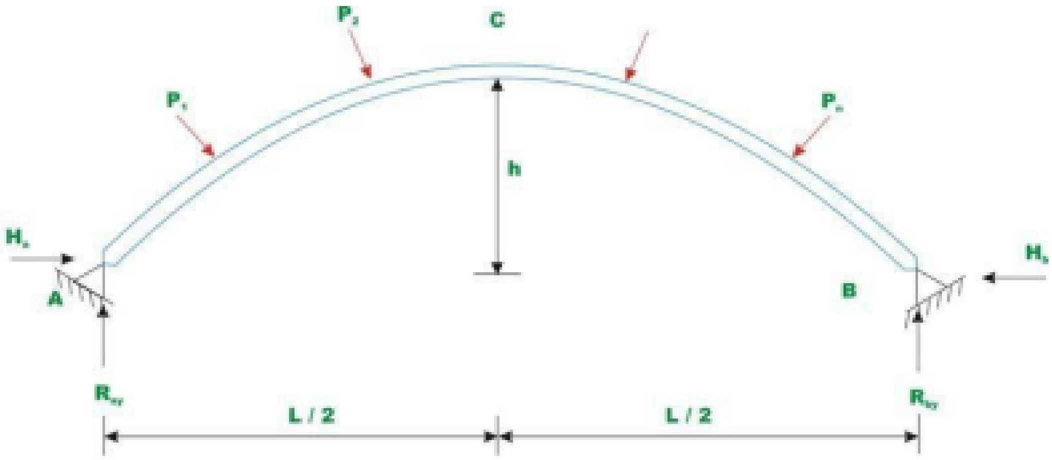
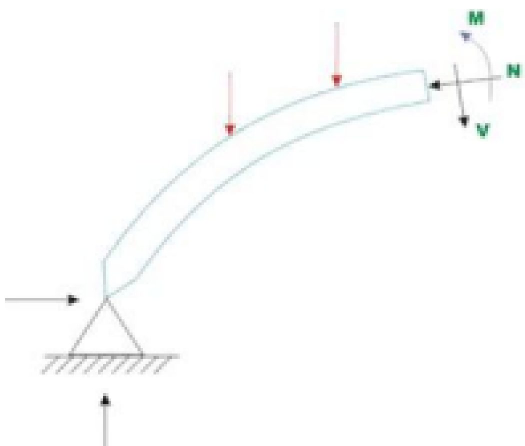


Fig. 33.1a Two - hinged arch.



confidence. However, with the development in structural analysis, for long span structures starting from late nineteenth century engineers adopted two-hinged and hingeless arches. Two-hinged arch is the statically indeterminate structure to degree one. Usually, the horizontal reaction is treated as the redundant and is evaluated by the method of least work. In this lesson, the analysis of two-hinged arches is discussed and few problems are solved to illustrate the procedure for calculating the internal forces.

ANALYSIS OF TWO-HINGED ARCH

A typical two-hinged arch is shown in Fig. 33.1a. In the case of two-hinged arch, we have four unknown reactions, but there are only three equations of equilibrium available. Hence, the degree of statical indeterminacy is one for two-hingedarch.

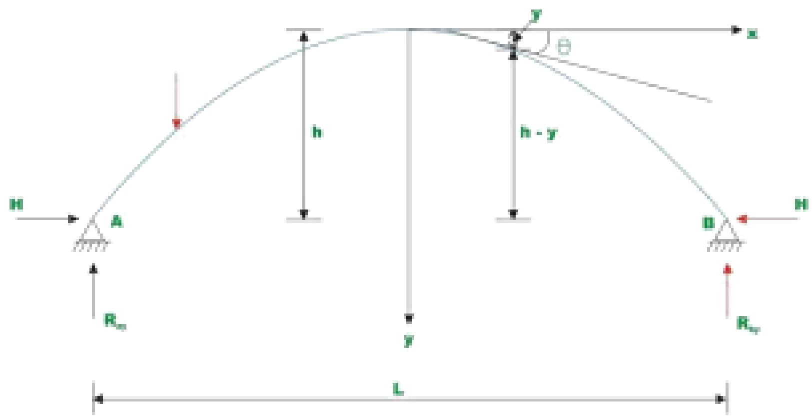


Fig. 33.2a

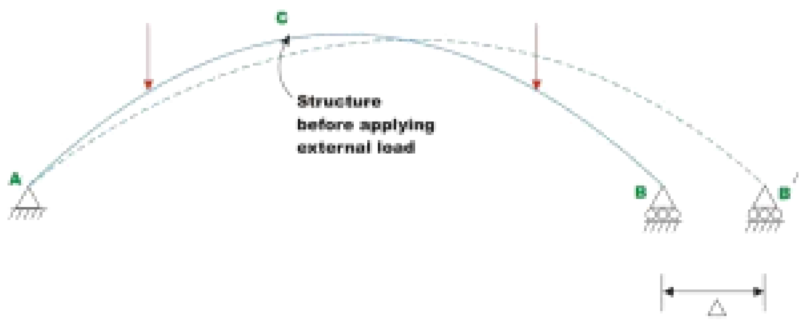


Fig. 33.2b.

$$U_b = \int_0^s \frac{M^2}{2EI} ds \qquad (33.1)$$

The fourth equation is written considering deformation of the arch. The unknown **15**
redundant reaction H_b is calculated by noting that the horizontal displacement of hinge B is zero. In general the horizontal reaction in the two hinged arch is evaluated by straightforward application of the theorem of least work (see module 1, lesson 4), which states that the partial derivative of the strain energy of a statically indeterminate structure with respect to statically indeterminate action should vanish. Hence to obtain, horizontal reaction, one must develop an expression for strain energy. Typically, any section of the arch (vide Fig 33.1b) is subjected to shear force V, bending moment M and the axial compression N . The strain energy due to bending U_b is calculated from the following expression.

The above expression is similar to the one used in the case of straight beams. However, in this case, the integration needs to be evaluated along the curved arch length. In the above equation, *s* is the length of the centerline of the arch, *I* is the moment of inertia of the arch cross section, *E* is the Young's modulus of the arch material. The strain energy due to shear is small as compared to the strain energy due to bending and is usually neglected in the analysis. In the case of flat arches, the strain energy due to axial compression can be appreciable and is givenby,

$$U_a = \int_0^s \frac{N^2}{2AE} ds \qquad (33.2)$$

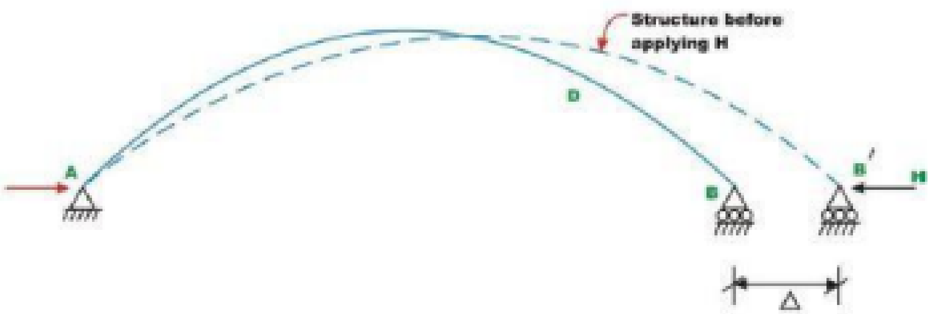


Fig. 33.2c.

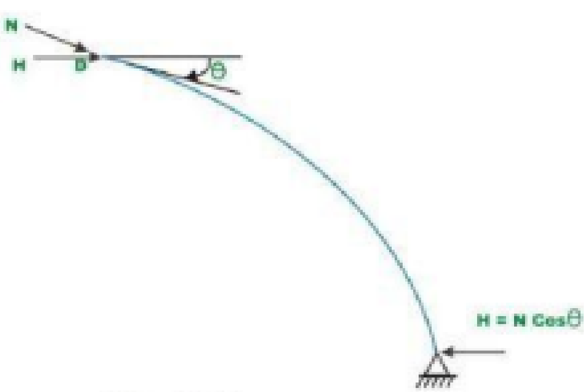
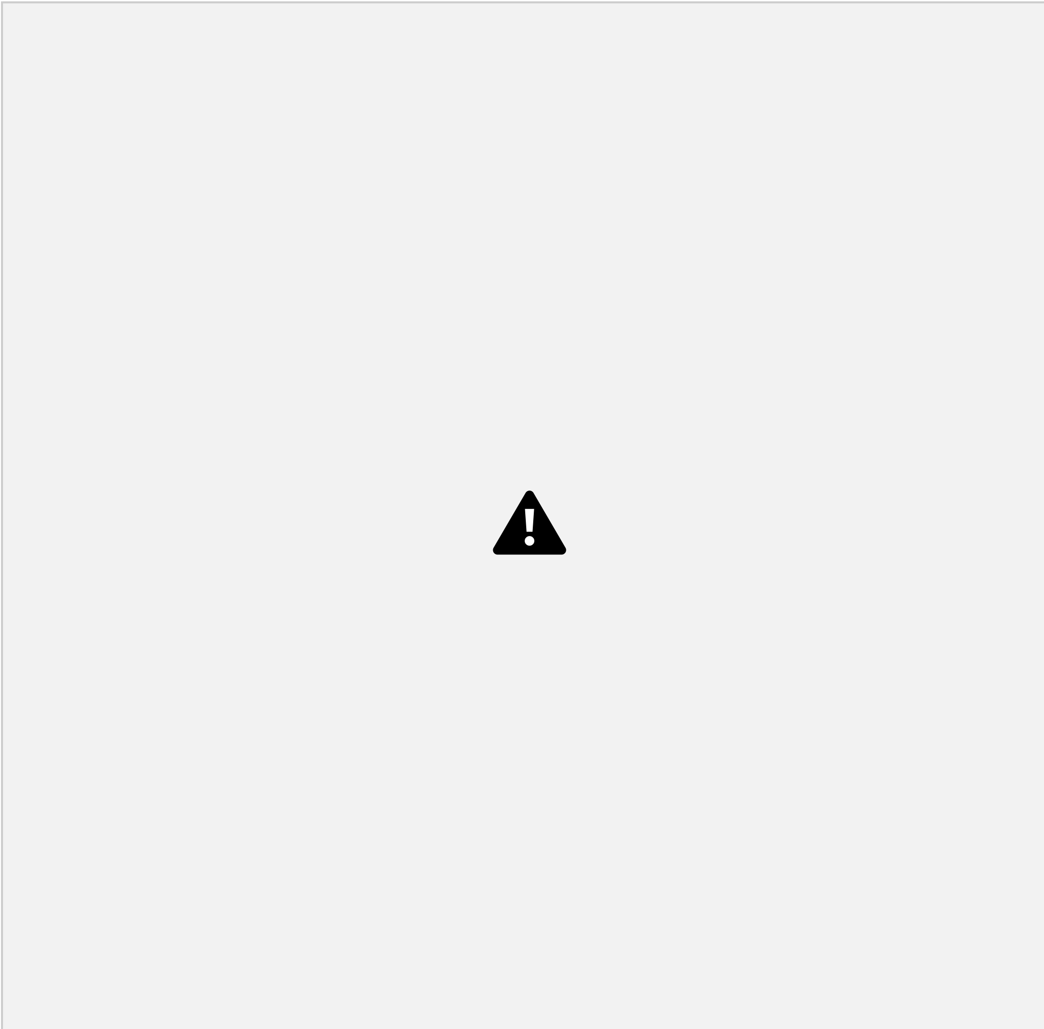


Fig. 33.2d.

The total strain energy of the arch is given by

$$U = \int_0^s \frac{M^2}{2EI} ds + \int_0^s \frac{N^2}{2AE} ds \qquad (33.3)$$



TEMPERATURE EFFECT

Consider an unloaded two-hinged arch of span L. When the arch undergoes a uniform temperature change of T, then its span would increase by $C^{\circ} T L \alpha$ if it were allowed to expand freely (vide Fig 33.3a). α is the co-efficient of thermal expansion of the arch material. Since the arch is restrained from the horizontal movement, a horizontal force is induced at the support as the temperature is increase

5

Now applying the Castigliano’s first theorem,

Solving for *H*,

The second term in the denominator may be neglected, as the axial rigidity is quite high. Neglecting the axial rigidity, the above equation can be written as

6

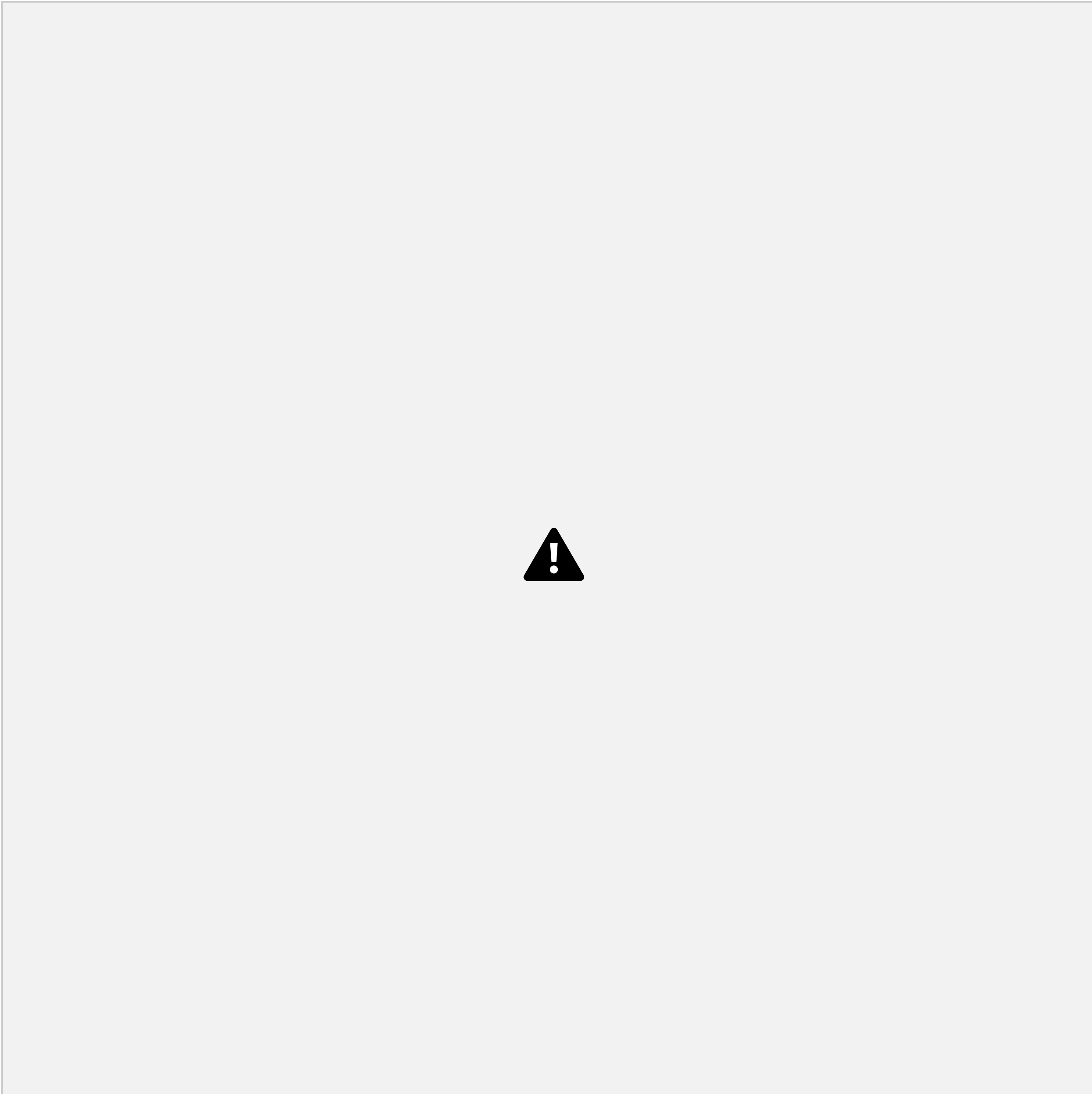
Example

A semicircular two hinged arch of constant cross section is subjected to a concentrated load as shown in Fig. Calculate reactions of the arch and draw bending moment diagram.

Solution:

Taking moment of all forces about hinge B leads to,

7



From figure,

8

Bending moment diagram

Bending moment M at any cross section of the arch is given by,

(9)

Using equations (8) and (9), bending moment at any angle θ can be computed. The bending moment diagram is shown in Fig.

Example

A two hinged parabolic arch of constant cross section has a span of 60m and a rise of 10m. It is subjected to loading as shown in Fig.. Calculate reactions of the arch if the temperature of the arch is raised by. Assume co-efficient of thermal expansion as

10

Taking A as the origin, the equation of two hinged parabolic arch may be written as,

The given problem is solved in two steps. In the first step calculate the horizontal reaction due to 40kN load applied at C. In the next step calculate the horizontal reaction due to rise in temperature. Adding both, one gets the horizontal reaction at the hinges due to 40kN combined external loading and temperature change. The horizontal reaction due to load may be calculated by the following equation,

Please note that in the above equation, the integrations are carried out along the x- axis instead of the curved arch axis. The error introduced by this change in the variables in the case of flat arches is negligible. Using equation (1), the above equation (3) can be easily evaluated. The vertical reaction A is calculated by taking moment of all forces about B. Hence,

12

G.Suryanarayana

Table 1. Numerical integration of equations (8) and (9)

124 14

UNIT - I

ANALYSIS OF PLANE FRAMES

PART-B MOMENT DISTRIBUTION METHOD

MOMENT DISTRIBUTION METHOD

This method of analyzing beams and frames was developed by Hardy Cross in 1930. Moment distribution method is basically a displacement method of analysis. But this method side steps the calculation of the displacement and instead makes it possible to apply a series of converging corrections that allow direct calculation of the end moments. This method of consists of solving slope deflection equations by successive approximation that may be carried out to any desired degree of accuracy. Essentially, the method begins by assuming each joint of a structure is fixed. Then by unlocking and locking each joint in succession, the internal moments at the joints are distributed and balanced until the joints have rotated to their final or nearly final positions. This method of analysis is both repetitive and easy to apply. Before explaining the moment distribution method certain definitions and concepts must beunderstood.

Sign convention: In the moment distribution table clockwise moments will be treated+ve and anticlockwise moments will be treated –ve. But for drawing BMD moments causing concavity upwards (sagging) will be treated +ve and moments causing convexity upwards (hogging) will be treated –ve.

Fixed end moments: The moments at the fixed joints of loaded member are called fixedend moment. FEM for few standards cases are given in previous chapter.

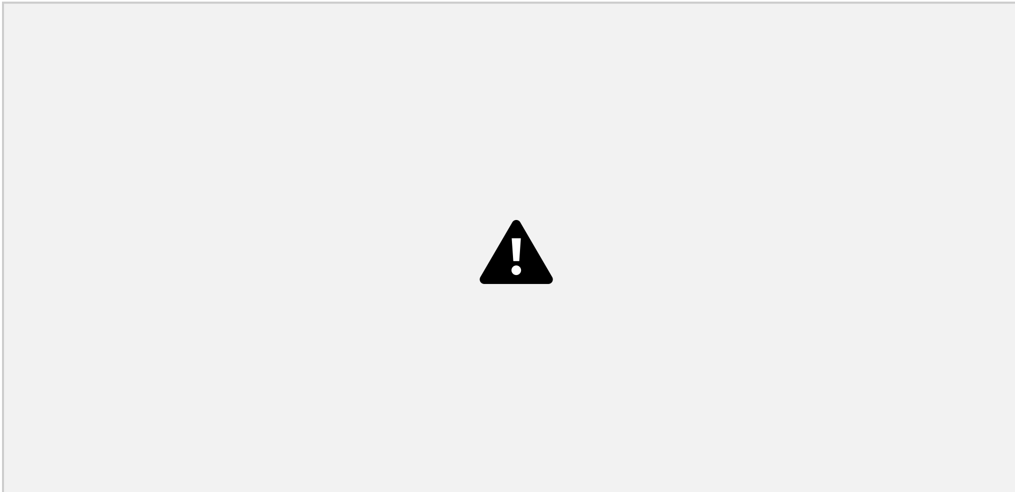
Distribution factors: If a moment 'M' is applied to a rigid joint 'o', as shown in figure, theconnecting members will each supply a portion of the resisting moment necessary to satisfy moment equilibrium at the joint. Distribution factor is that fraction which when multiplied with applied moment 'M' gives resisting moment supplied by the members. To obtain itsvalue imagine the joint is rigid joint connected to different members. If applied moment M cause the joint to rotate an amount 'θ' , then each member rotates by same amount.

From equilibrium requirement

$M = M_1 + M_2 + M_3 + \ldots\ldots$

Member relative stiffness factor: In majority of the cases continuous beams and frames willbe made from the same material so that their modulus of electricity E will be same for all members. It will be easier to determine member stiffness factor by removing term 4E & 3E from equation (4) and (5) then will be called as relative stiffness factor.

Carry over factors: Consider the beam shown infigure

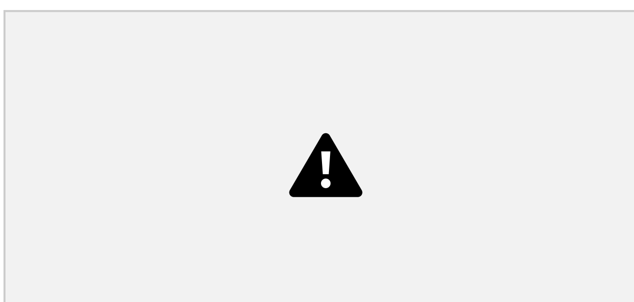


+ve BM of at A indicates clockwise moment of at A. In other words the moment 'M' at the pin induces a moment of at the fixed end. The carry over factor represents the fraction of M that is carried over from hinge to fixed end. Hence the carry over factor for the caseof far end fixed is +. The plus sign indicates both moments are in the samedirection.

MOMENT DISTRIBUTION FOR FRAMES WITH NO SIDE SWAY

The analysis of such a frame when the loading conditions and the geometry of the frame is such that there is no joint translation or sway, is similar to that given for beams.

Q. Analysis the frame shown in figure by moment distribution method and draw



BMD assume EI isconstant.

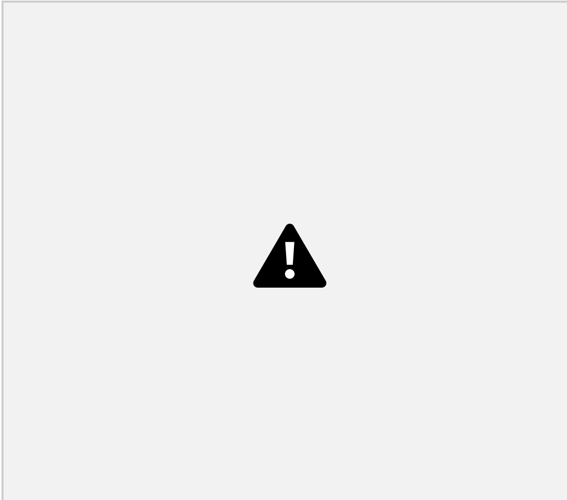
DISTRIBUTION

18

MOMENT DISTRIBUTION METHOD FOR FRAMES WITH SIDE SWAY

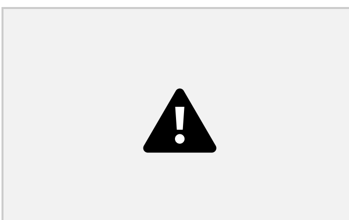
Frames that are non symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to non symmetrical loading have a tendency to side sway.

Analyze the frame shown in figure by moment distribution method. Assume EI is constant.

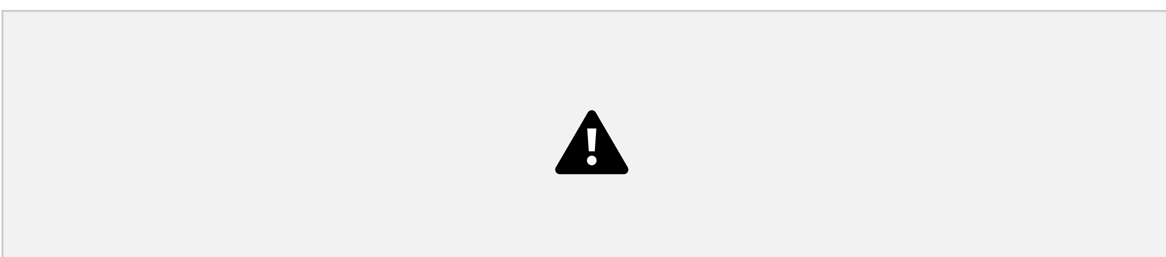


A. Non SwayAnalysis:

First consider the frame without side sway

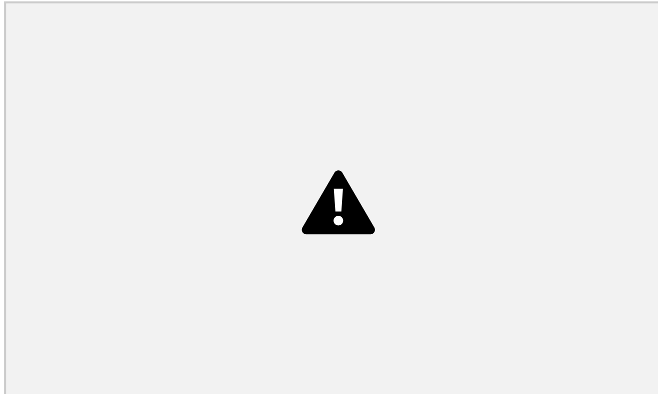


DISTRIBUTION FACTOR



DISTRIBUTION OF MOMENTS FOR NON-SWAY ANALYSIS

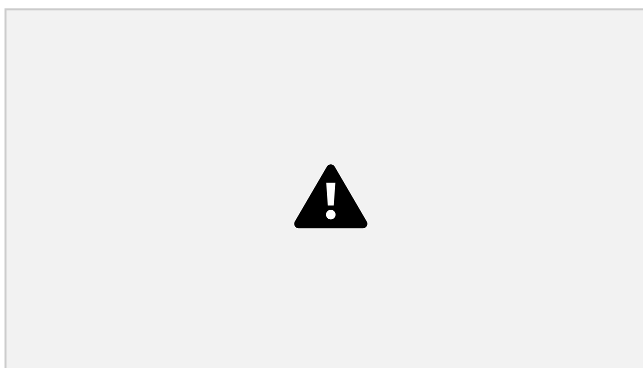
G.Suryanaraya

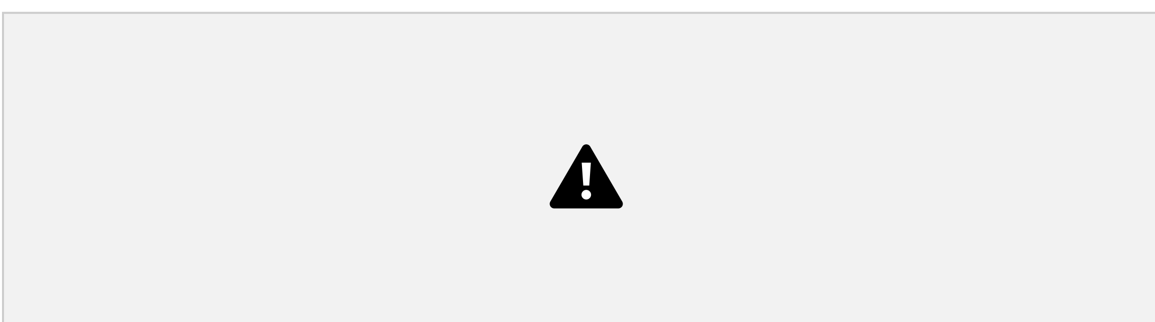


By seeing of the FBD of columns R = 1.73 – 0.82

(Using F_x =0 for entire frame) = 0.91 KN

Now apply R = 0.91 KN acting opposite as shown in the above figure for the sway analysis. Sway analysis: For this we will assume a force R' is applied at C causing the frame to deflect as shown in the following figure.





Since both ends are fixed, columns are of same length & I and assuming joints B &

C are temporarily restrained from rotating and resulting fixed end moment are

Assume

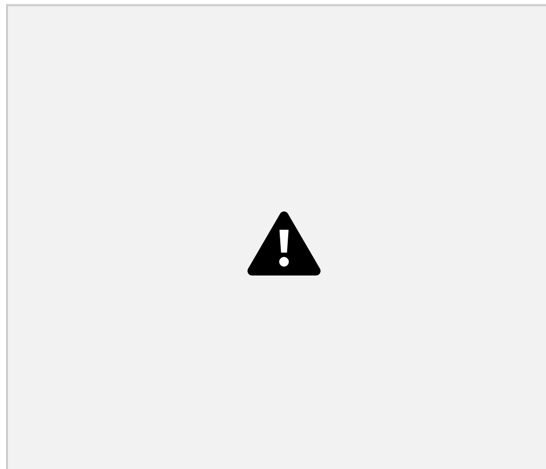
Moment distribution table for sway analysis:

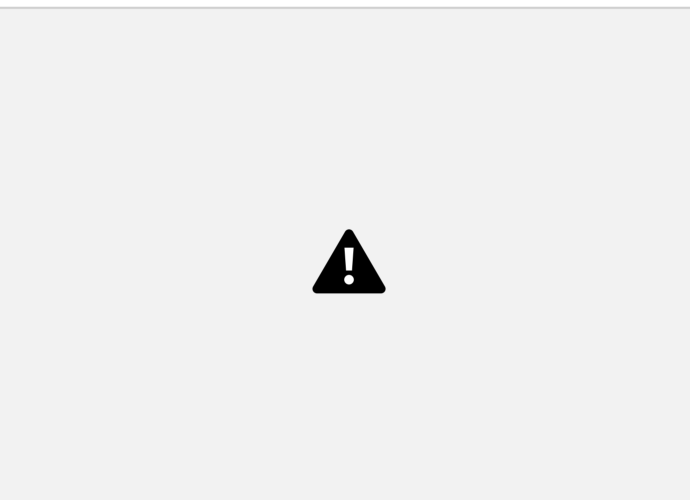
Free body diagram of columns

Using $F_x = 0$ for the entire frame R = 28 + 28 = 56 KN

Hence R' = 56KN creates the sway moments shown in above moment distribution table. Corresponding moments caused by R = 0.91KN can be determined by proportion. Thus final moments are calculated by adding non sway moments and sway.

Moments calculated for R = 0.91KN, as shown below.

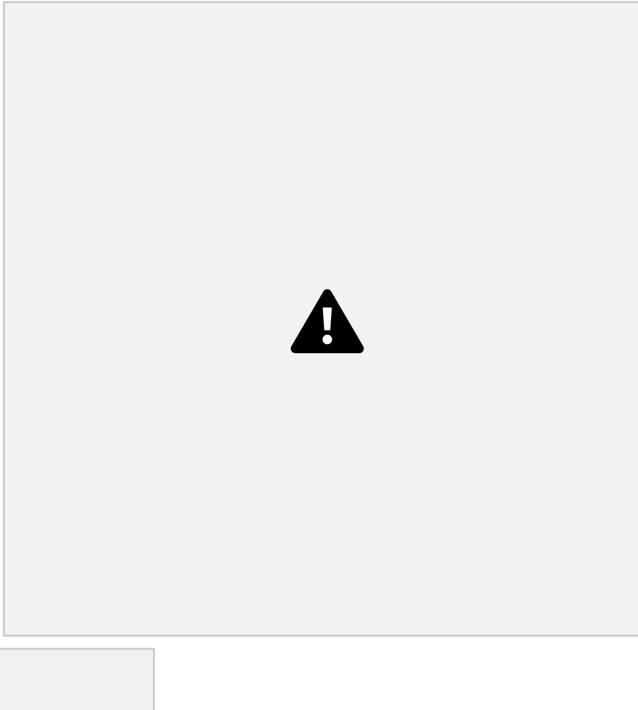


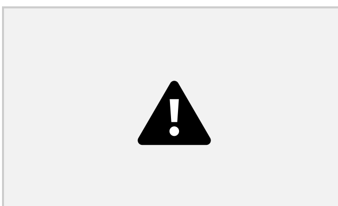


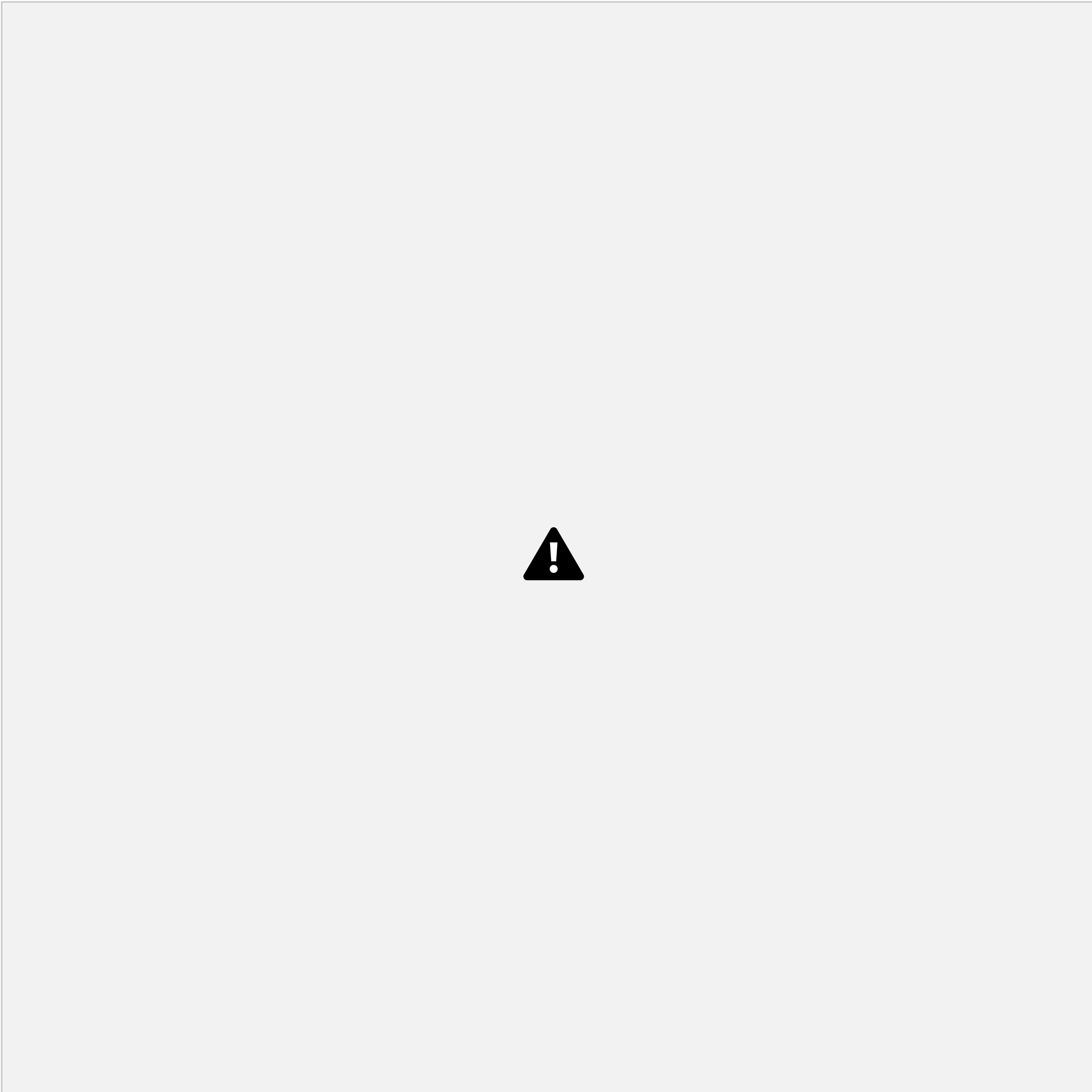
BMD

5.Q. Analysis the rigid frame shown in figure by moment distribution method and draw

BMD

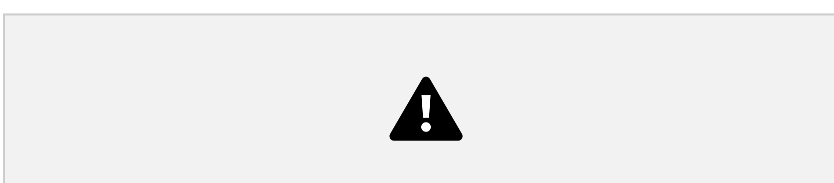




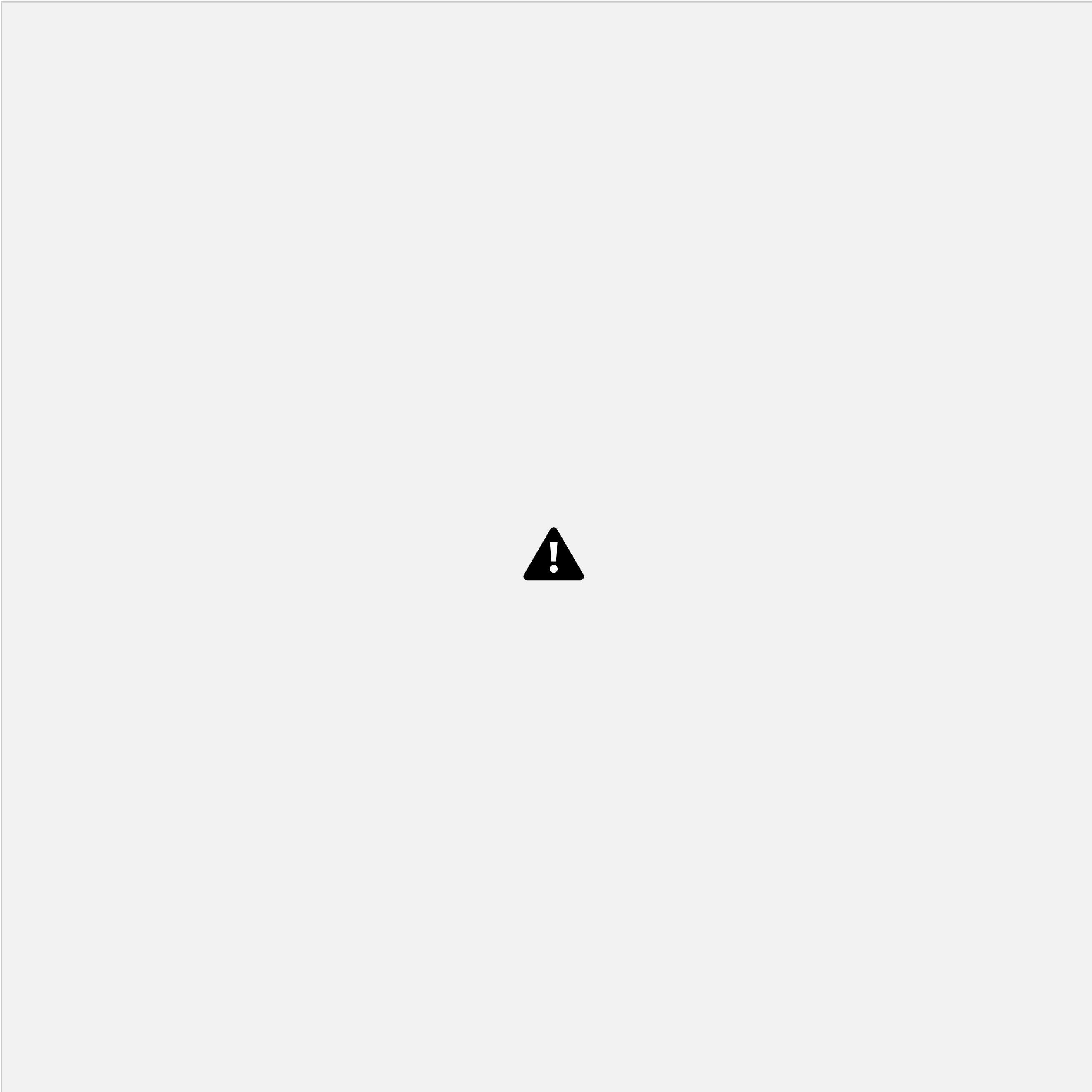


A. Non SwayAnalysis:

First consider the frame held from side sway



DISTRIBUTION FACTOR



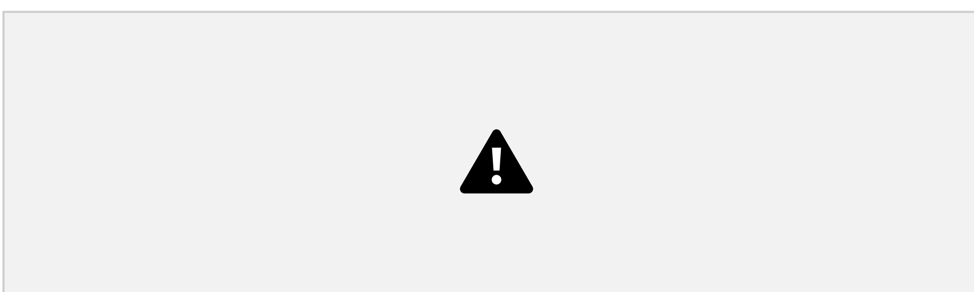
DISTRIBUTION OF MOMENTS FOR NON-SWAY ANALYSIS

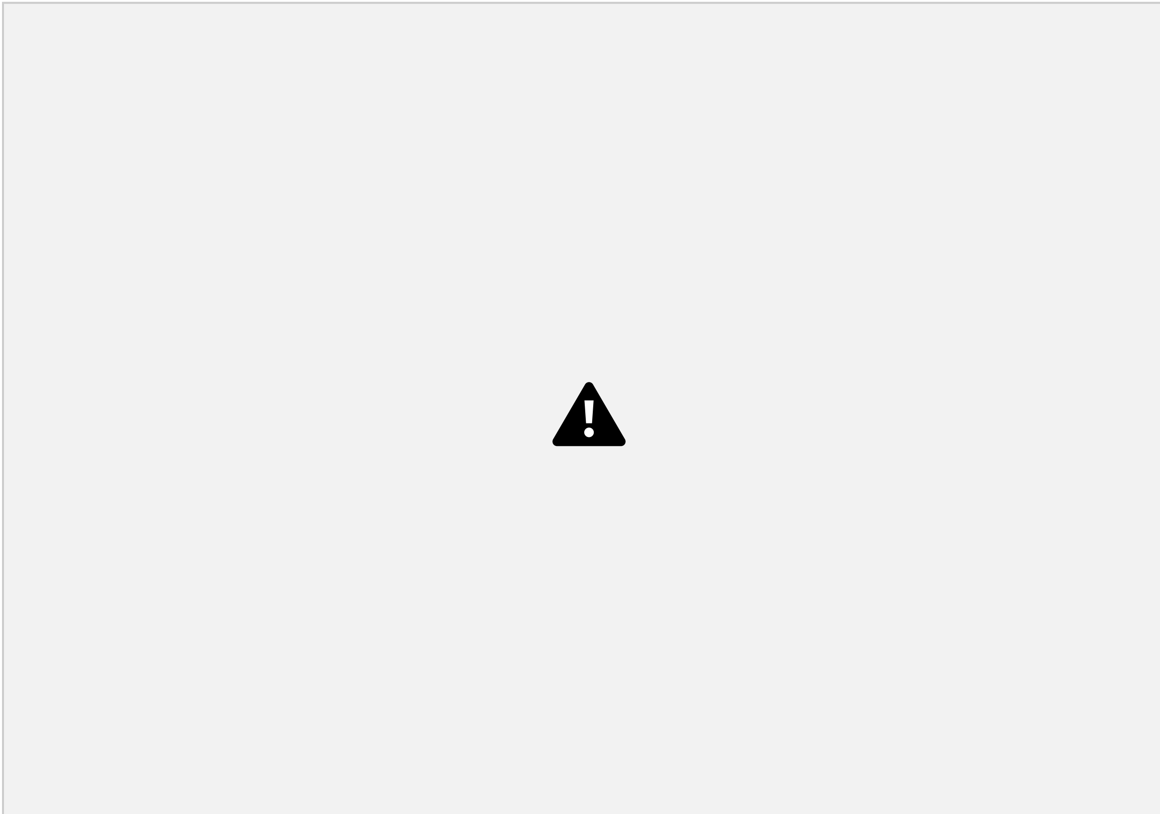
FREE BODY DIAGRAM OF COLUMNS

Applying Fx = 0 for frame as a Whole, R = 10 – 3.93 – 0.73 = 5.34 KN

Now apply R = 5.34KN acting opposite

Sway analysis: For this we will assume a force R' is applied at C causing the frame to deflect as shown in figure



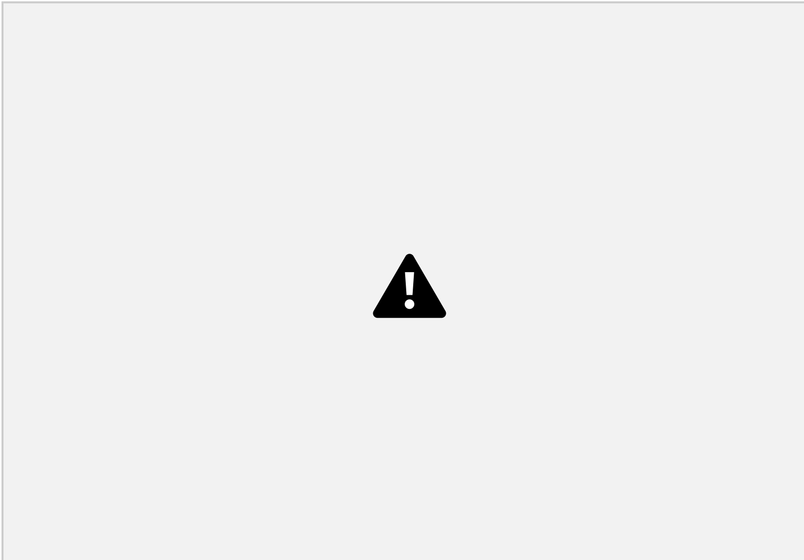


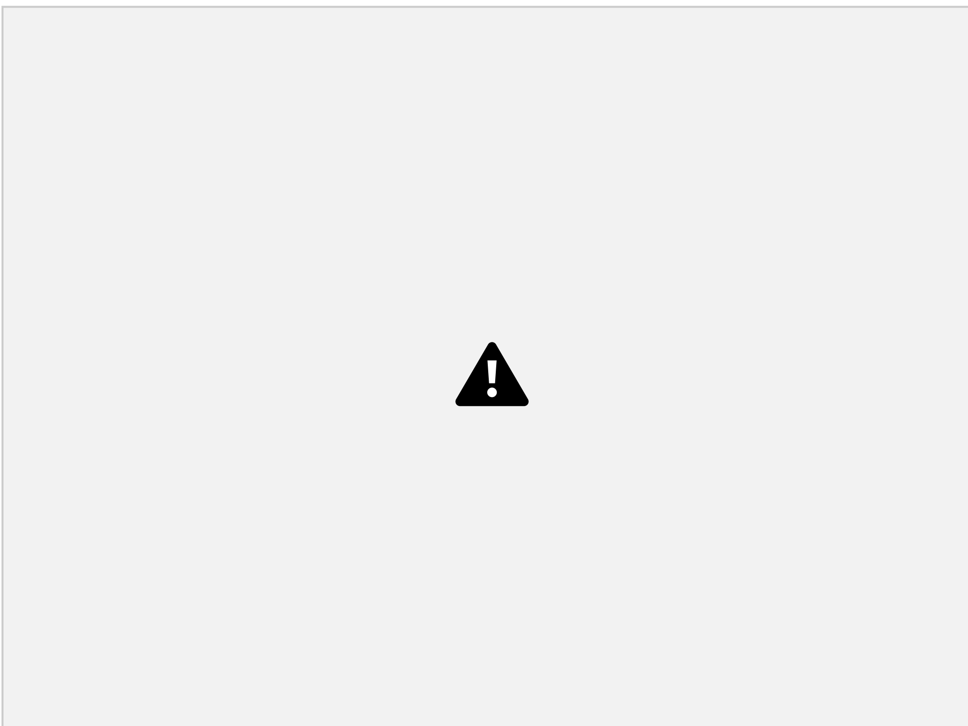
Assume

MOMENT DISTRIBUTION FOR SWAY ANALYSIS

Using Fx = 0 for the entire frame $R^{'}$= 11.12 kN

Hence $R^{'}$= 11.12 KN creates the sway moments shown in the above moment distribution table. Corresponding moments caused by R = 5.34 kN can be determined by proportion. Thus final moments are calculated by adding non-sway moments and sway moments determined for R = 5.34 KN as shown below.

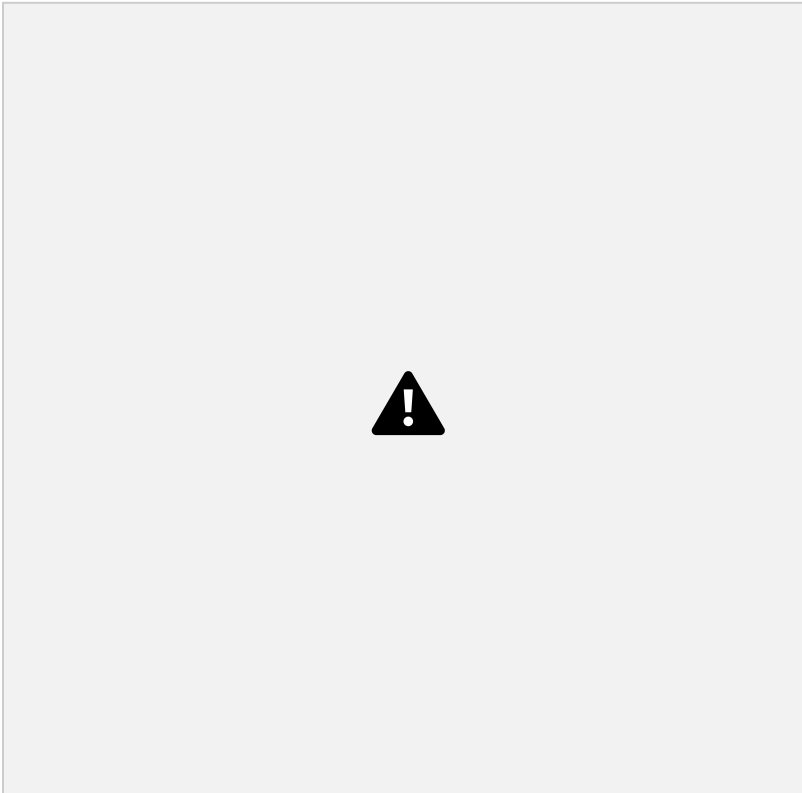




UNIT - II

PART A : KANI'S METHOD OF ANALYSIS

This method was developed by Dr. Gasper Kani of Germany in 1947. This method offers an iterative scheme for applying slope deflection method. We shall now see the application of

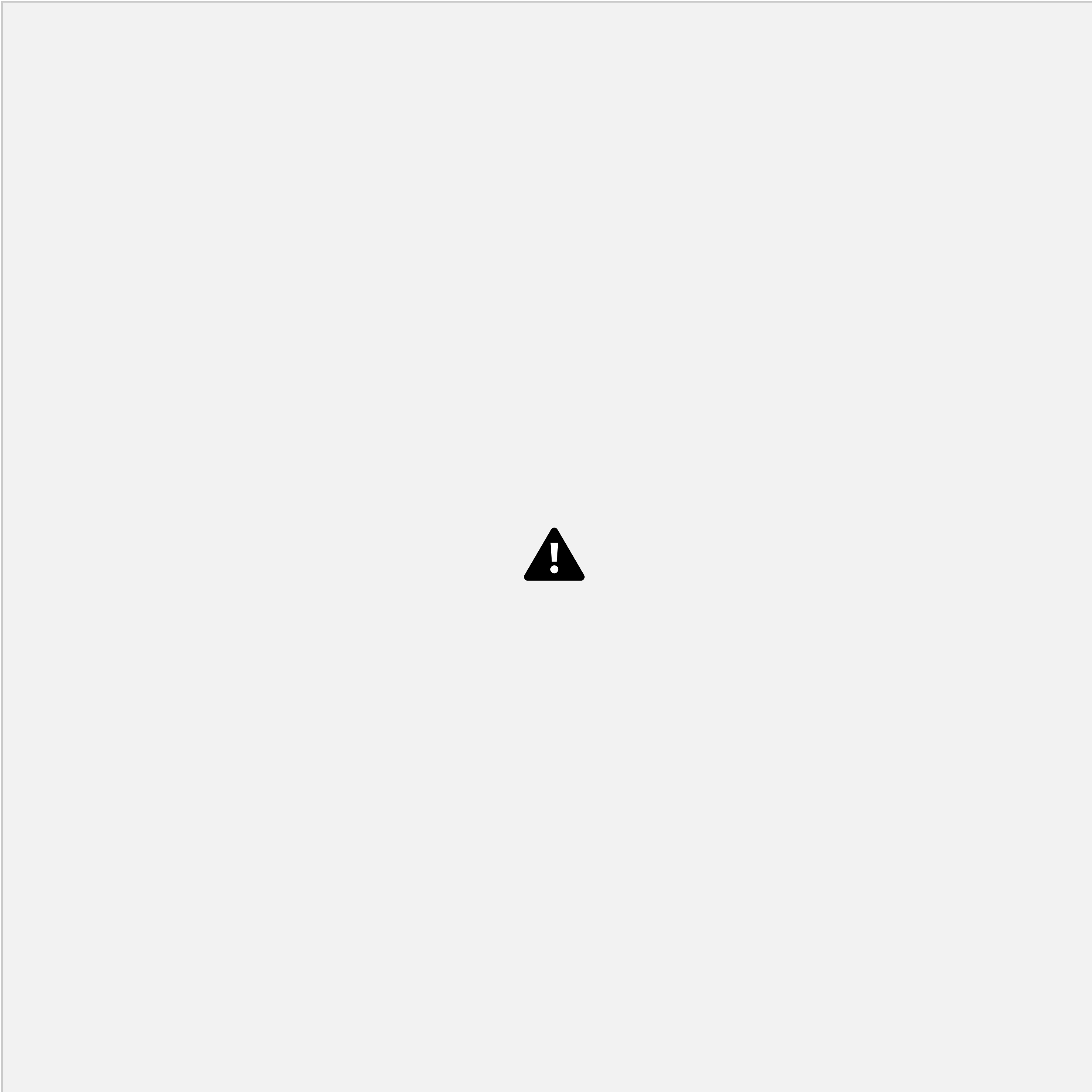


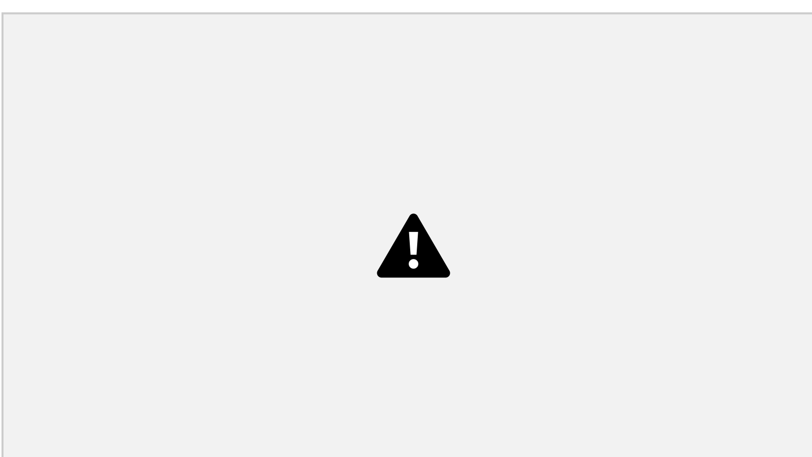
Kani’s method for different cases.

BEAMS WITH NO TRANSLATION OF JOINTS:

29

30

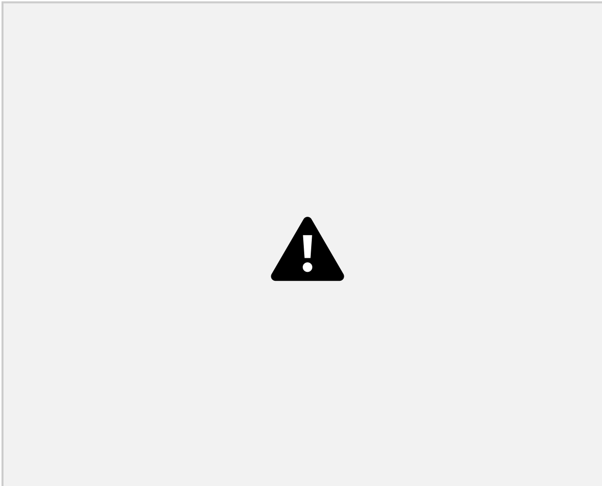




EXAMPLES:

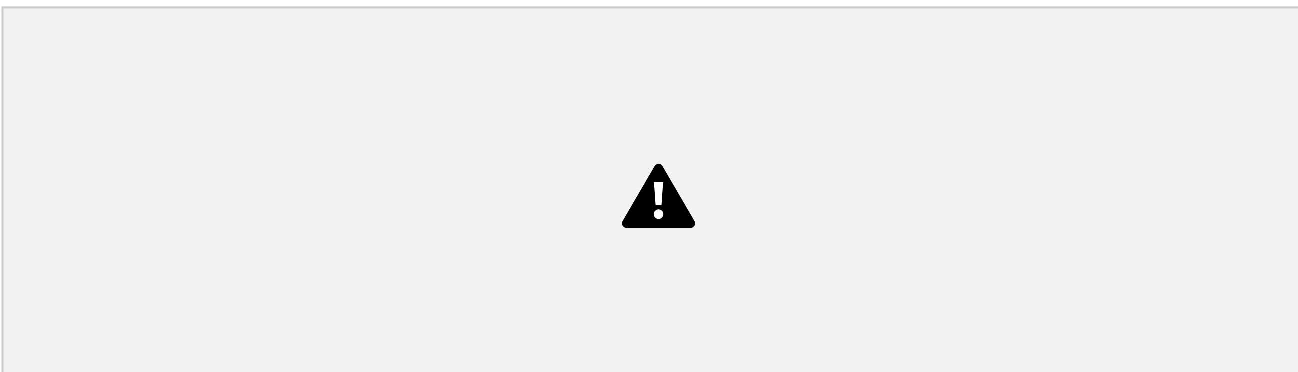
Ex.1:

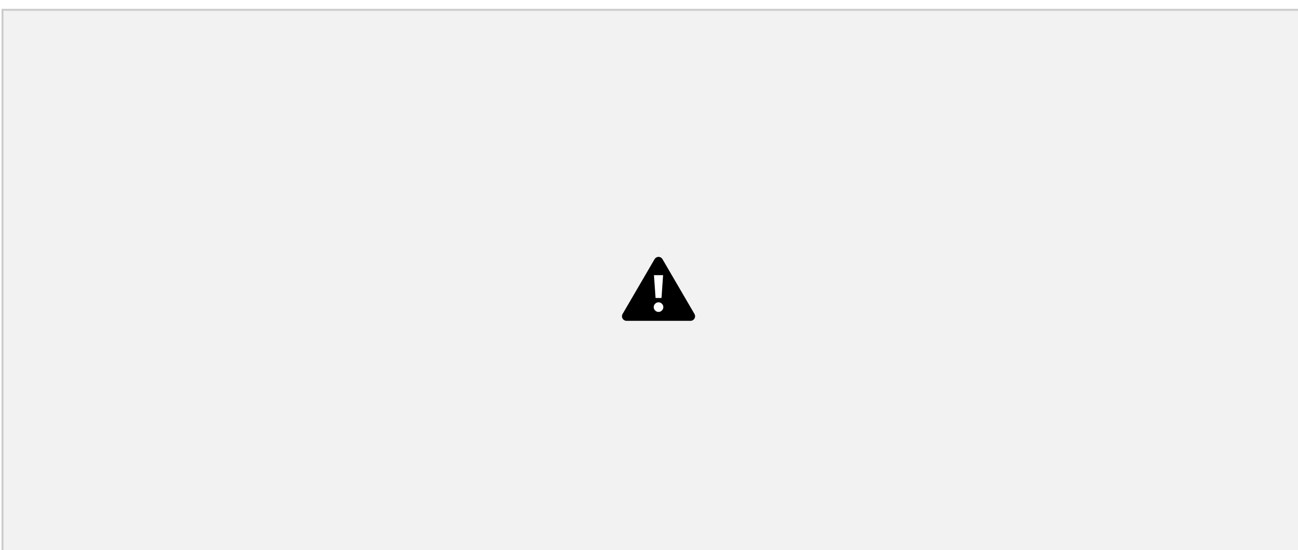
32



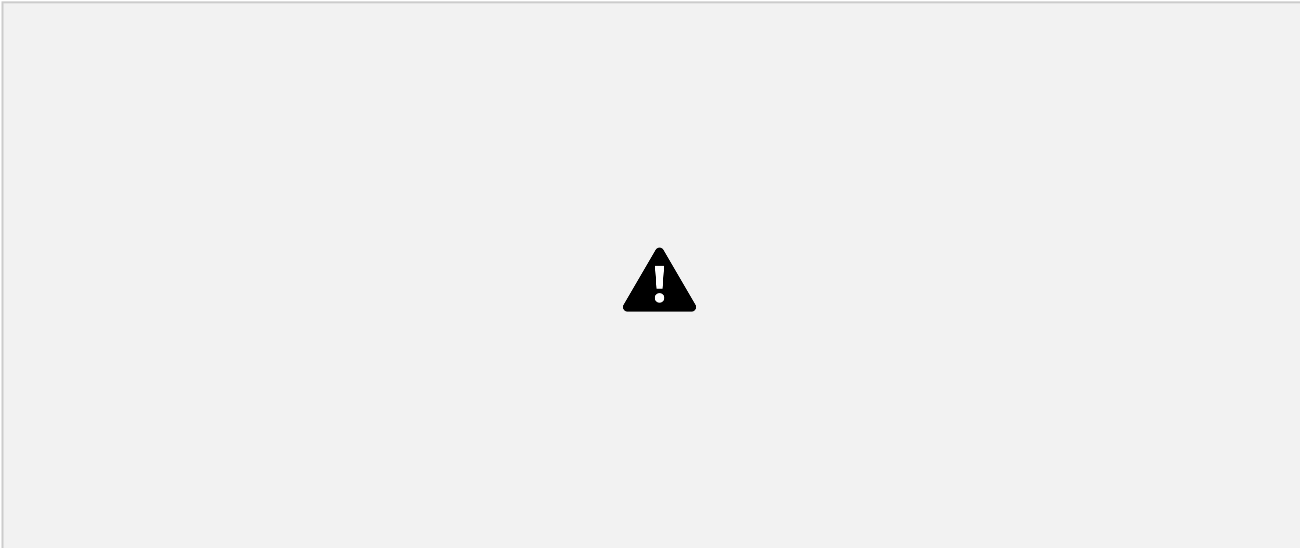
Ex.2:

34

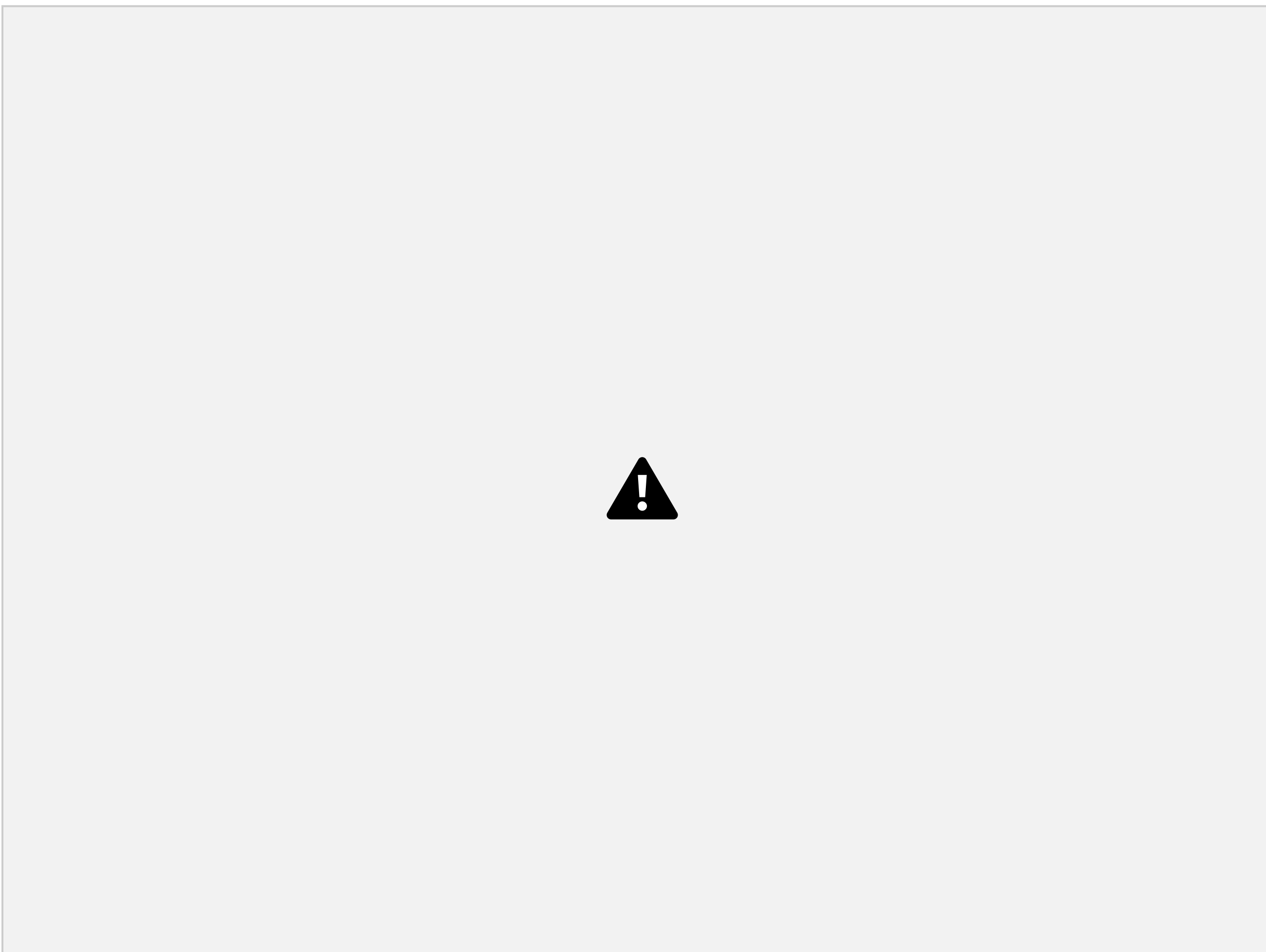




35

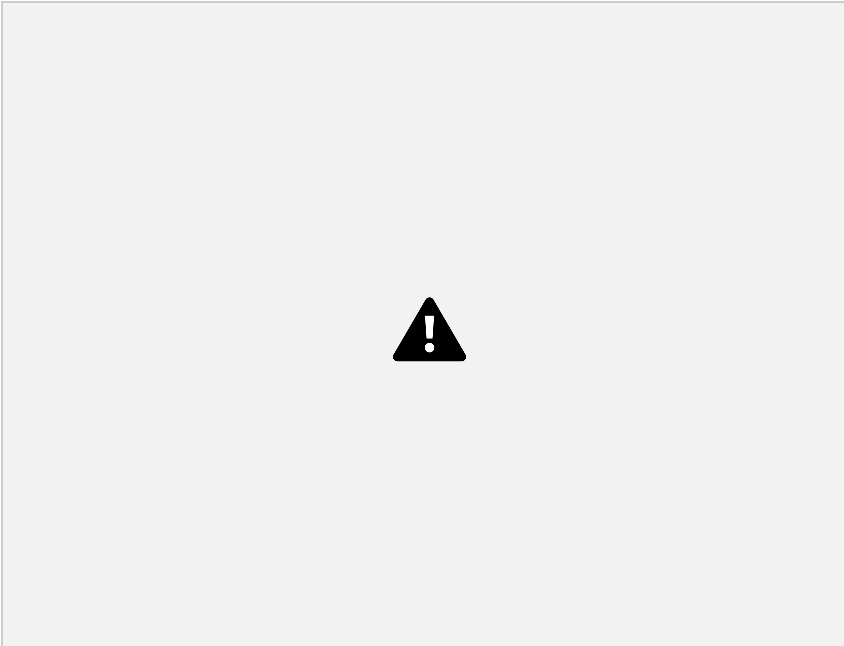


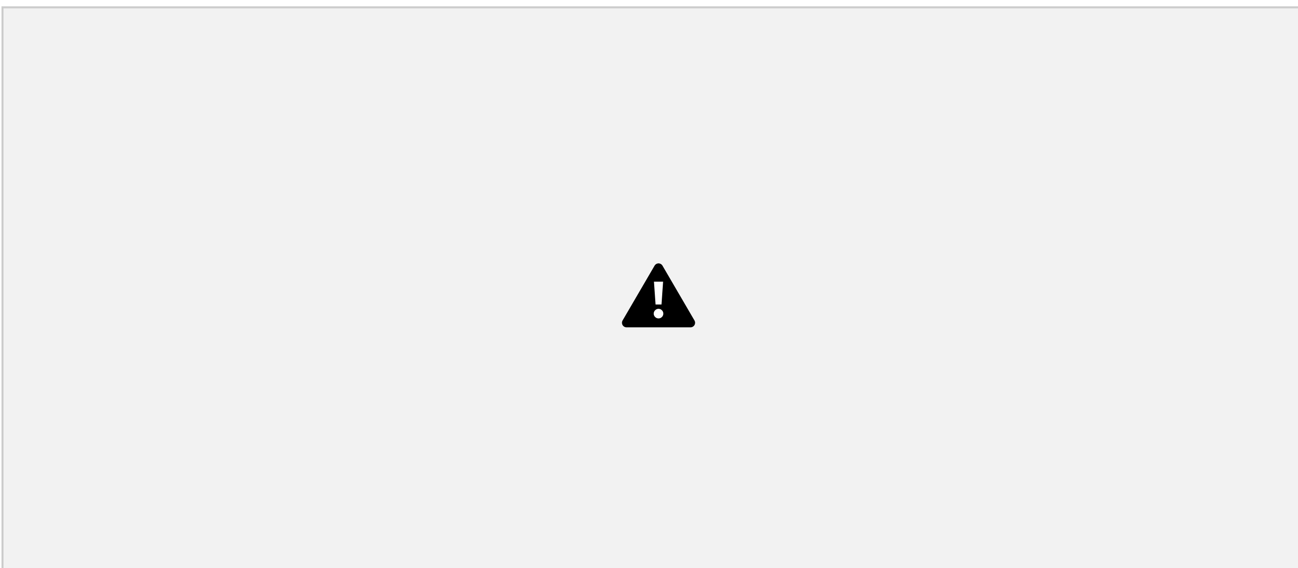
36



Ex.2:

In a continuous beam shown in fig. The support ‘B’ sinks by 10mm. Determine the moments by Kani’s method & draw BMD.





38

39

40

ANALYSIS OF FRAMES WITH NO TRANSLATION OF JOINTS

The frames, in which lateral translations are prevented, are analyzed in the same way as continuous beams. The lateral sway is prevented either due to symmetry of frame and loading or due to support conditions. The procedure is illustrated in following example.

Example-5. Analyze the frame shown in Figure 8 (a) by Kani’s method. Draw BMD.

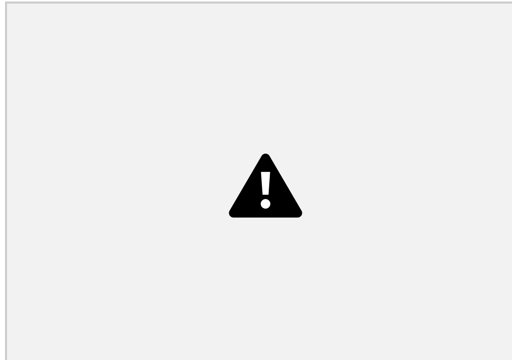


Fig-8(a)

Solution:

(a) Fixed endmoments:

(b) Rotationfactors:

Joint	**Member**	**Relative Stiffness (k)**	**k**	**Rotation factor = -½k/ k**
B	BC	3I/6 = 0.5I	0.83I	-0.3
	BA	I/3 = 0.33I		-0.2
C	CB	3I/6 = 0.5I	0.83I	-0.3
	CD	I/3 = 0.33I		-0.2

(c) Sum of FEM:

(d) Iteration process:

Joint	**B**		**C**	
Rotation Contribution	**M'$_{BA}$**	**M'$_{BC}$**	**M'$_{CB}$**	**M'$_{CD}$**

Rotation Factor	-0.2	-0.3	-0.3	-0.2
Iteration 1 Stated with	-0.2(-120+0) =24	-0.3(-120+0) =36	-0.2(120+36+0) = -46.8	-0.2(120+36+0) = -31.2
				120+50.0 -34.01
				+51.3) = 6
				+51.42) = 8

The fixed end

moments, sum of fixed and moments, rotation factors along with rotation contribution values at the end of each cycle in appropriate places is shown in figure 8(b).

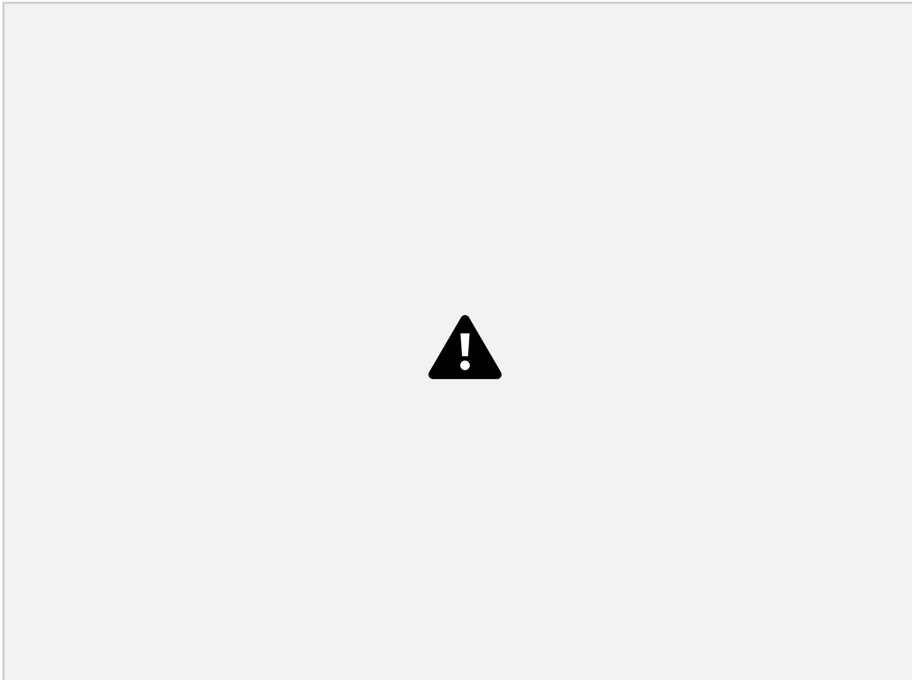
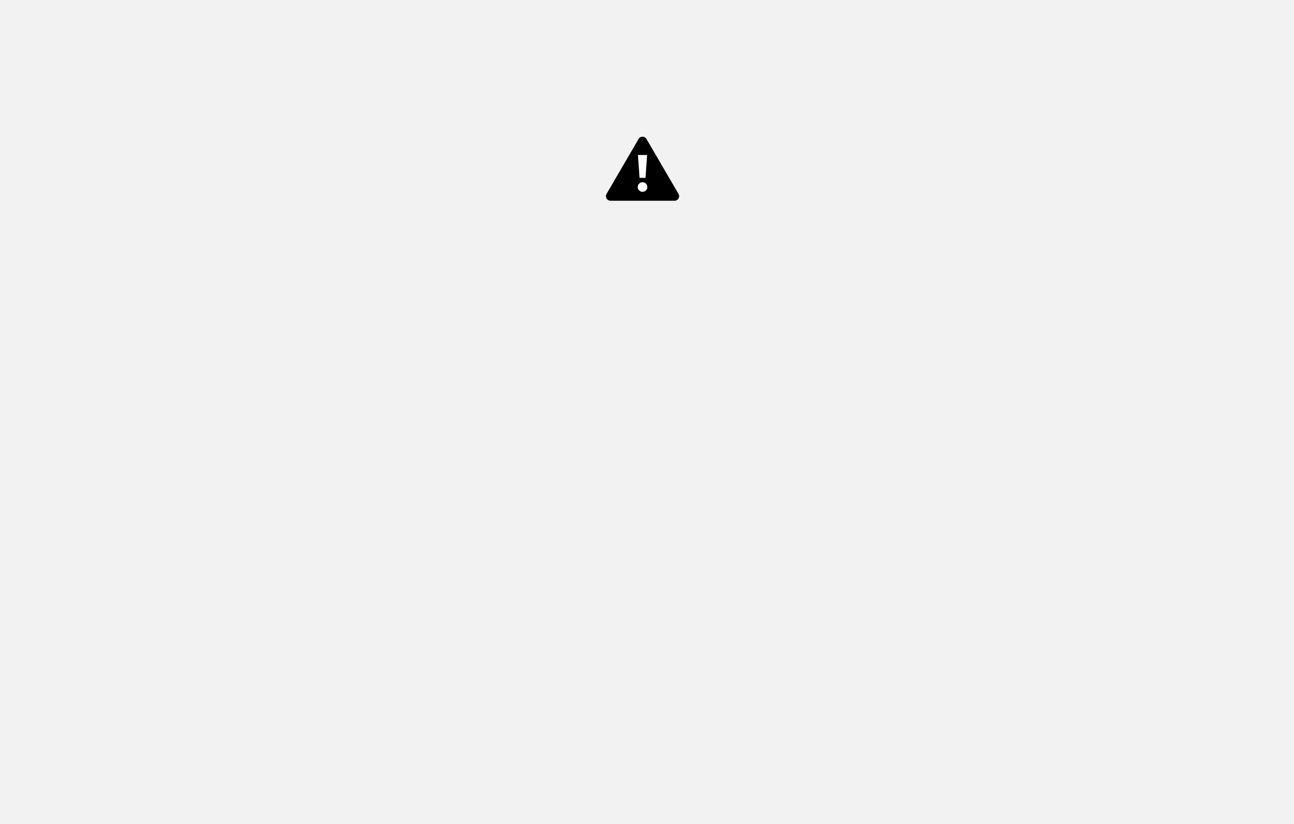


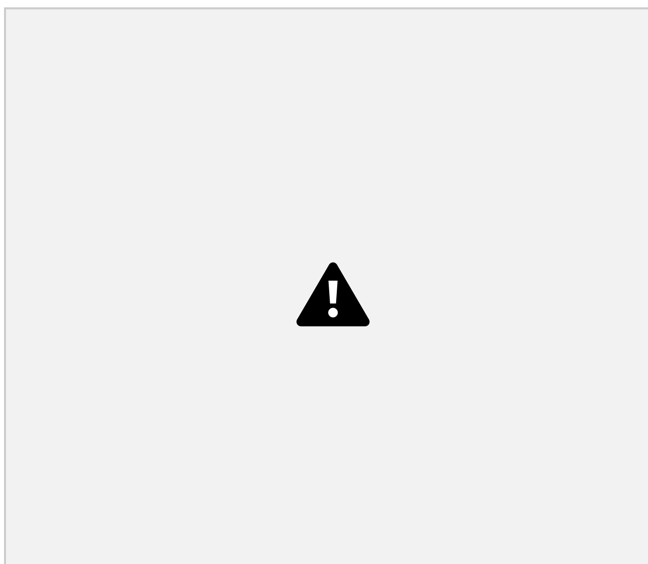
Fig-8(b)

(e) Final moments:

Member (ij)	**M_{Fij}**	**$2M'_{ij}$(kNm)**	**M'_{ji}(kNm)**	**(kNm) Final moment = $M_{Fij}+ 2M'_{ij}+ M'_{ji}$**
AB	0	0	34.28	34.28
BA	0	2 x 34.28	0	68.56
BC	-120	2 x 51.42	-51.43	-68.59
CB	120	2 x (-51.43)	51.42	68.56
CD	0	2 x (-34.28)	0	-68.56
DC	0	0	-34.28	-34.28



BMD is shown below in figure-8 (c)



PART - B CABLES AND BRIDGES

BY USING THE SLOPE DEFLECTION METHOD

In the slope-deflection method, the relationship is established between moments at the ends of the members and the corresponding rotations and displacements.

The slope-deflection method can be used to analyze statically determinate and indeterminate beams and frames. In this method it is assumed that all deformations are due to bending only. In other words deformations due to axial forces are neglected. In the force method of analysis compatibility equations are written in terms of unknown reactions. It must be noted that all the unknown reactions appear in each of the compatibility equations making it difficult to solve resulting equations. The slope deflection equations are not that lengthy in comparison. The basic idea of the slope deflection method is to write the equilibrium equations for each node in terms of the deflections and rotations. Solve for the generalized displacements. Using moment displacement relations, moments are then known. The structure is thus reduced to a determinate structure. The slope-deflection method was originally developed by Heinrich Manderla and Otto Mohr for computing secondary stresses in trusses. The method as used today was presented by G.A.Maney in 1915 for analyzing rigid jointed structures.

FUNDAMENTAL SLOPE-DEFLECTION EQUATIONS:

The slope deflection method is so named as it relates the unknown slopes and deflections to the applied load on a structure. In order to develop general form of slope deflection equations, we will consider the typical span AB of a continuous beam which is subjected to arbitrary loading and has a constant EI. We wish to relate the beams internal end moments in terms of its three degrees of freedom, namely its angular displacements and linear displacement which could be caused by relative settlements between the supports. Since we will be developing a formula, moments and angular displacements will be considered positive, when they act clockwise on the span. The linear displacement will be considered positive since this displacement causes the chord of the span and the span’s chord angle to rotate clockwise. The slope deflection equations can be obtained by using principle of superposition by considering separatelythemomentsdevelopedateachsupportsduetoeachofthedisplacements and then the load.

Case A: fixed-end moments

Case B: rotation at A, (angular displacement at A)

Consider node A of the member as shown in figure to rotate while its far end B is fixed. To determine the moment needed to cause the displacement, we will use conjugate beam ` method. The end shear at A acts downwards on the beam since is clockwise.

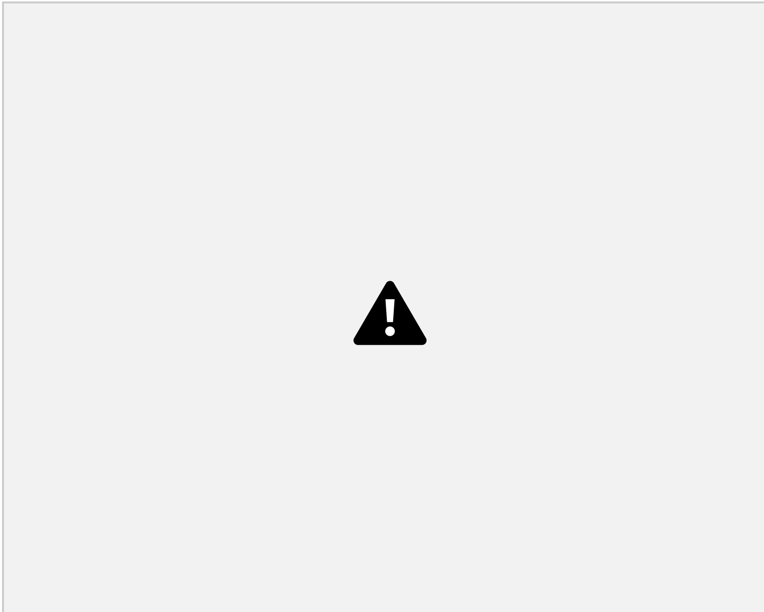
Case C: rotationatB, (angular displacement atB)

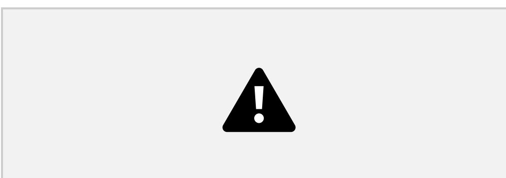
In a similar manner if the end B of the beam rotates to its final position, while end A is held fixed. We can relate the applied moment to the angular displacement and the reaction moment

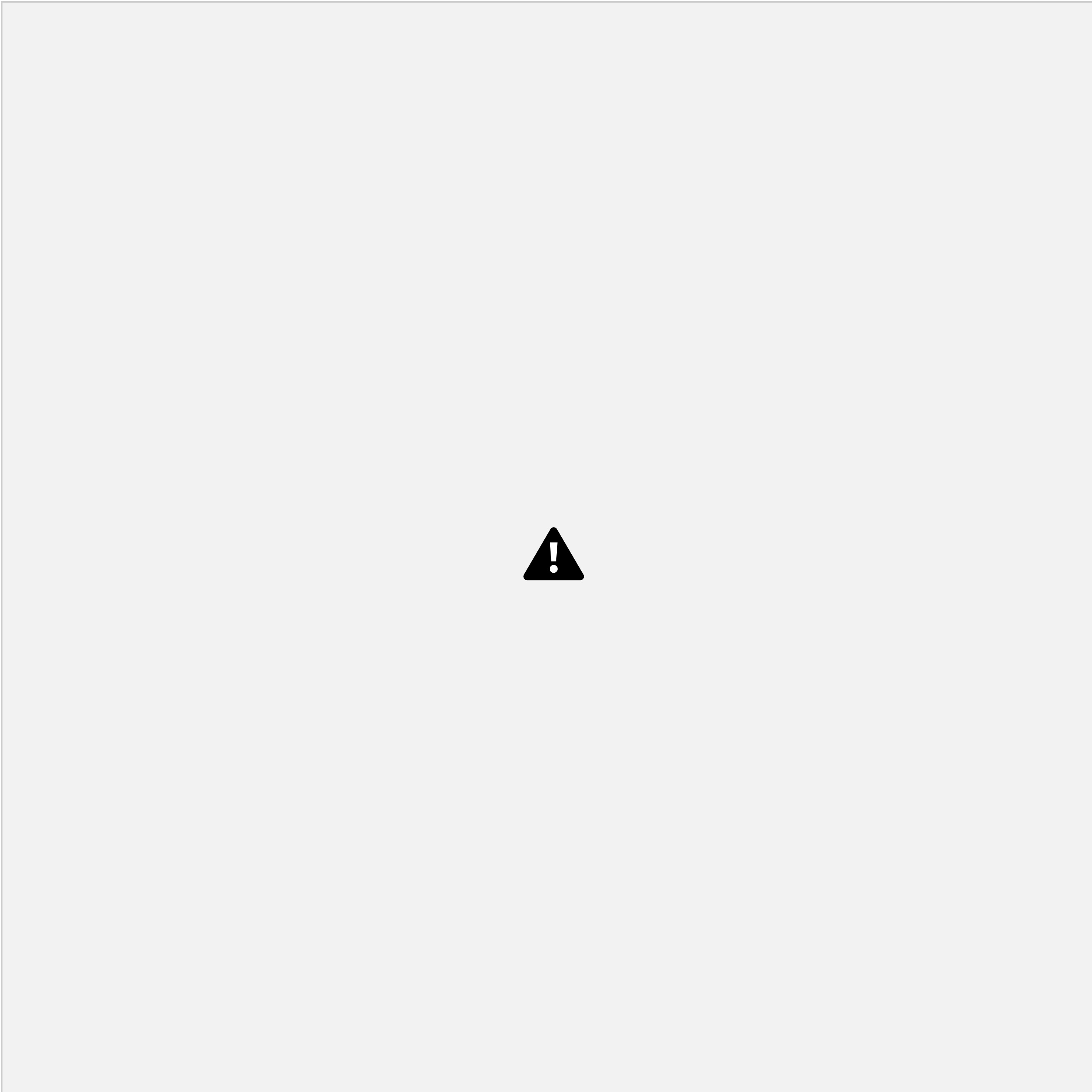
Case D: displacement of end B related to end A

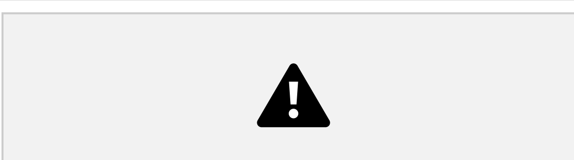
If the far node B of the member is displaced relative to A so that so that the chord of the member rotates clockwise (positive displacement) .The moment M can be related to displacement by using conjugate beam method. The conjugate beam is free at both the ends as the real beam is fixed supported. Due to displacement of the real beam at B, the momentat

the end B of the conjugate beam must have a magnitude of .Summing moments about B we have,







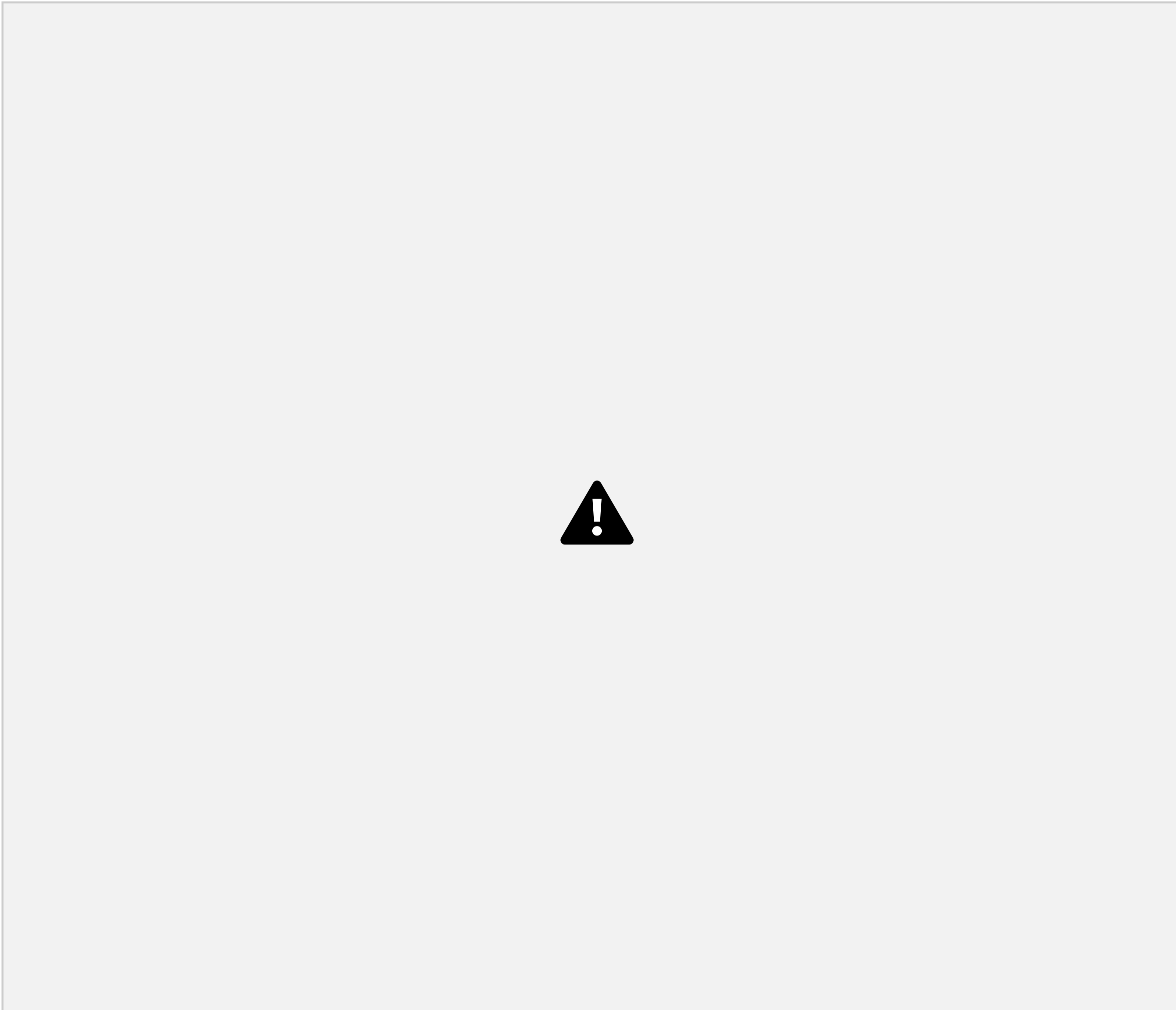


By our sign convention the induced moment is negative, since for equilibrium it acts counter clockwise on the member.

If the end moments due to the loadings and each displacements are added together, then the resultant moments at the ends can be writtenas,

FIXED END MOMENT TABLE

48



81 49

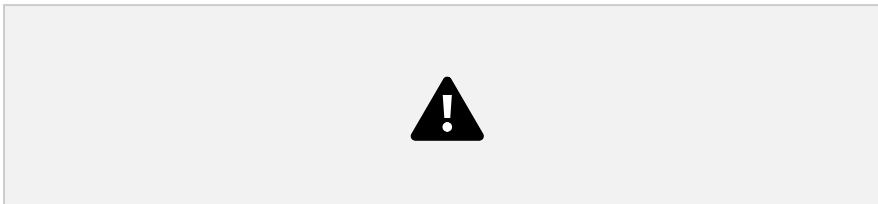
GENERAL PROCEDURE OF SLOPE-DEFLECTION METHOD

- Find the fixed end moments of each span (both ends left &right).
- Apply the slope deflection equation on each span & identify the unknowns.
- Write down the joint equilibrium equations.

- Solve the equilibrium equations to get the unknown rotation & deflections.
- Determine the end moments and then treat each span as simply supported beam subjected to given load & end moments so we can work out the reactions & draw the bending moment & shear force diagram.

Numerical Examples

1. Q. Analyze two span continuous beam ABC by slope deflection method.Then draw Bending moment & Shear force diagram. Take EIconstant.



Fixed end moments are

Slope deflection equations are

In all the above 4 equations there are only2unknowns and accordinglythe boundary

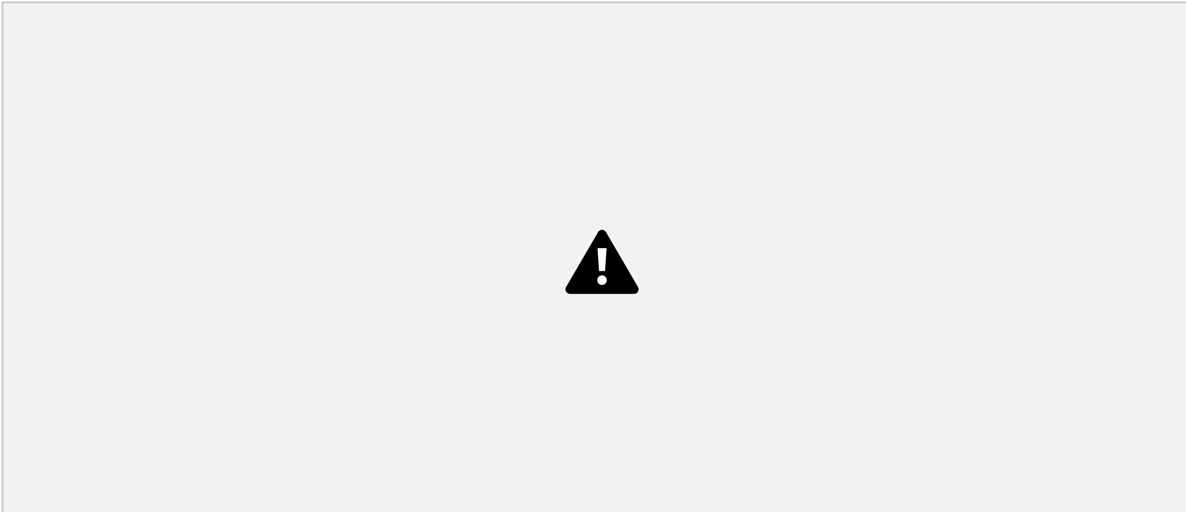
conditions are

Solving the equations (5) & (6), we get

Substituting the values in the slope deflections we have,

Reactions: Consider the free body diagram of the beam

Find reactions using equations of equilibrium.



Span AB: $M_A = 0$, $R_B \times 6 = 100 \times 4 + 75 - 51.38$

$R_B = 70.60$ KN

V =0, $R_A + R_B = 100$KN

$R_A = 100 - 70.60 = 29.40$ KN

Span BC**:** $M_C = 0$, $R_B \times 5 = 20 \times 5 \times +75$

$R_B = 65$ KN

V=0 $R_B + R_C = 20 \times 5 = 100$KN

$R_C = 100 - 65 = 35$ KN

Using these data BM and SF diagram can be drawn

84
52

G.Suryanaraya

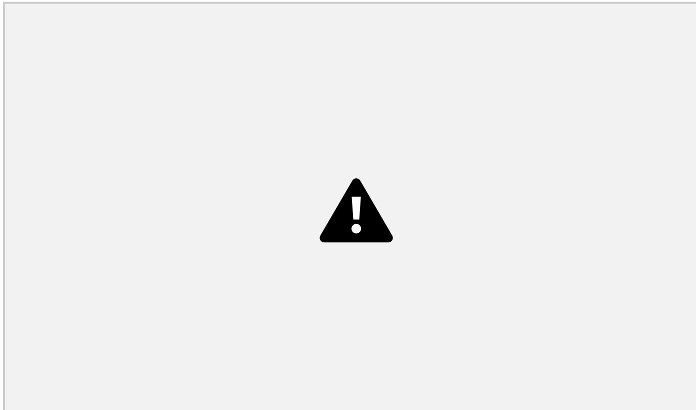
Max BM:

Span AB: Max BM in span AB occurs under point load and can be found geometrically,

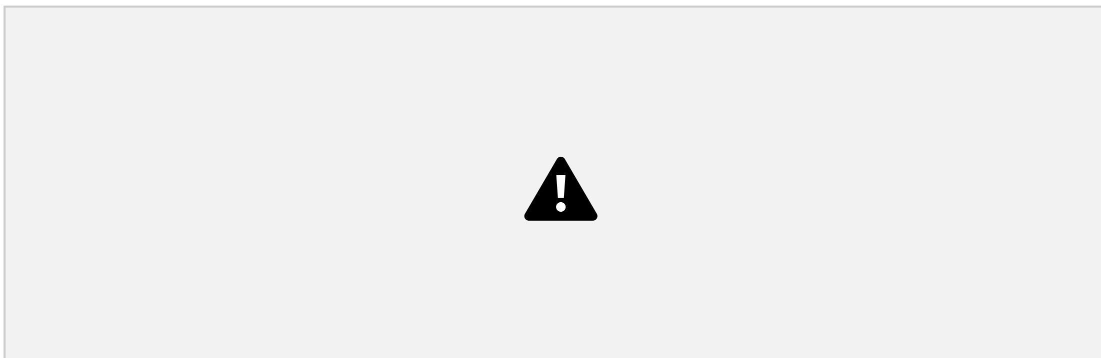
Span BC: Max BM in span BC occurs where shear force is zero or changes its sign. Henceconsider SF equation w.r.t C

Max BM occurs at 1.75m
from

2. Q. Analyze continuous beam ABCD by slope deflection method and thendraw bending moment diagram. Take EIconstant.



Slope deflection equations are



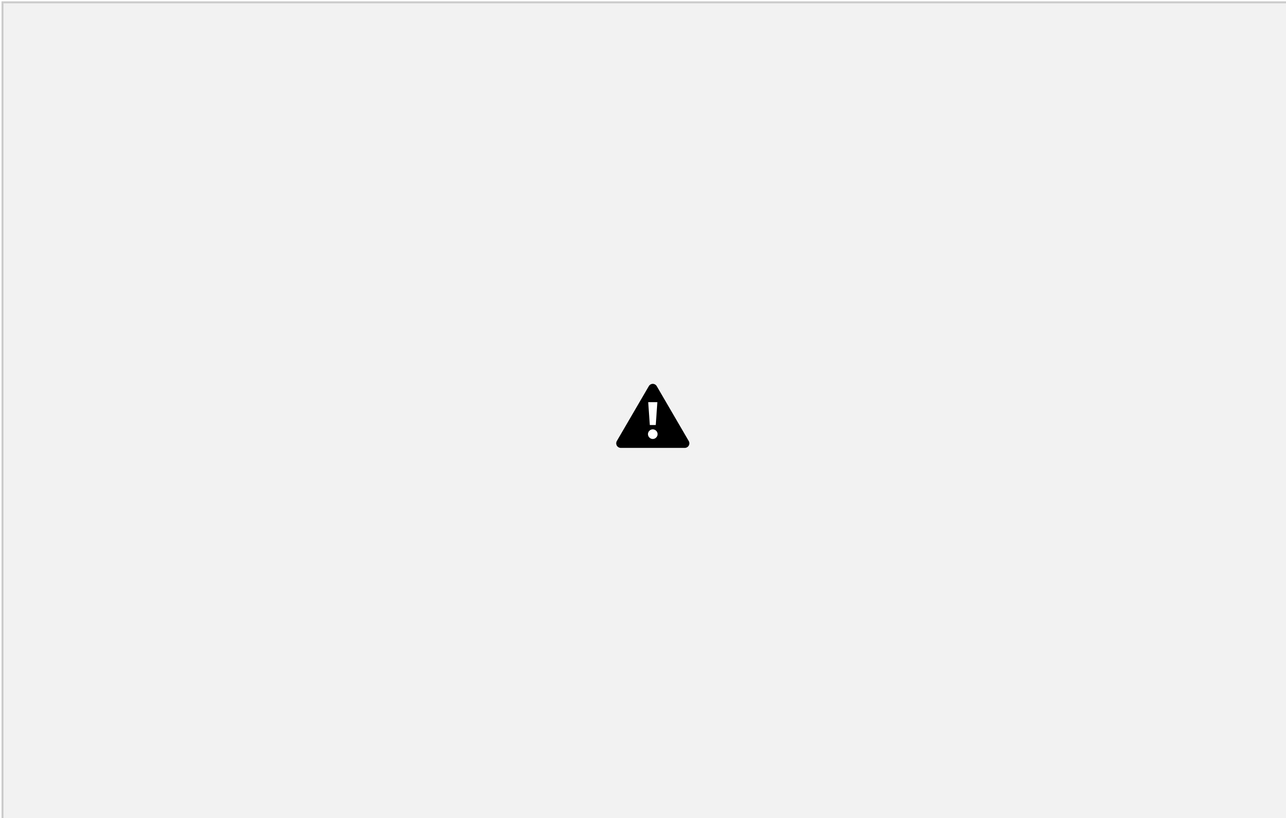
In all the above equations there are only 2 unknowns and accordingly the boundary conditions are

Solving equations (5) & (6),

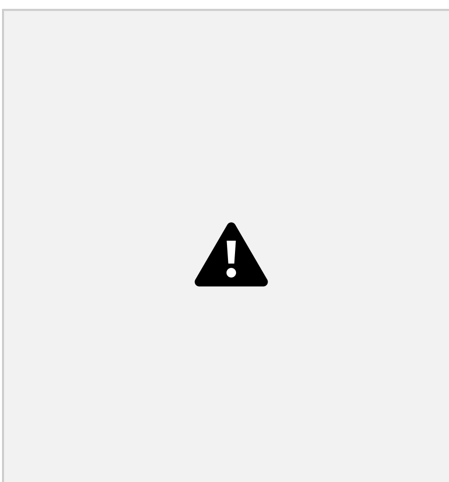
Substituting the values in the slope deflections we have,

Reactions: Consider free body diagram of beam AB, BC and CD as shown

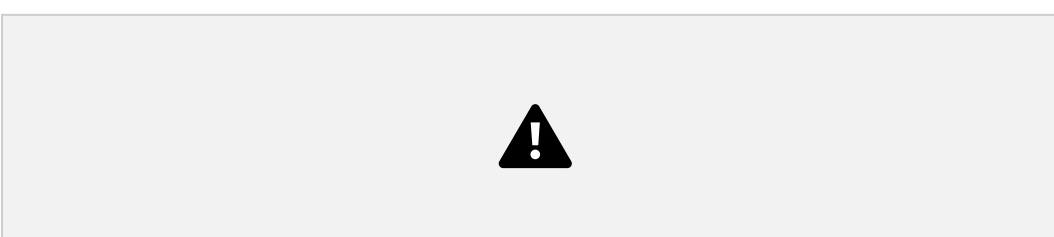
G.Suryana

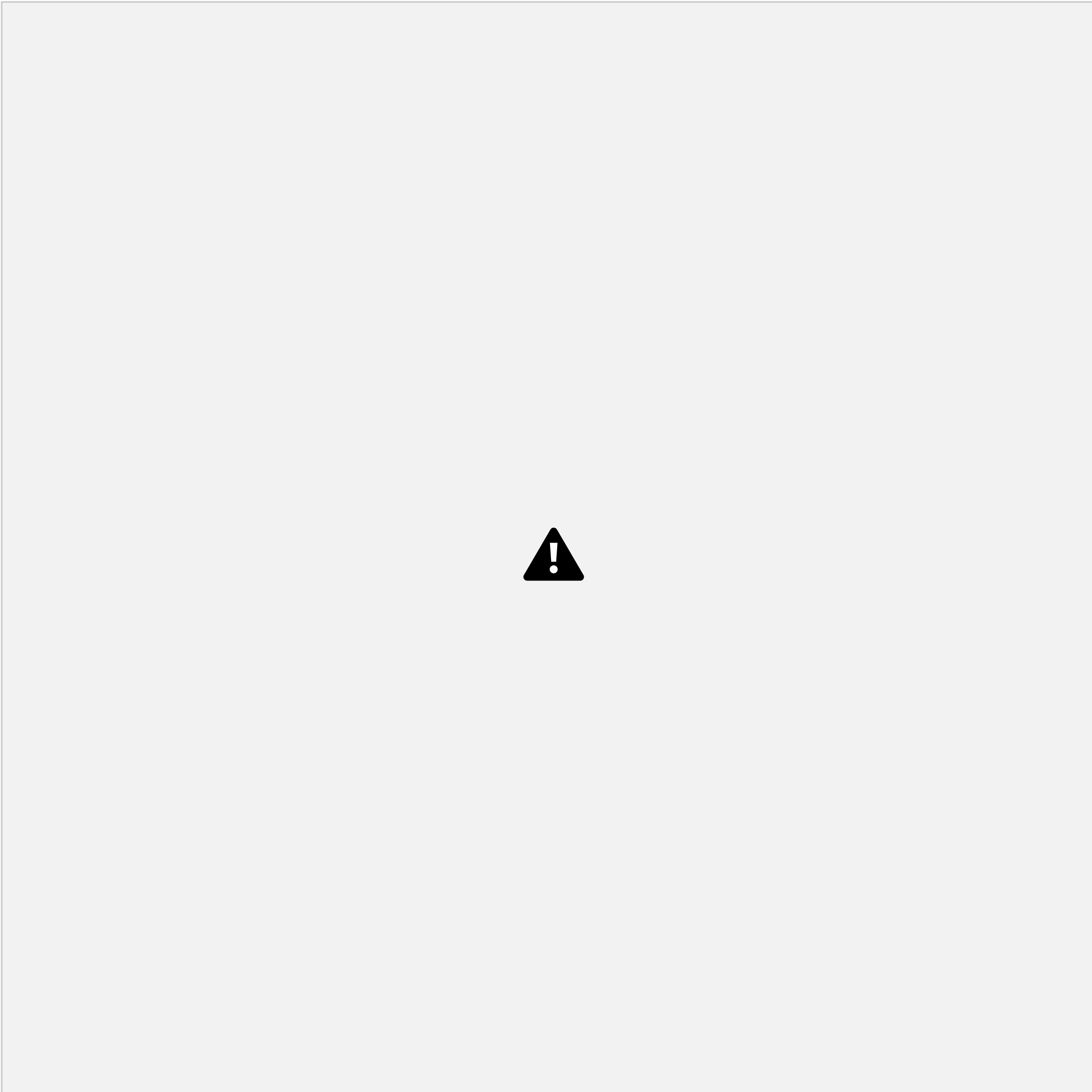


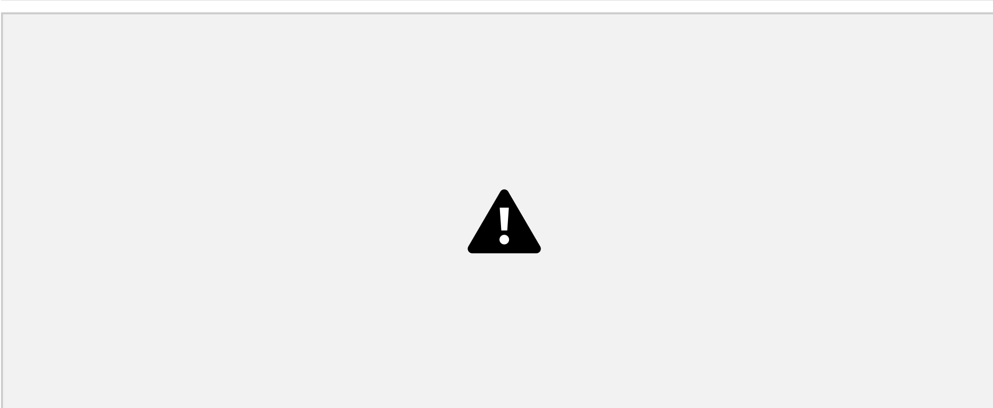
Span AB:



Maximum Bending Moments:







Span AB: Occurs under point load

Span BC: Where SF=0, consider SF equation with C as reference

3. Q. Analyse the continuous beam ABCD shown in figure by slope deflectionmethod. The support B sinks by 15mm.Take E =200 10^{5}KN/$m^{2}$and I =12010 $^{-6}$$m^{4}$

FEM due to yield of support B

For span AB:

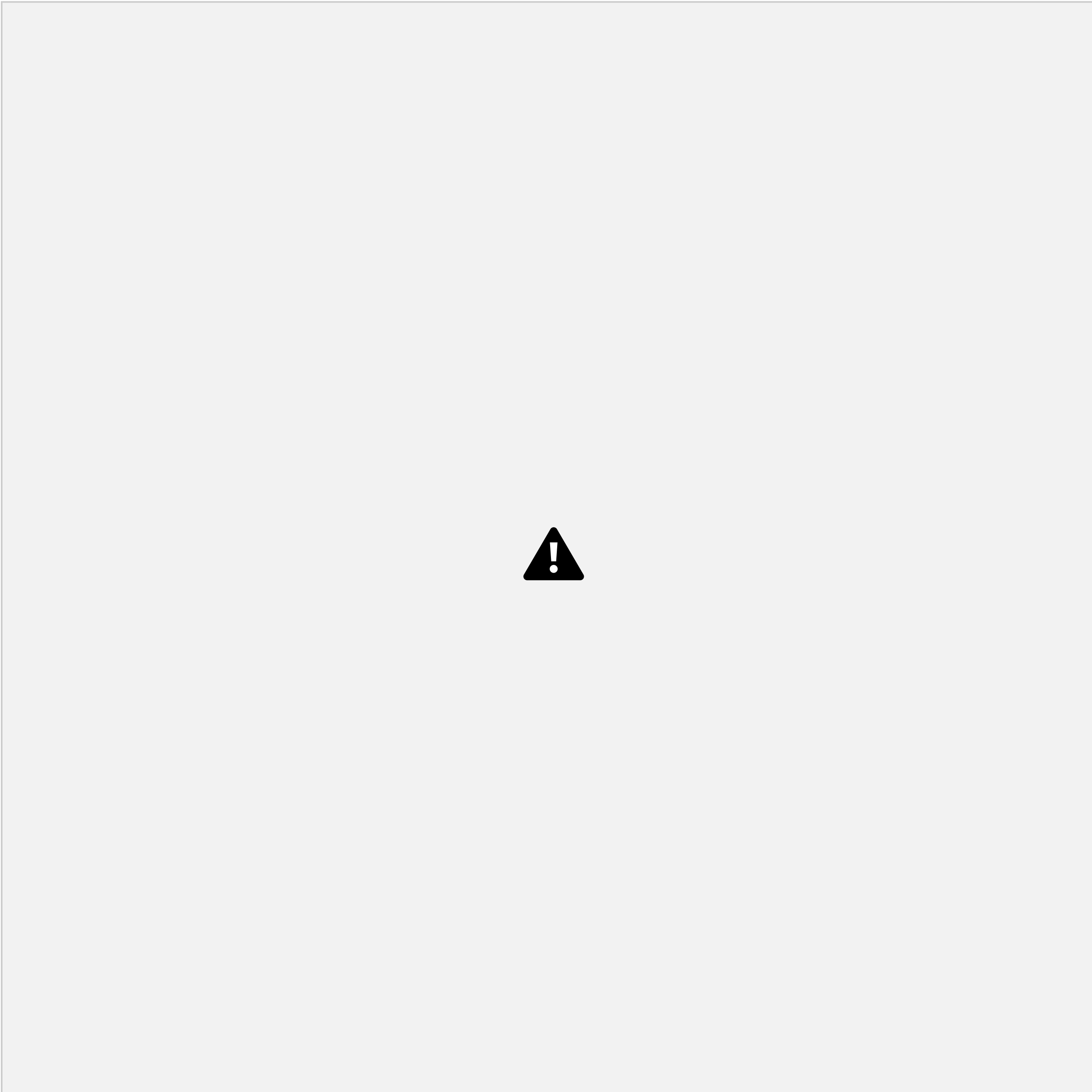
For span BC:

Slope deflection equations are

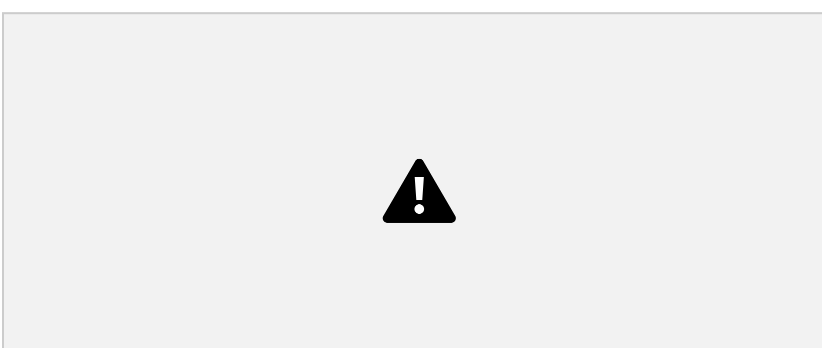
In all the above equations there are only2unknowns and accordinglythe boundary

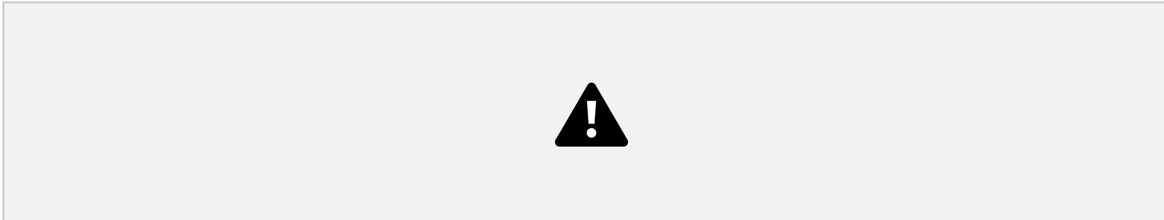
conditions are

Solving equations (5) & (6),



Substituting the values in the slope deflections we have,

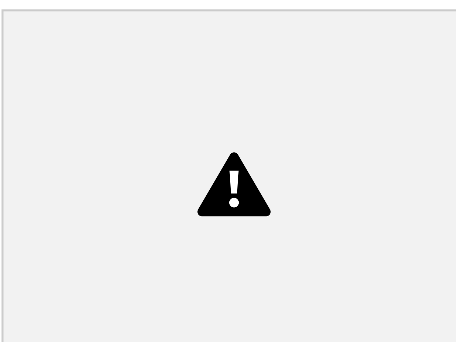


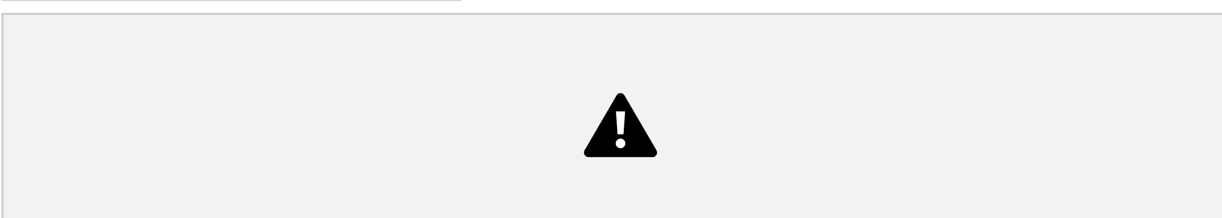


Consider the free body diagram of continuous beam for finding reactions

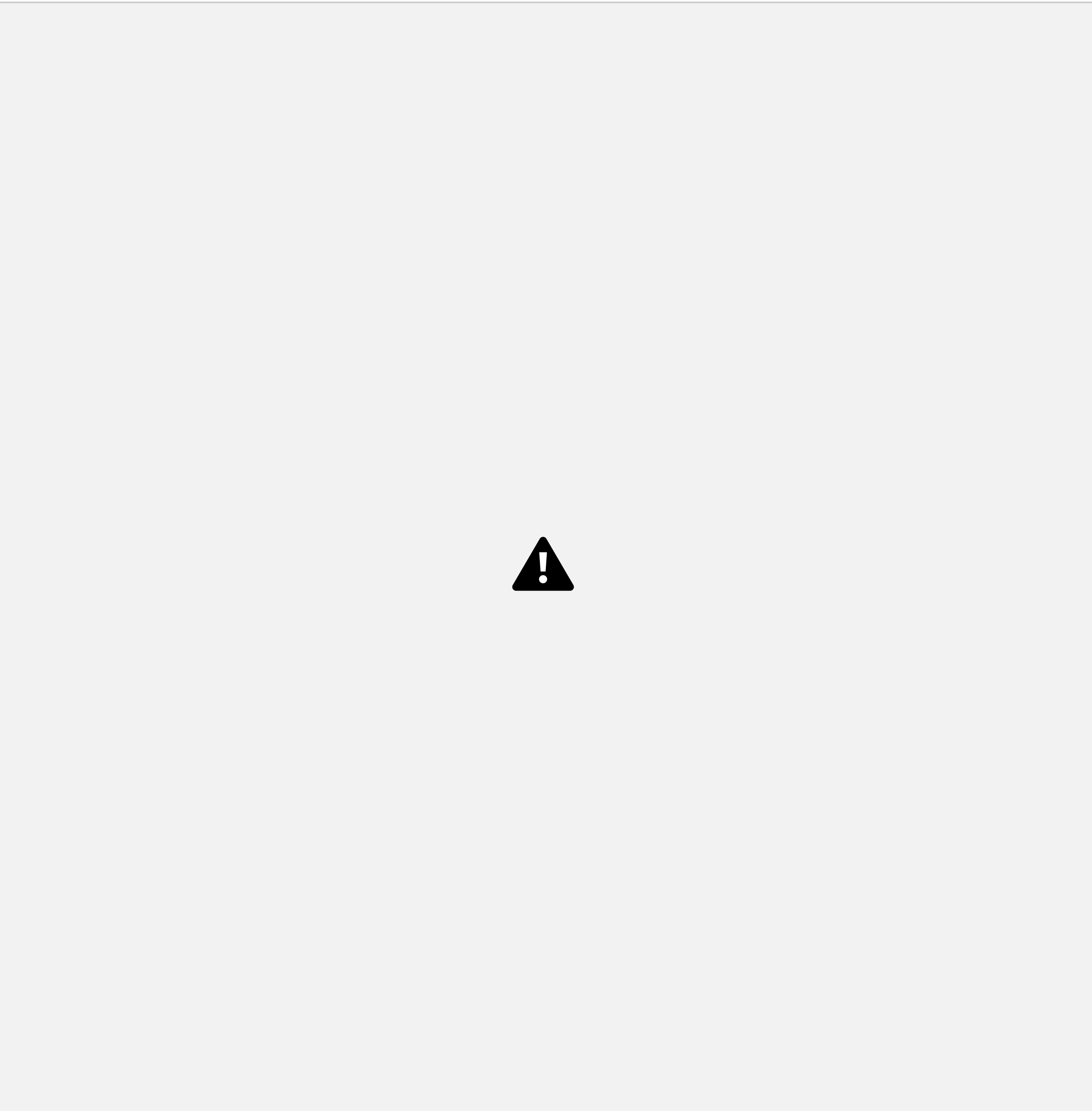
REACTIONS

Span AB:

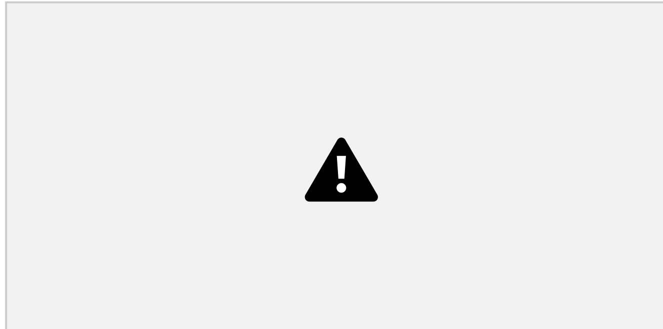




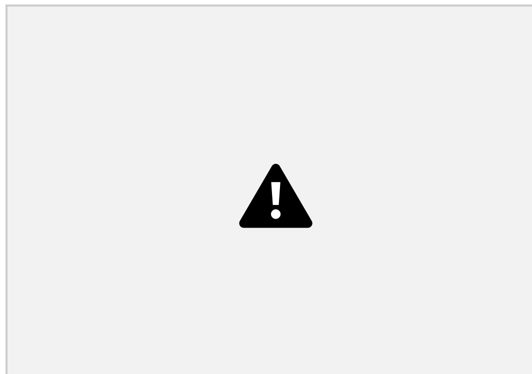
ANALYSIS OF FRAMES (WITHOUT & WITH SWAY)



The side movement of the end of a column in a frame is called sway. Sway can be prevented by unyielding supports provided at the beam level as well as geometric or load symmetry about vertical axis.

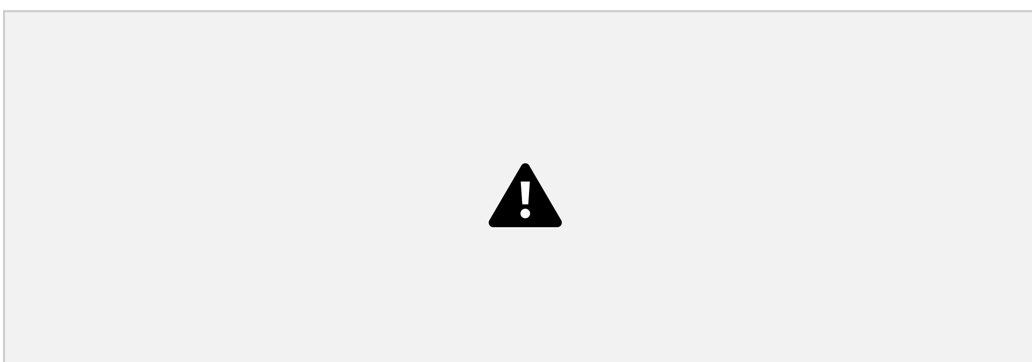


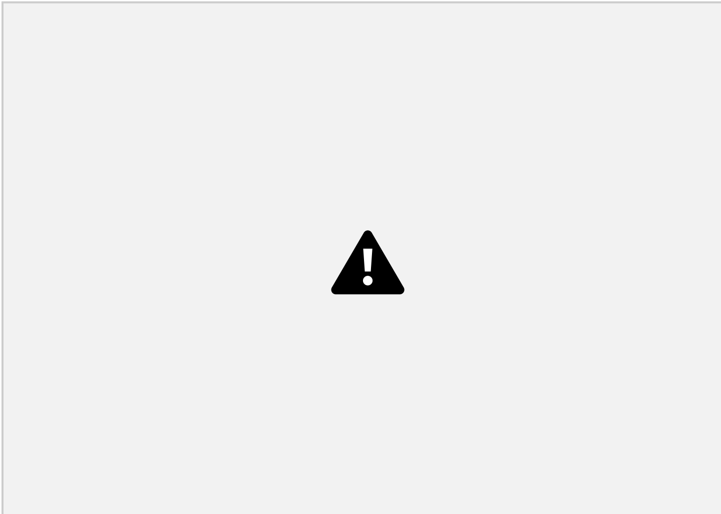
Frame with sway

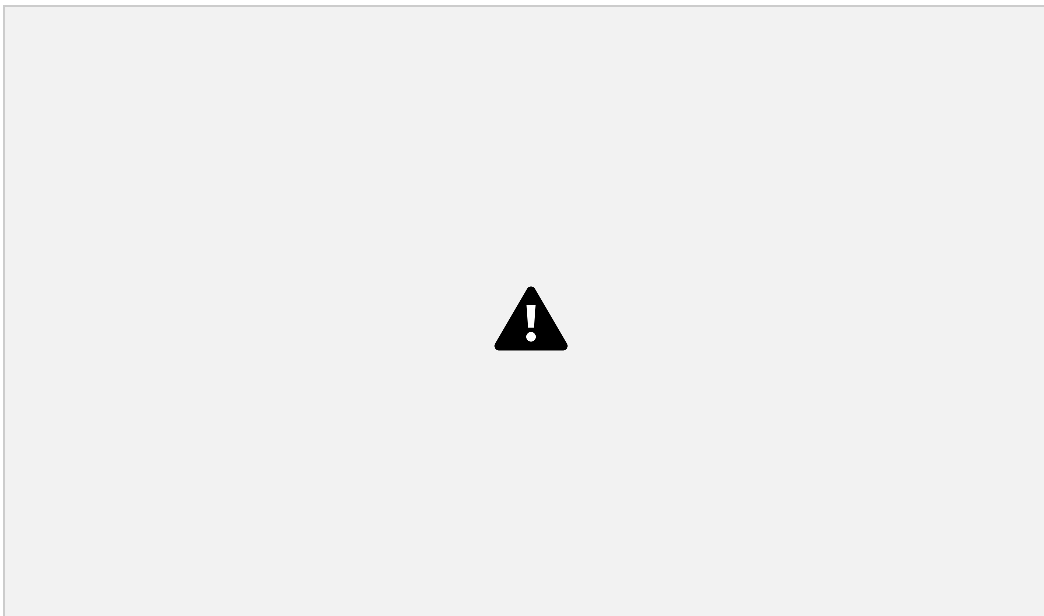


Sway prevented by unyielding support

4. Q. Analyse the simple frame shown in figure. End A is fixed and ends B & Care hinged. Draw the bending momentdiagram.




Slope deflection equations are



In all the above equations there are only 3 unknowns and accordingly the boundary conditions are

Solving equations (7) & (8) & (9),

Substituting the values in the slope deflections we have,

REACTIONS:

SPAN AB:

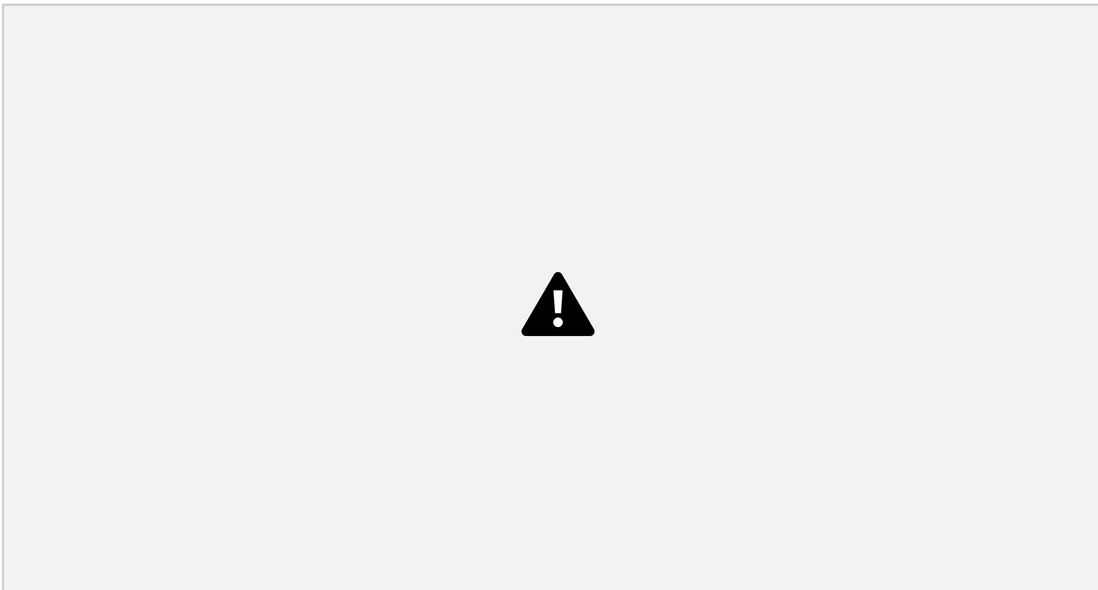
SPAN BC:

Column BD:

95

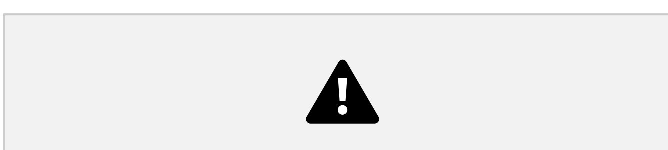
63

Analyse the portal frame and then draw the bending momentdiagram

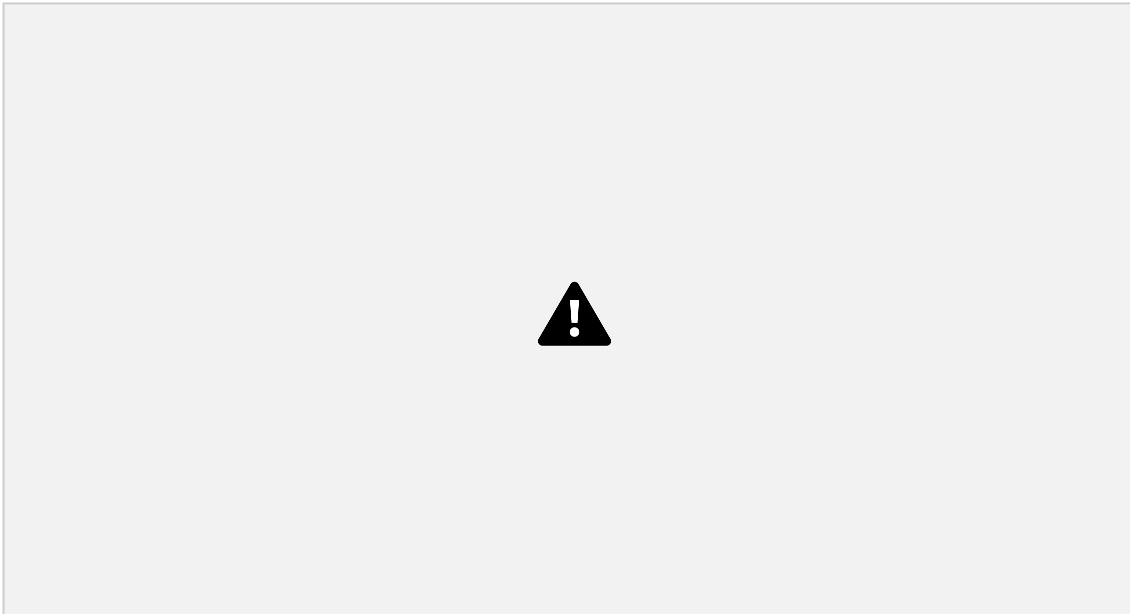


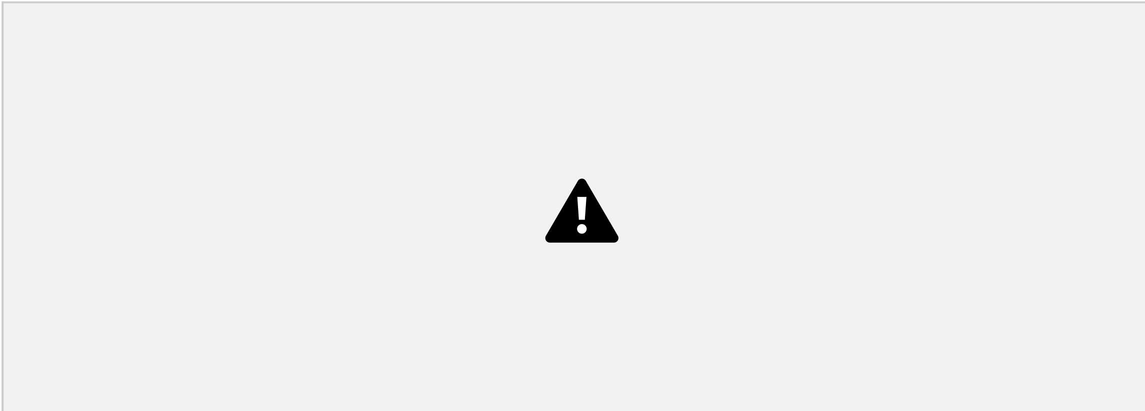
A. This is a symmetrical frame and unsymmetrically loaded, thus it is an unsymmetrical problem and there is a sway ,assume sway toright

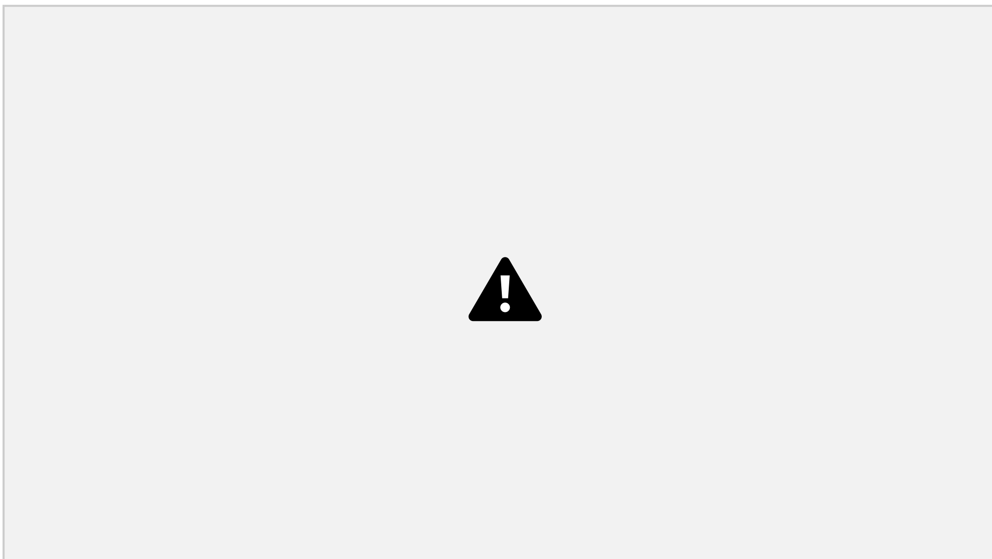
FEMS:



Slope deflection equations are







Reactions: consider the free body diagram of beam and columns

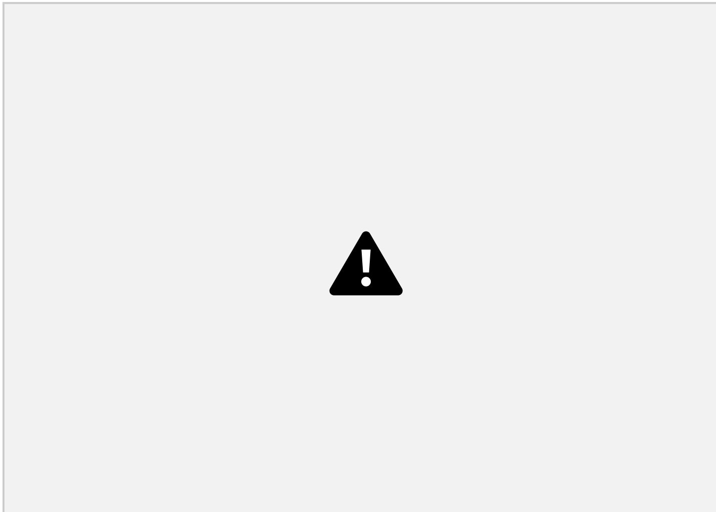
Column AB:

Span BC:

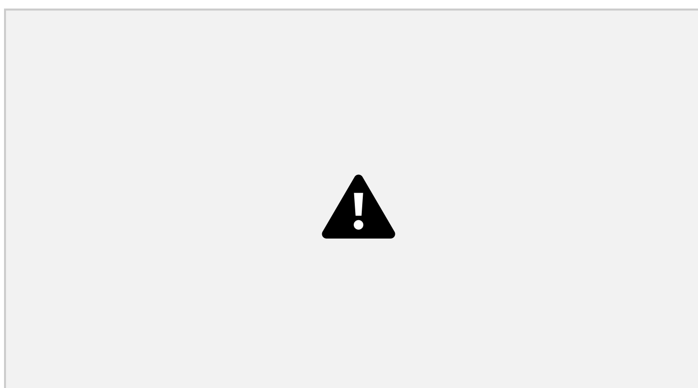
Column CD:

Check:

Hence okay

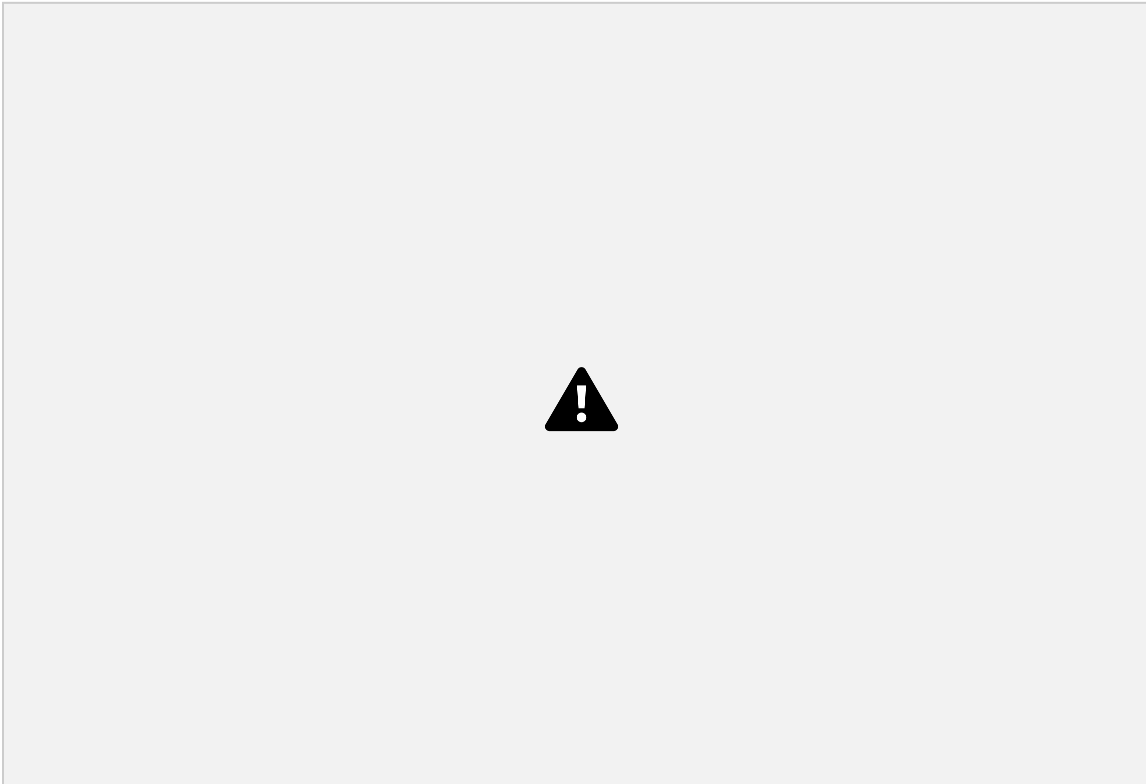


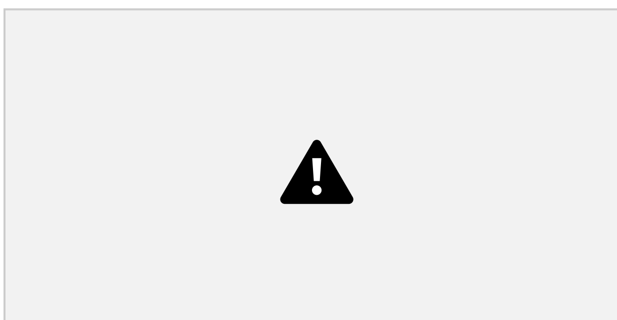
6. Q. Frame ABCD is subjected to a horizontal force of 20 KN at joint C as shown in figure. Analyse and draw bending moment diagram.



The frame is symmetrical but loading is unsymmetrical. Hence there is a sway, assume sway towards right. In this problem

FEMS





Slope deflection equations:

G.Suryanarayana

Reactions: Consider the free body diagram of various members

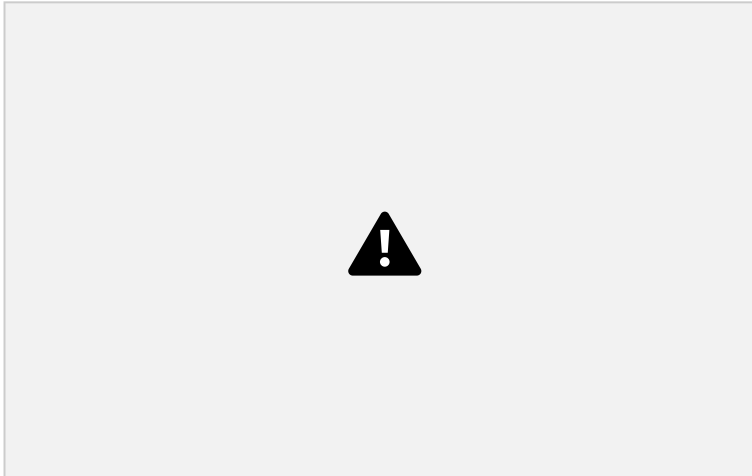
Member AB:

Span BC:

Column CD:

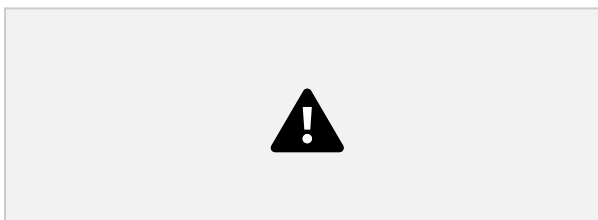
71

Analyse the portal frame and draw theB.M.D.

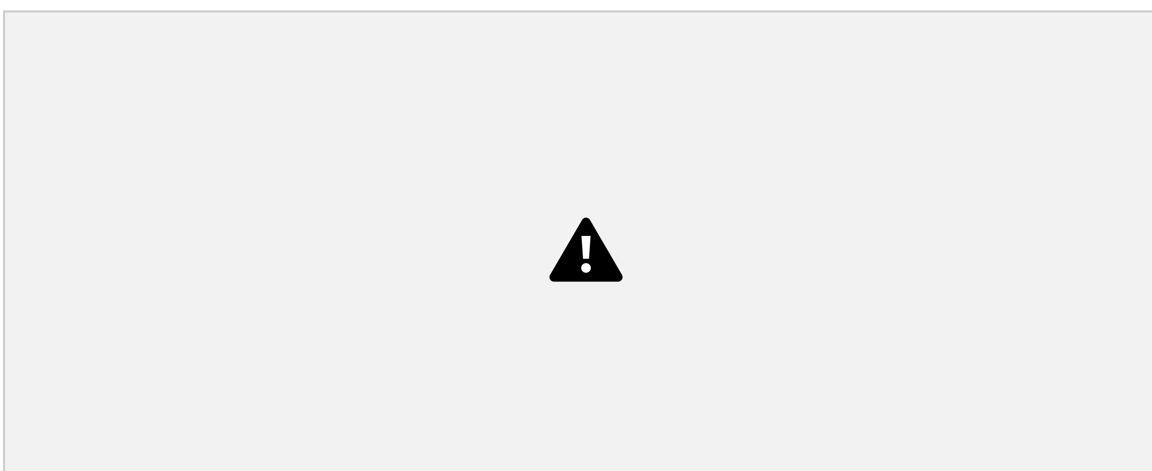


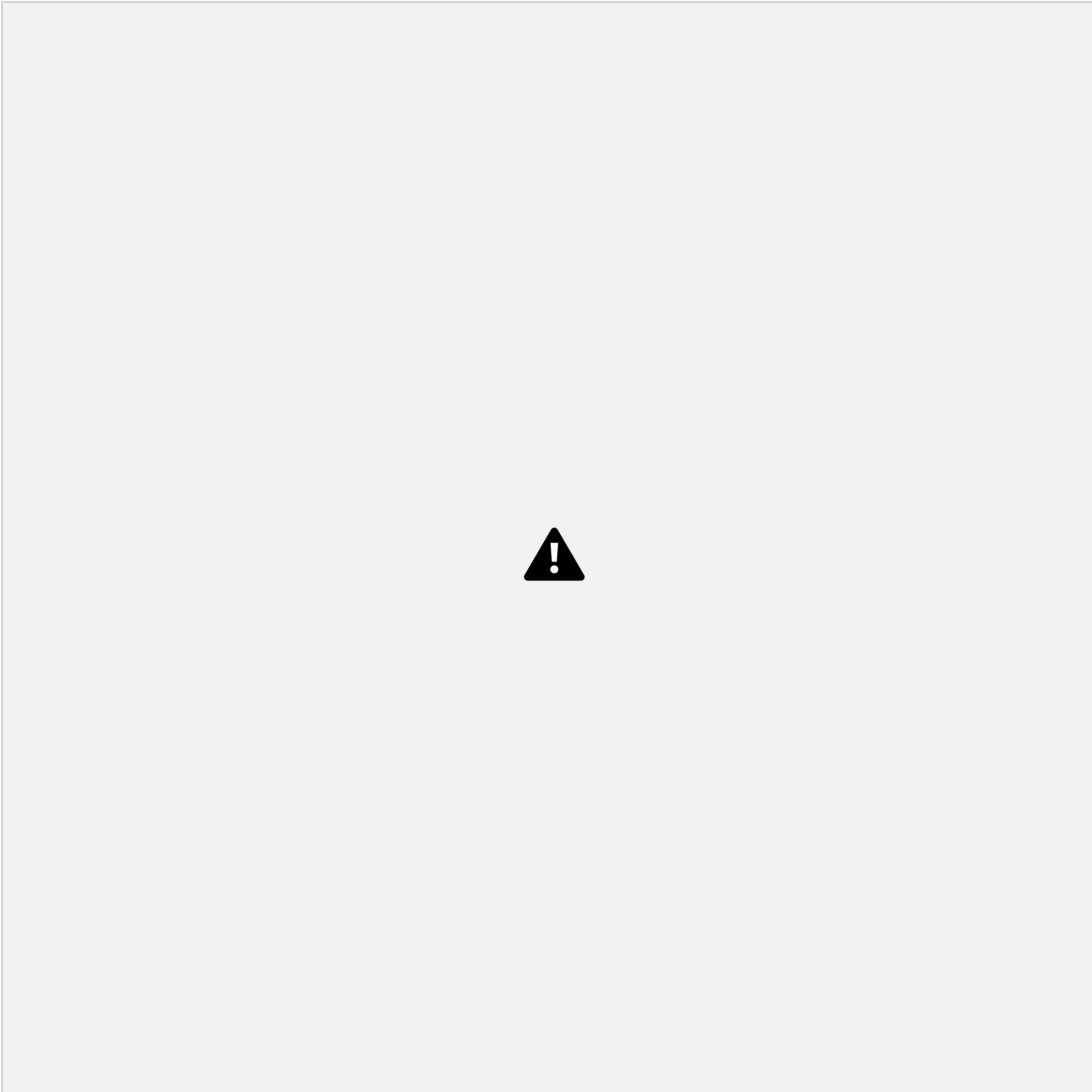
A. It is an unsymmetrical problem, hence there is a sway be towards right.

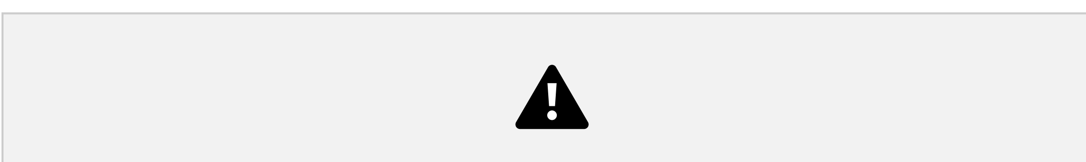
FEMS:



Slope deflection equations:







[106] 73

107 74

Reactions: Consider the free body diagram

Member AB:

Span BC:

Column CD:

Check:

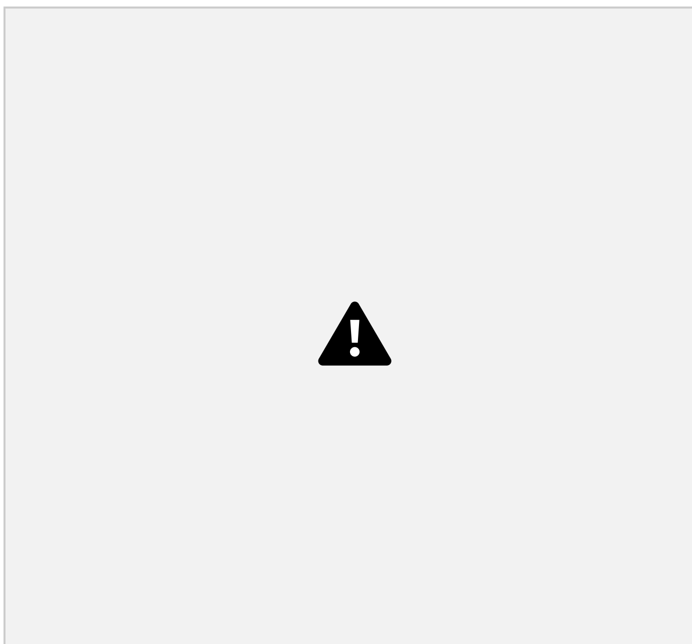
UNIT - III

APPROXIMATE METHODS OFANALYSIS OF BUILDING FRAMES

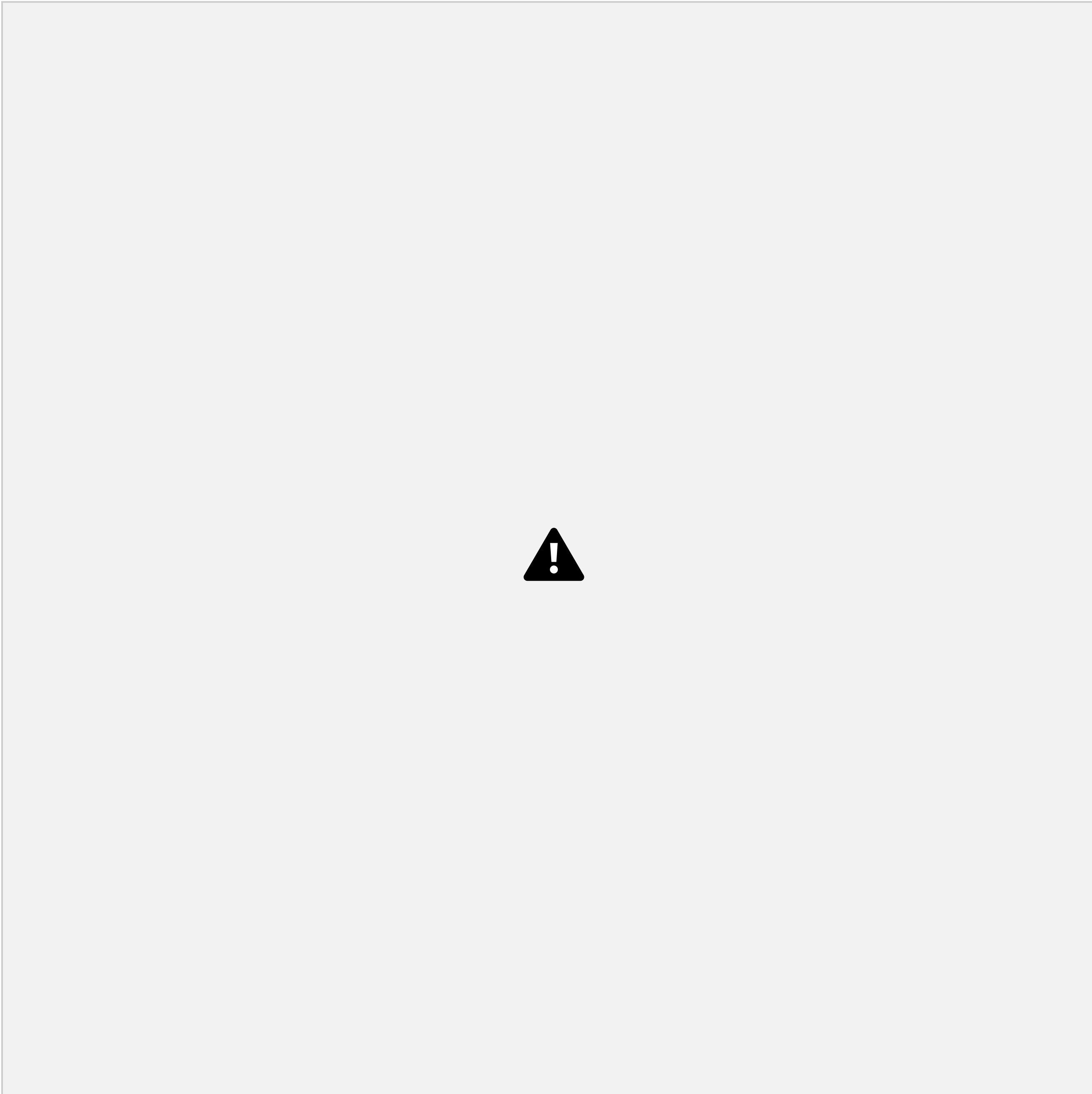
The

building frames are the most common structural form, an analyst/engineer encounters in practice. Usually the building frames are designed such that the beam column joints are rigid. A typical example of building frame is the reinforced concrete multistory frames. A two-bay, three-storey building plan and sectional elevation are shown in Fig. In principle this is a three dimensional frame.

However, analysis may be carried out by considering planar frame in two perpendicular directions separately for both vertical and horizontal loads as shown in Fig. 36.2 and finally superimposing moments appropriately. In the case of building frames, the beam column joints are monolithic and can resist bending moment, shear force and axial force. Any exact method, such as slope-deflection method, moment distribution method or direct stiffness method may be used to analyse this rigid frame. However, in order to estimate the preliminary size of different members, approximate methods are used to obtain approximate design values of moments, shear and axial forces in various members. Before applying approximate methods, it is necessary to reduce the given indeterminate structure to a determinate structure by suitable assumptions. These will be discussed in this lesson. In next section, analysis of building frames to vertical loads is discussed and in section after that, analysis of building frame to horizontal loads will be discussed.



[126] 77



SUBSTITUTE FRAME METHOD

Consider a building frame subjected to vertical loads as shown in Fig.36.3. Any typical beam, in this building frame is subjected to axial force, bending moment and shear force.

Hence each beam is statically indeterminate to third degree and hence 3 assumptions are required to reduce this beam to determinate beam.

Before we discuss the required three assumptions consider a simply supported beam. In this case zero moment (or point of inflexion) occurs at the supports as shown in Fig.36.4a. Next consider a fixed-fixed beam, subjected to vertical loads as shown in Fig. 36.4b. In this case, the point of inflexion or point of zero moment occurs at $0.21L$ from both ends of the support.

Now consider a typical beam of a building frame as shown in Fig.36.4c. In this case, the support provided by the columns is neither fixed nor simply supported.

For the purpose of approximate analysis the inflexion point or point of zero

the point of zero moment varies depending on the actual rigidity provided by the columns. Thus the beam is approximated for the analysis as shown in Fig.