UNIT-IV :ELECTRIC TRACTION - I

CONTENTS:-

1. System of Electric Traction.
2. Track Electrification.
3. Special Features of Traction Motor.
4. Methods of Electric Braking.
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   - Rheostat Braking and
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5. Mechanics of Train Movement(Unit-5)
6. Types of Services In India.
7. Speed-time Curves For Different Services.
   - Trapezoidal Speed Time Curves.
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Electric Traction

7.1 Basic definition

(a) Traction System

- Propulsion of vehicle is called the traction.

(b) Electric Traction System

- The system of traction involving the use of electricity is called the electric traction system.

7.2 Ideal traction system

(the force which is exerted by powered equipment)

- High starting tractive effort in order to have rapid acceleration.
- Self contained and compact locomotive (train unit) so that it may be able to run on any route
- Equipment capable of withstanding large temporary overloads.
- Minimum wear on the track.
- Braking should be such that minimum wear is caused on the brake shoes, and if possible the energy should be regenerated and returned to the supply.
- Equipment required should be minimum, high efficiency, low initial and maintenance cost.
- No interference to the communication line running along the track.
- Easy speed control.
- It should be pollution free.

Since the invention of locomotives in the 1800s, the way trains are powered has changed beyond all recognition. Formerly powered by burning solid fuel to generate steam, trains today run on a mix of either pure electric, diesel-electric, or gas-turbine engines.

Electrical trains rose to prominence in the early-20th century. Some of the first appeared in around 1910 with the opening of the Hudson River Tunnels on the New York mainline.

As these tunnels were so long, steam locomotives were prohibited from being used, due to the dense fumes they generate. An alternative way of moving trains was needed, and the electric train was born.

Over the next few decades, electrical trains became more popular around the world and were notably used for various high-speed projects around the world.

Different system of traction

Non-electric Traction System

Direct Steam Engine Drive

Direct Internal Combustion Engine Drive

Electric Traction System

Steam Electric Drive

Internal Combustion Engine Electric Drive

Battery Electric Drive

Electric Traction Drive

[Note: Trains are the mode of transport of land that is used to cover long distances, there are several carriages and coaches attached to it to increase its capacity.

Trams are also a land mode of transportation and are used to cover shorter distances as compared to the distance covered by a train.

Regenerative braking is a way of taking the wasted energy from the process of slowing down a car and using it to recharge the car's batteries.]

TRAM

Trams have flanged wheels and run on rails like a train [whether on reserved track like most railways or in streets on grooved track]

Trolley buses or tramways supplied with DC supply (i.e., battery electric drives)

TROLLEY BUS

Trolleybuses have conventional rubber tyres for ordinary road surface and are essentially electrically powered buses.

Drawbacks of Electric Traction

- Electric traction system involves high erection cost of power system.
- Interference causes to the communication lines due to the overhead distribution networks.
- The failure of power supply brings whole traction system to stand still.
- In an electric traction system, the electrically operated vehicles have to move only on the electrified routes. Additional equipment should be needed for the provision of regenerative braking, it will increase the overall cost of installation.

1.2 System of Traction In India

Traction system is normally classified into two types based on the type of energy given as input to drive the system and they are:

1. Non-electric traction system.
2. Electric traction system.

3. Non-electrictraction system:

Traction system develops the necessary propelling torque, which do not involve the use of electrical energy at any stage to drive the traction vehicle known as electric traction system.

Ex: Direct steam engine drive and direct internal combustion engine drive.

2. Electric traction system:

Traction system develops the necessary propelling torque, which involves the use of electrical energy at any stage to drive the traction vehicle, known as electric traction system. Based upon the type of sources used to feed electric supply for traction system, electric traction may be classified into two groups:

A) Self-contained locomotives.

B) Electric vehicle fed from the distribution networks.

2. Track Electrification

Now a day, based on the available supply, the track electrification system is categorized into:

1. DC system.
2. Single-phaseAC system.
3. Three-phase AC system.
4. Composite system.

1. DC system:

In this system of traction, the electric motors employed for getting necessary propelling torque should be selected in such a way that they should be able to operate on DC supply. **Examples** for such vehicles operating based on DC system are tramways and trolley buses.

The operating voltages of vehicles for DC track electrification system are 600, 750, 1,500, and 3,000 V. Direct current at 600–750 V is universally employed for tramways in the urban areas and for many sub-urban and main line railways 1,500–3,000 V is used.

****The DC system is preferred for suburban services and road transport where stops are frequent and distance between the stops is small*.***

2. Single-phaseAC system:

In this system of track electrification, usually AC series motors are used for getting the necessary propelling power. The distribution network employed for such traction systems is normally 15–25 kV at reduced frequency of 25 Hz. The main reason of operating at reduced frequencies is AC series motors that are more efficient and show better performance at low frequency.

These high voltages are stepped down to suitable low voltage of 300–400 V by means of step-down transformer. Low frequency can be obtained from normal supply frequency with the help of frequency converter.

****AC system is mainly preferred for main line services where the cost of overhead structure is not much importance moreover rapid acceleration and retardation is not required for main line services*.***

3. Three-phaseAC system:

In this system of track electrification, 3φ induction motors are employed for getting the necessary propelling power. The operating voltage of induction motors is normally 3,000–3,600-V AC at either normal supply frequency or 16⅔-Hz frequency.

Usually 3-φ induction motors are preferable because they have simple and robust construction, high operating efficiency, provision of regenerative braking without placing any additional equipment.

The main disadvantage of such track electrification system is high cost of overhead distribution structure. This distribution system consists of two overhead wires and track rail for the third phase and receives power either directly from the generating station or through transformer substation.

Table 7.2 DC System

S. No.	Operating Voltage in Volts	Spacing between Sub-Station in km	Application
1.	600	3 to 5	Tramways, Trolley Bus
2.	1500 to 3000	30 to 40	Main Line Services

- The distribution system consists of one contact wire in case of tramways and two contact wires in case of trolley buses.
- The spacing of sub-stations depends upon the operating voltage and the traffic density of the route.

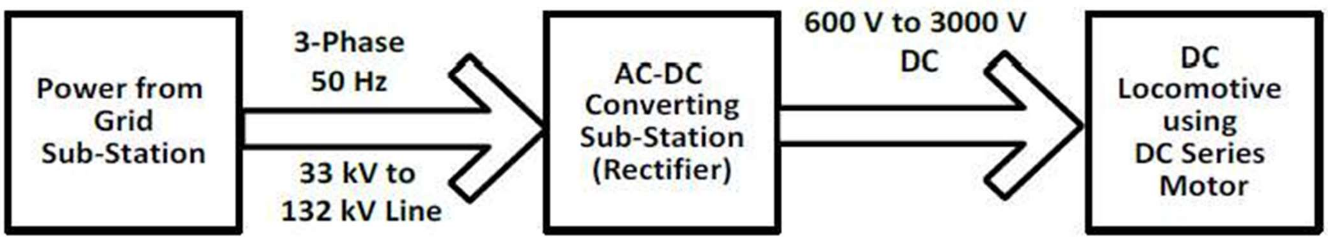


Figure 7.1 DC System

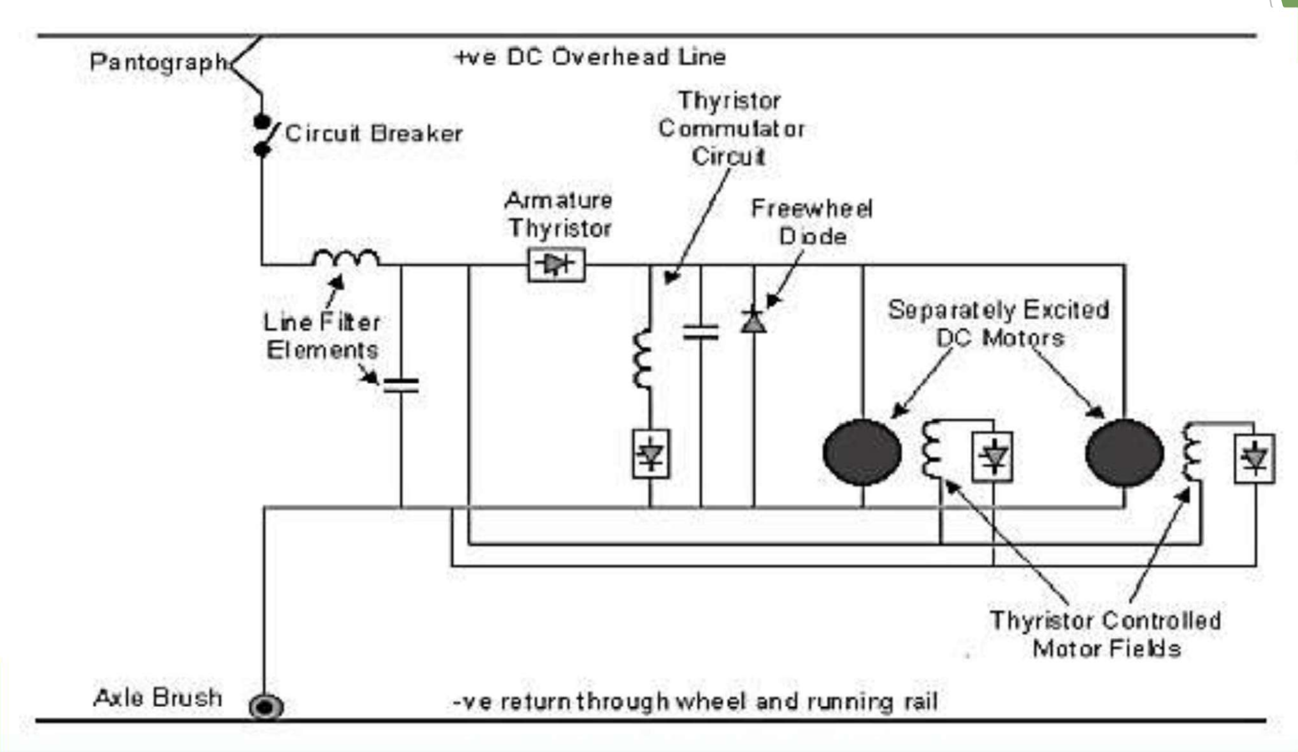
With two wires, they have to connect to them with poles, because the two sides of the circuit have to be kept apart.
Trains, by contrast, are grounded through the rails and therefore need only one wire above.

Power conversion for DC systems tends to take place at a railway substation using large, heavy, and more efficient hardware compared to AC systems.

Power conversion for AC systems, on the other hand, tend to convert current to AC onboard the train where space is limited and losses can be significantly higher.

Whichever is chosen is usually a trade-off between these considerations, but can also be dictated by existing infrastructure.

DC consumes less energy compared to an AC unit for operating the same service conditions. The equipment in the DC traction system is less costly, lighter, and more efficient than an AC traction system. It also causes no electrical interference with nearby communication lines.


Pantograph
+ve DC Overhead Line
Circuit Breaker
Thyristor
Commutator
Circuit
Armature
Thyristor
Freewheel
Diode
Line Filter
Elements
Separately Excited
DC Motors
Thyristor Controlled
Motor Fields
Axle Brush
-ve return through wheel and running rail

(a) Advantages

- DC series motor has better speed torque characteristics and smooth speed control.
- It offers high starting torque.
- It has low maintenance cost.
- Smaller weight per kW output.
- Batter speed control.
- Efficient braking system.

(b) Disadvantages

- This system has high cost of sub-station due to converting equipments.
- More number of sub-stations is required as they are spaced at shorter distance.
- Additional equipments like negative boosters are also required to maintain return voltage within specified limit.

7.12 The single phase AC system

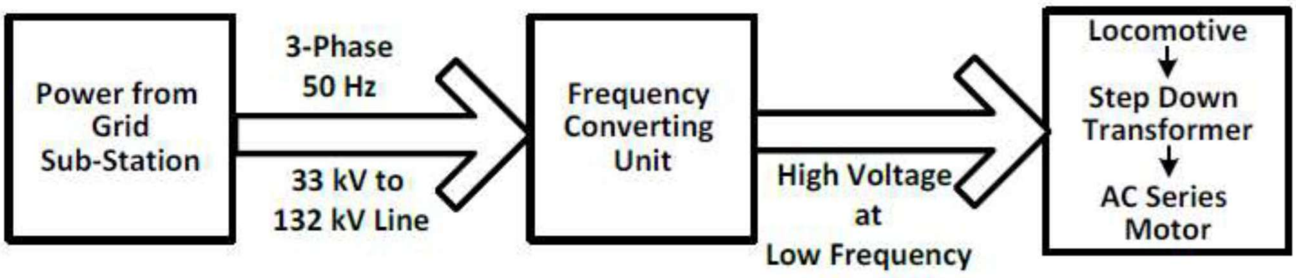
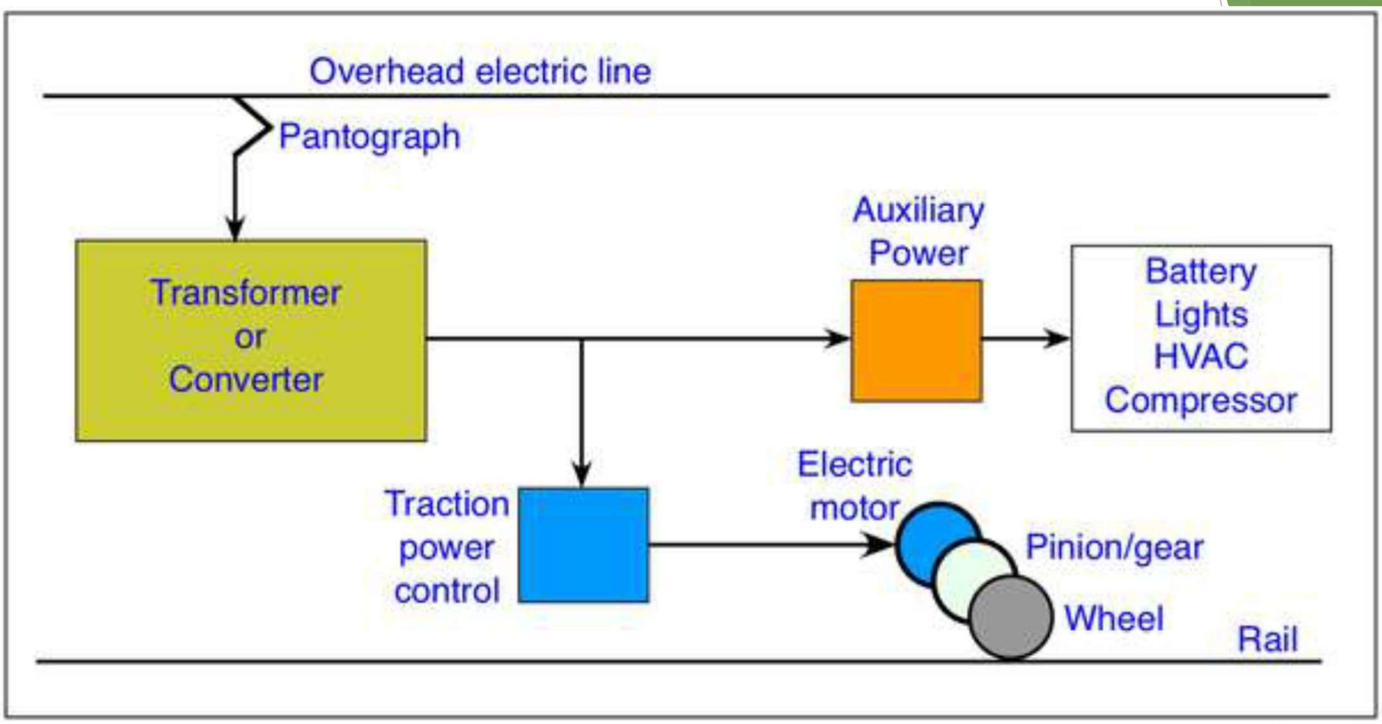


Figure 7.2 Single Phase AC System

- In single phase AC system ac series motors are used for getting necessary motive power.
- The voltage employed for distribution network is 15 to 25 kV at $6\frac{2}{3}$ or 25 Hz, which is stepped down on locomotive to a low voltage suitable for supplying to single ac series motor.
- The spacing of substation is 50 to 80 km.
- The change of supply frequency become necessary because of
  - Batter performance.
  - Improves its commutation properties, power factor and efficiency.
  - Reduces the line reactance and hence the voltage drop.
- AC single phase system is invariably adopted for main line service.


Overhead electric line
Pantograph
Transformer
or
Converter
Auxiliary
Power
Battery
Lights
HVAC
Compressor
Traction
power
control
Electric
motor
Pinion/gear
Wheel
Rail

7.13 The three phase AC system

- In this system 3-phase induction motor operating at 300 to 3600 V and low frequency are employed for getting the required motive power.
- The 3-phase induction motor
  - Simple
  - Robust in construction
  - High operating efficiency
  - Automatic regenerative braking without required any additional equipment.

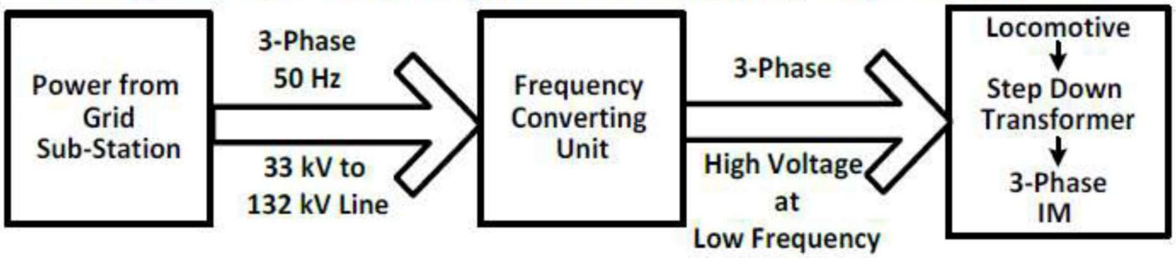
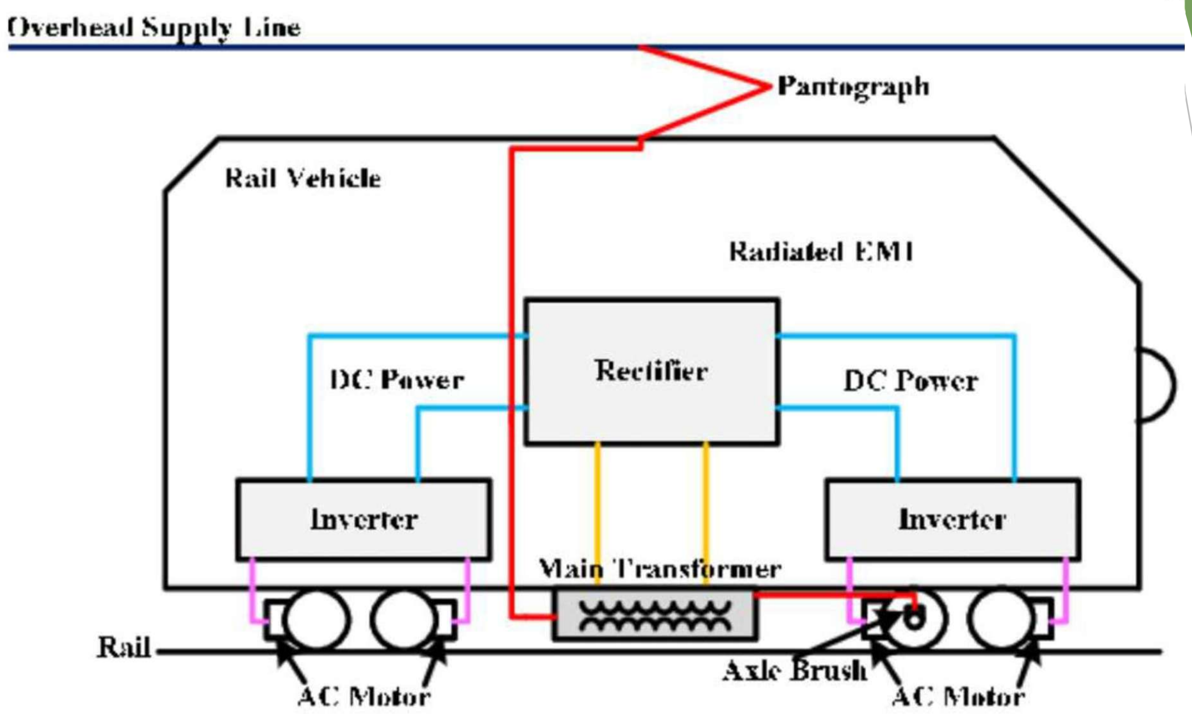


Figure 7.3 Three Phase AC System

- Drawbacks
  - Low starting torque
  - High starting current
  - Two overhead contact wires

7.14 The kando system (single phase to three phase system)

- In this system single phase high voltage (25 kV) at normal supply frequency is used to distribute power.
- The locomotive which carries a phase convertor which converts single phase AC to three-phase AC. The three-phase power is then fed to three-phase induction motors for getting necessary motive force.
- In this system only one contact wire of overhead system which is overcome the disadvantage of 3 phase AC system
- This system was adopted in Hungary in 1932.


Overhead Supply Line
Pantograph
Rail Vehicle
Radiated EMI
Rectifier
DC Power
DC Power
Inverter
Inverter
Main Transformer
Rail
Axle Brush
AC Motor
AC Motor

7.15 The single phase AC to DC system

- In this system of track electrification single phase AC 25 kV at normal frequency is fed to overhead distribution.
- The AC locomotive carries transformer to step down high input voltage and rectifying equipments to convert AC into DC This system is adopted in India for track electrification.
- This system becomes most popular because of various salient advantages over other systems particularly DC system.
- This system has got numerous advantages over dc system
  - The line current for a given demand of power is reduced on account of high system voltage
  - On account of high voltage the substations can be spaced at longer distances (50 to 80 km) whereas the substations are spaced at 12 to 30 km in case of 3000 V DC system and at 5 to 12 km in case of 1500 V DC system.
  - Since the dc series motors having ideal traction characteristics are employed in this system for getting the required propelling power, therefore, this system have got the advantages of the dc system.

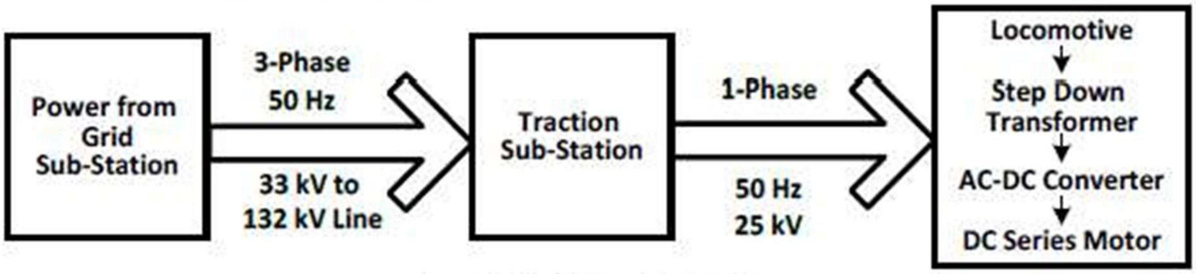


Figure 7.5 Single Phase AC to DC System

Pantograph

The main function of pantograph is to maintain link between overhead conductor and power circuit of locomotive at different speeds of the vehicle under all wind conditions. It collects the current from overhead conductor and supplies to rest circuit.

Circuit Breaker

It protects the power circuit in the event of any fault by isolating it from the supply. It also isolates the circuit during maintenance.

Transformer

It receives the high voltage from overhead conductor via pantograph and circuit breaker and then step-down the voltage to desired level required by the rest circuit.

Rectifier

It converts a low voltage AC supply from the secondary of transformer to a DC supply.

DC Link

It connects the rectifier and inverter circuits. It consists of filter arrangement (capacitor and inductor arrangement) that filters the output from rectifier (by removing the harmonics form it) and then supplies it to the inverter.

Main Inverter

It converts the DC power to three phase AC power in order to drive three phase AC motors.

Axle Brush

It acts as a return path for the supply. Once the power is drawn to the locomotive from overhead system, the current complete its path through axle brush and one of running tacks.

Auxiliary Inverter

This inverter supplies the power to other parts in the locomotive unit including fans, motor blowers, compressors, etc.

Battery

It supplies the necessary starting current and also power up the essential circuits such as emergency lighting.

Compressor

It maintains the cooling/heating requirement in the locomotive unit.

Cooling Fans

These fans maintain the necessary cooling for the power circuits. Modern locomotive systems use electronically controlled air management systems to keep the desired temperature.

7.16 Comparison of DC and AC system of railway electrification from the point of view of main line and suburban line railway service

(a) Main Line Railway Service

- The essential requirements of main line railway service are :
  - Higher maximum speed. (V_m)
  - Minimum cost of track electrification.
- Single phase AC system is preferred for main line service because of following features
  - 25 kV overhead systems reduce conductor section and hence simplified structure design due to high voltage.
  - Higher spacing of sub-station reduces number of sub-station and increases flexibility of selecting cheaper, land Maintenance cost is less due to cheap and efficient equipment of AC system.

(b) Sub-urban Railway Service

- The essential requirements of sub-urban railway service are :
  - Rapid acceleration and retardation rates due to frequent starting and stopping.
  - Motor performance should not affected by voltage fluctuations.
  - Less chances of interference in the telecommunication lines running along the track.

Table 7.4 Characteristics of Various type of Services

S. No.	Type of Service	Acceleration in kmphps	Retardation in kmphps	Maximum Speed in kmph	Distance between Stations in km
1.	Urban	1.5 to 4.00	3 to 4	60	1
2.	Suburban	1.5 to 4.00	3 to 4	75	1 to 8
3.	Main Line	0.6 to 0.8	1.5	110	More than 10

Table 7.3 Comparison of DC and AC Traction

Factor	DC Traction	AC Traction
Motor	DC series motor.	AC series motor.
Performance	Good performance.	Not as good as that used for DC traction.
Starting torque	More.	Less.
Speed control	The speed control of DC series Motor is limited.	Wide range of speed control is Possible.
Interference	DC system causes less interference with Communication lines.	It will produce more interference with Communication lines.
Overhead distribution	Heavier and more costly Comparatively.	Lighter and less costly.
Substations	The number of substations required for a given track distance on DC traction is More.	The number of substations required in AC traction is less.
Weight of cu	Weight of cu required per track km is more.	Weight of cu required per track km is less.
Application	Tramway, Trolley bus.	Main Line Service.

7.25 Mechanics of train movement

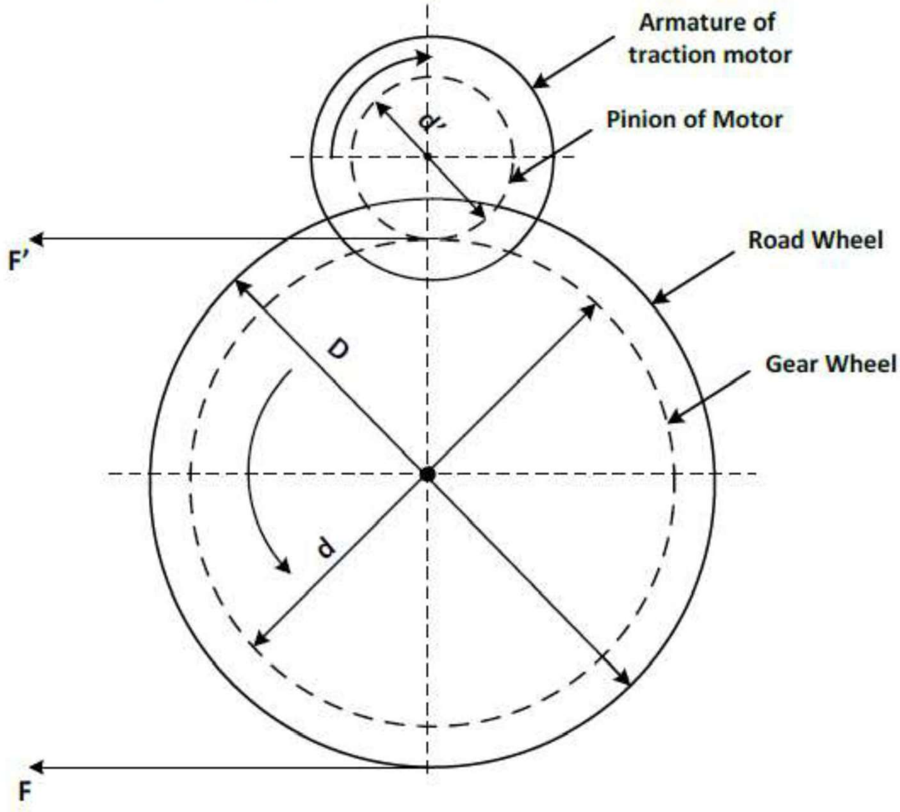


Figure 7.15 Transmission of Tractive Effort

$T =$ *Torque exerted by Motor*
$F' =$ *Tractive effort at Pinion*
$F =$ *Tractive effort at Wheel*
$d' =$ *Diameter of Pinion*
$d =$ *Diameter of gear Wheel*
$D =$ *Diameter of Road Wheel*

$$\gamma = \text{gear ratio} = \frac{d}{d'}$$

$\eta =$ *Efficiency of Transmission*

Let, driving Motor exert a torque T in Nm
Tractive Effort at edge of pinion is given by

$$T = F'\frac{d'}{2} \text{ or } \boxed{F' = \frac{2T}{d'}}$$

Tractive effort transferred the driving wheel

$$F = \eta F'\left(\frac{d}{D}\right)$$

$$F = \eta\frac{2T}{d'}\left(\frac{d}{D}\right)$$

$$\boxed{F = \eta T\frac{2\gamma}{D}}$$

7.26 Co-efficient of adhesion(μ)

- Maximum friction force between driving wheel and track=μW
  Where, μ=co-efficient of adhesion between driving wheel and track
  W=Weight of train on driving axles
- Motion of train without slipping

$$\boxed{F \leq \mu W}$$

- Co-efficient of adhesion (μ)

$$\mu = \frac{\text{Tractive effort to slip the wheels}}{\text{Adhesive Weight}}$$

Table 7.5 Co-efficient of Adhesion

Speed in kmph	0	15	30	45	60	75
μ	0.25	0.18	0.14	0.12	0.10	0.09

- For clean dry rails μ=0.25
- For wet or greasy rails μ=0.08
- Depends upon
  - Friction between wheels and the rail track.
  - Series-Parallel connection of Motor.
  - Speed of Response of drive.
  - Smoothness with which torque can be controlled.
  - Nature of motor torque-speed characteristics.

7.18 Typical speed-time curve for main line service

- The Curve drawn between Speed and Time, taking Speed in km/hr on Y-axis and Time in Sec or min on X-axis is known as speed-time curve.
- It's give complete information of the motion of the train. (*Distance=Speed × Time*)
- Speed time curve mainly consists of
  - Acceleration.
  - Free Run or Constant Speed Run.
  - Coasting.
  - Retardation or Braking.

(a) Acceleration

- Constant Acceleration of Acceleration During Notching up
  - Nothing up period. (0 to t_1)
  - Current will remain constant.
  - Supply voltage will be increase. (By cutting out the starting resistance)
  - Tractive effort will remain constant.
  - Acceleration will be constant.
- Speed curve Running or Acceleration on speed curve
  - Speed curve running period. (t_1 to t_2)
  - Voltage will be constant. (Approx.)
  - Current will be decreasing and become constant.

Table 7.4 Characteristics of Various type of Services

S. No.	Type of Service	Acceleration in kmphps	Retardation in kmphps	Maximum Speed in kmph	Distance between Stations in km
1.	Urban	1.5 to 4.00	3 to 4	60	1
2.	Suburban	1.5 to 4.00	3 to 4	75	1 to 8
3.	Main Line	0.6 to 0.8	1.5	110	More than 10

- Acceleration will be decreases and finally become zero.
- Tractive effort is equal to resistance to motion of train.

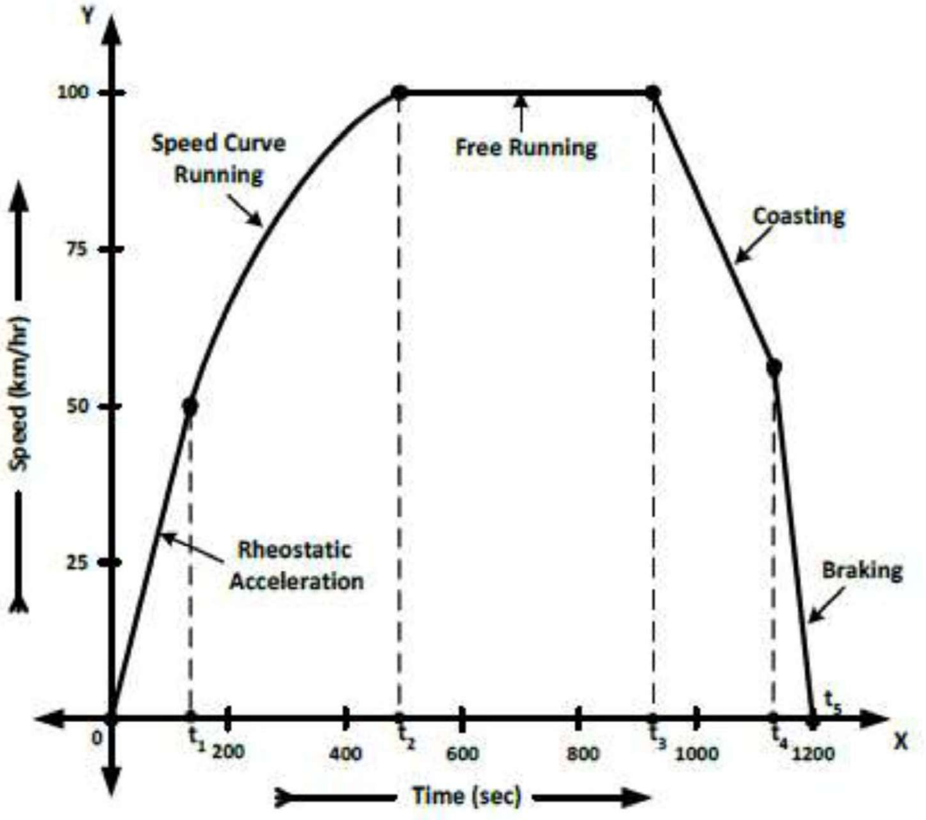


Figure 7.8 Typical Speed-Time curve for Main Line Service

(b) Free Run or Constant Speed Run

- Free run period. (t_2 to t_3)
- The train attains the maximum speed.
- During this period the train runs with constant speed and constant power is drawn.

(c) Coasting

- At the end of free running period (t_3 to t_4), power supply is cut off and the train is allowed to run under its own momentum.
- The speed of train starts decreasing on account of resistance to the motion of train.
- The rate of decrease of speed during coasting period is known as coasting retardation.

(d) Retardation or Braking Period

- At the end of coasting period (t_4 to t_5), the brakes are applied to bring the train to rest.
- During this period speed decreases rapidly and finally reduces to zero.

7.19 Typical speed-time curve for suburban and urban service

(a) Suburban Service

- In this service the distance between the stops is little longer than urban service but smaller than main line service (between 1 to 8 km).
- Free run is not possible.
- Coasting is for comparatively longer period.
- Acceleration and retardation required are as high as for urban service.

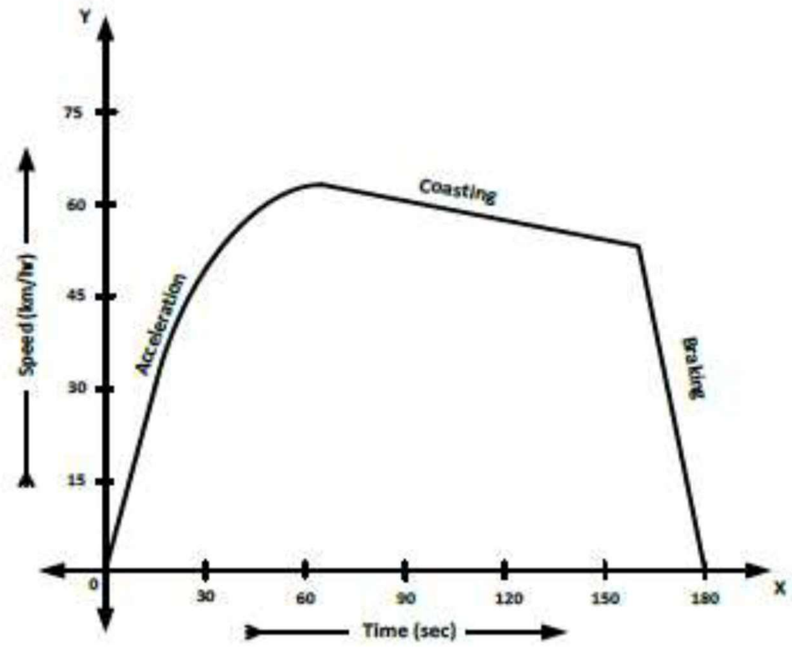


Figure 7.9 Typical Speed-Time curve for Suburban Service

(b) **Urban Service or City Service**

- In this service the distance between the stops is comparatively very short. (say 1 km or so)
- Time required for this run is very small.
- The acceleration as well as retardation is required to be high so that high average speed and short time run is obtained.
- Free run is not possible.
- Coasting period is also small.

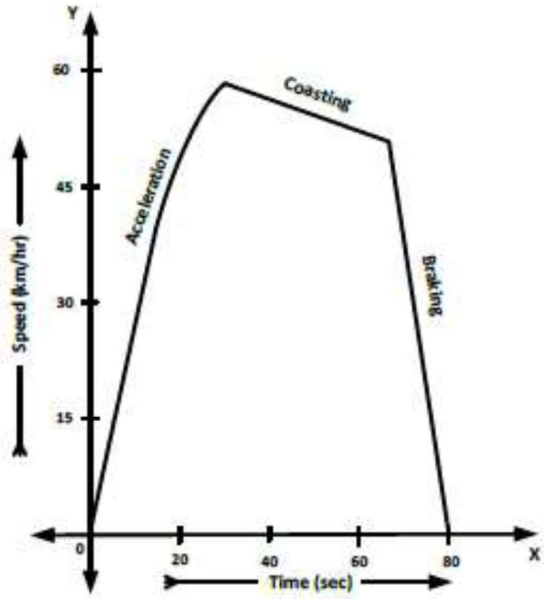


Figure 7.10 Typical Speed-Time curve for Urban Service

Table 7.4 Characteristics of Various type of Services

S. No.	Type of Service	Acceleration in kmphps	Retardation in kmphps	Maximum Speed in kmph	Distance between Stations in km
1.	Urban	1.5 to 4.00	3 to 4	60	1
2.	Suburban	1.5 to 4.00	3 to 4	75	1 to 8
3.	Main Line	0.6 to 0.8	1.5	110	More than 10

7.20 Crest speed, average speed and schedule speed

(a) Crest Speed

- The maximum speed attained by the vehicle during the run is known as crest speed.

(b) Average Speed

- The distance covered between two stops divided by the actual time of run is known as average speed.

$$Average\ Speed = \frac{Distance\ between\ Stops}{Actual\ Time\ of\ run, T}$$

(c) Schedule Speed

- The ratio of distance covered between two stops and total time of run including time of stop is known as schedule speed.

$$Schedule\ Speed = \frac{Distance\ between\ Stops}{Actual\ Time\ of\ run + Stop\ Time}$$

- Schedule Speed < Average Speed
- Difference is large in case of urban & sub-urban, negligibly small in case of main line service.

7.21 Factor affecting on schedule speed

- The Schedule speed of a given is affected by the following factors:

(a) Effect of acceleration and Braking Retardation

- For a given run and with fixed crest speed the increase in acceleration will result in decrease in actual time of run and therefore increase in schedule speed.
- Similarly increase in braking retardation will affect the schedule speed.
- Variation in acceleration and retardation will have more effect on schedule speed in case of shorter distance run in comparison to longer distance run.

(b) Effect on maximum speed

- For constant distance run and with fixed acceleration and retardation the actual time of run will decrease, and therefore schedule speed will increase with the increase in crest speed.
- The effect of variation in crest speed on schedule speed in considerable in case of long distance run.

(c) Effect on duration of stop

- For a given average speed the schedule will increase by reducing the duration of stop.
- The variation in duration of stop will affect the schedule speed more in case of shorter distance run as compared to longer distance run.

7.22 Simplified speed-time curve

- "Convert or Replace typical speed-time curve to simple geometric shaped curve is known as simplified speed-time curve."
- From these simplified curves, the relationships between acceleration, retardation, average speed and distance can be easily workout.
- Trapezoidal curve can be used for main line service and quadrilateral curve can be used for urban and sub-urban service.

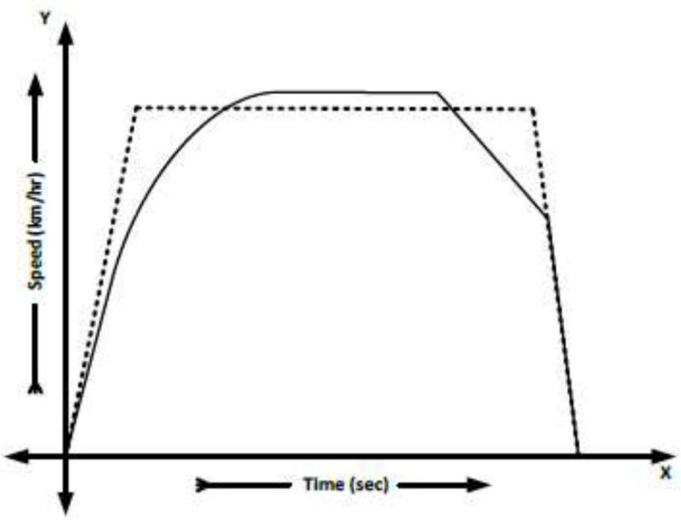


Figure 7.11 Approximate Trapezoidal Speed-Time Curves

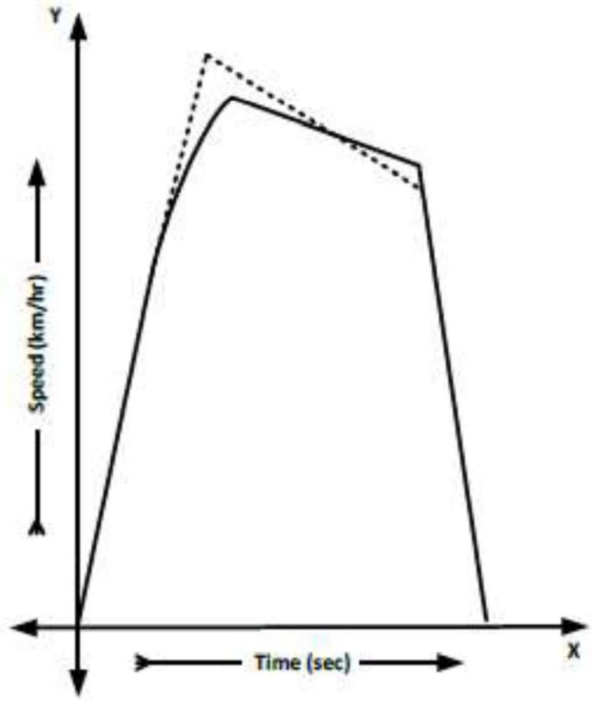


Figure 7.12 Approximate Quadrilateral Speed-Time Curves

7.23 Calculation by trapezoidal speed-time curve

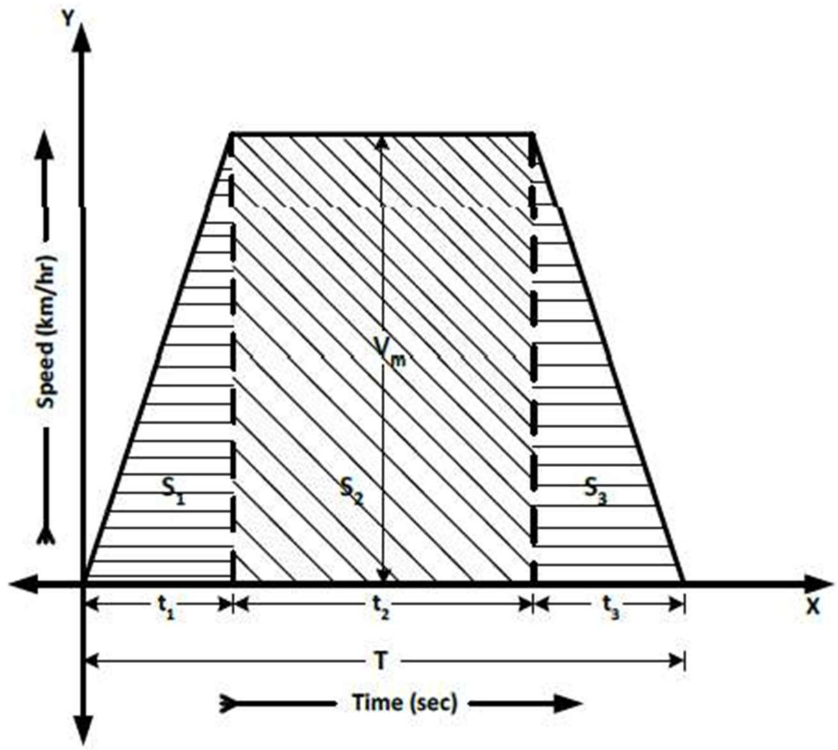


Figure 7.13 Trapezoidal Speed-Time Curves

α = *Acceleration in kmphps*

β = *Retardation in kmphps*

V_m = *Crest speed in kmph*

T = *Total time of run in* sec

$$Time\ for\ Acceleration\ in\ \sec,\ t_1 = \frac{V_m}{\alpha}$$

$$Time\ for\ Retardation\ in\ \sec,\ t_3 = \frac{V_m}{\beta}$$

$$Time\ for\ Free\ Run\ in\ \sec,\ t_2 = T - (t_1 + t_3)$$

$$= T - \left[\frac{V_m}{\alpha} + \frac{V_m}{\beta}\right]$$

Total Distance of Run, in km(S) = Distance travelled during acceleration + Distance travelled during free run + Distance travelled during retardation

$$S = S_1 + S_2 + S_3$$

$$S_1 = \frac{1}{2}\frac{V_m t_1}{3600}$$

$$put,\ t_1 = \frac{V_m}{\alpha}$$

$$S_1 = \frac{V_m^2}{7200\alpha}$$

$$S_2 = \frac{V_m t_2}{3600}$$

$$put,\ t_2 = T - \left[\frac{V_m}{\alpha} + \frac{V_m}{\beta}\right]$$

$$S_2 = \frac{V_m}{3600}\left[T - \left(\frac{V_m}{\alpha} + \frac{V_m}{\beta}\right)\right]$$

$$S_2 = \frac{V_m T}{3600} - \frac{V_m^2}{3600\alpha} - \frac{V_m^2}{3600\beta}$$

$$S_3 = \frac{1}{2}\frac{V_m t_3}{3600}$$

$$put,\ t_3 = \frac{V_m}{\beta}$$

$$S_3 = \frac{V_m^2}{7200\beta}$$

Now,

$$S = S_1 + S_2 + S_3$$

$$= \frac{V_m^2}{7200\alpha} + \frac{V_m T}{3600} - \frac{V_m^2}{3600\alpha} - \frac{V_m^2}{3600\beta} + \frac{V_m^2}{7200\beta}$$

$$= \frac{V_m^2}{\alpha}\left[\frac{1}{7200} - \frac{1}{3600}\right] + \frac{V_m^2}{\beta}\left[\frac{1}{7200} - \frac{1}{3600}\right] + \frac{V_m T}{3600}$$

$$= \frac{V_m^2}{\alpha}\left[\frac{1-2}{7200}\right] + \frac{V_m^2}{\beta}\left[\frac{1-2}{7200}\right] + \frac{V_m T}{3600}$$

$$S = \frac{V_m T}{3600} - \frac{V_m^2}{7200\alpha} - \frac{V_m^2}{7200\beta}$$

$$\therefore \frac{V_m^2}{3600}\left[\frac{1}{2\alpha} + \frac{1}{2\beta}\right] - \frac{V_m T}{3600} + S = 0$$

$$\therefore V_m^2\left[\frac{1}{2\alpha} + \frac{1}{2\beta}\right] - V_m T + 3600S = 0$$

$$take,\ K = \left[\frac{1}{2\alpha} + \frac{1}{2\beta}\right]$$

$$\therefore V_m^2 K - V_m T + 3600S = 0$$

Using quadratic equation,
$a = K, b = -T, c = 3600S$

$$V_m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$V_m = \frac{T \pm \sqrt{T^2 - 4K3600S}}{2K}$$

$$V_m = \frac{T}{2K} \pm \sqrt{\frac{T^2}{4K^2} - \frac{3600S}{K}}$$

for +Ve sign will be much higher value than that is possible in practical value.
so, take -Ve sign for calculation

$$\boxed{V_m = \frac{T}{2K} - \sqrt{\frac{T^2}{4K^2} - \frac{3600S}{K}}}$$

7.24 Calculation by quadrilateral speed-time curve

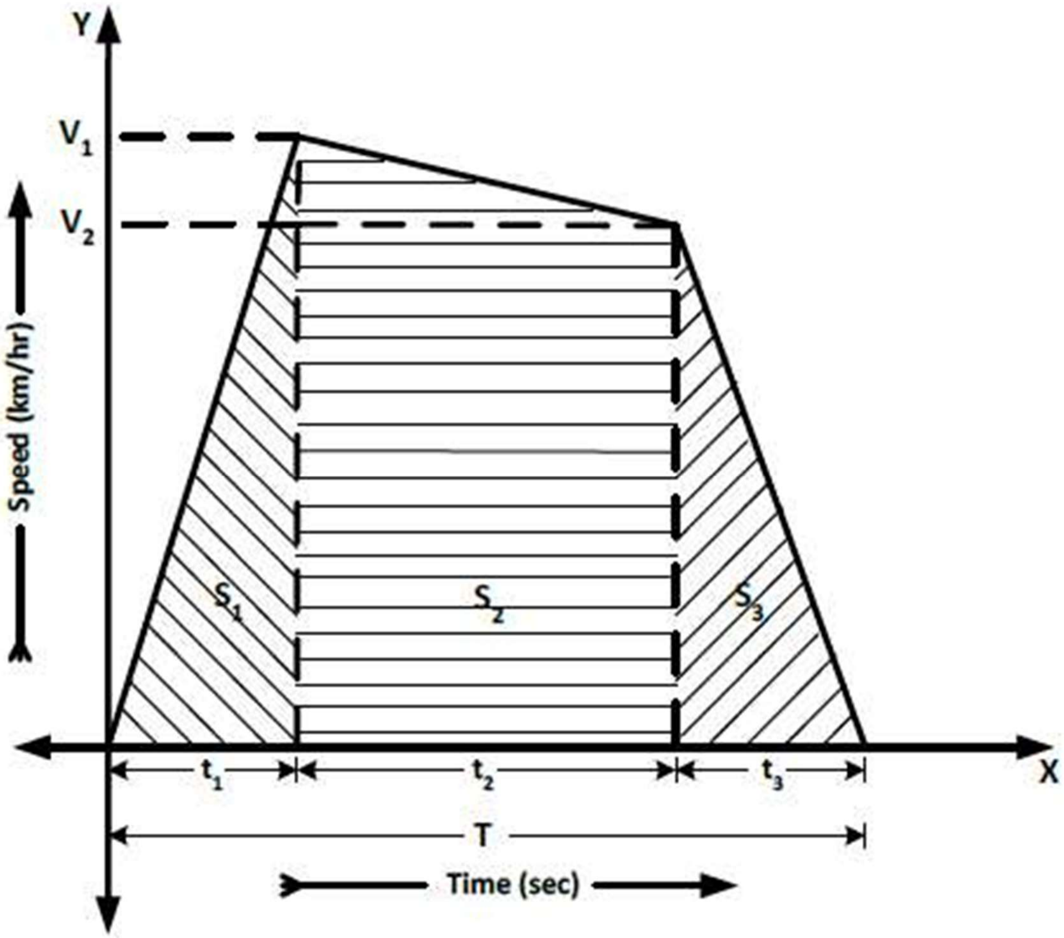


Figure 7.14 Quadrilateral Speed-Time Curves

α = Acceleration in kmphps
β_c = Coasting Retardation in kmphps
β = Braking Retardation in kmphps
V_1 = Maximum speed at the end of acceleration in kmph
V_2 = Maximum speed at the end of Coasting in kmph
T = Total time of run in sec

Time for Acceleration in sec, $t_1 = \frac{V_1}{\alpha}$

Time for Retardation in sec, $t_3 = \frac{V_2}{\beta}$

Time for Free Run in sec, $t_2 = \frac{V_1 - V_2}{\beta_c}$

Total Distance of Run, in km(S) = Distance travelled during acceleration + Distance travelled during coasting + Distance travelled during braking

$$S = S_1 + S_2 + S_3$$

$$S_1 = \frac{1}{2}\frac{V_1 t_1}{3600}$$

$$S_2 = \frac{V_2 t_2}{3600} + \frac{1}{2}\frac{(V_1 - V_2)t_2}{3600}$$
$$= \left(\frac{V_1 + V_2}{2}\right)\frac{t_2}{3600}$$

$$S_3 = \frac{1}{2}\frac{V_2 t_3}{3600}$$

Now,

$$S = S_1 + S_2 + S_3$$
$$S = \frac{1}{2}\frac{V_1 t_1}{3600} + \left(\frac{V_1 + V_2}{2}\right)\frac{t_2}{3600} + \frac{1}{2}\frac{V_2 t_3}{3600}$$
$$= \frac{V_1 t_1}{7200} + \frac{V_1 t_2}{7200} + \frac{V_2 t_2}{7200} + \frac{V_2 t_3}{7200}$$
$$S = \frac{V_1}{7200}(t_1 + t_2) + \frac{V_2}{7200}(t_2 + t_3)$$

$$Put \;\; t_1 + t_2 + t_3 = T$$

$$S = \frac{V_1}{7200}(T - t_3) + \frac{V_2}{7200}(T - t_1)$$
$$= \frac{T}{7200}(V_1 + V_2) - \frac{V_1 t_3}{7200} - \frac{V_2 t_1}{7200}$$

$$Put, \; t_1 = \frac{V_1}{\alpha} \;\; \& \;\; t_3 = \frac{V_2}{\beta}$$

$$S = \frac{T}{7200}(V_1 + V_2) - \frac{V_1}{7200}\frac{V_2}{\beta} - \frac{V_2}{7200}\frac{V_1}{\alpha}$$

$$\boxed{7200S = T(V_1 + V_2) - V_1 V_2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right]}$$

Now,

$$t_2 = \frac{V_1 - V_2}{\beta_c}$$

$$V_2 = V_1 - t_2\beta_c$$

$$V_2 = V_1 - \beta_c\,(T - t_1 - t_3)$$

$$V_2 = V_1 - \beta_c\left(T - \frac{V_1}{\alpha} - \frac{V_2}{\beta}\right)$$

$$\left[V_2 - \frac{\beta_c}{\beta}V_2\right] = V_1 - \beta_c\,(T - \frac{V_1}{\alpha})$$

$$\boxed{V_2 = \frac{V_1 - \beta_c\,\mathrm{T} + \frac{\beta_c}{\alpha}V_1}{1 - \frac{\beta_c}{\beta}}}$$