

FLUID MECHANICS

Topic: UNIT IV - Hardy Cross Method for Pipe Network Analysis

Reynolds Experiment & Flow Regimes

- Investigates the physical transition mechanics between laminar and turbulent flow states.
- Reynolds Number (Re): Dimensionless inertia-to-viscous force ratio ($Re = \rho V d / \mu$).
- $Re < 2000$: Laminar state; fluid moves in distinct parallel layers without mixing.
- $Re > 4000$: Turbulent state; chaotic eddies drive continuous structural mixing loops.

Friction Head Losses: Darcy-Weisbach Form

- Major energy losses in pipelines are driven by fluid wall friction shear forces.
- Darcy-Weisbach Equation: $h_f = (4fLV^2) / (2gd)$ [or $h_f = f_D \cdot L V^2 / (2gd)$].
- Where f represents the friction factor, L is length, V is velocity, and d is diameter.
- Laminar friction factor depends solely on Reynolds number parameters: $f = 16/Re$.

Minor Losses in Pipeline Layouts

- Caused by localized geometric flow disruptions that alter fluid streamlines.
- Sudden Expansion Loss: $h_e = (V_1 - V_2)^2 / 2g$, driven by eddy formations.
- Sudden Contraction Loss: $h_c = 0.5 \cdot V_2^2 / 2g$, caused by vena contracta effects.
- Minor losses also encompass entry effects ($0.5V^2/2g$), exit zones ($V_2/2g$), and valve fittings.

Energy Line Grids: TEL & HGL Profiles

- Total Energy Line (TEL): Graphical plot tracking total head availability: $(p/\gamma) + (V^2/2g) + z$.
- Hydraulic Gradient Line (HGL): Graphical plot tracking piezometric head levels: $(p/\gamma) + z$.
- TEL drops continuously in the direction of flow due to ongoing friction energy losses.
- HGL sits below the TEL by a distance equal to the local dynamic velocity head $(V^2/2g)$.

Pipe Configurations: Series and Parallel

- Series Systems: Discharges match ($Q = Q_1 = Q_2 = Q_3$); total head loss equals individual losses.
- Dupuit's Equivalent Pipe Equation: $L_{eq} / d_{eq}^5 = \sum (L_i / d_i^5)$, modeling complex channels.
- Parallel Systems: Head losses balance out across branches ($h_{f1} = h_{f2} = h_{f3}$).
- Total system capacity equals the sum of branch flow rates ($Q = Q_1 + Q_2 + Q_3$).

Siphon Systems & Three-Reservoir Problems

- Siphons: Inverted long conduits used to lift liquids over high intermediate ridges.
- Summit: The highest physical point where internal pressure drops to sub-atmospheric levels.
- Vapor pressure limitations at the summit restrict maximum operational heights to $\approx 7.5\text{m}$.
- Three-Reservoir Problems: Complex networks analyzed by setting continuity equations at the central junction.

Introduction to Complex Pipe Networks

- Municipal water distribution grids feature closed, intersecting pipe loops.
- Manual network balance requires solving simultaneous non-linear head loss equations.
- Head loss relationships are summarized via the Darcy form: $h_f = r \cdot Q^n$ (where $n = 2.0$).
- Analysis balances internal nodal flows alongside loop pressure equilibriums.

Hardy Cross Balancing Principles

- Core Constraint 1: The algebraic sum of all inflows and outflows at any node must equal zero.
- Core Constraint 2: The algebraic sum of all head losses around any closed loop must equal zero.
- Starts with an initial guess of flow distribution that satisfies nodal continuity.
- Applies successive loop iterations to eliminate head loss imbalances.

Derivation of Flow Correction (ΔQ)

- Let actual flow $Q = Q_0 + \Delta Q$, where Q_0 is the estimated flow and ΔQ is the correction factor.
- Expand using Taylor Series: $\sum r(Q_0 + \Delta Q)^2 \approx \sum rQ_0^2 + \sum 2rQ_0\Delta Q = 0$.
- Solving for the loop correction factor yields: $\Delta Q = - \sum rQ_0|Q_0| / \sum 2r|Q_0|$.
- For general equations where $h_f = rQ^n$, the correction factor becomes: $\Delta Q = - \sum rQ_0^n / \sum n r Q_0^{(n-1)}$.

Hardy Cross Sign Conventions & Iterations

- Clockwise flows around a loop are considered positive; counter-clockwise flows are negative.
- Evaluated corrections (ΔQ) are added algebraically to update estimated pipe flows.
- Pipes shared between adjacent loops receive corrections from both loops simultaneously.
- Iterations repeat until calculated adjustments converge close to zero ($\Delta Q \rightarrow 0$).

Water Hammer Phenomena in Pipes

- Sudden closure of control valves generates high pressure shock waves.
- Kinetic energy converts into elastic strain energy, causing acoustic pressure oscillations.
- Instantaneous rigid closure pressure rise: $\Delta p = \rho \cdot C \cdot V$, where C is wave speed.
- Surge tanks and pressure relief valves are deployed to protect systems from structural bursting.

UNIT IV Summary & High-Yield Viva-Voce Key

- Comprehensive tracking of Unit IV core syllabus parameters and theoretical frameworks.
- High-yield focus on governing mathematical derivations and mechanical configurations.

□ VIVA-VOCE ASSESSMENT CORE QUESTIONS:

1. State the two primary physical laws that must be balanced in the Hardy Cross method.
2. Why does the Total Energy Line (TEL) drop continuously in the direction of flow?
3. Explain how surge tanks protect cross-country pipelines from water hammer failures.

Thank You..