NARSIMHA REDDY ENGINEERING COLLEGE
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UNIT-II AC-DC Converters(Phase Controlled Rectifiers)

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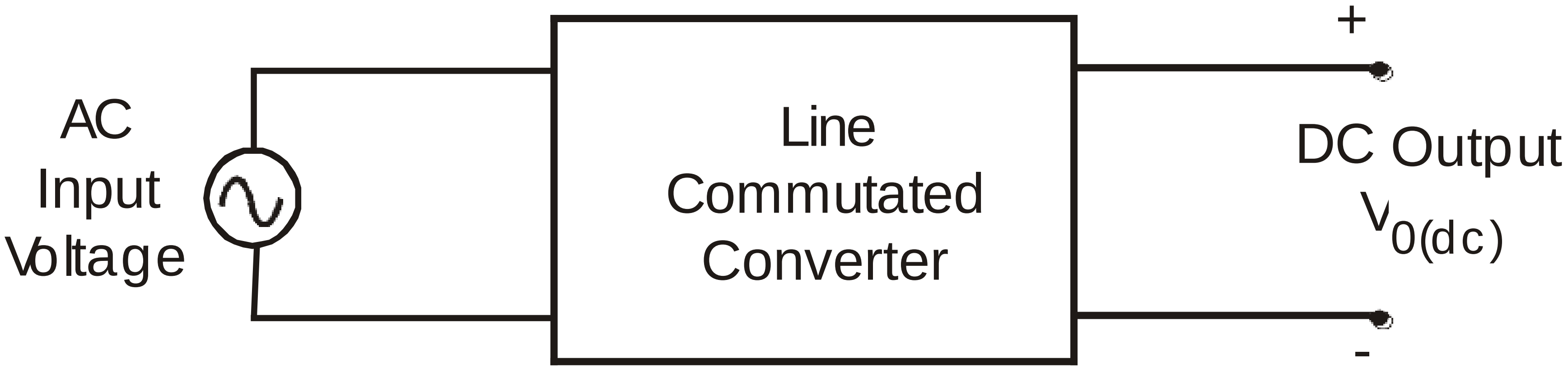


- Type of input: Fixed voltage, fixed frequency ac power supply.
- Type of output: Variable dc output voltage
- Type of commutation: Natural / AC line commutation

Different types of Line Commutated Converters

- AC to DC Converters (Phase controlled rectifiers)
- AC to AC converters (AC voltage controllers)
- AC to AC converters (Cyclo-converters) at low output frequency.

Differences Between
Diode Rectifiers
&
Phase Controlled Rectifiers

Cntd…

- The diode rectifiers are referred to as uncontrolled rectifiers .
- The diode rectifiers give a fixed dc output voltage .
- Each diode conducts for one half cycle.
- Diode conduction angle = 180^0 or π radians.
- We can not control the dc output voltage or the average dc load current in a diode rectifier circuit.

Cntd…

Single phase half wave diode rectifier gives an

$$\text{Average dc output voltage } V_{O(dc)} = \frac{V_m}{\pi}$$

Single phase full wave diode rectifier gives an

$$\text{Average dc output voltage } V_{O(dc)} = \frac{2V_m}{\pi}$$

Applications of Phase Controlled Rectifiers

- DC motor control in steel mills, paper and textile mills employing dc motor drives.
- AC fed traction system using dc traction motor.
- Electro-chemical and electro-metallurgical processes.
- Magnet power supplies.
- Portable hand tool drives.

Classification of Phase Controlled Rectifiers

- Single Phase Controlled Rectifiers.
- Three Phase Controlled Rectifiers.

Different types of Single Phase Controlled Rectifiers.

- Half wave controlled rectifiers.
- Full wave controlled rectifiers.
  - Using a center tapped transformer.
  - Full wave bridge circuit.
    - Semi converter.
    - Full converter.

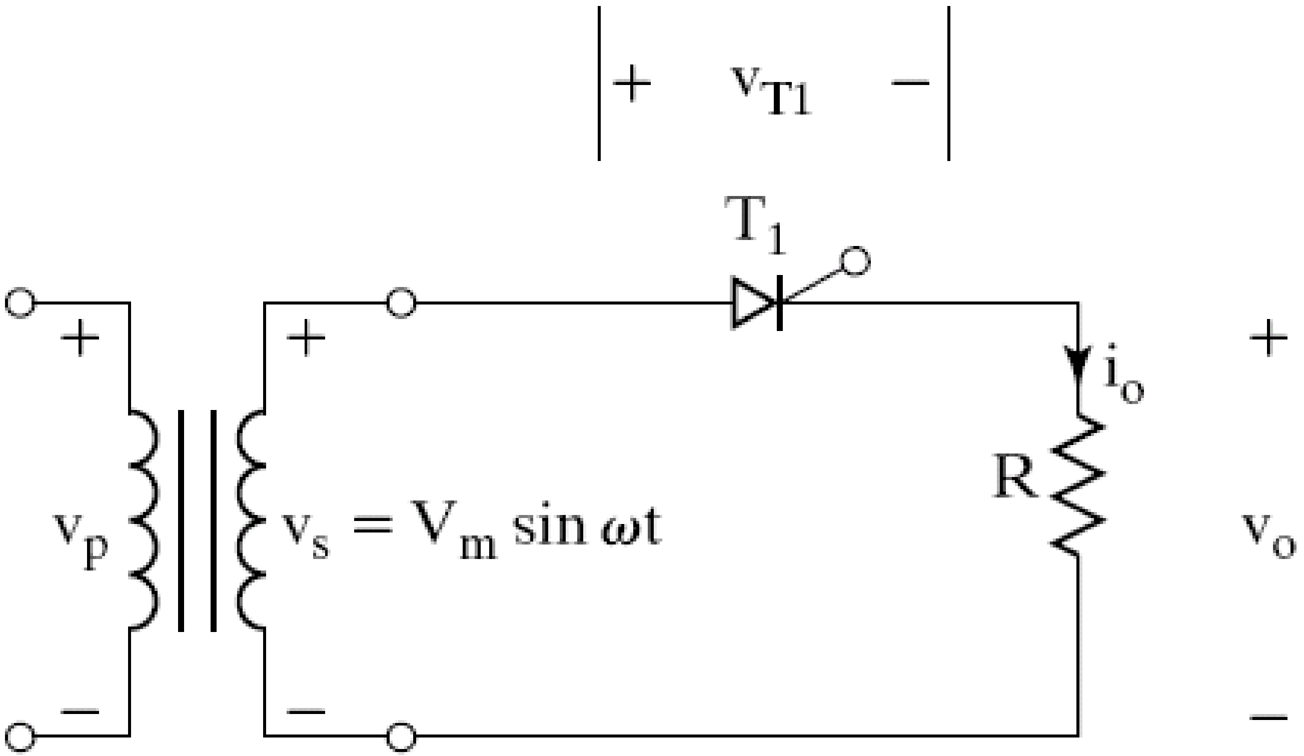
Different Types of Three Phase Controlled Rectifiers

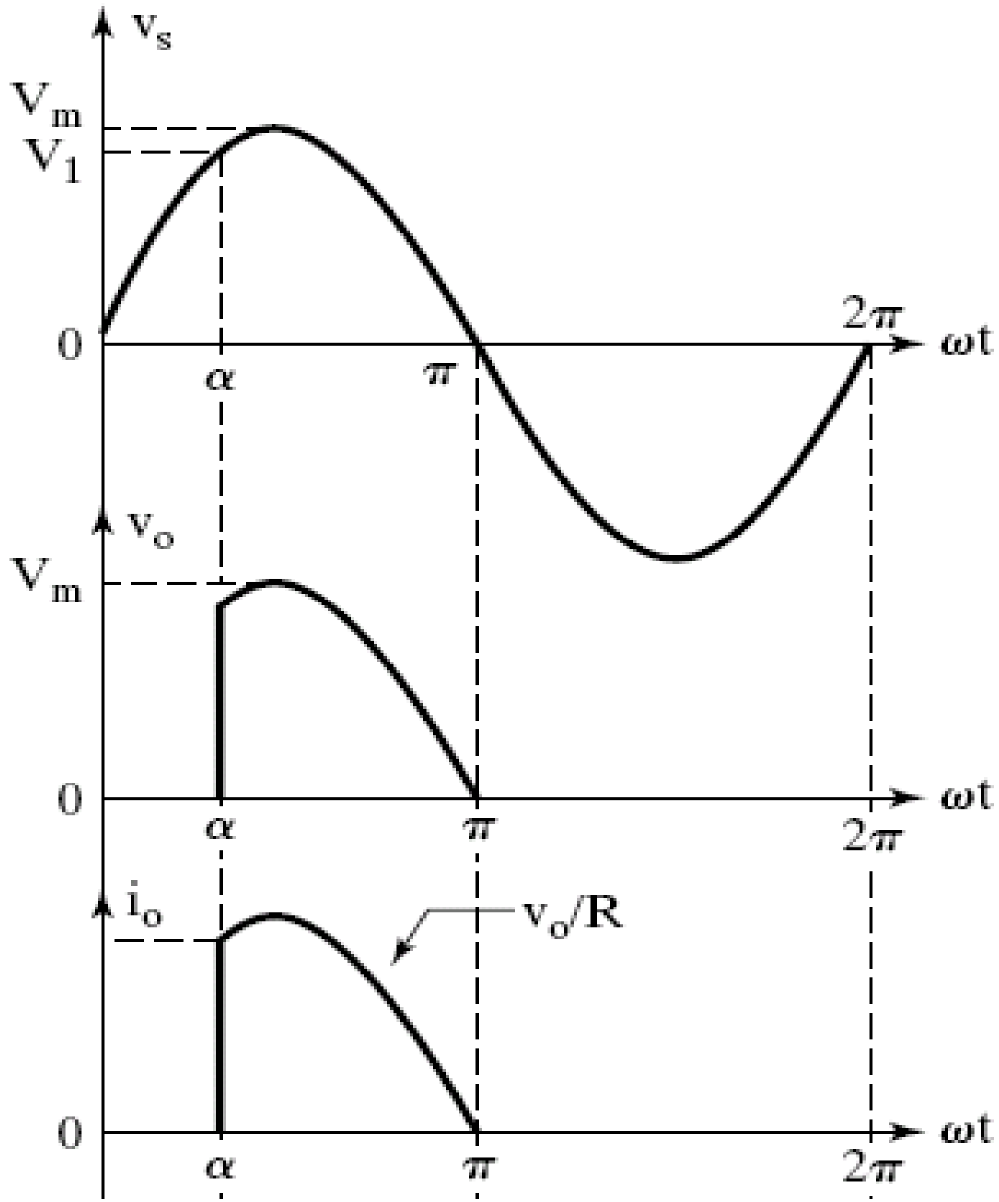
- Half wave controlled rectifiers.
- Full wave controlled rectifiers.
  - Semi converter (half controlled bridge converter).
  - Full converter (fully controlled bridge converter).

Principle of Phase Controlled Rectifier Operation

Principle of Phase Controlled Rectifier Operation

Single Phase Half-Wave Thyristor Converter with Resistive Load

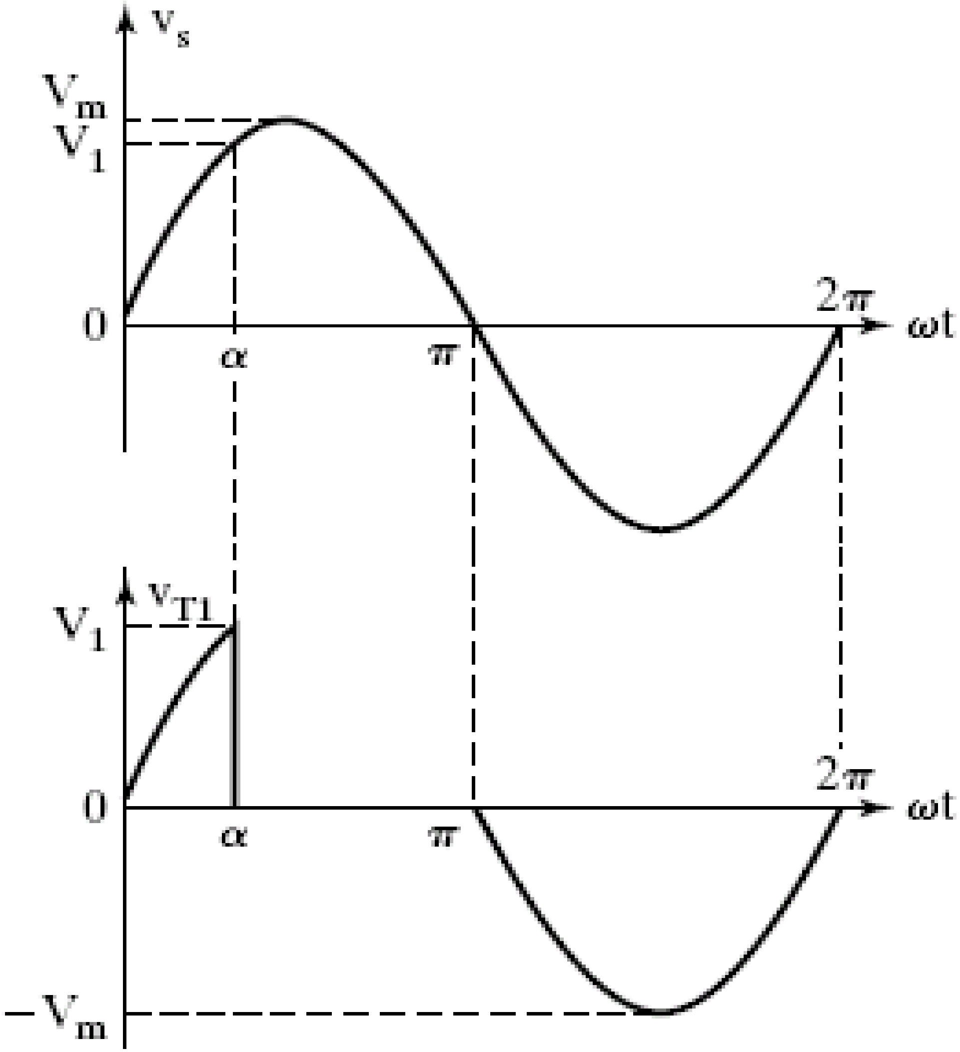




Supply Voltage

Output Voltage

Output (load) Current



Supply Voltage

Thyristor Voltage

Equations

$$v_s = V_m \sin\omega t = \text{ i/p ac supply voltage}$$

$$V_m = \text{max. value of i/p ac supply voltage}$$

$$V_S = \frac{V_m}{\sqrt{2}} = \text{RMS value of i/p ac supply voltage}$$

$$v_O = v_L = \text{ output voltage across the load}$$

When the thyristor is triggered at $\omega t = \alpha$

$$v_O = v_L = V_m \sin\omega t; \quad \omega t = \alpha \text{ to } \pi$$

$$i_O = i_L = \frac{v_O}{R} = \text{Load current}; \quad \omega t = \alpha \text{ to } \pi$$

$$i_O = i_L = \frac{V_m \sin\omega t}{R} = I_m \sin\omega t; \omega t = \alpha \text{ to } \pi$$

$$\text{Where } I_m = \frac{V_m}{R} = \text{ max. value of load current}$$

To Derive an Expression for the Average (DC) Output Voltage Across The Load

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi}\int_{0}^{2\pi} v_O . d(\omega t);$$

$$v_O = V_m \sin \omega t \ \ for \ \ \omega t = \alpha \ \ to \ \ \pi$$

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin \omega t . d(\omega t)$$

$$V_{O(dc)} = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin \omega t . d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\int_{\alpha}^{\pi} \sin\omega t.d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[-\cos\omega t \Big/_{\alpha}^{\pi}\right]$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[-\cos\pi + \cos\alpha\right]; \ \cos\pi = -1$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[1 + \cos\alpha\right]; \ V_m = \sqrt{2}V_S$$

Maximum average (dc) o/p

voltage is obtained when $\alpha = 0$

and the maximum dc output voltage

$$V_{dc(\max)} = V_{dm} = \frac{V_m}{2\pi}\left(1 + \cos 0\right); \quad \cos\left(0\right) = 1$$

$$\therefore V_{dc(\max)} = V_{dm} = \frac{V_m}{\pi}$$

Cntd…

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[1+\cos\alpha\right] \; ; \; V_m = \sqrt{2}V_S$$

The average dc output voltage can be varied by varying the trigger angle α from 0 to a maximum of 180^0 $(\pi$ radians$)$

We can plot the control characteristic $\left(V_{O(dc)} \text{ vs } \alpha\right)$ by using the equation for $V_{O(dc)}$

Control Characteristic of Single Phase Half Wave Phase Controlled Rectifier with Resistive Load

Cntd…

The average dc output voltage is given by the expression

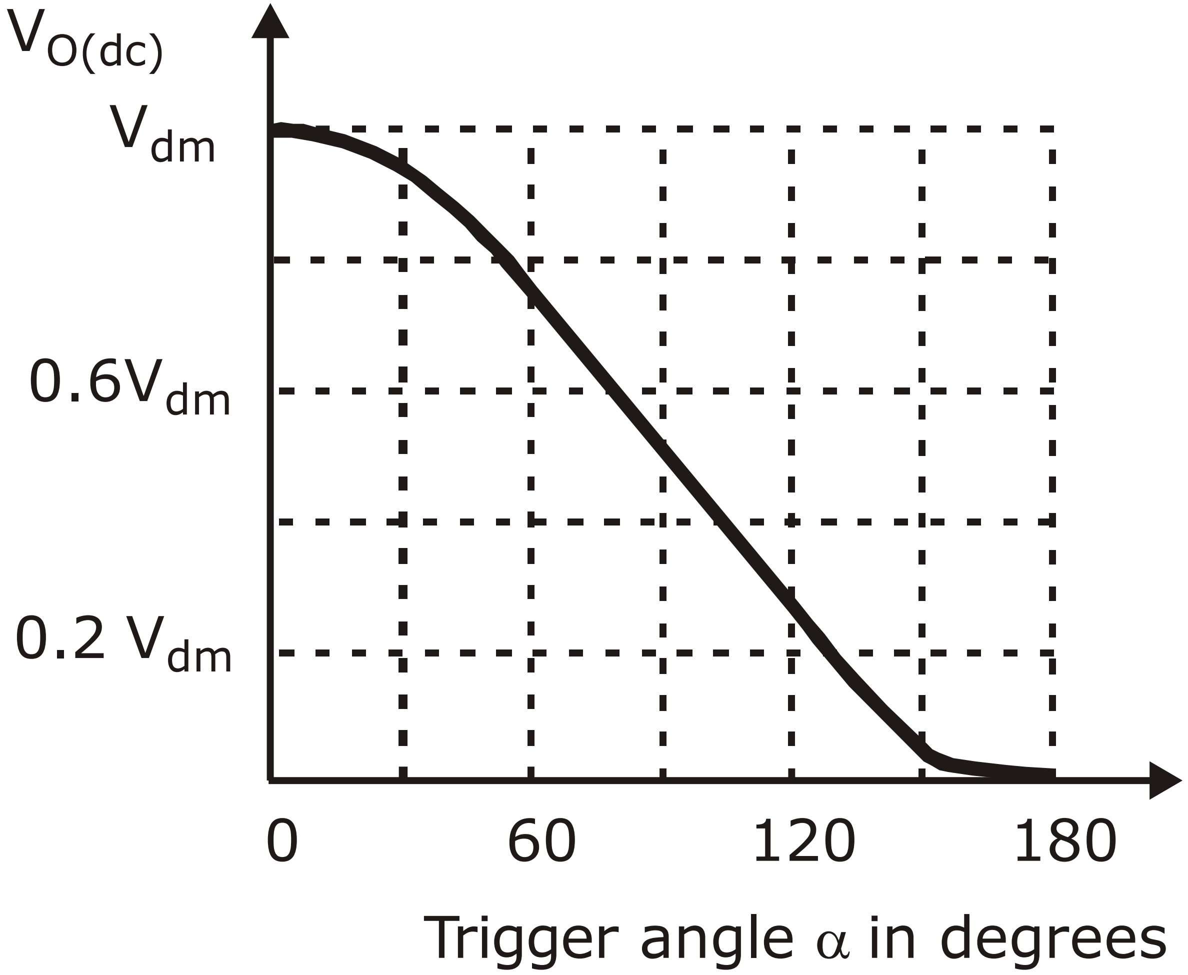
$$V_{O(dc)} = \frac{V_m}{2\pi}\left[1 + \cos\alpha\right]$$

We can obtain the control characteristic by plotting the expression for the dc output voltage as a function of trigger angle α

Trigger angle α in degrees	$V_{O(dc)}$	**%**
0	$V_{dm} = \frac{V_m}{\pi}$	100% V_{dm}
30^0	0.933 V_{dm}	93.3 % V_{dm}
60^0	0.75 V_{dm}	75 % V_{dm}
90^0	0.5 V_{dm}	50 % V_{dm}
120^0	0.25 V_{dm}	25 % V_{dm}
150^0	0.06698 V_{dm}	6.69 % V_{dm}
180^0	0	0

$$V_{dm} = \frac{V_m}{\pi} = V_{dc(\max)}$$

Control Characteristic



Normalizing the dc output voltage with respect to V_{dm}, the Normalized output voltage

$$V_n = \frac{V_{dc}}{V_{dm}} = \frac{\frac{V_m}{2\pi}\left(1+\cos\alpha\right)}{\frac{V_m}{\pi}}$$

$$V_n = \frac{V_{dc}}{V_{dm}} = \frac{1}{2}\left(1+\cos\alpha\right) = V_{dcn}$$

To Derive An Expression for the RMS Value of Output Voltage of a Single Phase Half Wave Controlled Rectifier With Resistive Load

Cntd…

The RMS output voltage is given by

$$V_{O(RMS)} = \left[\frac{1}{2\pi} \int_{0}^{2\pi} v_O^2 . d(\omega t) \right]$$

Output voltage $v_O = V_m \sin \omega t$; for $\omega t = \alpha$ to π

$$V_{O(RMS)} = \left[\frac{1}{2\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t . d(\omega t) \right]^{\frac{1}{2}}$$

Cntd…

By substituting $\sin^2 \omega t = \frac{1 - \cos 2\omega t}{2}$, we get

$$V_{O(RMS)} = \left[\frac{1}{2\pi} \int_{\alpha}^{\pi} V_m^2 \frac{(1 - \cos 2\omega t)}{2} . d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{4\pi} \int_{\alpha}^{\pi} (1 - \cos 2\omega t) . d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{4\pi} \left\{ \int_{\alpha}^{\pi} d(\omega t) - \int_{\alpha}^{\pi} \cos 2\omega t . d(\omega t) \right\} \right]^{\frac{1}{2}}$$

Cntd…

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left\{(\omega t)\Big/_{\alpha}^{\pi} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{\pi}\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left((\pi - \alpha) - \frac{(\sin 2\pi - \sin 2\alpha)}{2}\right)\right]^{\frac{1}{2}} ;\sin 2\pi = 0$$

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left((\pi - \alpha) + \frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\left((\pi - \alpha) + \frac{\sin 2\alpha}{2}\right)^{\frac{1}{2}}$$

Performance Parameters Of Phase Controlled Rectifiers

Cntd…

Output dc power (avg. or dc o/p power delivered to the load)

$$P_{O(dc)} = V_{O(dc)} \times I_{O(dc)} \; ; \; i.e., \; P_{dc} = V_{dc} \times I_{dc}$$

Where

$V_{O(dc)} = V_{dc}$ = avg./ dc value of o/p voltage.

$I_{O(dc)} = I_{dc}$ = avg./dc value of o/p current

Cntd…

Output ac power

$$P_{O(ac)} = V_{O(RMS)} \times I_{O(RMS)}$$

Efficiency of Rectification (Rectification Ratio)

$$\text{Efficiency } \eta = \frac{P_{O(dc)}}{P_{O(ac)}}; \ \% \text{ Efficiency } \eta = \frac{P_{O(dc)}}{P_{O(ac)}} \times 100$$

The o/p voltage consists of two components

The dc component $V_{O(dc)}$

The ac /ripple component $V_{ac} = V_{r(rms)}$

Cntd…

The total RMS value of output voltage is given by

$$V_{O(RMS)} = \sqrt{V_{O(dc)}^2 + V_{r(rms)}^2}$$

$$\therefore \qquad V_{ac} = V_{r(rms)} = \sqrt{V_{O(RMS)}^2 - V_{O(dc)}^2}$$

Form Factor (FF) which is a measure of the shape of the output voltage is given by

$$FF = \frac{V_{O(RMS)}}{V_{O(dc)}} = \frac{\text{RMS output (load) voltage}}{\text{DC load output (load) voltage}}$$

The Ripple Factor (RF) w.r.t. o/p voltage w/f

$$r_v = RF = \frac{V_{r(rms)}}{V_{O(dc)}} = \frac{V_{ac}}{V_{dc}}$$

$$r_v = \frac{\sqrt{V^2_{O(RMS)} - V^2_{O(dc)}}}{V_{O(dc)}} = \sqrt{\left[\frac{V_{O(RMS)}}{V_{O(dc)}}\right]^2 - 1}$$

$$\therefore \qquad r_v = \sqrt{FF^2 - 1}$$

Cntd…

Current Ripple Factor $r_i = \frac{I_{r(rms)}}{I_{O(dc)}} = \frac{I_{ac}}{I_{dc}}$

Where $I_{r(rms)} = I_{ac} = \sqrt{I^2_{O(RMS)} - I^2_{O(dc)}}$

$V_{r(pp)} =$ peak to peak ac ripple output voltage

$$V_{r(pp)} = V_{O(\max)} - V_{O(\min)}$$

$I_{r(pp)} =$ peak to peak ac ripple load current

$$I_{r(pp)} = I_{O(\max)} - I_{O(\min)}$$

Transformer Utilization Factor (TUF)

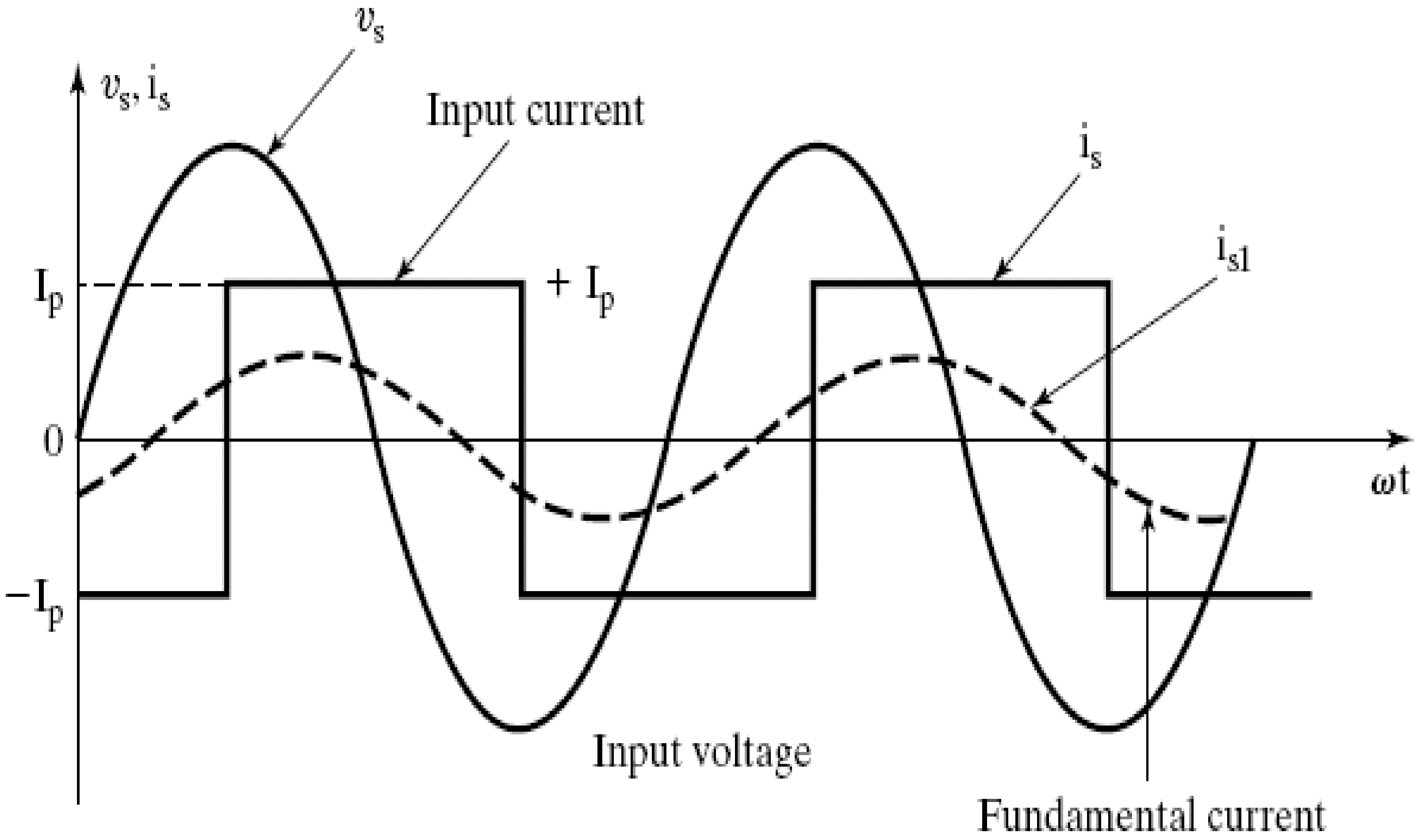
$$TUF = \frac{P_{O(dc)}}{V_S \times I_S}$$

Where

$$V_S = \text{RMS supply (secondary) voltage}$$

$$I_S = \text{RMS supply (secondary) current}$$

Cntd…



v_s
v_s, i_s
Input current
i_s
i_{s1}
I_p
$+ I_p$
0
ωt
$-I_p$
Input voltage
Fundamental current

Cntd…

Wher
e

$v_S =$ Supply voltage at the transformer secondary side

$i_S =$ i/p supply current
(transformer secondary winding current)

$i_{S1} =$ Fundamental component of the i/p supply current

$I_P =$ Peak value of the input supply current

$\phi =$ Phase angle difference between (sine wave components) the fundamental components of i/p supply current & the input supply voltage.

Cntd…

ϕ = Displacement angle (phase angle) For an RL load

ϕ = Displacement angle = Load impedance angle

$$\therefore \quad \phi = \tan^{-1}\left(\frac{\omega L}{R}\right) \text{ for an RL load}$$

Displacement Factor (DF) or

Fundamental Power Factor

$$DF = Cos\phi$$

Cntd…

Harmonic Factor (HF) or

Total Harmonic Distortion Factor ; THD

$$HF = \left[\frac{I_S^2 - I_{S1}^2}{I_{S1}^2}\right]^{\frac{1}{2}} = \left[\left(\frac{I_S}{I_{S1}}\right)^2 - 1\right]^{\frac{1}{2}}$$

Where

$I_S =$ RMS value of input supply current.

$I_{S1} =$ RMS value of fundamental component of the i/p supply current.

Cntd…

Input Power Factor (PF)

$$PF = \frac{V_S I_{S1}}{V_S I_S}\cos\phi = \frac{I_{S1}}{I_S}\cos\phi$$

The Crest Factor (CF)

$$CF = \frac{I_{S(peak)}}{I_S} = \frac{\text{Peak input supply current}}{\text{RMS input supply current}}$$

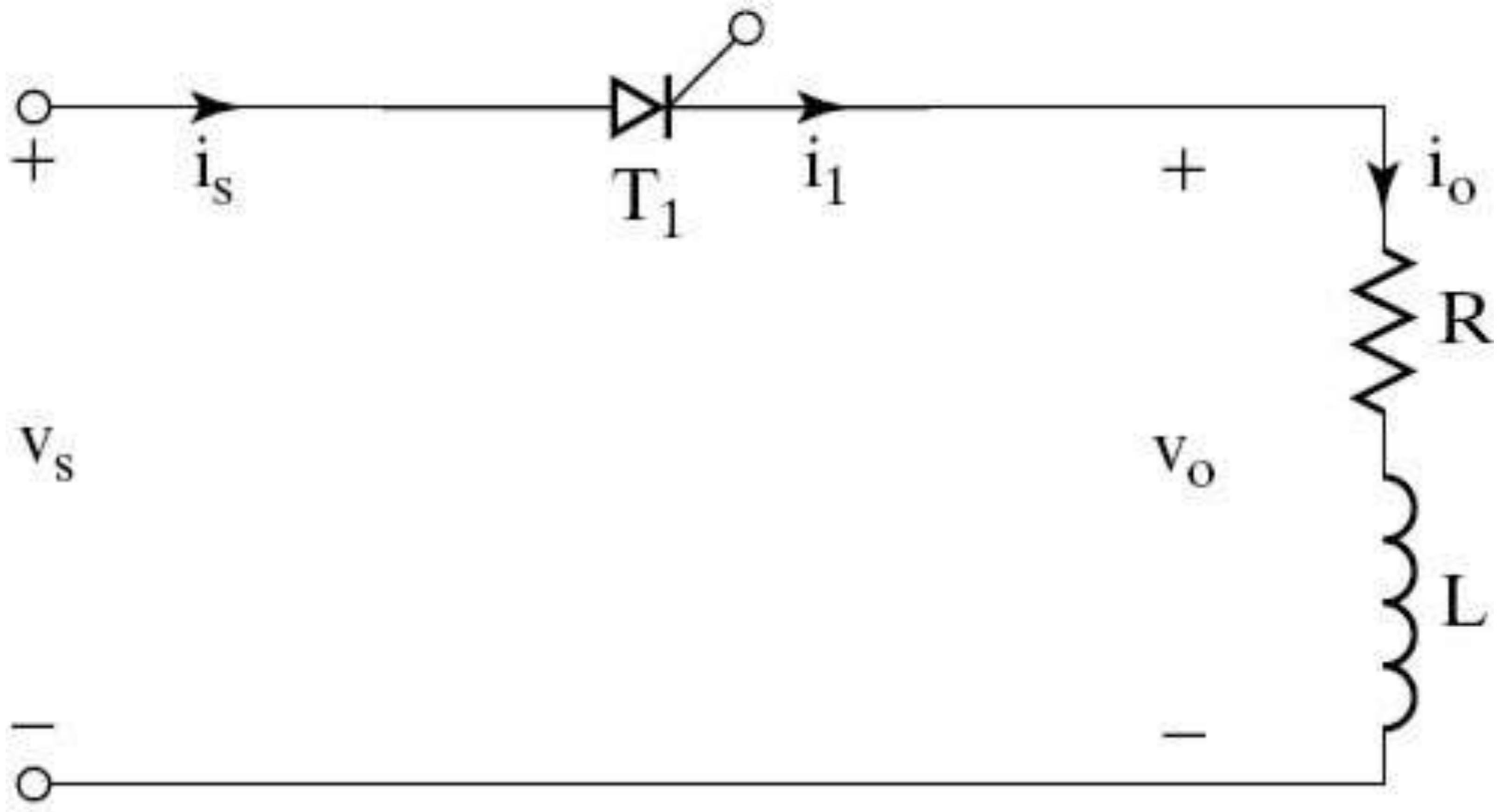
For an Ideal Controlled Rectifier

$$FF = 1;\ \eta = 100\%\ ;\ V_{ac} = V_{r(rms)} = 0\ ;\ TUF = 1;$$

$$RF = r_v = 0\ ;\ HF = THD = 0;\ \ PF = DPF = 1$$

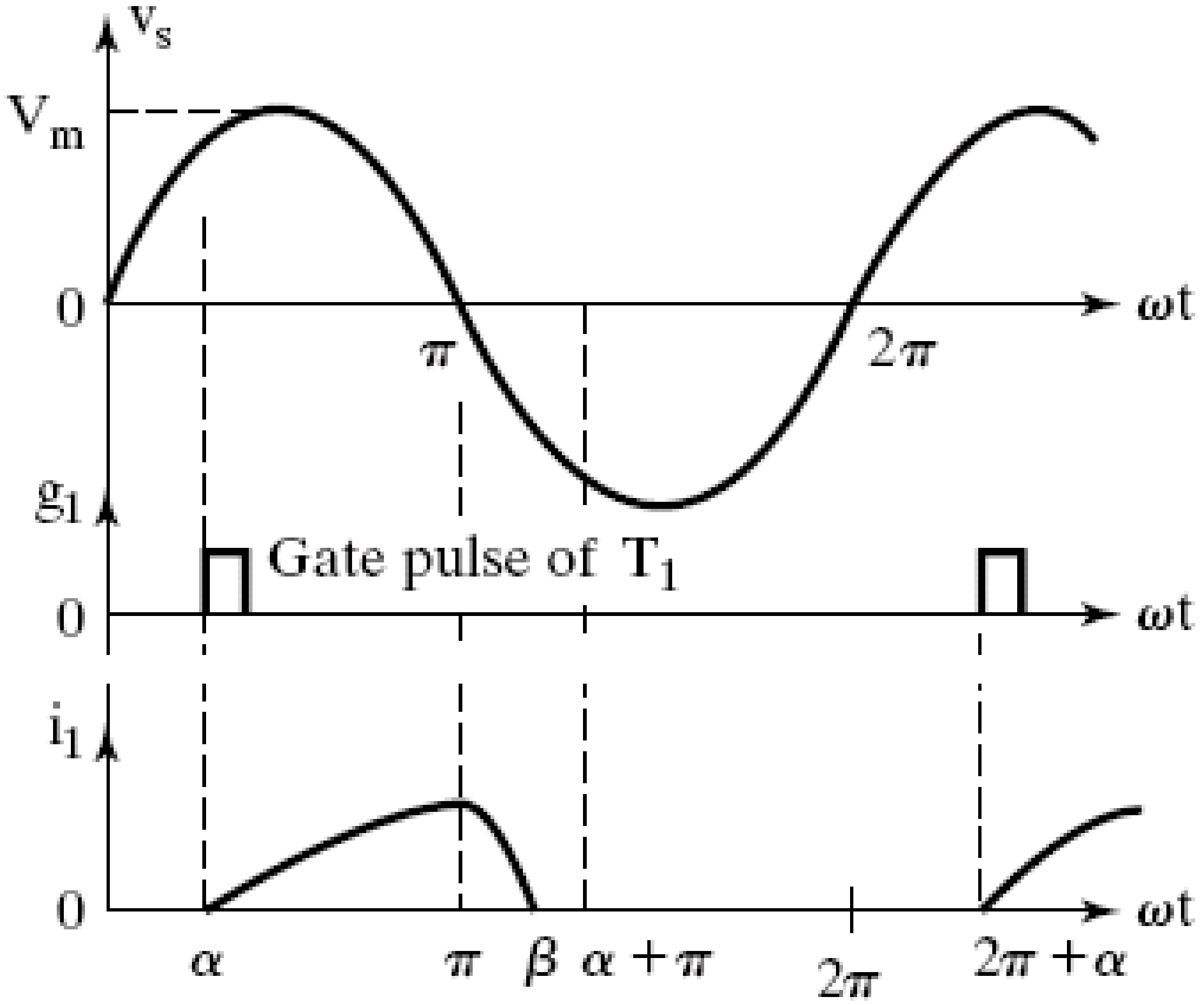
Single Phase Half Wave Controlled Rectifier With An RL Load

Cntd…



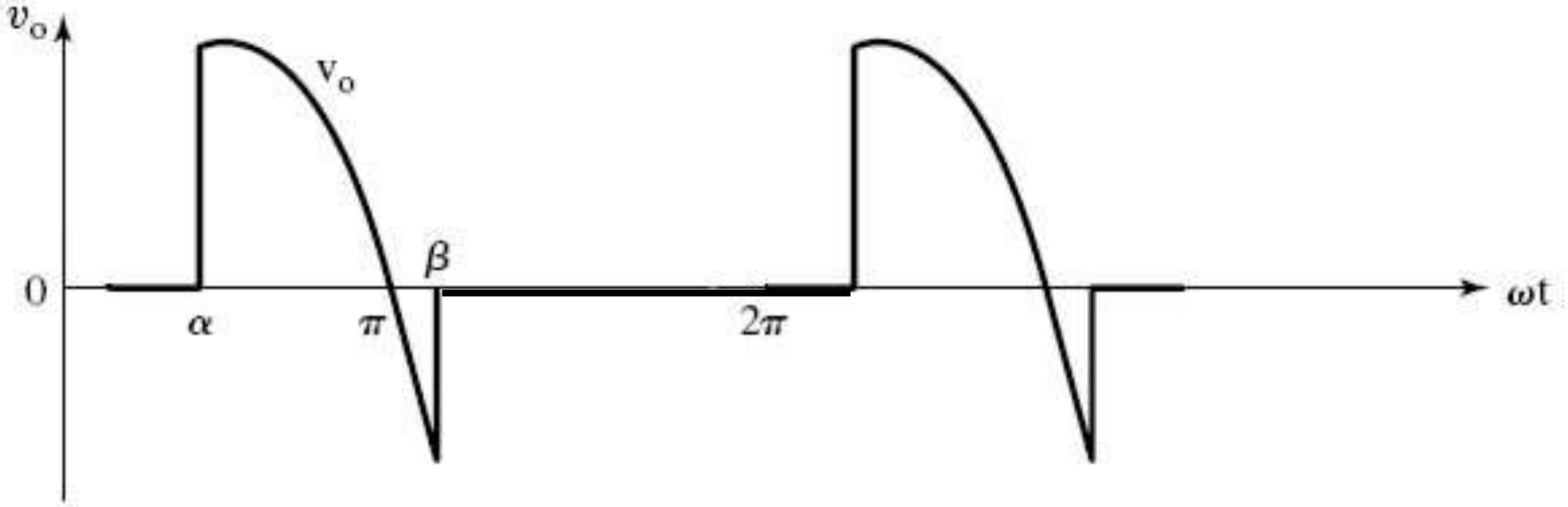
Input Supply Voltage (V_s)
&
Thyristor (Output) Current Waveforms

Cntd…



v_s
V_m
0
ωt
π
2π
g_1
Gate pulse of T_1
0
ωt
i_1
0
ωt
α
π
β
$\alpha+\pi$
2π
$2\pi+\alpha$

Output (Load) Voltage Waveform



Cntd…

To Derive An Expression For The Output (Load) Current, During $\omega t = \alpha$ to β When Thyristor T_1 Conducts

Cntd…

Assuming T_1 is triggered $\omega t = \alpha$,

we can write the equation,

$$L\left(\frac{di_O}{dt}\right) + Ri_O = V_m \sin \omega t \;\; ; \alpha \leq \omega t \leq \beta$$

General expression for the output current,

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + A_1 e^{\frac{-t}{\tau}}$$

Cntd…

$$V_m = \sqrt{2}V_S = \text{maximum supply voltage.}$$

$$Z = \sqrt{R^2 + (\omega L)^2} = \text{Load impedance.}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \text{Load impedance angle.}$$

$$\tau = \frac{L}{R} = \text{Load circuit time constant.}$$

$\therefore$ general expression for the output load current

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) + A_1 e^{\frac{-R}{L}t}$$

Cntd…

Constant A_1 is calculated from

initial condition $i_O = 0$ at $\omega t = \alpha$; t= $\left(\dfrac{\alpha}{\omega}\right)$

$$i_O = 0 = \frac{V_m}{Z}\sin(\alpha - \phi) + A_1 e^{\frac{-R}{L}t}$$

$$\therefore \qquad A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z}\sin(\alpha - \phi)$$

We get the value of constant A_1 as

$$A_1 = e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin(\alpha - \phi)\right]$$

Cntd…

Substituting the value of constant A_1 in the general expression for i_O

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) + e^{\frac{-R}{\omega L}(\omega t - \alpha)}\left[\frac{-V_m}{Z}\sin(\alpha - \phi)\right]$$

$\therefore$ we obtain the final expression for the inductive load current

$$i_O = \frac{V_m}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right];$$

Where $\alpha \le \omega t \le \beta$

Cntd…

Extinction angle β can be calculated by using the condition that $i_O = 0 \; at \; \omega t = \beta$

$$i_O = \frac{V_m}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right] = 0$$

$$\therefore \sin(\beta - \phi) = e^{\frac{-R}{\omega L}(\beta - \alpha)} \times \sin(\alpha - \phi)$$

β can be calculated by solving the above eqn.

To Derive An Expression For Average (DC) Load Voltage of a Single Half Wave Controlled Rectifier with RL Load

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\int_0^{2\pi} v_O . d(\omega t)$$

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_0^{\alpha} v_O . d(\omega t) + \int_{\alpha}^{\beta} v_O . d(\omega t) + \int_{\beta}^{2\pi} v_O . d(\omega t)\right]$$

$$v_O = 0 \text{ for } \omega t = 0 \text{ to } \alpha \text{ \& for } \omega t = \beta \text{ to } 2\pi$$

$$\therefore V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} v_O . d(\omega t)\right];$$

$$v_O = V_m \sin\omega t \text{ for } \omega t = \alpha \text{ to } \beta$$

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} V_m \sin\omega t.d(\omega t)\right]$$

$$V_{O(dc)} = V_L = \frac{V_m}{2\pi}\left[-\cos\omega t \Big/_{\alpha}^{\beta}\right]$$

$$V_{O(dc)} = V_L = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta)$$

$$\therefore V_{O(dc)} = V_L = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta)$$

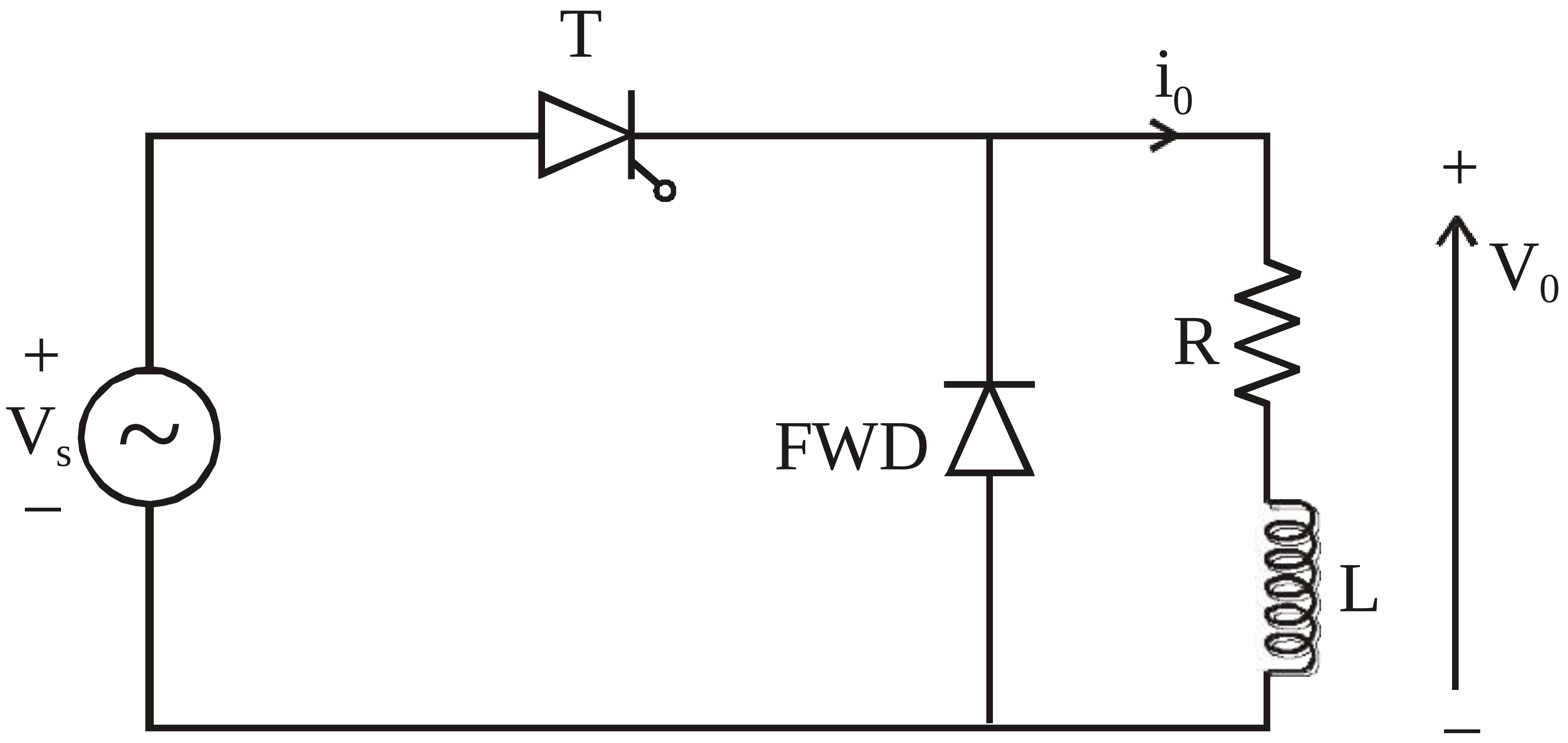
Effect of Load Inductance on the Output

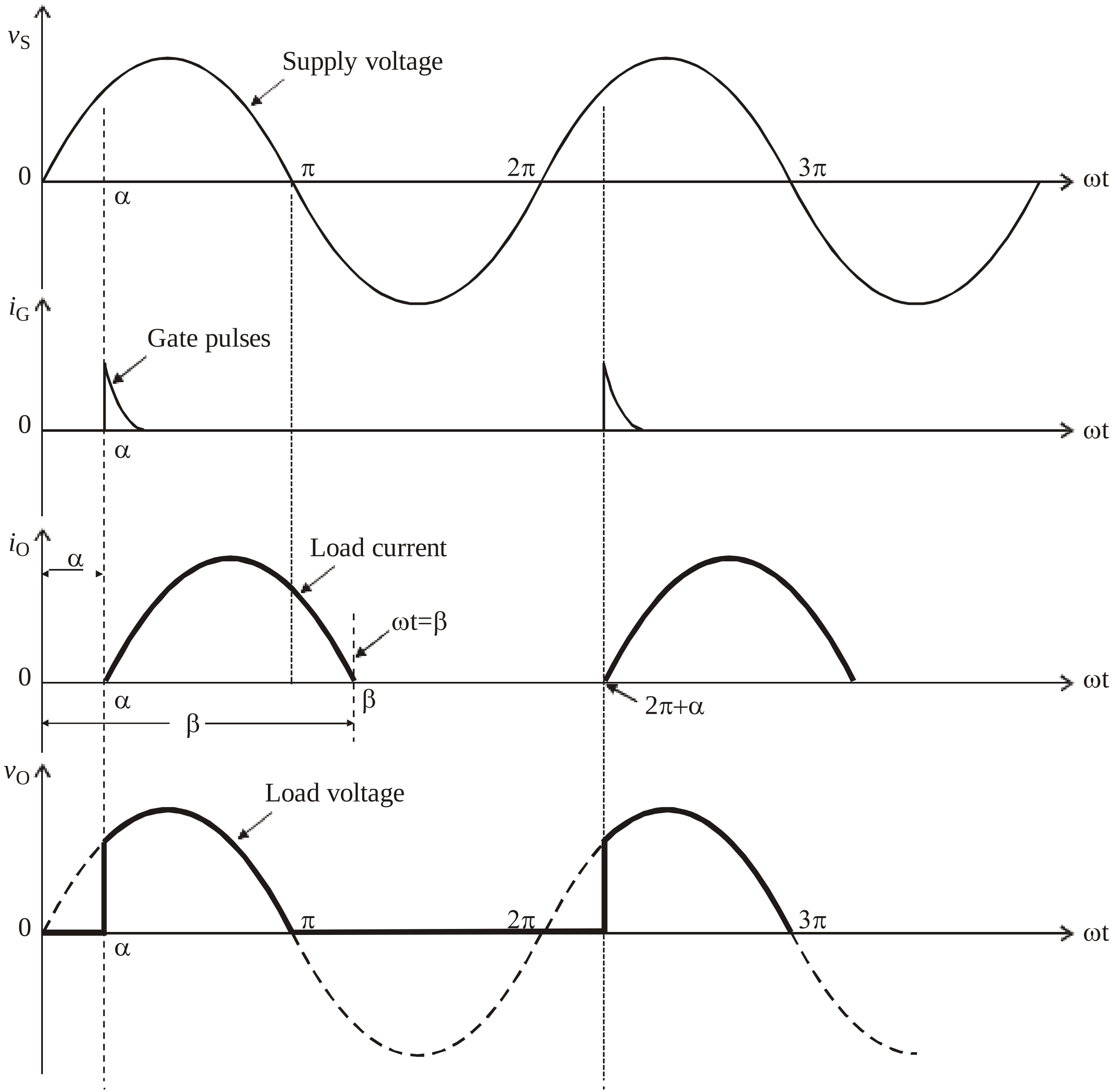
During the period $\omega t = \pi$ to β the instantaneous o/p voltage is negative and this reduces the average or the dc output voltage when compared to a purely resistive load.

Average DC Load Current

$$I_{O(dc)} = I_{L(Avg)} = \frac{V_{O(dc)}}{R_L} = \frac{V_m}{2\pi R_L}\left(\cos\alpha - \cos\beta\right)$$

Single Phase Half Wave Controlled Rectifier With RL Load & Free Wheeling Diode


T
i_0
+
V_0
R
+
V_s
−
FWD
L
−



v_S
Supply voltage
0
α
π
2π
3π
ωt
i_G
Gate pulses
0
α
ωt
i_O
α
Load current
ωt=β
0
α
β
β
2π+α
ωt
v_O
Load voltage
0
α
π
2π
3π
ωt

The average output voltage

$V_{dc} = \frac{V_m}{2\pi}\left[1 + \cos\alpha\right]$ which is the same as that of a purely resistive load.

The following points are to be noted

For low value of inductance, the load current tends to become discontinuous.

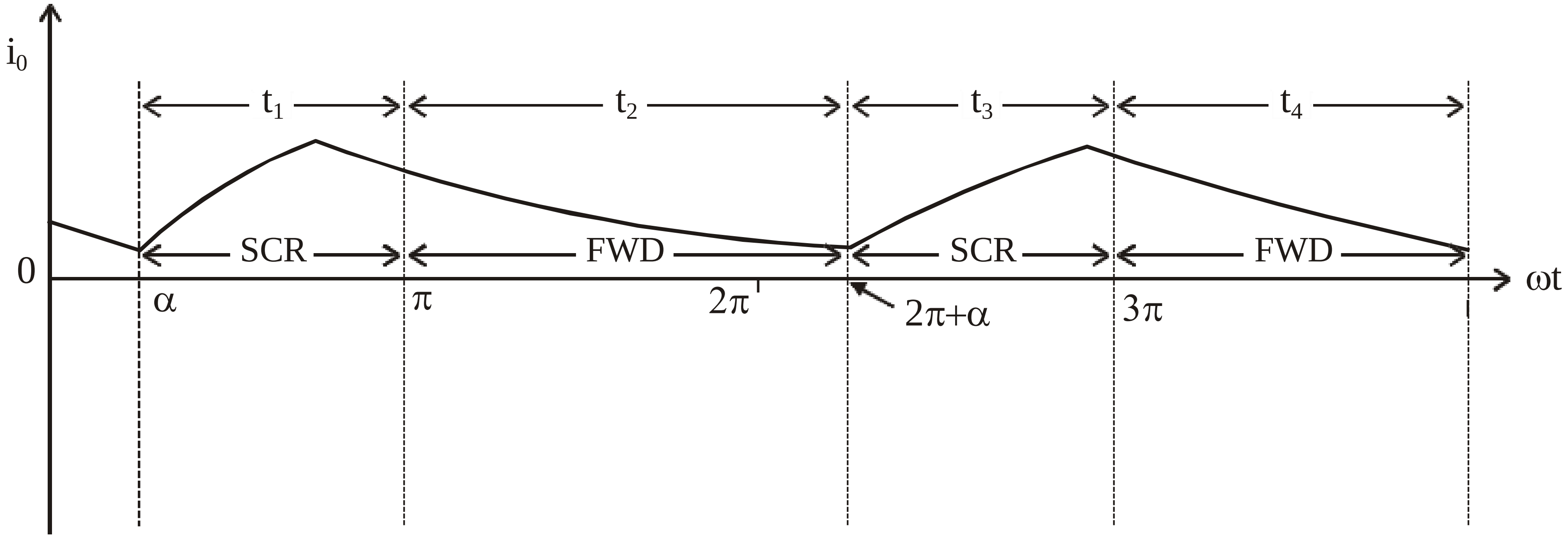
During the period α to π

the load current is carried by the SCR.

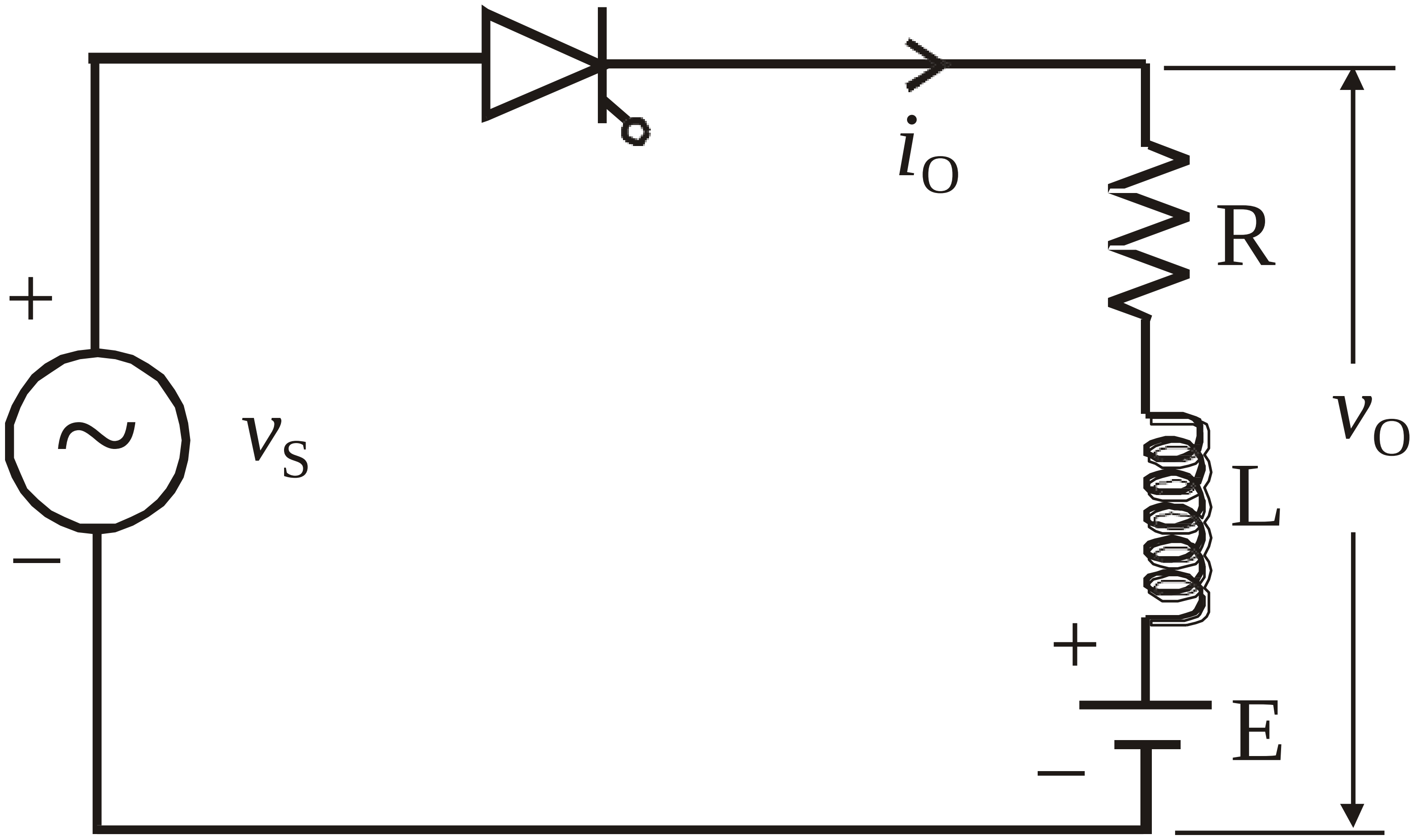
During the period π to β load current is carried by the free wheeling diode.

The value of β depends on the value of R and L and the forward resistance of the FWD.

For Large Load Inductance
the load current does not reach zero,
& we obtain continuous load current



Single Phase Half Wave Controlled Rectifier With A General Load



i_O
R
+
v_S
−
L
v_O
+
E
−

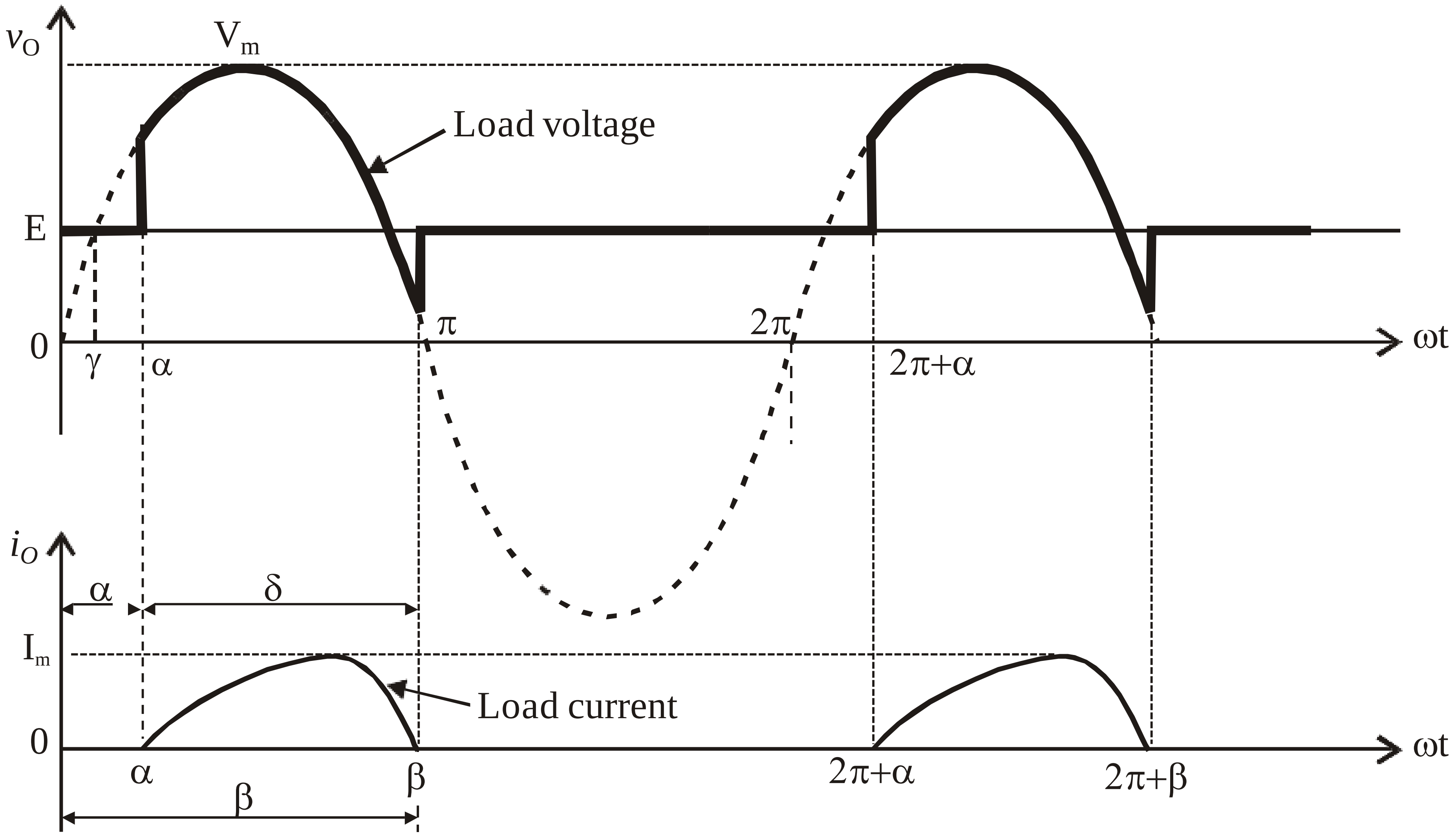
$$\gamma = \sin^{-1}\left(\frac{E}{V_m}\right)$$

For trigger angle $\alpha < \gamma$,

the Thyristor conducts from $\omega t = \gamma$ to β

For trigger angle $\alpha > \gamma$,

the Thyristor conducts from $\omega t = \alpha$ to β



v_O
V_m
Load voltage
E
0
γ
α
π
2π
ωt
2π+α
i_O
α
δ
I_m
Load current
0
α
β
2π+α
2π+β
ωt
β

Equations

$$v_S = V_m \sin \omega t = \text{ Input supply voltage.}$$

$$v_O = V_m \sin \omega t = \text{ o/p } (\text{load}) \text{ voltage}$$

$$\text{for } \omega t = \alpha \text{ to } \beta.$$

$$v_O = E \quad \text{for } \omega t = 0 \text{ to } \alpha \text{ \& }$$

$$\text{for } \omega t = \beta \text{ to } 2\pi.$$

Expression for the Load Current

When the thyristor is triggered at a delay angle of $\alpha > \gamma$, the eqn. for the circuit can be written as

$$V_m \sin\omega t = i_O \times R + L\left(\frac{di_O}{dt}\right) + \mathrm{E}\ ;\ \alpha \leq \omega t \leq \beta$$

The general expression for the output load current can be written as

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) - \frac{E}{R} + Ae^{\frac{-t}{\tau}}$$

Where

$$Z = \sqrt{R^2 + (\omega L)^2} = \text{Load Impedance.}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \text{ Load impedance angle.}$$

$$\tau = \frac{L}{R} = \text{ Load circuit time constant.}$$

The general expression for the o/p current can be written as $i_O = \frac{V_m}{Z}\sin(\omega t - \phi) - \frac{E}{R} + Ae^{\frac{-R}{L}t}$

To find the value of the constant 'A' apply the initial conditions at $\omega t = \alpha$, load current $i_O = 0$, Equating the general expression for the load current to zero at $\omega t = \alpha$, we get

$$i_O = 0 = \frac{V_m}{Z}\sin(\alpha - \phi) - \frac{E}{R} + Ae^{\frac{-R}{L} \times \frac{\alpha}{\omega}}$$

We obtain the value of constant 'A' as

$$A = \left[\frac{E}{R} - \frac{V_m}{Z}\sin(\alpha - \phi)\right] e^{\frac{R}{\omega L}\alpha}$$

Substituting the value of the constant 'A' in the expression for the load current; we get the complete expression for the output load current as

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) - \frac{E}{R} + \left[\frac{E}{R} - \frac{V_m}{Z}\sin(\alpha - \phi)\right] e^{\frac{-R}{\omega L}(\omega t - \alpha)}$$

To Derive
An
Expression For The Average
Or
DC Load Voltage

$$V_{O(dc)} = \frac{1}{2\pi} \int_0^{2\pi} v_O . d(\omega t)$$

$$V_{O(dc)} = \frac{1}{2\pi} \left[\int_0^{\alpha} v_O . d(\omega t) + \int_{\alpha}^{\beta} v_O . d(\omega t) + \int_{\beta}^{2\pi} v_O . d(\omega t) \right]$$

$v_O = V_m \sin \omega t =$ Output load voltage for $\omega t = \alpha$ to β

$v_O = E$ for $\omega t = 0$ to α & for $\omega t = \beta$ to 2π

$$V_{O(dc)} = \frac{1}{2\pi} \left[\int_0^{\alpha} E . d(\omega t) + \int_{\alpha}^{\beta} V_m \sin \omega t + \int_{\beta}^{2\pi} E . d(\omega t) \right]$$

$$V_{O(dc)} = \frac{1}{2\pi}\left[E\left(\omega t\right)\Big/_{0}^{\alpha} + V_m\left(-\cos\omega t\right)\Big/_{\alpha}^{\beta} + E\left(\omega t\right)\Big/_{\beta}^{2\pi} \right]$$

$$V_{O(dc)} = \frac{1}{2\pi}\left[E\left(\alpha - 0\right) - V_m\left(\cos\beta - \cos\alpha\right) + E\left(2\pi - \beta\right) \right]$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[\left(\cos\alpha - \cos\beta\right) \right] + \frac{E}{2\pi}\left[\left(2\pi - \beta + \alpha\right) \right]$$

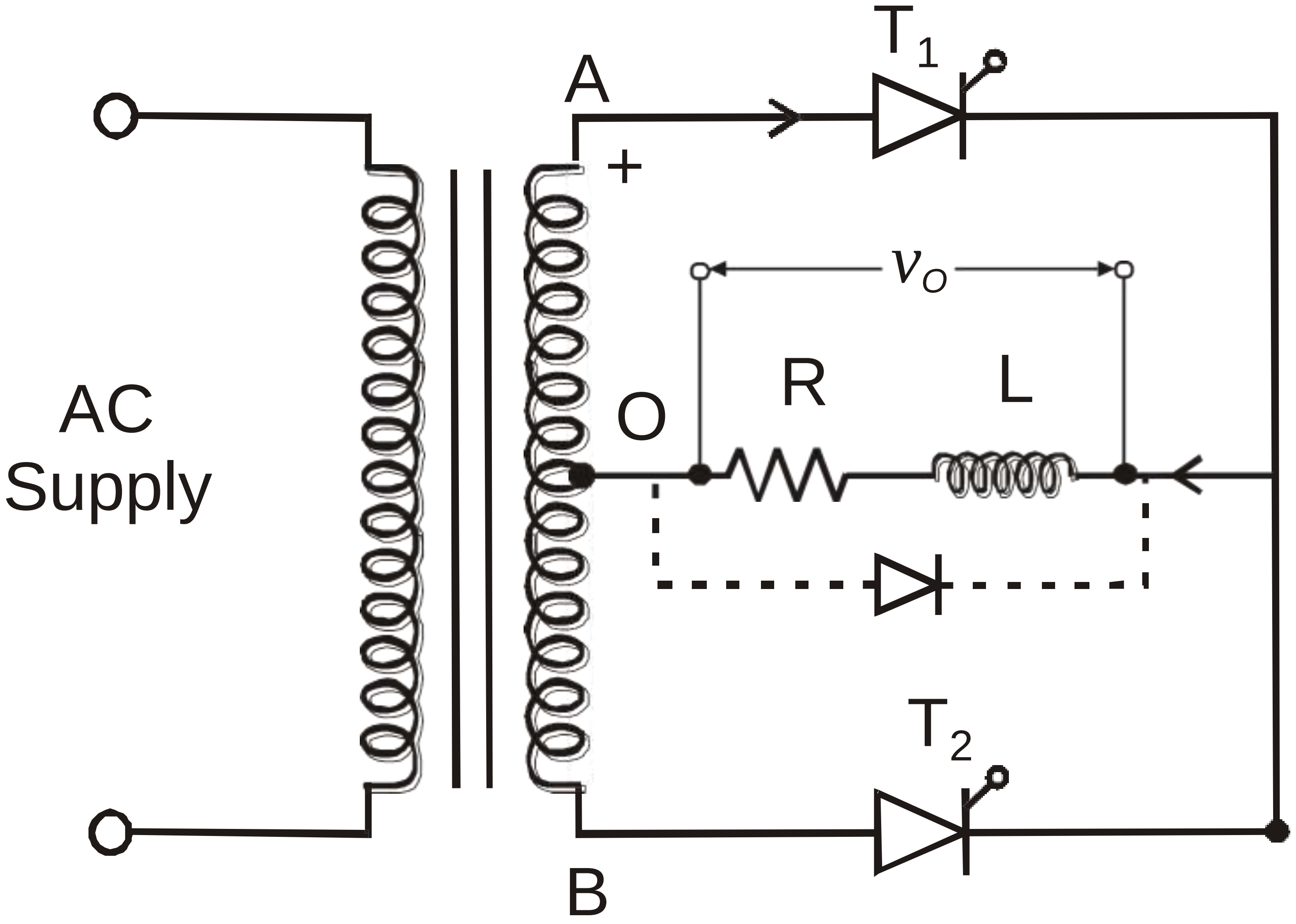
$$V_{O(dc)} = \frac{V_m}{2\pi}\left(\cos\alpha - \cos\beta\right) + \left[\frac{2\pi - \left(\beta - \alpha\right)}{2\pi} \right] E$$

Conduction angle of thyristor $\delta = (\beta - \alpha)$

RMS Output Voltage can be calculated by using the expression

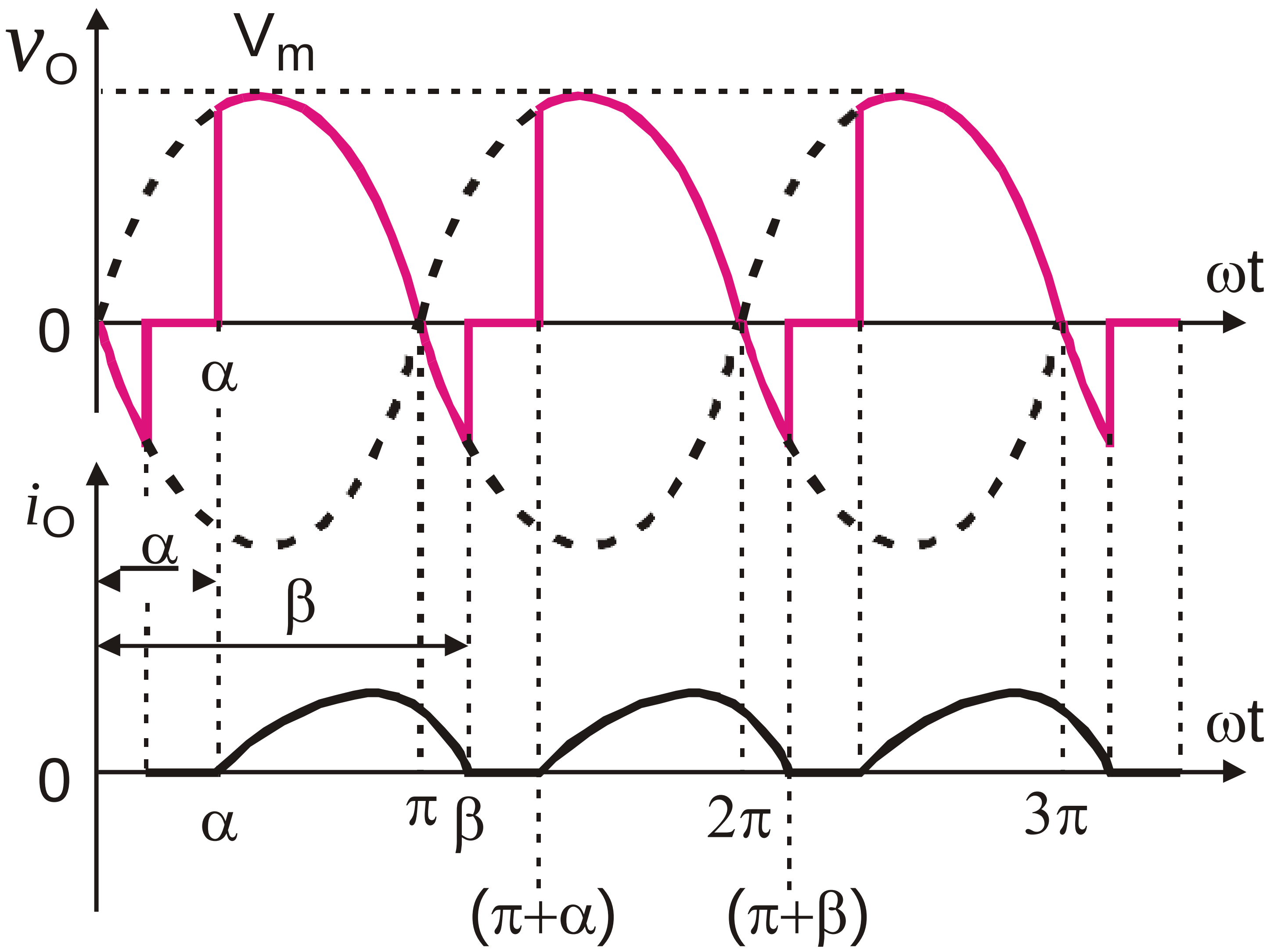
$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_0^{2\pi} v_O^2 . d(\omega t)\right]}$$

Single Phase Full Wave Controlled Rectifier Using A Center Tapped Transformer


T1
A
+
v_O
R
L
AC
Supply
O
T2
B

Discontinuous Load Current Operation without FWD for

- $\pi < \beta < (\pi + \alpha)$



v_O
V_m
ωt
0
α
i_O
α
β
ωt
0
α
π
β
2π
3π
(π+α)
(π+β)

To Derive An Expression For The Output (Load) Current, During $\omega t = \alpha$ to β When Thyristor T_1 Conducts

Assuming T_1 is triggered $\omega t = \alpha$,

we can write the equation,

$$L\left(\frac{di_O}{dt}\right) + Ri_O = V_m \sin\omega t \;\; ; \alpha \le \omega t \le \beta$$

General expression for the output current,

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + A_1 e^{\frac{-t}{\tau}}$$

$$V_m = \sqrt{2}V_S = \text{maximum supply voltage.}$$

$$Z = \sqrt{R^2 + (\omega L)^2} = \text{Load impedance.}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \text{Load impedance angle.}$$

$$\tau = \frac{L}{R} = \text{Load circuit time constant.}$$

$\therefore$ general expression for the output load current

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) + A_1 e^{\frac{-R}{L}t}$$

Constant A_1 is calculated from

initial condition $i_O = 0$ at $\omega t = \alpha$; t= $\left(\dfrac{\alpha}{\omega}\right)$

$$i_O = 0 = \frac{V_m}{Z}\sin(\alpha - \phi) + A_1 e^{\frac{-R}{L}t}$$

$$\therefore \quad A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z}\sin(\alpha - \phi)$$

We get the value of constant A_1 as

$$A_1 = e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin(\alpha - \phi)\right]$$

Substituting the value of constant A_1 in the general expression for i_O

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) + e^{\frac{-R}{\omega L}(\omega t - \alpha)}\left[\frac{-V_m}{Z}\sin(\alpha - \phi)\right]$$

$\therefore$ we obtain the final expression for the inductive load current

$$i_O = \frac{V_m}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right];$$

Where $\alpha \le \omega t \le \beta$

Extinction angle β can be calculated by using the condition that $i_O = 0$ at $\omega t = \beta$

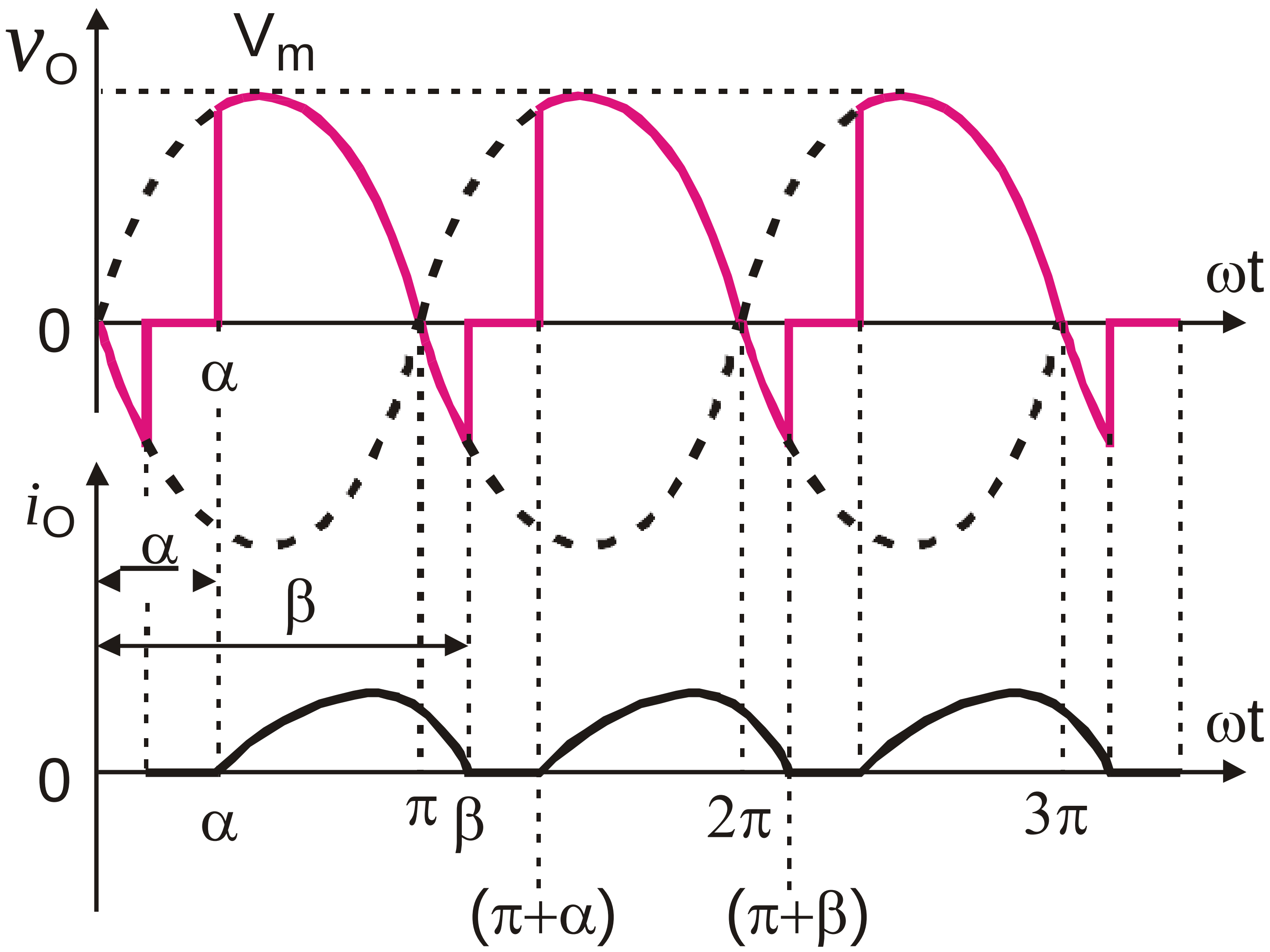
$$i_O = \frac{V_m}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right] = 0$$

$$\therefore \sin(\beta - \phi) = e^{\frac{-R}{\omega L}(\beta - \alpha)} \times \sin(\alpha - \phi)$$

β can be calculated by solving the above eqn.

To Derive An Expression For The DC Output Voltage Of A Single Phase Full Wave Controlled Rectifier With RL Load

(Without FWD)



v_O
V_m
ωt
0
α
i_O
α
β
ωt
0
α
π β
2π
3π
(π+α)
(π+β)

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=\alpha}^{\beta} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \left[\int_{\alpha}^{\beta} V_m \sin \omega t . d(\omega t) \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\beta} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (\cos \alpha - \cos \beta)$$

When the load inductance is negligible $\left(\text{ i.e., } L \approx 0\right)$

Extinction angle $\beta = \pi$ radians

Hence the average or dc output voltage for R load

$$V_{O(dc)} = \frac{V_m}{\pi}\left(\cos\alpha - \cos\pi\right)$$

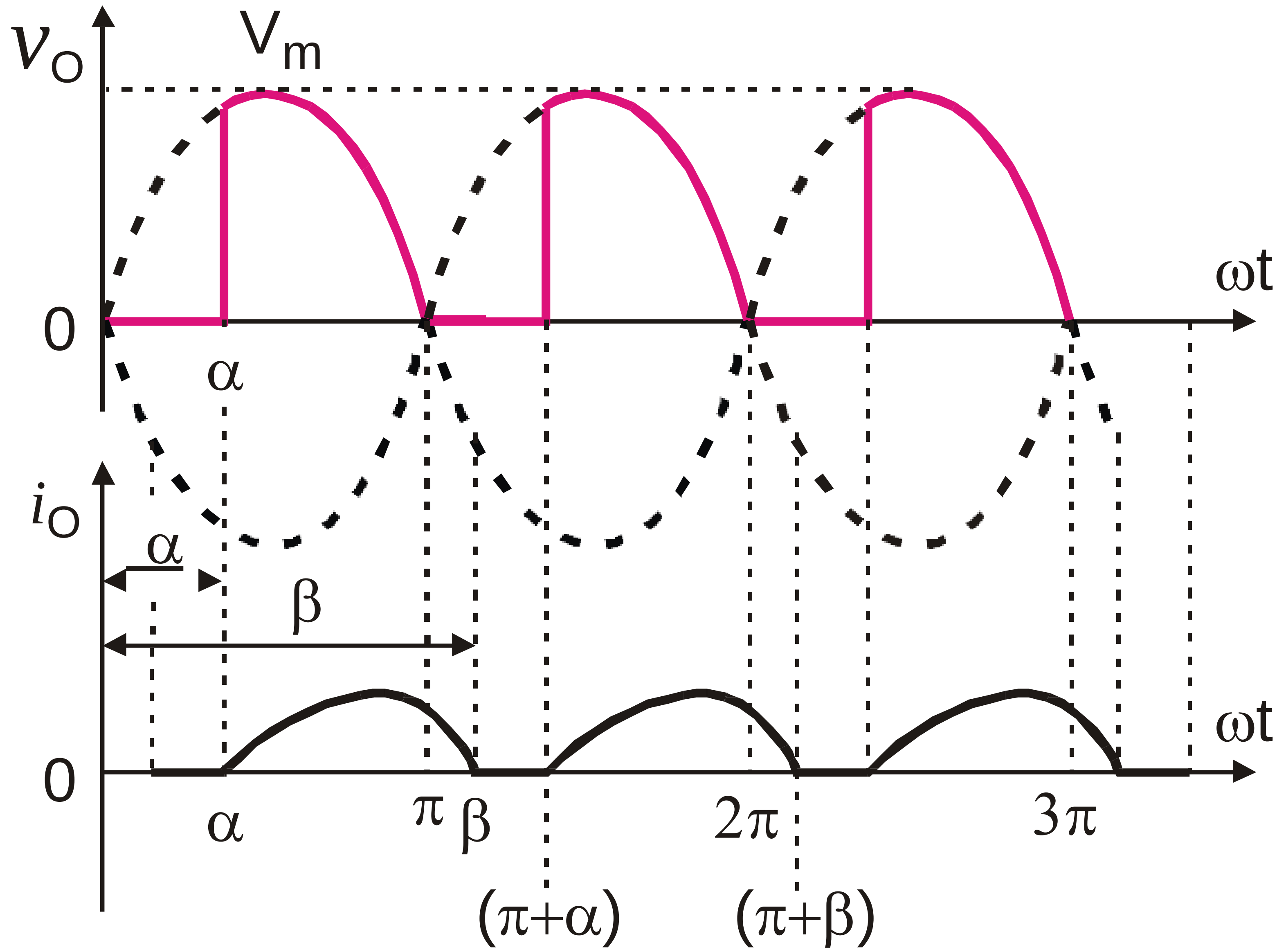
$$V_{O(dc)} = \frac{V_m}{\pi}\left(\cos\alpha - \left(-1\right)\right)$$

$$V_{O(dc)} = \frac{V_m}{\pi}\left(1 + \cos\alpha\right); \text{ for R load, when } \beta = \pi$$

To calculate the RMS output voltage we use the expression

$$V_{O(RMS)} = \sqrt{\frac{1}{\pi}\left[\int_{\alpha}^{\beta} V_m^2 \sin^2 \omega t . d\left(\omega t\right)\right]}$$

Discontinuous Load Current Operation with FWD



v_O
V_m
ωt
0
α
i_O
α
β
ωt
0
α
π β
2π
3π
(π+α)
(π+β)

Thyristor T_1 is triggered at $\omega t = \alpha$;

T_1 conducts from $\omega t = \alpha$ to π

Thyristor T_2 is triggered at $\omega t = (\pi + \alpha)$;

T_2 conducts from $\omega t = (\pi + \alpha)$ to 2π

FWD conducts from $\omega t = \pi$ to β &

$v_O \approx 0$ during discontinuous load current.

To Derive an Expression For The DC Output Voltage For A Single Phase Full Wave Controlled Rectifier With RL Load & FWD

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=0}^{\pi} v_O . d(\omega t)$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t . d(\omega t)$$

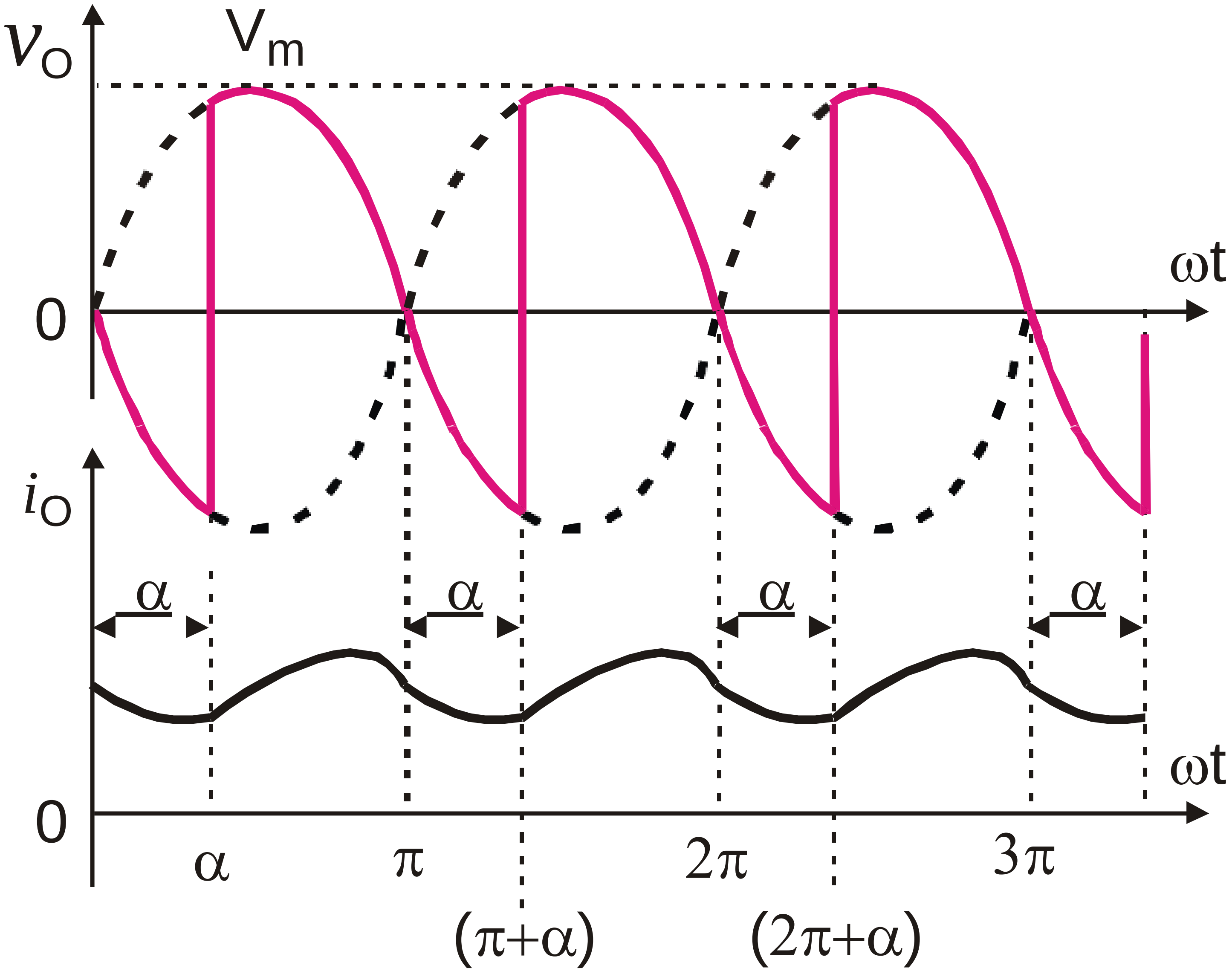
$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\pi} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos\pi + \cos\alpha \right] \;;\; \cos\pi = -1$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (1 + \cos\alpha)$$

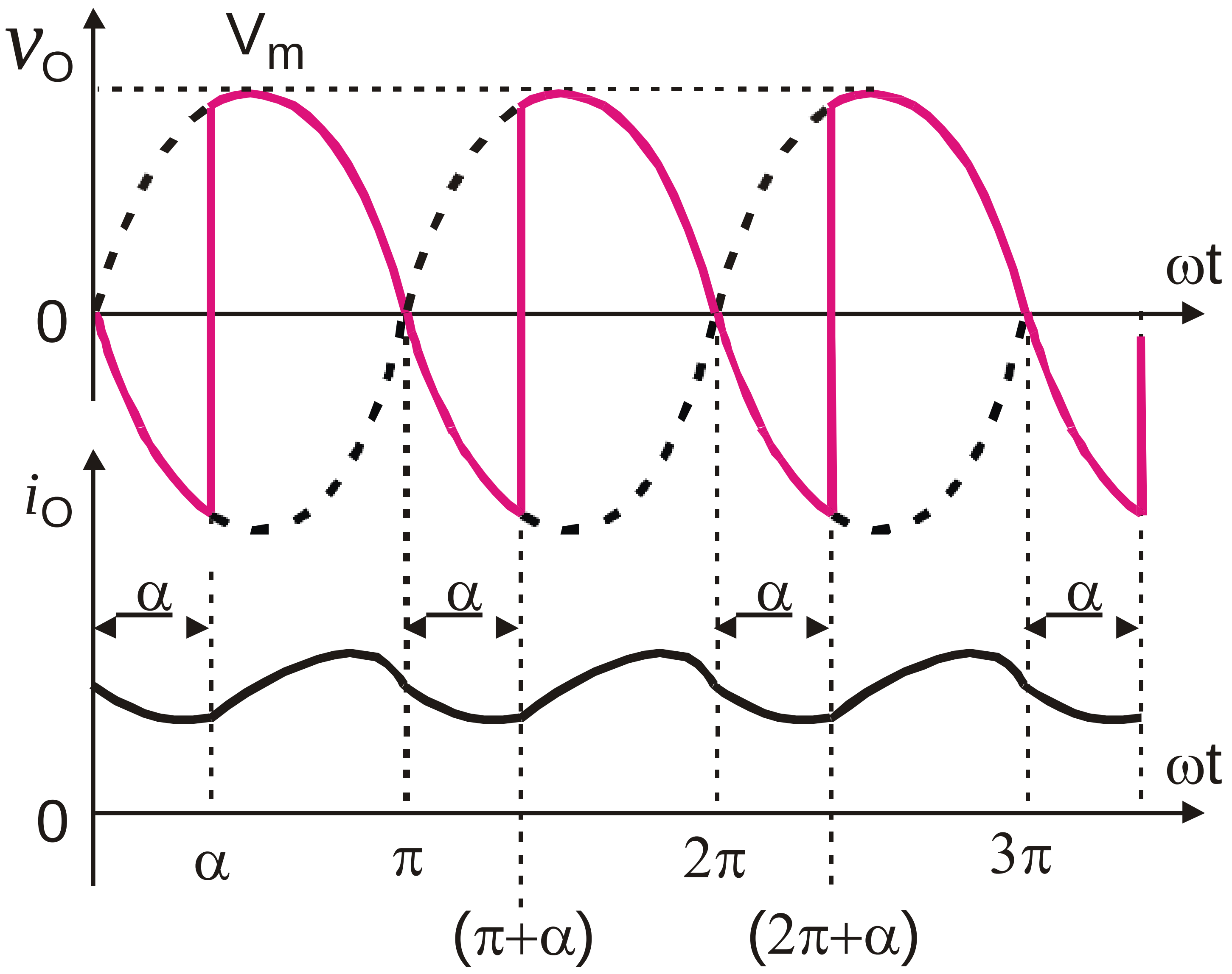
- The load current is discontinuous for low values of load inductance and for large values of trigger angles.
- For large values of load inductance the load current flows continuously without falling to zero.
- Generally the load current is continuous for large load inductance and for low trigger angles.

Continuous Load Current Operation (Without FWD)



v_O
V_m
ωt
0
i_O
α
α
α
α
ωt
0
α
π
2π
3π
(π+α)
(2π+α)

To Derive
An Expression For
Average / DC Output Voltage
Of
Single Phase Full Wave Controlled Rectifier For Continuous Current Operation without FWD



vO
Vm
ωt
0
iO
α
α
α
α
ωt
0
α
π
2π
3π
(π+α)
(2π+α)

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=\alpha}^{(\pi+\alpha)} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \left[\int_{\alpha}^{(\pi+\alpha)} V_m \sin \omega t . d(\omega t) \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{(\pi+\alpha)} \right]$$

$$V_{O(dc)} = V_{dc}$$

$$= \frac{V_m}{\pi}\left\lfloor\left\lceil \cos\alpha - \cos(\pi + \alpha)\right\rceil\right\rfloor \;;$$

$$\cos(\pi + \alpha) = -\cos\alpha$$

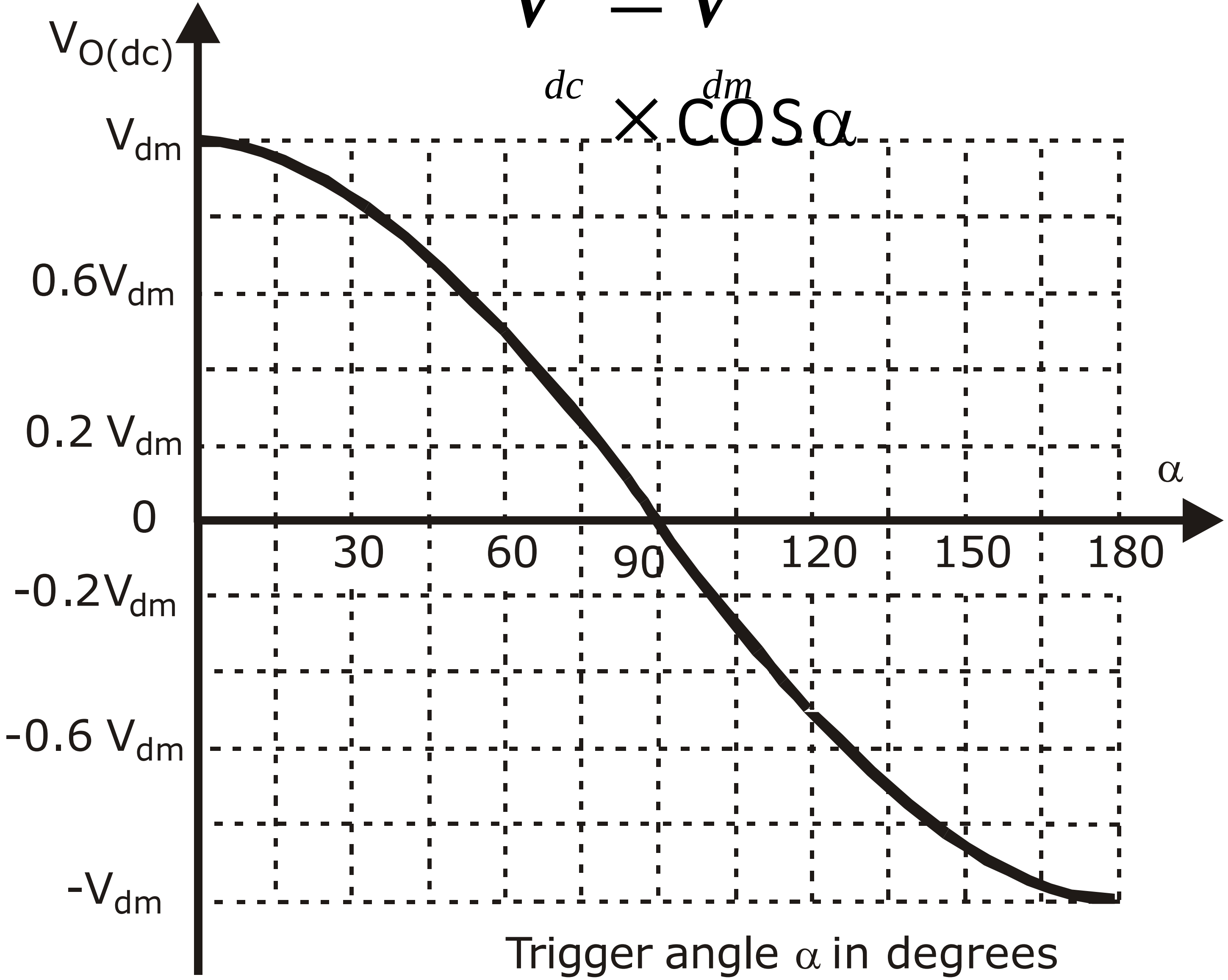
$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[\cos\alpha + \cos\alpha\right]$$

$$\therefore \qquad V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi}\cos\alpha$$

- By plotting $V_{O(dc)}$ *versus* α,
  we obtain the control characteristic of a single phase full wave controlled rectifier with RL load for continuous load current operation without FWD

Trigger angle α in degrees	$V_{O(dc)}$	Remarks
0	$V_{dm} = \left(\frac{2V_m}{\pi} \right)$	Maximum dc output voltage $V_{dc(\max)} = V_{dm} = \left(\frac{2V_m}{\pi} \right)$
30^0	$0.866\ V_{dm}$	$V_{dc} = V_{dm} \times \cos\alpha$
60^0	$0.5\ V_{dm}$	
90^0	$0\ V_{dm}$	
120^0	$-0.5\ V_{dm}$	
150^0	$-0.866\ V_{dm}$	
180^0	$-V_{dm} = -\left(\frac{2V_m}{\pi} \right)$	

$$V_{dc} = V_{dm} \times \cos\alpha$$



By varying the trigger angle we can vary the output dc voltage across the load. Hence we can control the dc output power flow to the load.

For trigger angle α, 0 to 90^0 $\left(i.e.,\ 0 \le \alpha \le 90^0\right)$;

$\cos\alpha$ is positive and hence V_{dc} is positive

V_{dc} & I_{dc} are positive ; $P_{dc} = \left(V_{dc} \times I_{dc}\right)$ is positive

Converter operates as a Controlled Rectifier.

Power flow is from the ac source to the load.

For trigger angle α, 90^0 to 180^0

$$\left(i.e.,\ 90^0 \le \alpha \le 180^0\right),$$

$\cos\alpha$ is negative and hence

V_{dc} is negative; I_{dc} is positive ;

$$P_{dc} = \left(V_{dc} \times I_{dc}\right) \text{ is negative.}$$

In this case the converter operates

as a Line Commutated Inverter.

Power flows from the load ckt. to the i/p ac source.

The inductive load energy is fed back to the

i/p source.

Drawbacks Of Full Wave Controlled Rectifier With Centre Tapped Transformer

- We require a centre tapped transformer which is quite heavier and bulky.
- Cost of the transformer is higher for the required dc output voltage & output power.
- Hence full wave bridge converters are preferred.

Single Phase
Full Wave Bridge Controlled Rectifier

Single Phase Full Wave Bridge Controlled Rectifier

2 types of FW Bridge Controlled Rectifiers are

- Half Controlled Bridge Converter

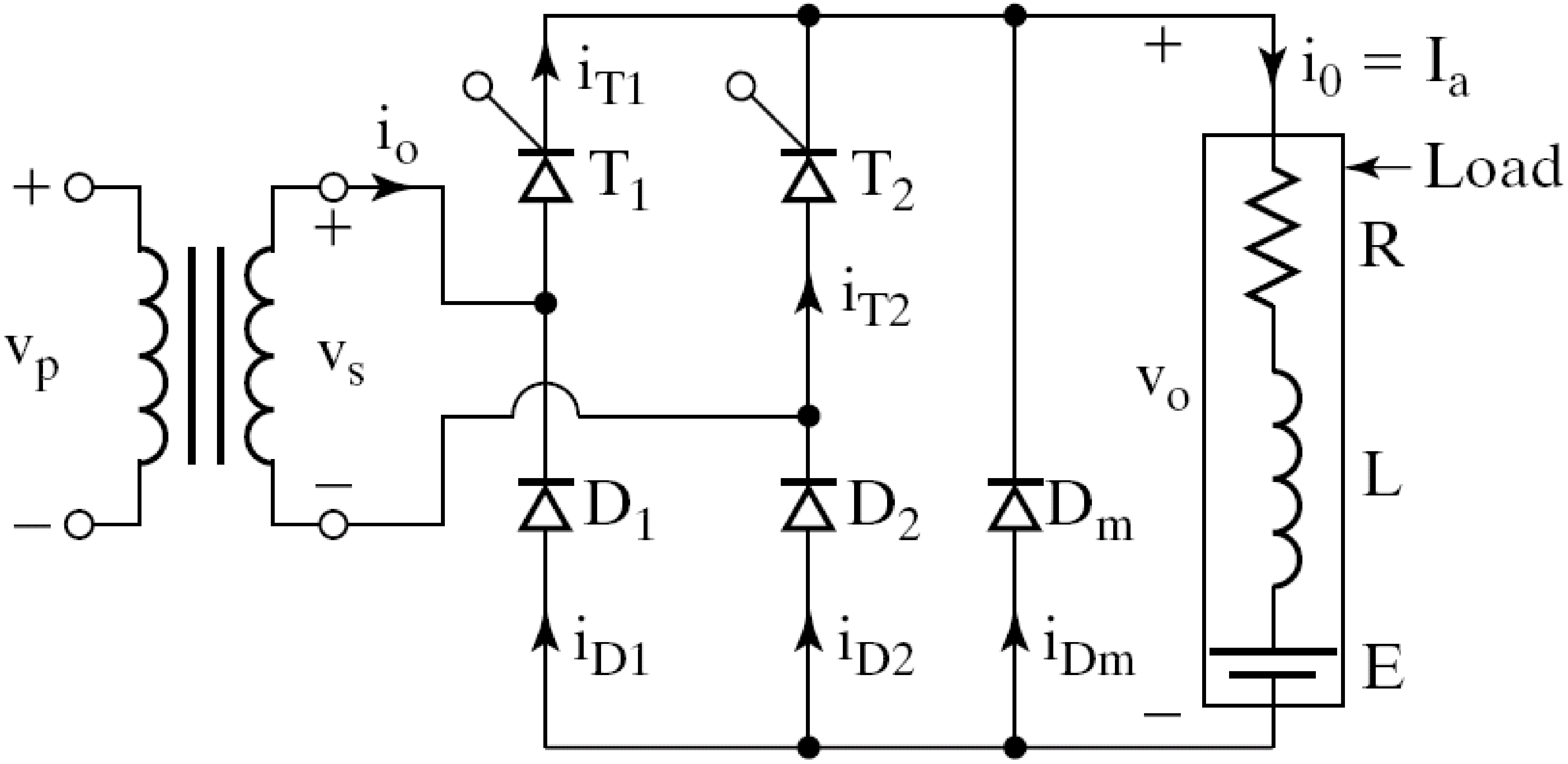
  (Semi-Converter)

- Fully Controlled Bridge Converter

  (Full Converter)

The bridge full wave controlled rectifier does not require a centre tapped transformer

Single Phase
Full Wave Half Controlled Bridge Converter
(Single Phase Semi Converter)



+
−
vp
vs
+
−
io
iT1
T1
T2
iT2
D1
D2
Dm
iD1
iD2
iDm
+
i0 = Ia
Load
R
vo
L
E
−

Trigger Pattern of Thyristors

$Thyristor\ T_1\ is\ triggered\ at$

$$\omega t = \alpha,\ at\ \omega t = (2\pi + \alpha),...$$

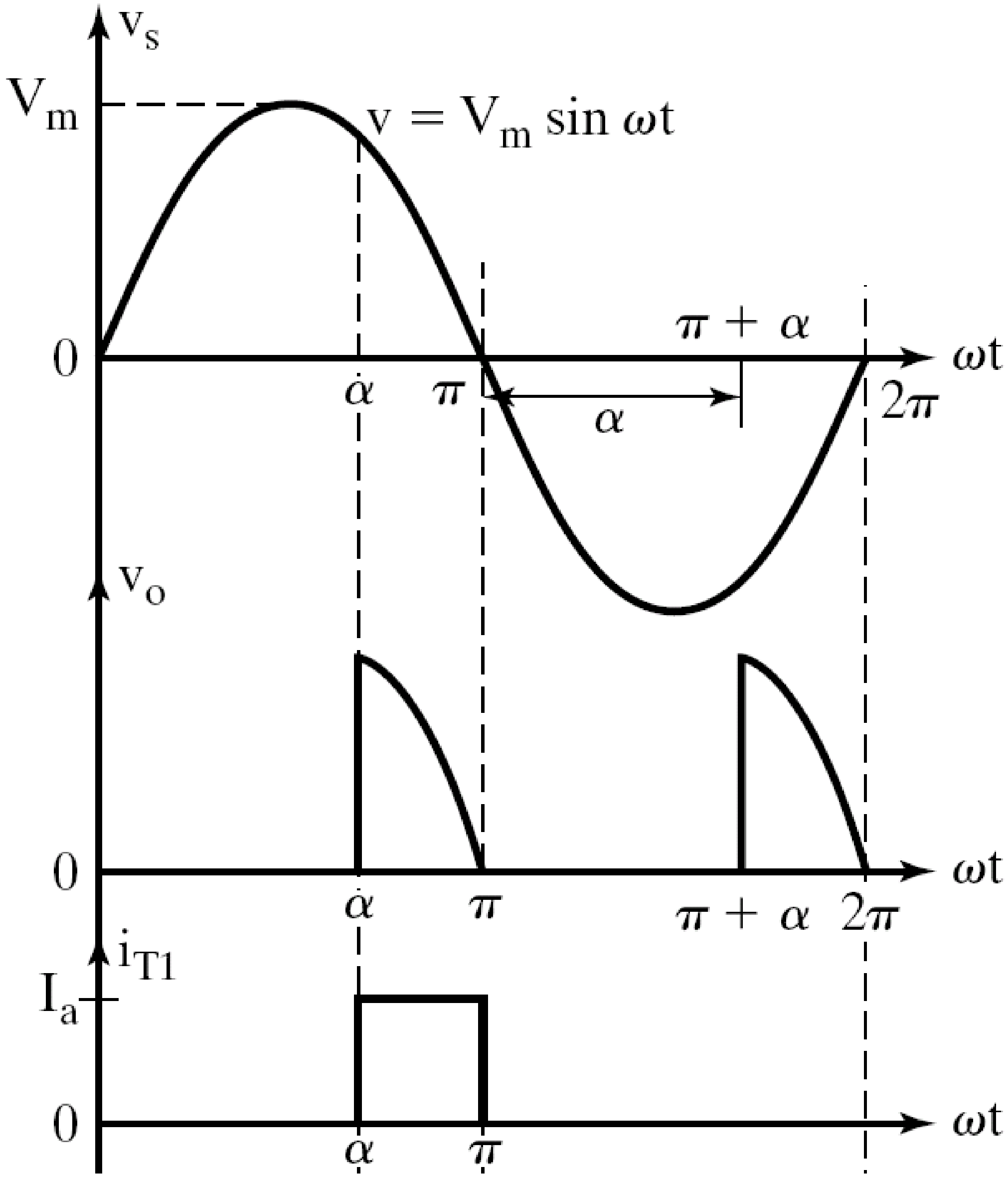
$Thyristor\ T_2\ is\ triggered\ at$

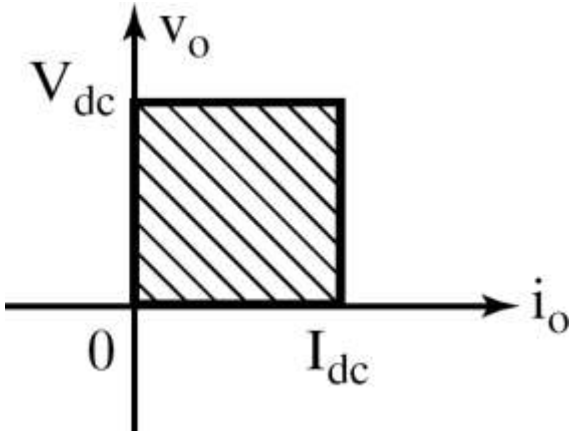
$$\omega t = (\pi + \alpha),\ at\ \omega t = (3\pi + \alpha),...$$

$The\ time\ delay\ between\ the\ gating$

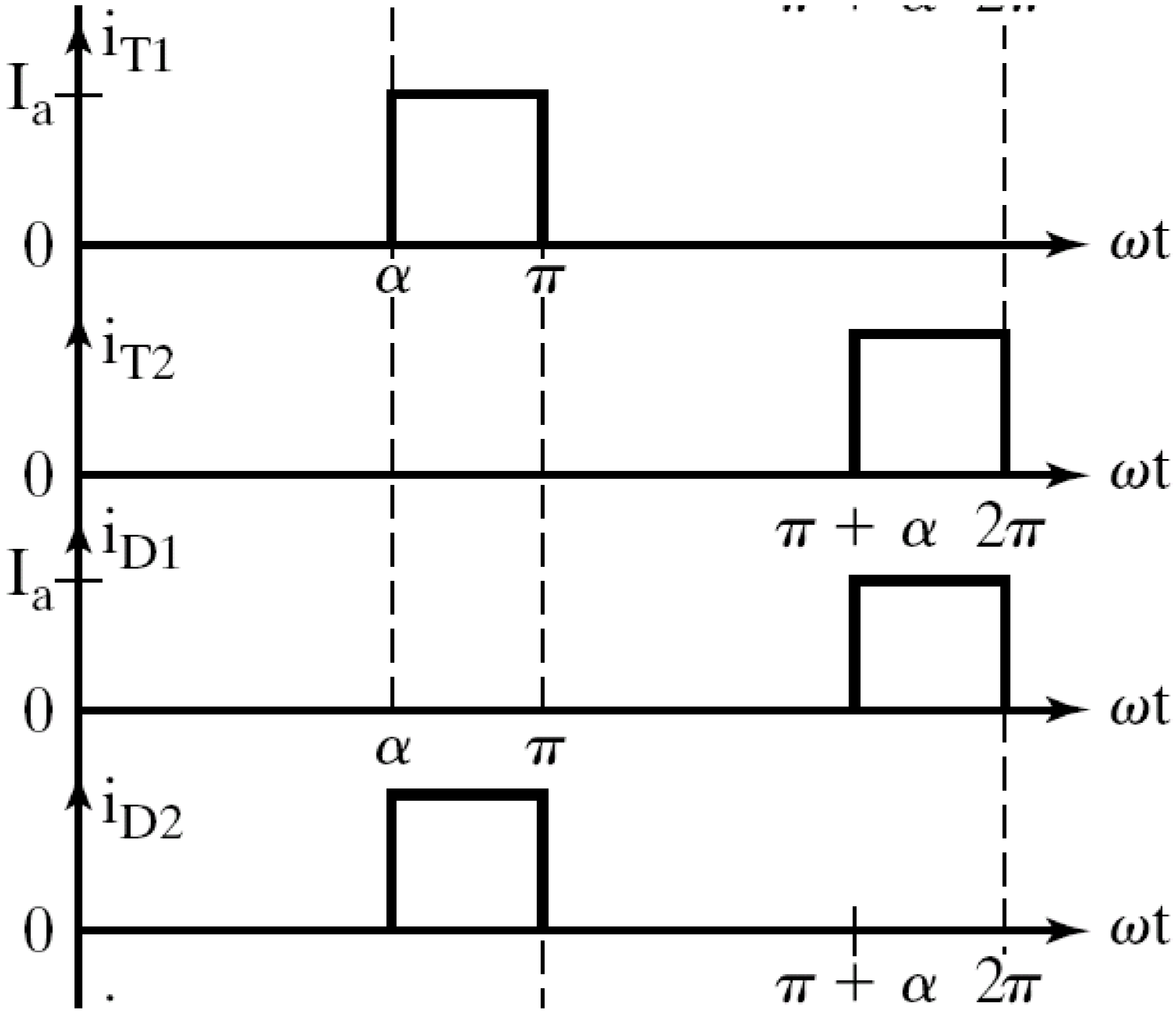
$signals\ of\ T_1\ \&\ T_2 = \pi\ radians\ or\ 180^0$

Waveforms of
single phase semi-converter
with general load & FWD
for $\alpha > 90^0$

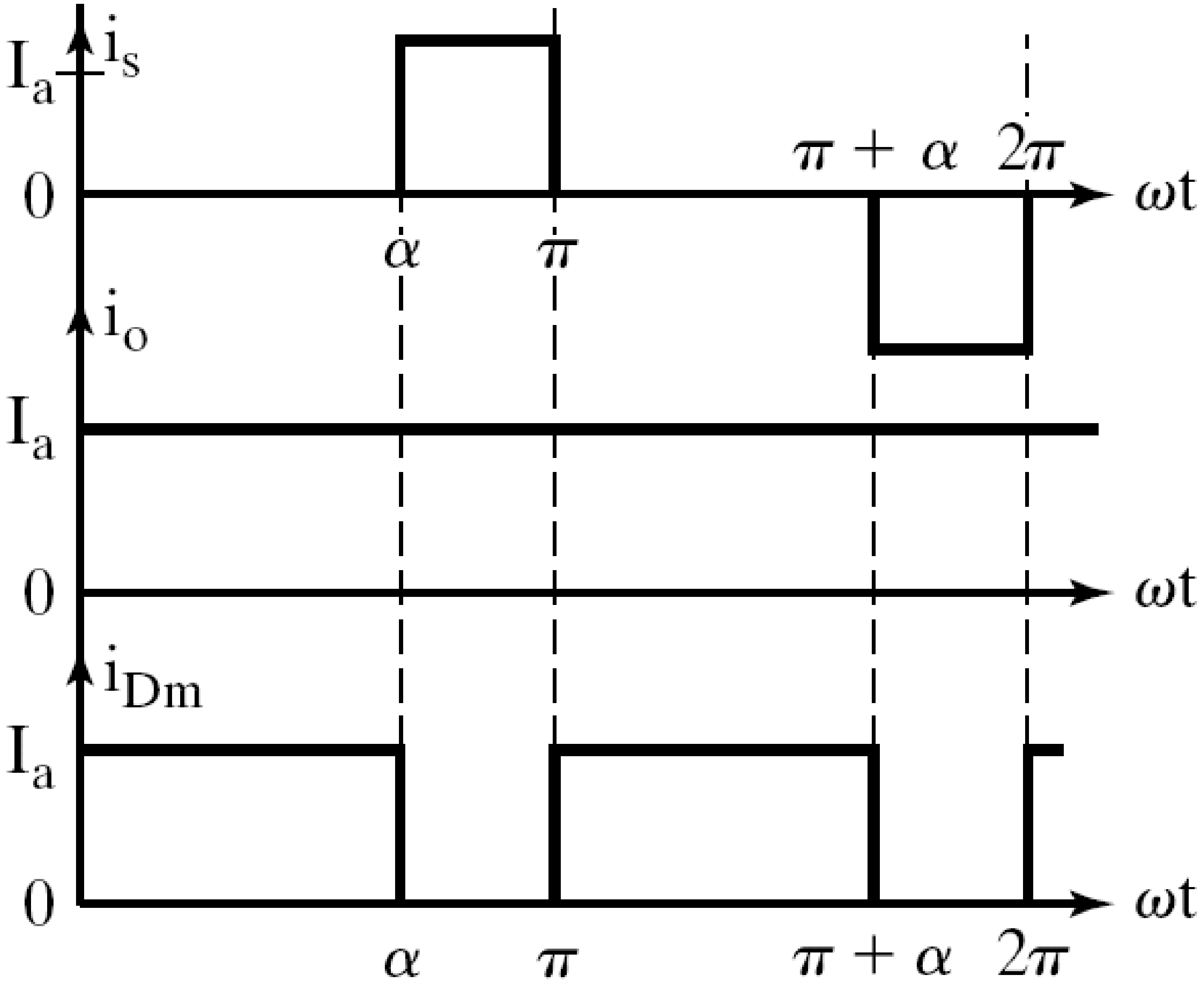




Single Quadrant Operation



i_{T1}
I_a
0
ωt
α
π
i_{T2}
0
ωt
$\pi + \alpha$
2π
i_{D1}
I_a
0
ωt
α
π
i_{D2}
0
ωt
$\pi + \alpha$
2π



I_a
i_s
0
α
π
$\pi + \alpha$
2π
ωt
i_o
I_a
0
ωt
i_{Dm}
I_a
0
ωt
α
π
$\pi + \alpha$
2π

Thyristor T_1 & D_1

conduc

t from $\omega t = \alpha\, to$

π

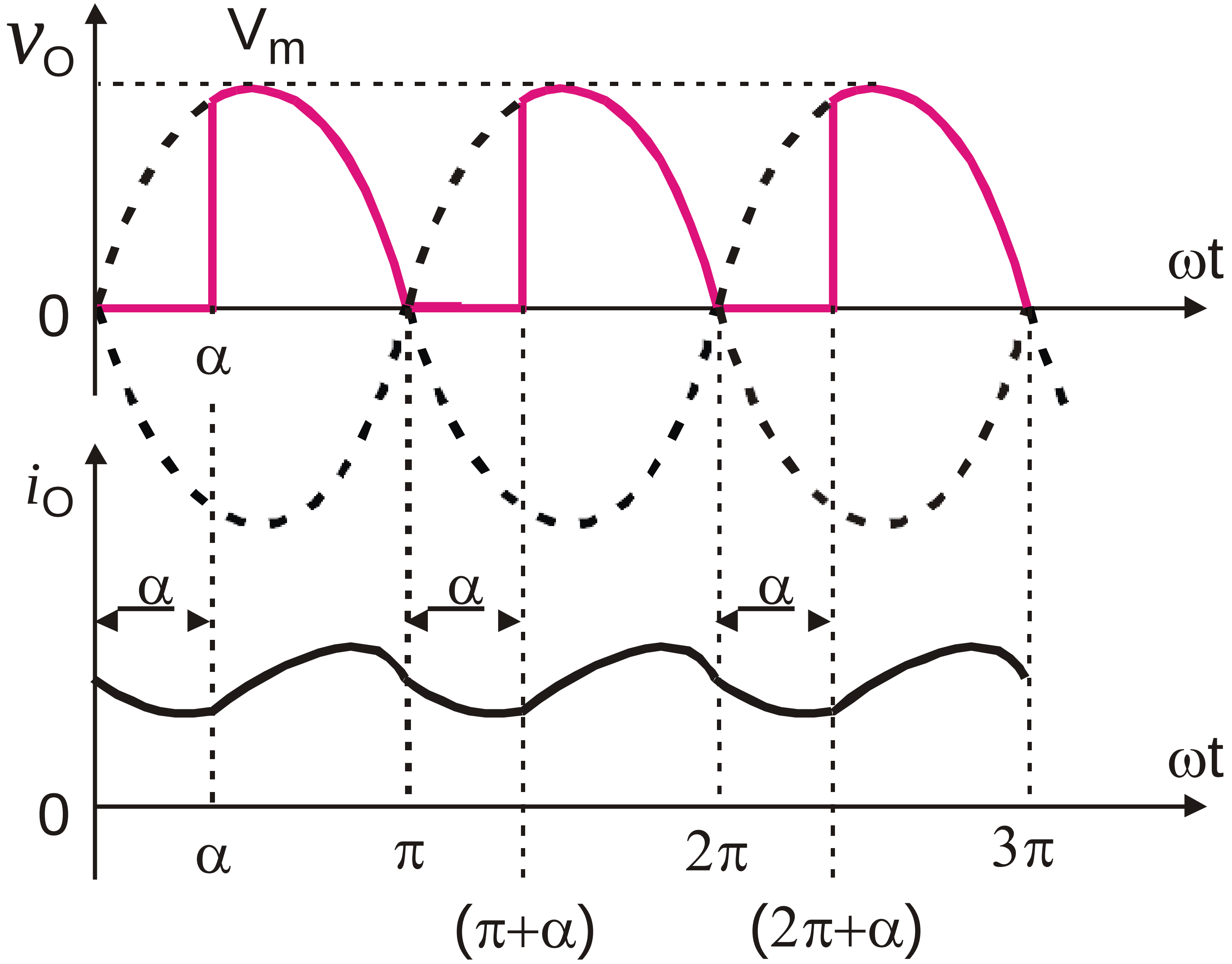
Thyristor T_2 & D_2 conduct

from $\omega t = (\pi + \alpha)\ to\ 2\pi$

FWD conducts during

$\omega t = 0$ to $\alpha,\ \pi\ to\ (\pi + \alpha), \ldots$

Load Voltage & Load Current Waveform of Single Phase Semi Converter for $\alpha < 90^0$ & Continuous load current operation


v_O
V_m
ωt
0
α
i_O
α
α
α
ωt
0
α
π
2π
3π
(π+α)
(2π+α)

To Derive an Expression For The DC Output Voltage of A Single Phase Semi-Converter With R,L, & E Load & FWD For Continuous, Ripple Free Load Current Operation

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=0}^{\pi} v_O . d(\omega t)$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\pi} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \pi + \cos \alpha \right] \ ; \ \cos \pi = -1$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (1 + \cos \alpha)$$

V_{dc} can be varied from a max.

value of $\dfrac{2V_m}{\pi}$ *to* 0 by varying α from 0 to π.

For $\alpha = 0$, The max. dc o/p voltage obtained is

$$V_{dc(\max)} = V_{dm} = \frac{2V_m}{\pi}$$

Normalized dc o/p voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dn}} = \frac{\frac{V_m}{\pi}\left(1+\cos\alpha\right)}{\left(\frac{2V_m}{\pi}\right)} = \frac{1}{2}\left(1+\cos\alpha\right)$$

RMS O/P Voltage $V_{O(RMS)}$

$$V_{O(RMS)} = \left[\frac{2}{2\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t . d(\omega t) \right]^{\frac{1}{2}}$$

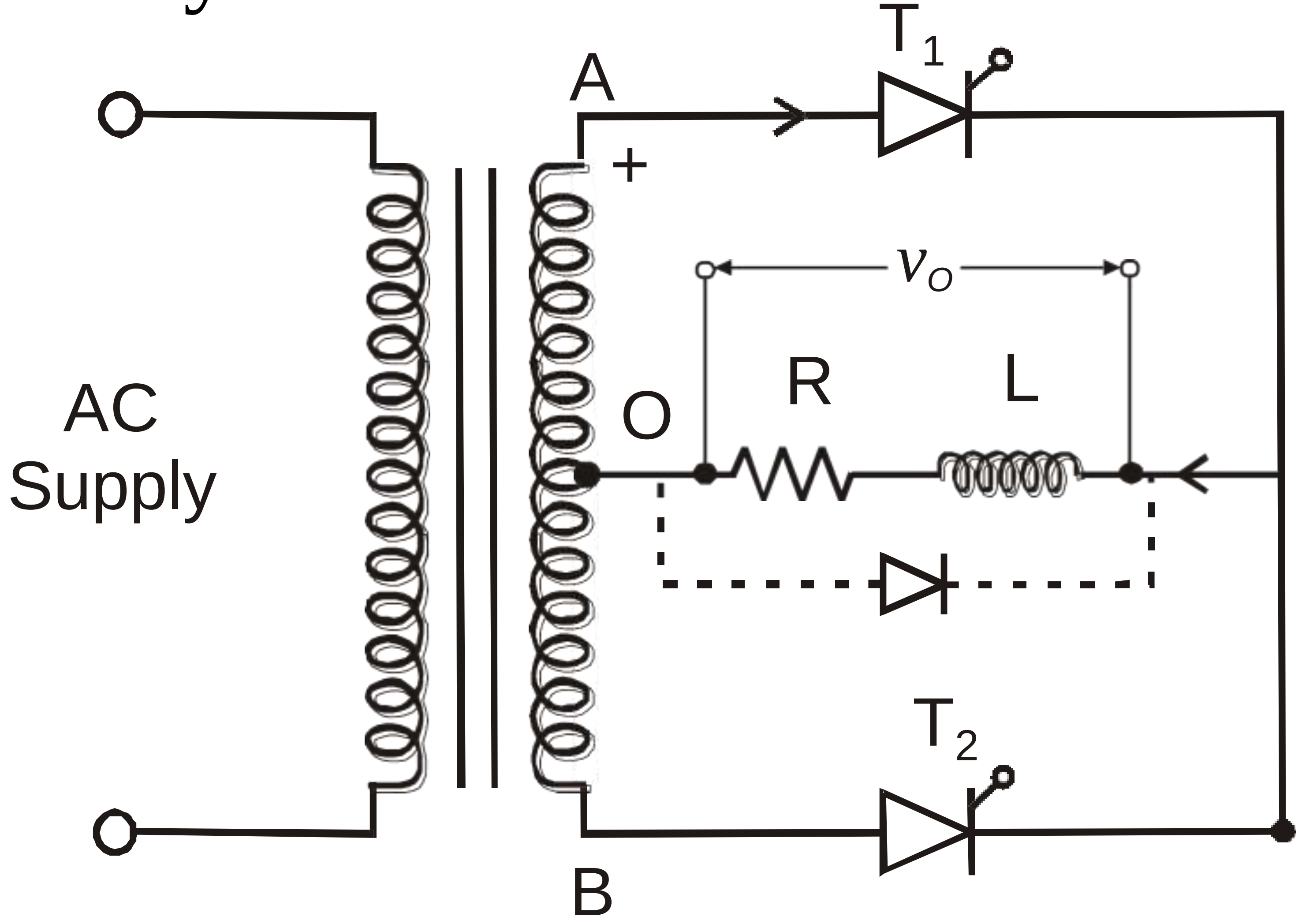
$$V_{O(RMS)} = \left[\frac{V_m^2}{2\pi} \int_{\alpha}^{\pi} (1 - \cos 2\omega t).d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \left[\frac{1}{\pi} \left(\pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{\frac{1}{2}}$$

Single Phase Full Wave Controlled Rectifier

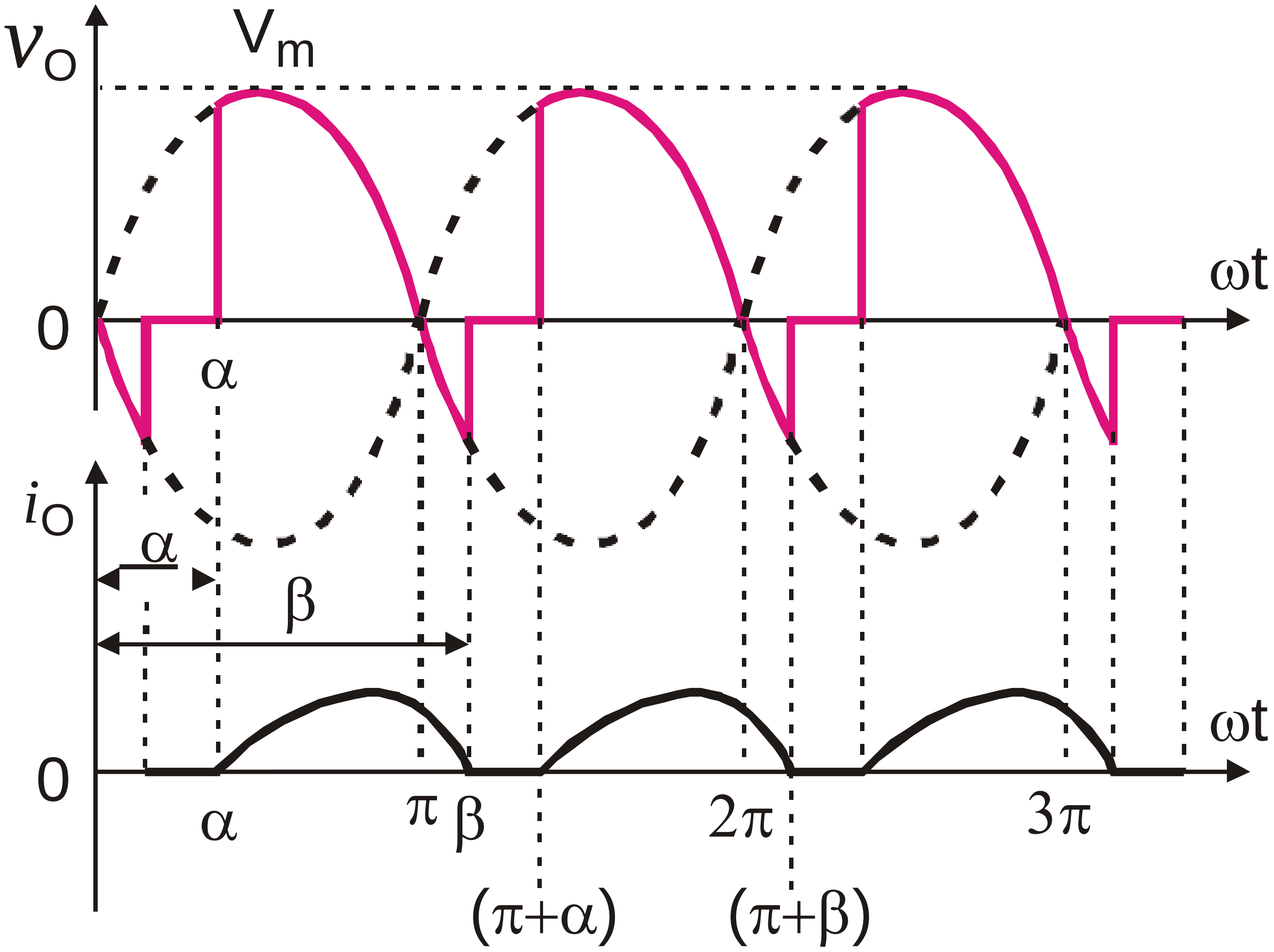
Single Phase Full Wave Controlled Rectifier Using A Center Tapped Transformer

Single Phase Midpoint type
Fully controlled Rectifier



Discontinuous Load Current Operation without FWD for

- $\pi < \beta < (\pi + \alpha)$



v_O
V_m
ωt
0
α
i_O
α
β
ωt
0
α
π
β
2π
3π
(π+α)
(π+β)

To Derive An Expression For The Output (Load) Current, During $\omega t = \alpha$ to β When Thyristor T_1 Conducts

Assuming T_1 is triggered $\omega t = \alpha$,

we can write the equation,

$$L\left(\frac{di_O}{dt}\right) + Ri_O = V_m \sin \omega t \ ; \alpha \le \omega t \le \beta$$

General expression for the output current,

$$i_O = \frac{V_m}{Z} \sin\left(\omega t - \phi\right) + A_1 e^{\frac{-t}{\tau}}$$

$$V_m = \sqrt{2}V_S = \text{maximum supply voltage.}$$

$$Z = \sqrt{R^2 + (\omega L)^2} = \text{Load impedance.}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \text{Load impedance angle.}$$

$$\tau = \frac{L}{R} = \text{Load circuit time constant.}$$

$\therefore$ general expression for the output load current

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) + A_1 e^{\frac{-R}{L}t}$$

Constant A_1 is calculated from

initial condition $i_O = 0$ at $\omega t = \alpha$; t= $\left(\frac{\alpha}{\omega}\right)$

$$i_O = 0 = \frac{V_m}{Z}\sin(\alpha - \phi) + A_1 e^{\frac{-R}{L}t}$$

$$\therefore \quad A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z}\sin(\alpha - \phi)$$

We get the value of constant A_1 as

$$A_1 = e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin(\alpha - \phi)\right]$$

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Substituting the value of constant A_1 in the general expression for i_O

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + e^{\frac{-R}{\omega L}\left(\omega t - \alpha\right)}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

$\therefore$ we obtain the final expression for the inductive load current

$$i_O = \frac{V_m}{Z}\left[\sin\left(\omega t - \phi\right) - \sin\left(\alpha - \phi\right)e^{\frac{-R}{\omega L}\left(\omega t - \alpha\right)}\right];$$

Where $\alpha \le \omega t \le \beta$

Extinction angle β can be calculated by using the condition that $i_O = 0 \; at \; \omega t = \beta$

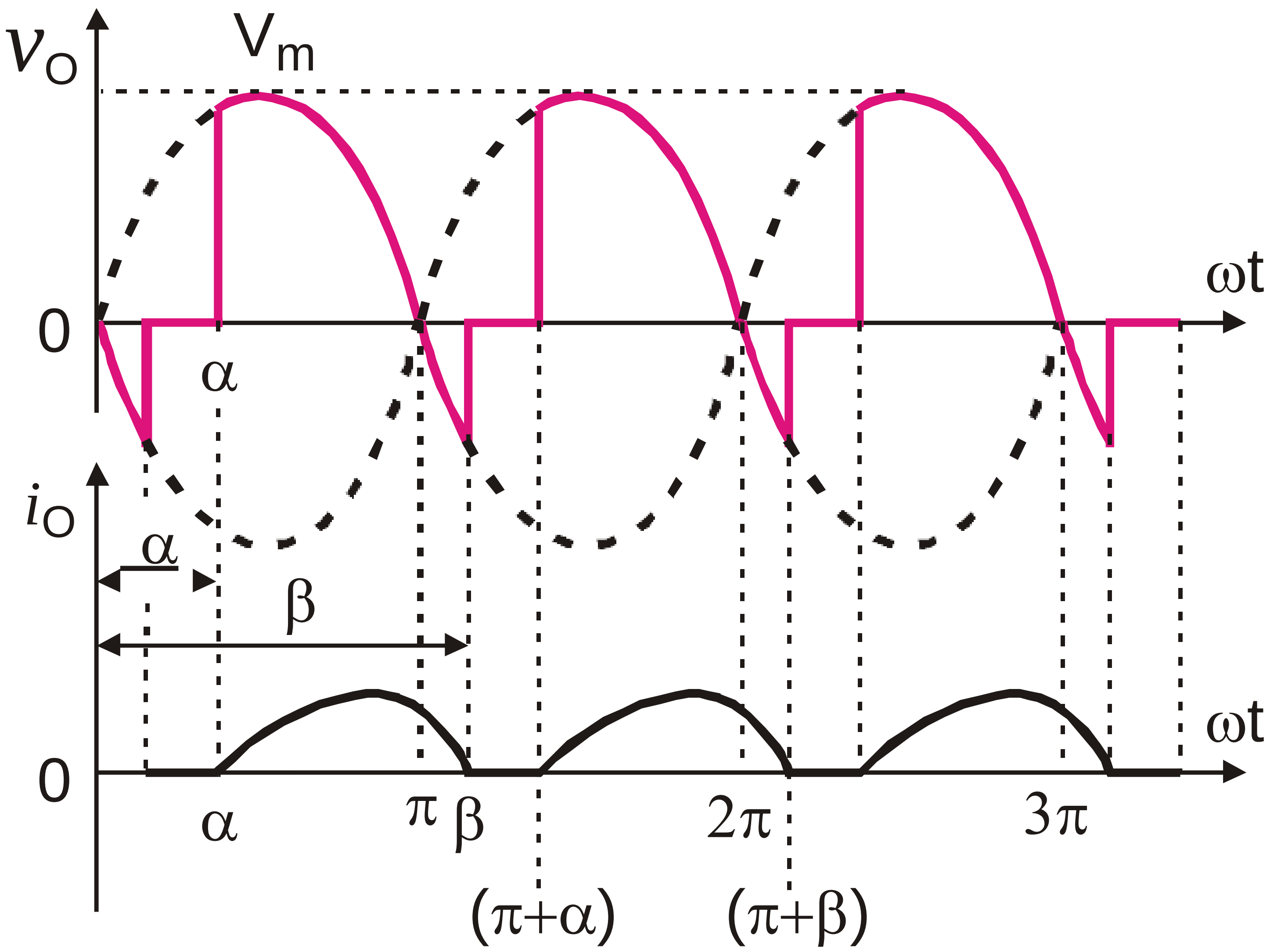
$$i_O = \frac{V_m}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi) e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right] = 0$$

$$\therefore \sin(\beta - \phi) = e^{\frac{-R}{\omega L}(\beta - \alpha)} \times \sin(\alpha - \phi)$$

β can be calculated by solving the above eqn.

To Derive An Expression For The DC Output Voltage Of A Single Phase Full Wave Controlled Rectifier With RL Load

(Without FWD)



vO
Vm
ωt
0
α
α
β
ωt
iO
0
α
π β
2π
3π
(π+α)
(π+β)

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=\alpha}^{\beta} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \left[\int_{\alpha}^{\beta} V_m \sin \omega t . d(\omega t) \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\beta} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (\cos \alpha - \cos \beta)$$

When the load inductance is negligible $\left(\text{i.e., } L \approx 0 \right)$

Extinction angle $\beta = \pi$ radians

Hence the average or dc output voltage for R load

$$V_{O(dc)} = \frac{V_m}{\pi}\left(\cos\alpha - \cos\pi\right)$$

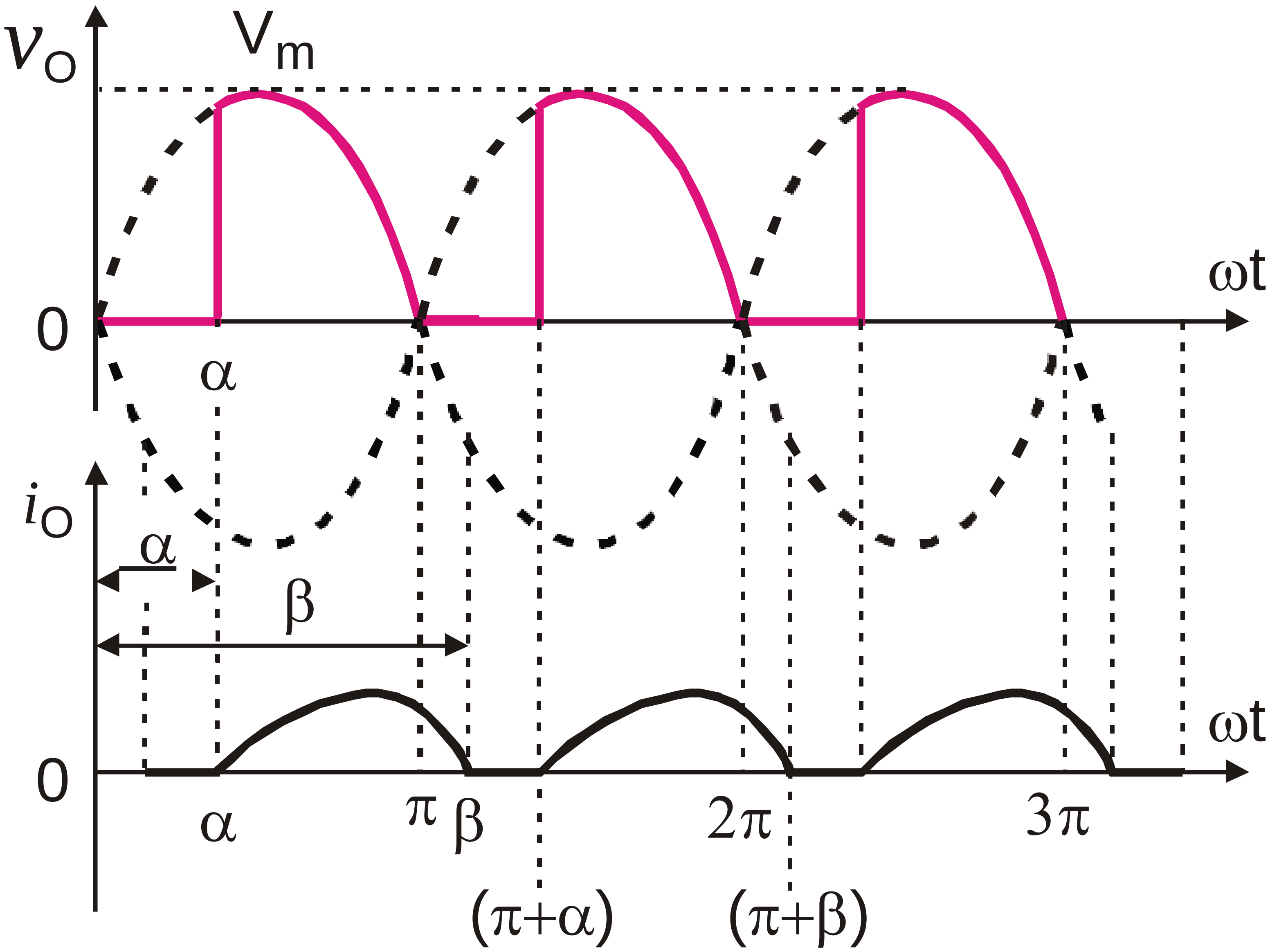
$$V_{O(dc)} = \frac{V_m}{\pi}\left(\cos\alpha - (-1)\right)$$

$$V_{O(dc)} = \frac{V_m}{\pi}\left(1 + \cos\alpha\right); \text{ for R load, when } \beta = \pi$$

To calculate the RMS output voltage we use the expression

$$V_{O(RMS)} = \sqrt{\frac{1}{\pi}\left[\int_{\alpha}^{\beta} V_m^2 \sin^2 \omega t . d\left(\omega t\right)\right]}$$

Discontinuous Load Current Operation with FWD


v_O
V_m
ωt
0
α
i_O
α
β
ωt
0
α
π
β
2π
3π
(π+α)
(π+β)

Thyristor T_1 is triggered at $\omega t = \alpha$;

T_1 conducts from $\omega t = \alpha$ to π

Thyristor T_2 is triggered at $\omega t = (\pi + \alpha)$;

T_2 conducts from $\omega t = (\pi + \alpha)$ to 2π

FWD conducts from $\omega t = \pi$ to β &

$v_O \approx 0$ during discontinuous load current.

To Derive an Expression For The DC Output Voltage For A Single Phase Full Wave Controlled Rectifier With RL Load & FWD

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=0}^{\pi} v_O . d(\omega t)$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t . d(\omega t)$$

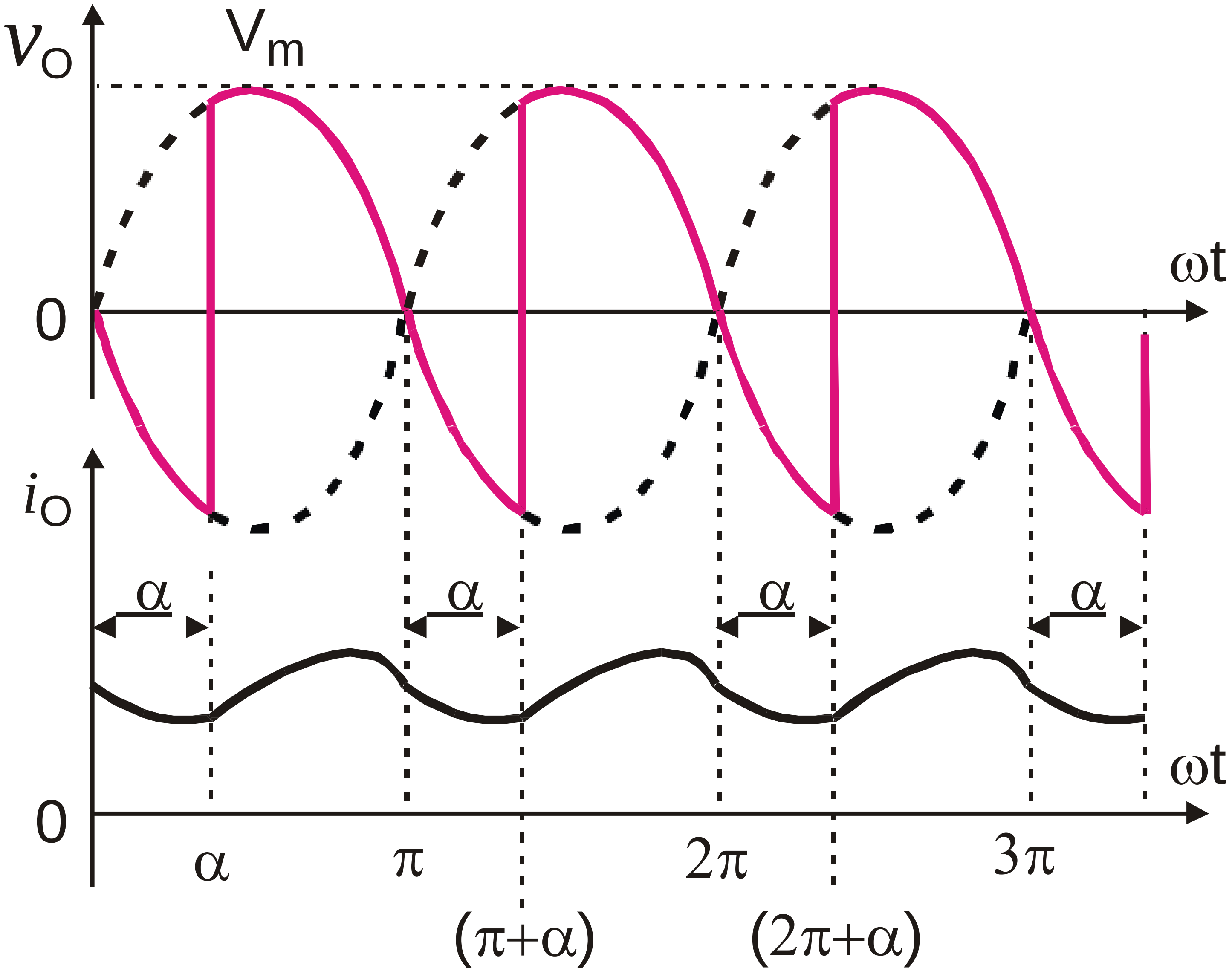
$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\pi} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos\pi + \cos\alpha \right] \; ; \; \cos\pi = -1$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (1 + \cos\alpha)$$

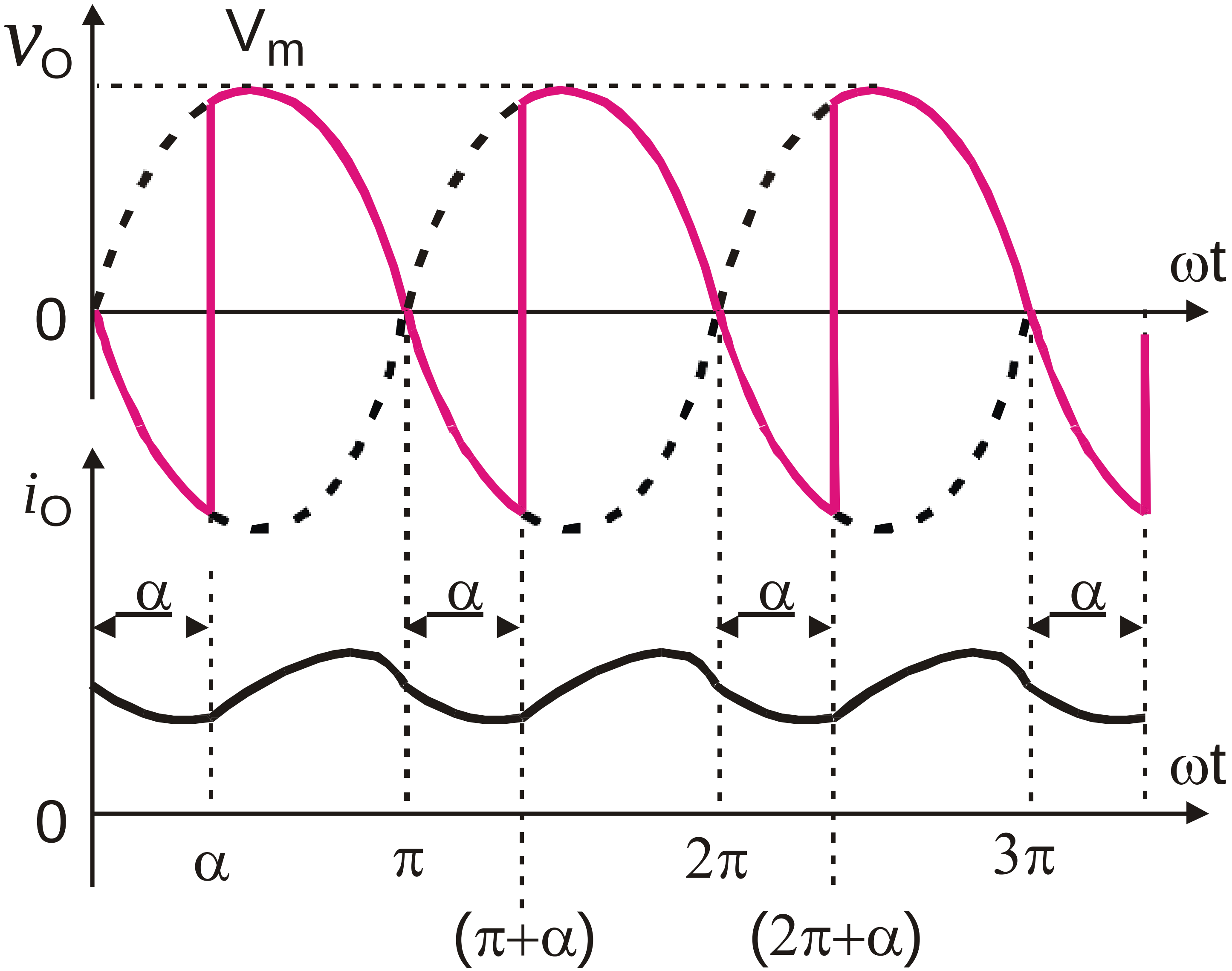
- The load current is discontinuous for low values of load inductance and for large values of trigger angles.
- For large values of load inductance the load current flows continuously without falling to zero.
- Generally the load current is continuous for large load inductance and for low trigger angles.

Continuous Load Current Operation (Without FWD)



15120
3

To Derive
An Expression For
Average / DC Output Voltage
Of
Single Phase Full Wave Controlled Rectifier For Continuous Current Operation without FWD


v_O
V_m
ωt
0
i_O
α
α
α
α
ωt
0
α
π
2π
3π
(π+α)
(2π+α)

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=\alpha}^{(\pi+\alpha)} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \left[\int_{\alpha}^{(\pi+\alpha)} V_m \sin \omega t . d(\omega t) \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{(\pi+\alpha)} \right]$$

$$V_{O(dc)} = V_{dc}$$

$$= \frac{V_m}{\pi} \lfloor \lceil \cos\alpha - \cos(\pi + \alpha) \rceil \rfloor \ ;$$

$$\cos(\pi + \alpha) = -\cos\alpha$$

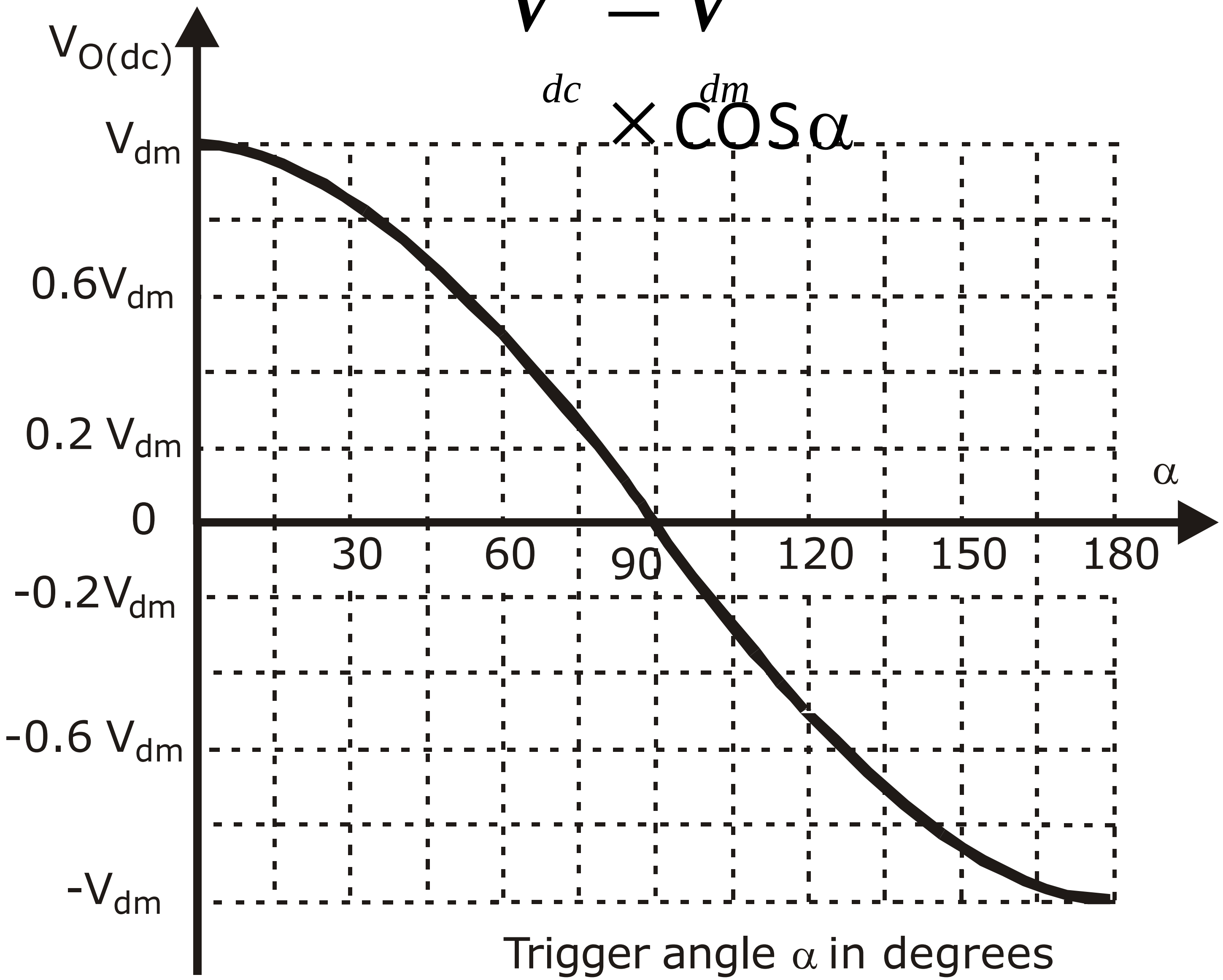
$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} [\cos\alpha + \cos\alpha]$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi} \cos\alpha$$

- By plotting $V_{O(dc)}$ *versus* α,
  we obtain the control characteristic of a single phase full wave controlled rectifier with RL load for continuous load current operation without FWD

Trigger angle α in degrees	$V_{O(dc)}$	Remarks
0	$V_{dm} = \left(\frac{2V_m}{\pi}\right)$	Maximum dc output voltage $V_{dc(\max)} = V_{dm} = \left(\frac{2V_m}{\pi}\right)$
30^0	$0.866\ V_{dm}$	$V_{dc} = V_{dm} \times \cos\alpha$
60^0	$0.5\ V_{dm}$	
90^0	$0\ V_{dm}$	
120^0	$-0.5\ V_{dm}$	
150^0	$-0.866\ V_{dm}$	
180^0	$-V_{dm} = -\left(\frac{2V_m}{\pi}\right)$	

$$V_{dc} = V_{dm} \times \cos\alpha$$



By varying the trigger angle we can vary the output dc voltage across the load. Hence we can control the dc output power flow to the load.

For trigger angle α, 0 to 90^0 $\left(i.e., \ 0 \le \alpha \le 90^0\right)$;

$\cos\alpha$ is positive and hence V_{dc} is positive

V_{dc} & I_{dc} are positive ; $P_{dc} = \left(V_{dc} \times I_{dc}\right)$ is positive

Converter operates as a Controlled Rectifier.

Power flow is from the ac source to the load.

For trigger angle α, 90^0 to 180^0

$$\left(i.e.,\ 90^0 \le \alpha \le 180^0\right),$$

$\cos\alpha$ is negative and hence

V_{dc} is negative; I_{dc} is positive ;

$$P_{dc} = \left(V_{dc} \times I_{dc}\right) \text{ is negative.}$$

In this case the converter operates

as a Line Commutated Inverter.

Power flows from the load ckt. to the i/p ac source.

The inductive load energy is fed back to the

i/p source.

Single Phase
Full Wave Bridge Controlled Rectifier

Drawbacks Of Full Wave Controlled Rectifier With Centre Tapped Transformer

- We require a centre tapped transformer which is quite heavier and bulky.
- Cost of the transformer is higher for the required dc output voltage & output power.
- Hence full wave bridge converters are preferred.

Single Phase Full Wave Bridge Controlled Rectifier

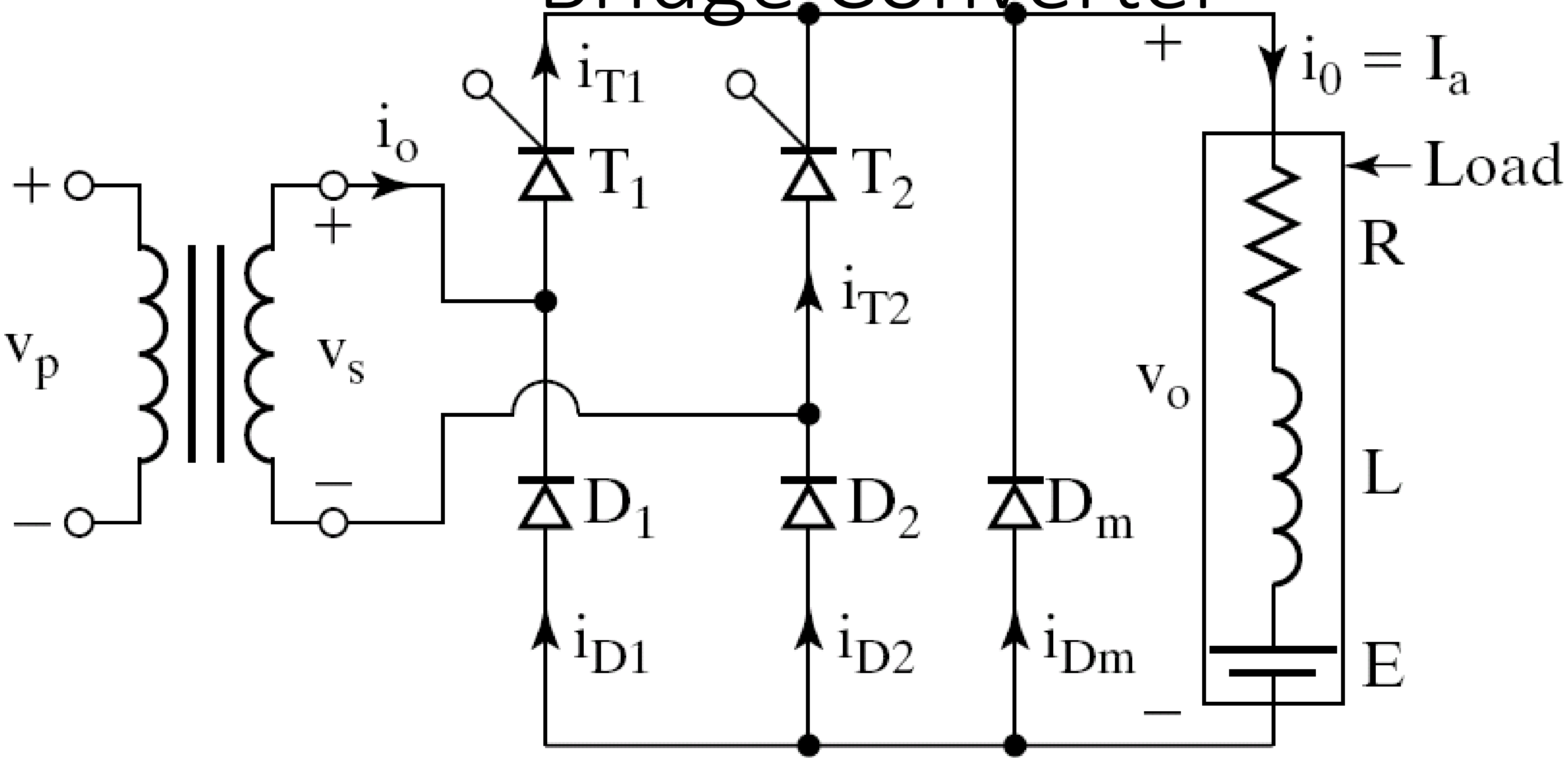
2 types of FW Bridge Controlled Rectifiers are

- Half Controlled Bridge Converter
  (Semi-Converter)
- Fully Controlled Bridge Converter
  (Full Converter)

The bridge full wave controlled rectifier does not require a centre tapped transformer

Single Phase
Full Wave Half Controlled Bridge Converter
(Single Phase Semi Converter)

Single Phase Full Wave Half Controlled Bridge Converter



Trigger Pattern of Thyristors

$$Thyristor\ T_1\ is\ triggered\ at$$

$$\omega t = \alpha,\ at\ \omega t = (2\pi + \alpha),...$$

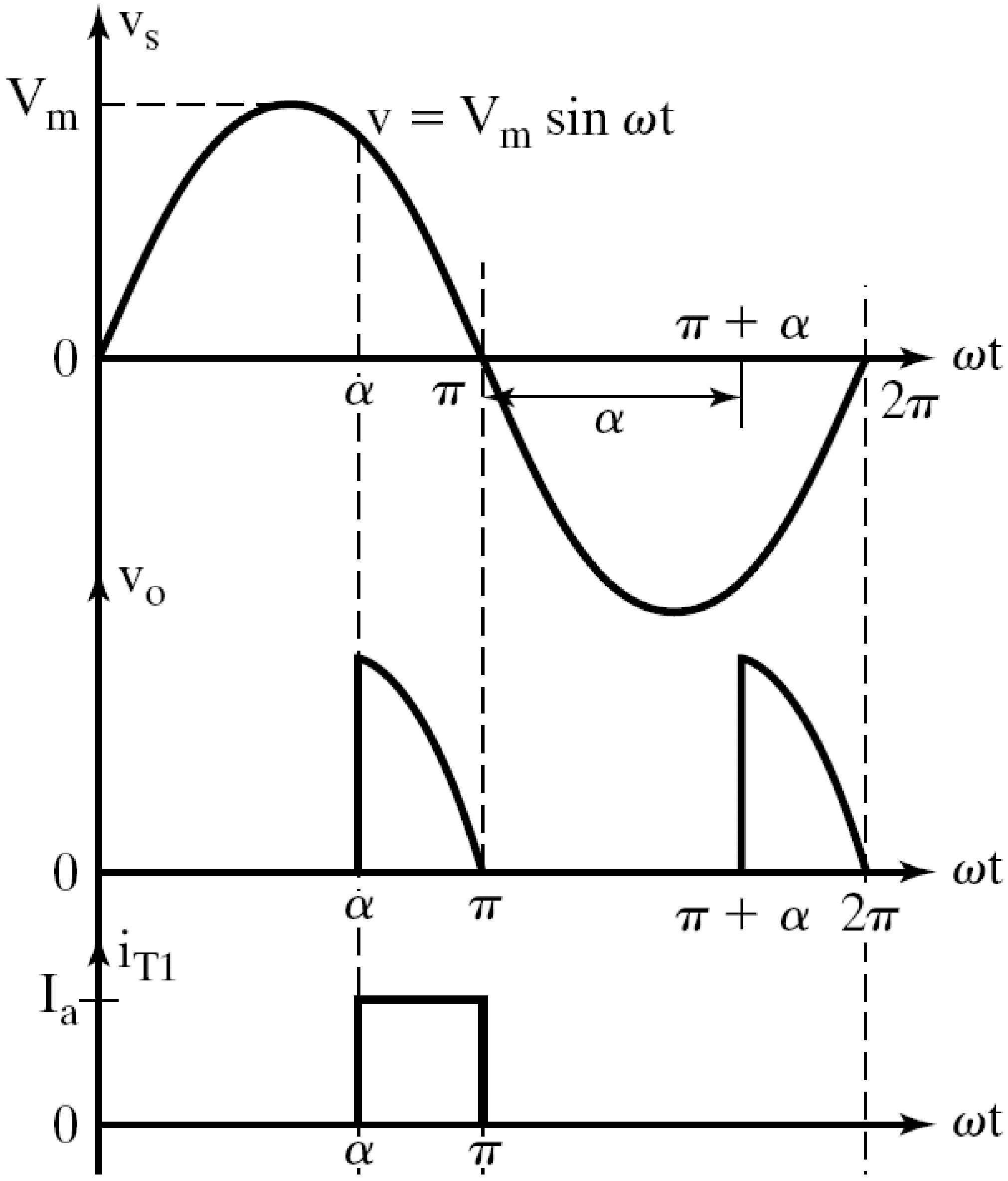
$$Thyristor\ T_2\ is\ triggered\ at$$

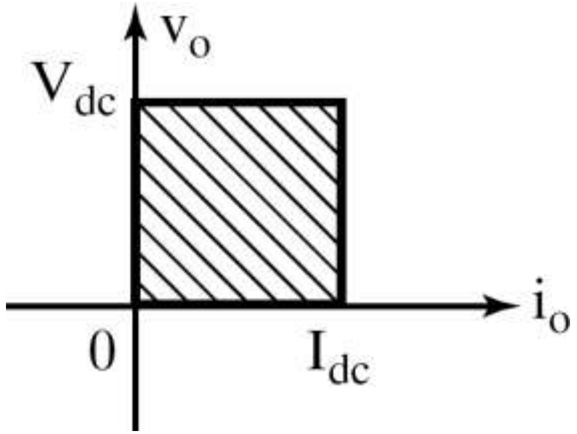
$$\omega t = (\pi + \alpha),\ at\ \omega t = (3\pi + \alpha),...$$

$$The\ time\ delay\ between\ the\ gating$$

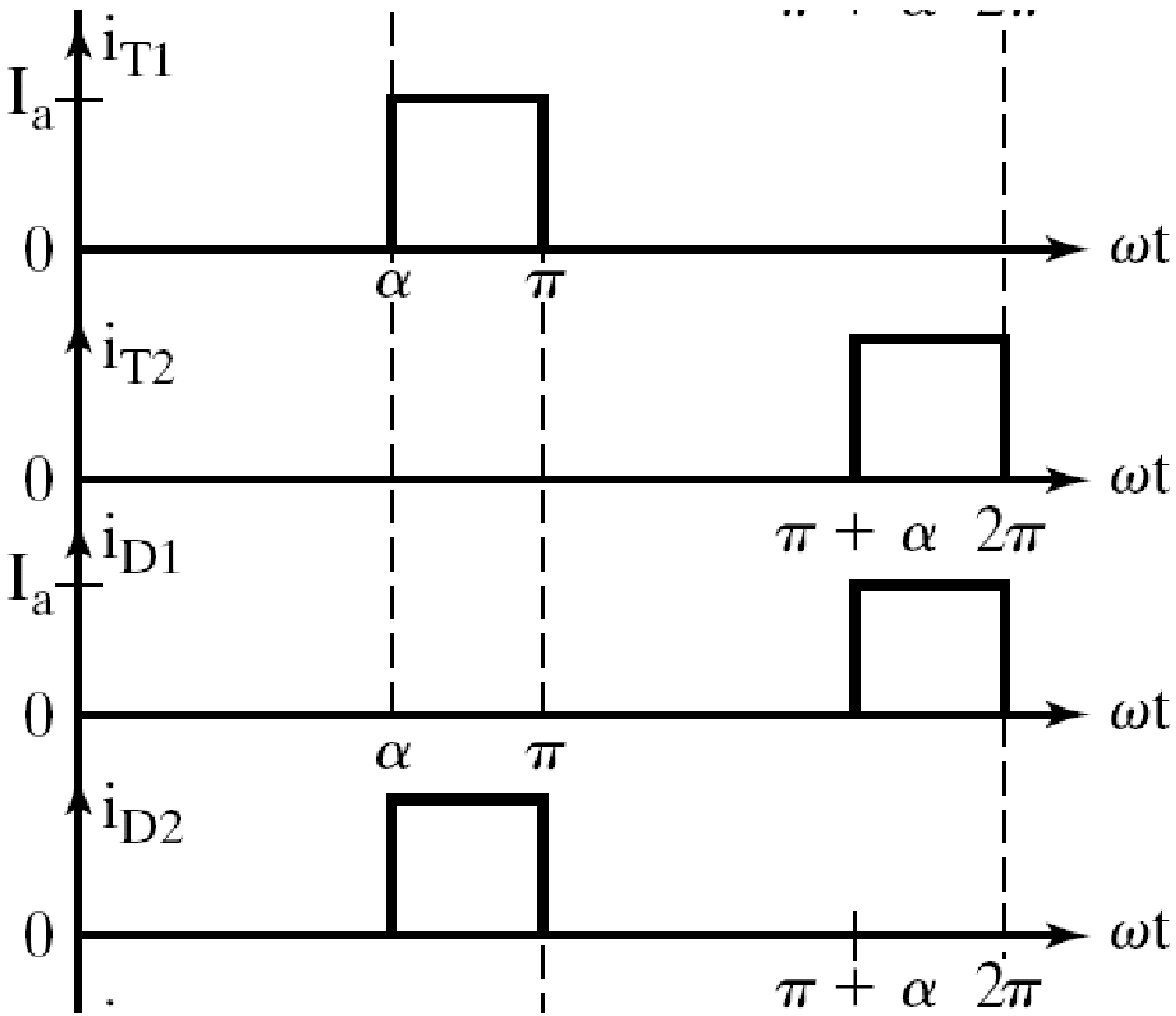
$$signals\ of\ T_1\ \&\ T_2 = \pi\ radians\ or\ 180^0$$

Waveforms of single phase semi-converter with general load & FWD for $\alpha > 90^0$

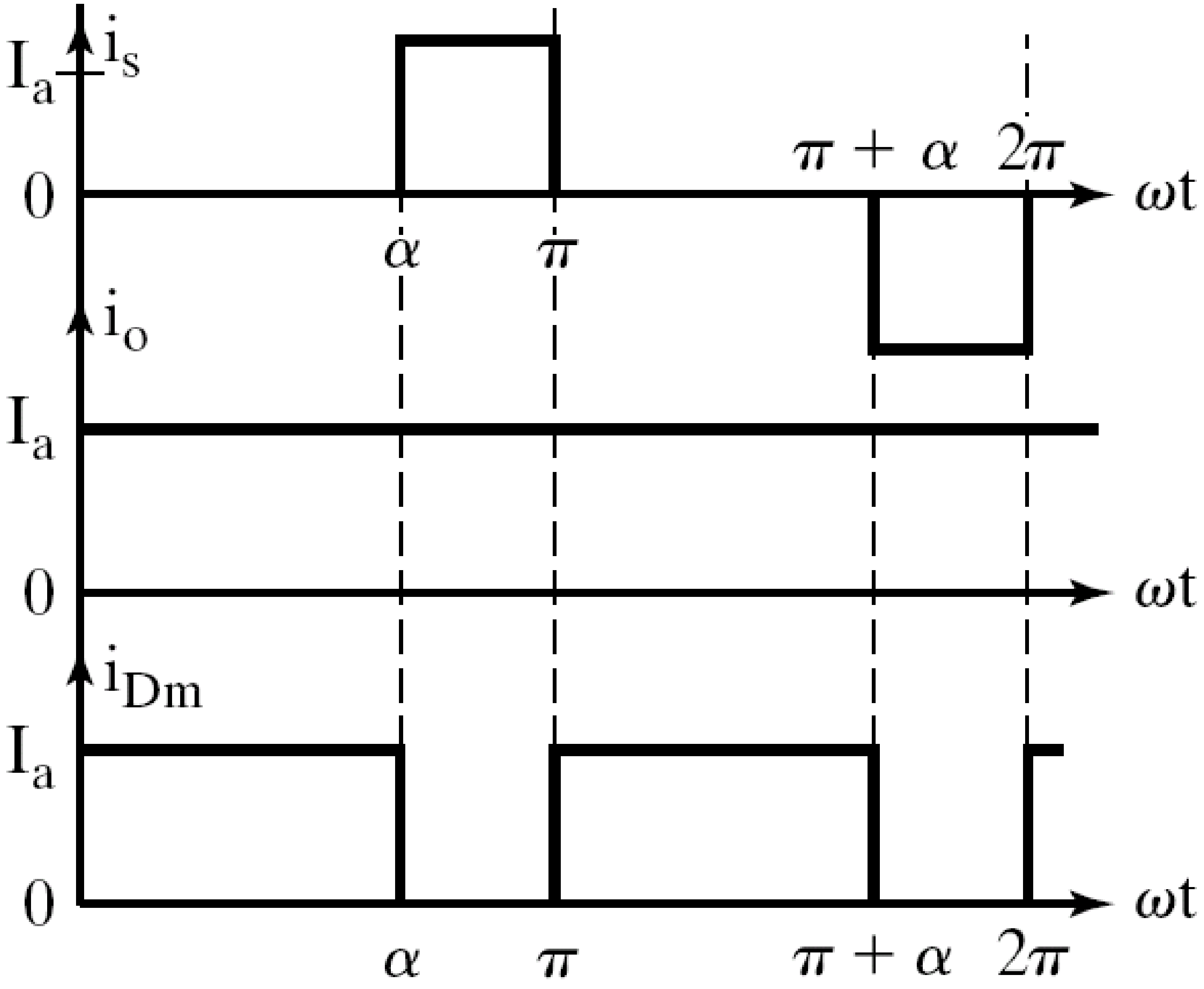




Single Quadrant Operation



i_{T1}
I_a
0
ωt
α
π
i_{T2}
0
ωt
$\pi + \alpha$
2π
i_{D1}
I_a
0
ωt
α
π
i_{D2}
0
ωt
$\pi + \alpha$
2π



I_a
i_s
0
$\pi + \alpha$
2π
ωt
α
π
i_o
I_a
0
ωt
i_{Dm}
I_a
0
ωt
α
π
$\pi + \alpha$
2π

Thyristor T_1 & D_1

conduc

t from $\omega t = \alpha\, to$

π

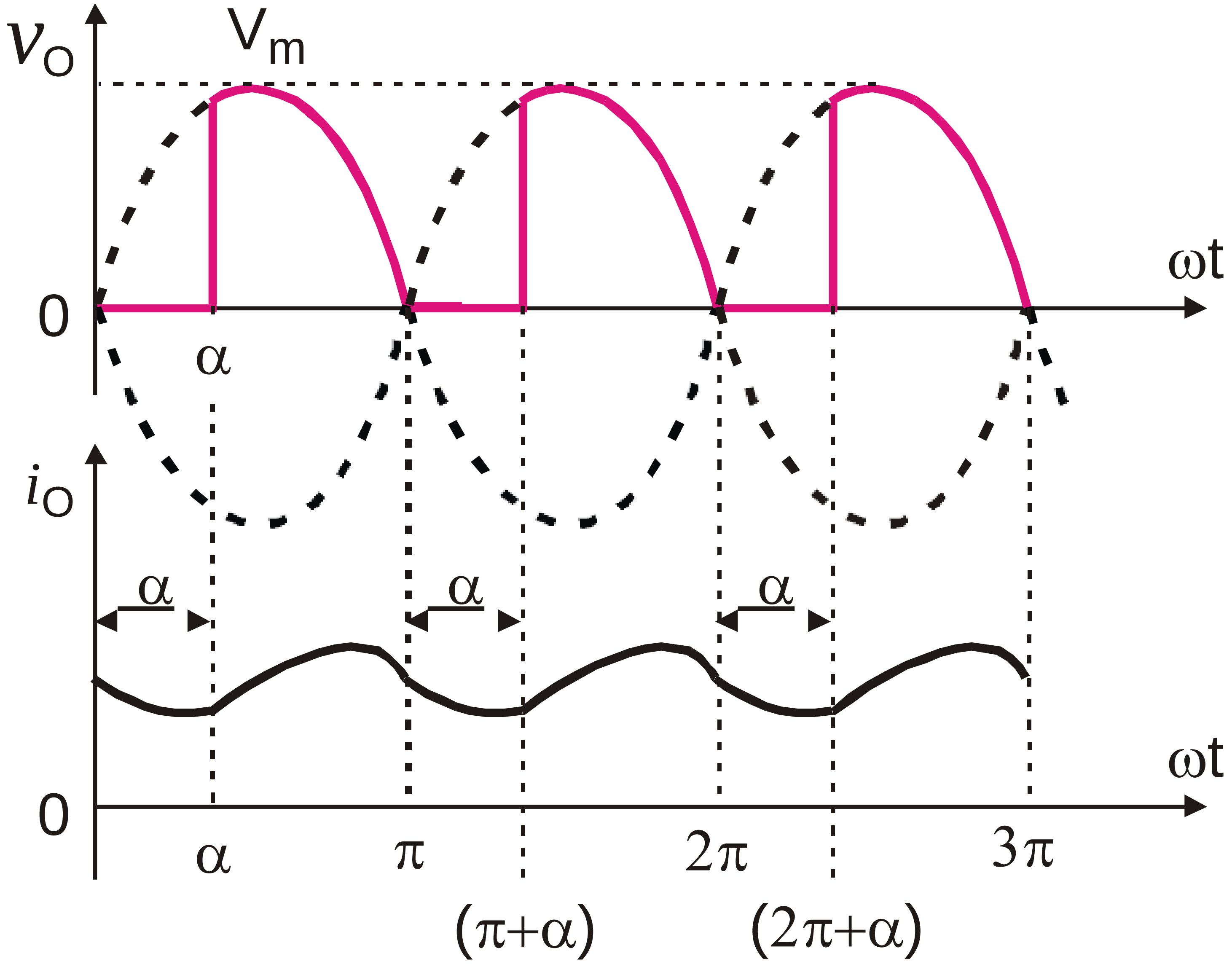
Thyristor T_2 & D_2 conduct

from $\omega t = (\pi + \alpha)\ to\ 2\pi$

FWD conducts during

$\omega t = 0$ to $\alpha,\ \pi\ to\ (\pi + \alpha), \ldots$

Load Voltage & Load Current Waveform of Single Phase Semi Converter for $\alpha < 90^0$ & Continuous load current operation


vO
Vm
0
ωt
α
iO
α
α
α
ωt
0
α
π
2π
3π
(π+α)
(2π+α)

To Derive an Expression For The DC Output Voltage of A Single Phase Semi-Converter With R,L, & E Load & FWD For Continuous, Ripple Free Load Current Operation

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=0}^{\pi} v_O . d(\omega t)$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\pi} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \pi + \cos \alpha \right] \; ; \; \cos \pi = -1$$

$$\therefore \quad V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (1 + \cos \alpha)$$

V_{dc} can be varied from a max.

value of $\dfrac{2V_m}{\pi}$ *to* 0 by varying α from 0 to π.

For $\alpha = 0$, The max. dc o/p voltage obtained is

$$V_{dc(\max)} = V_{dm} = \frac{2V_m}{\pi}$$

Normalized dc o/p voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dn}} = \frac{\frac{V_m}{\pi}(1+\cos\alpha)}{\left(\frac{2V_m}{\pi}\right)} = \frac{1}{2}(1+\cos\alpha)$$

RMS O/P Voltage $V_{O(RMS)}$

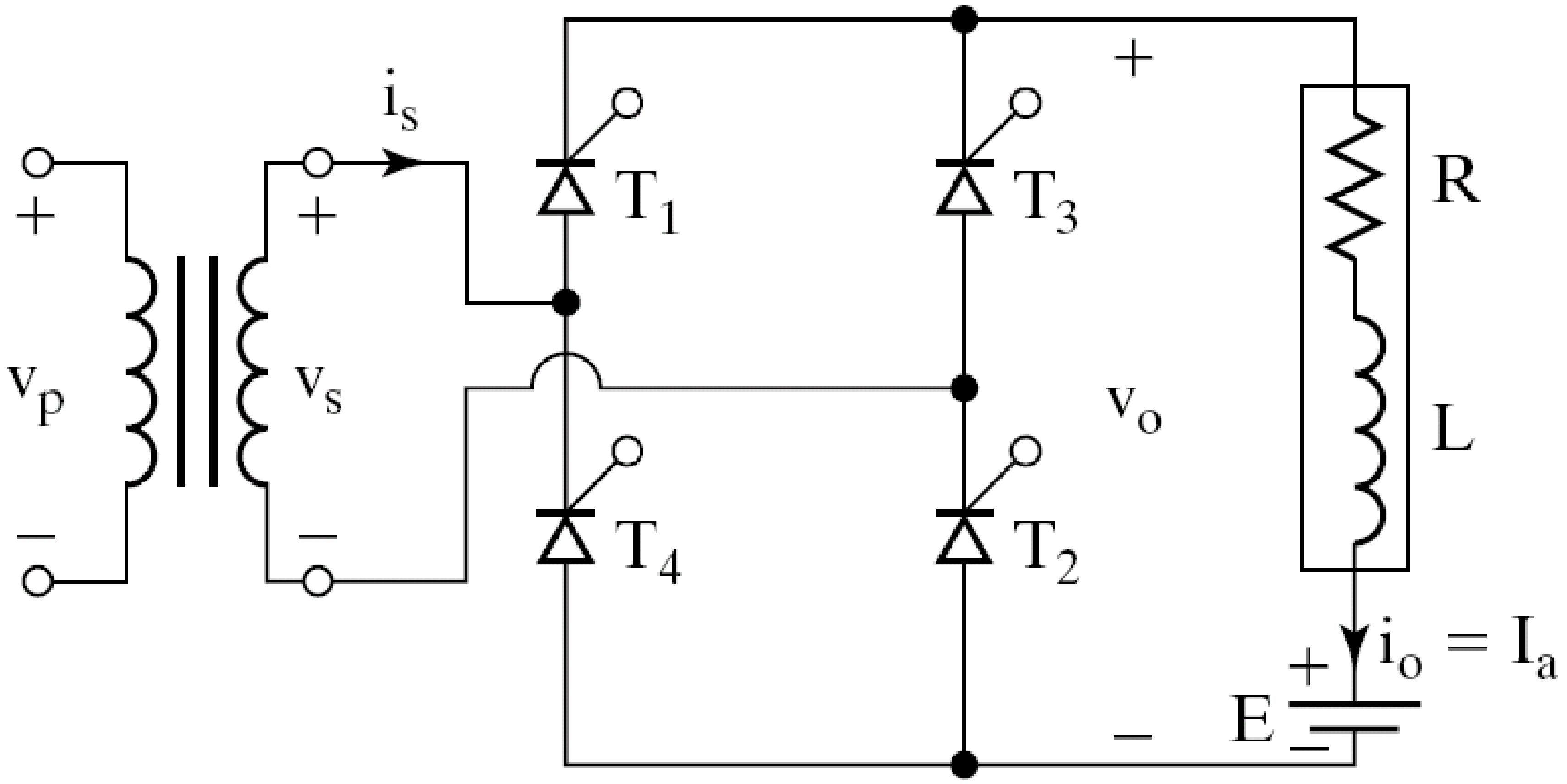
$$V_{O(RMS)} = \left[\frac{2}{2\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t . d\left(\omega t\right) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{2\pi} \int_{\alpha}^{\pi} \left(1 - \cos 2\omega t\right) . d\left(\omega t\right) \right]^{\frac{1}{2}}$$

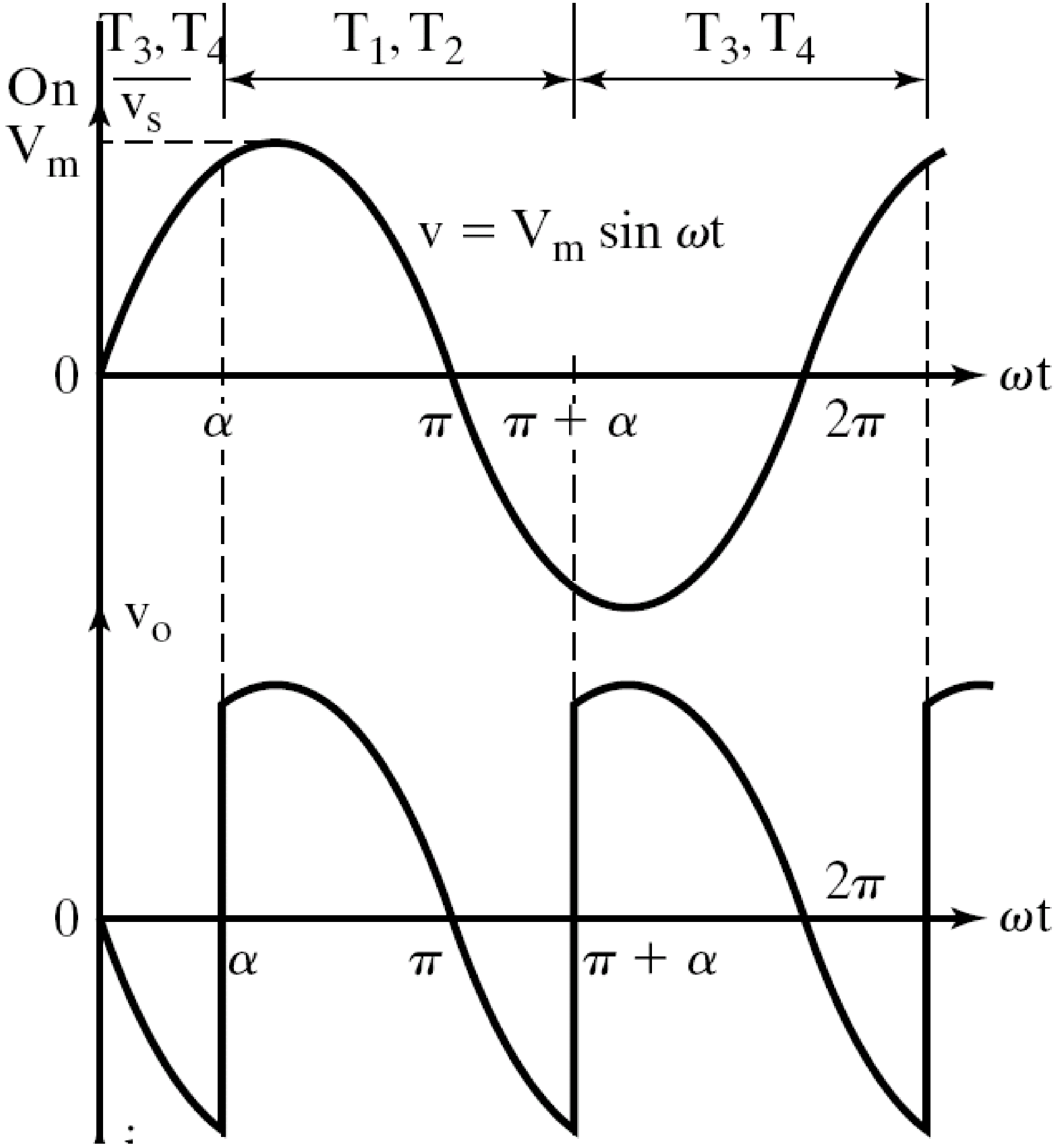
$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \left[\frac{1}{\pi} \left(\pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{\frac{1}{2}}$$

Single Phase Full Converter

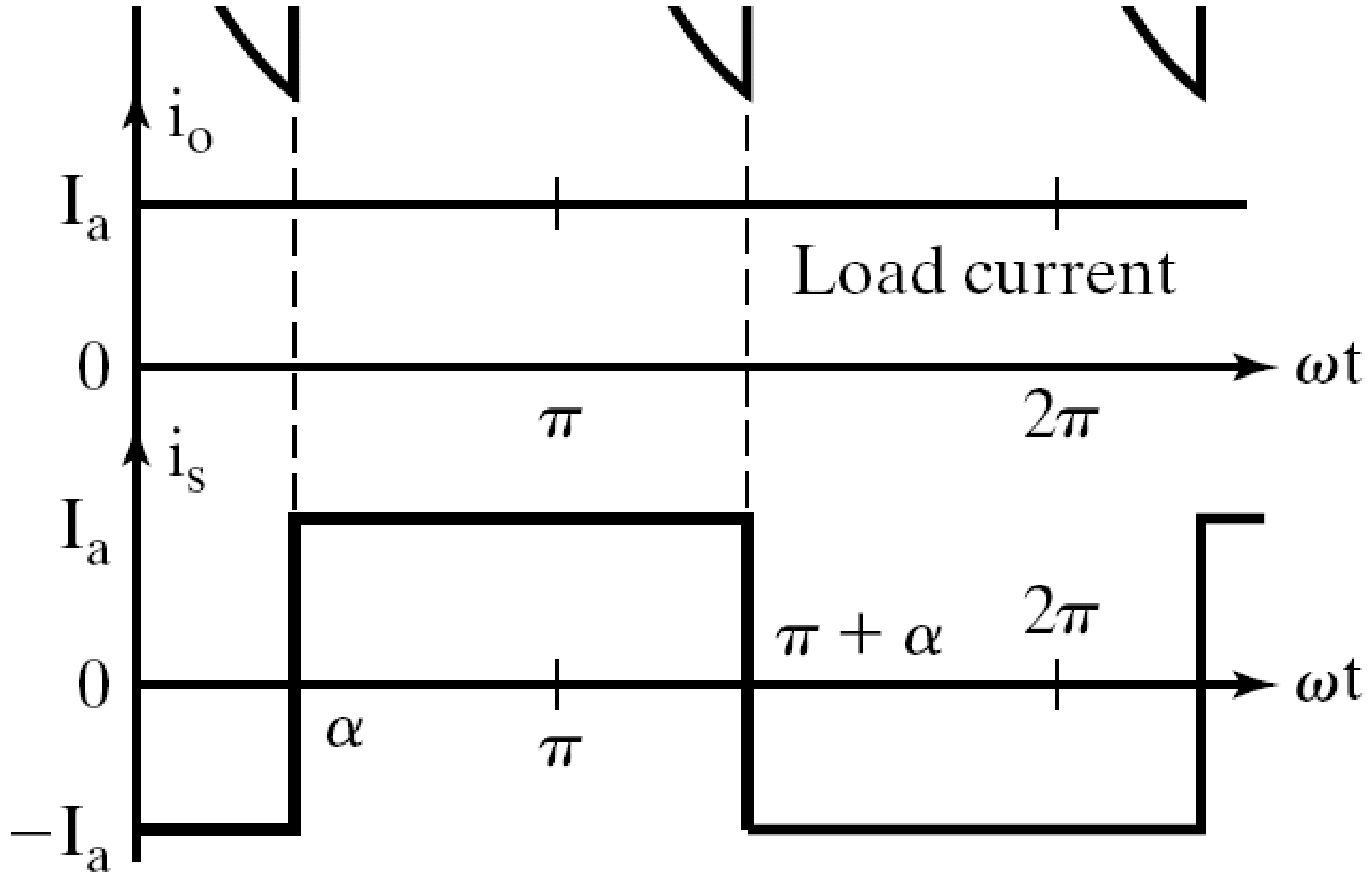
Single Phase Full Converter



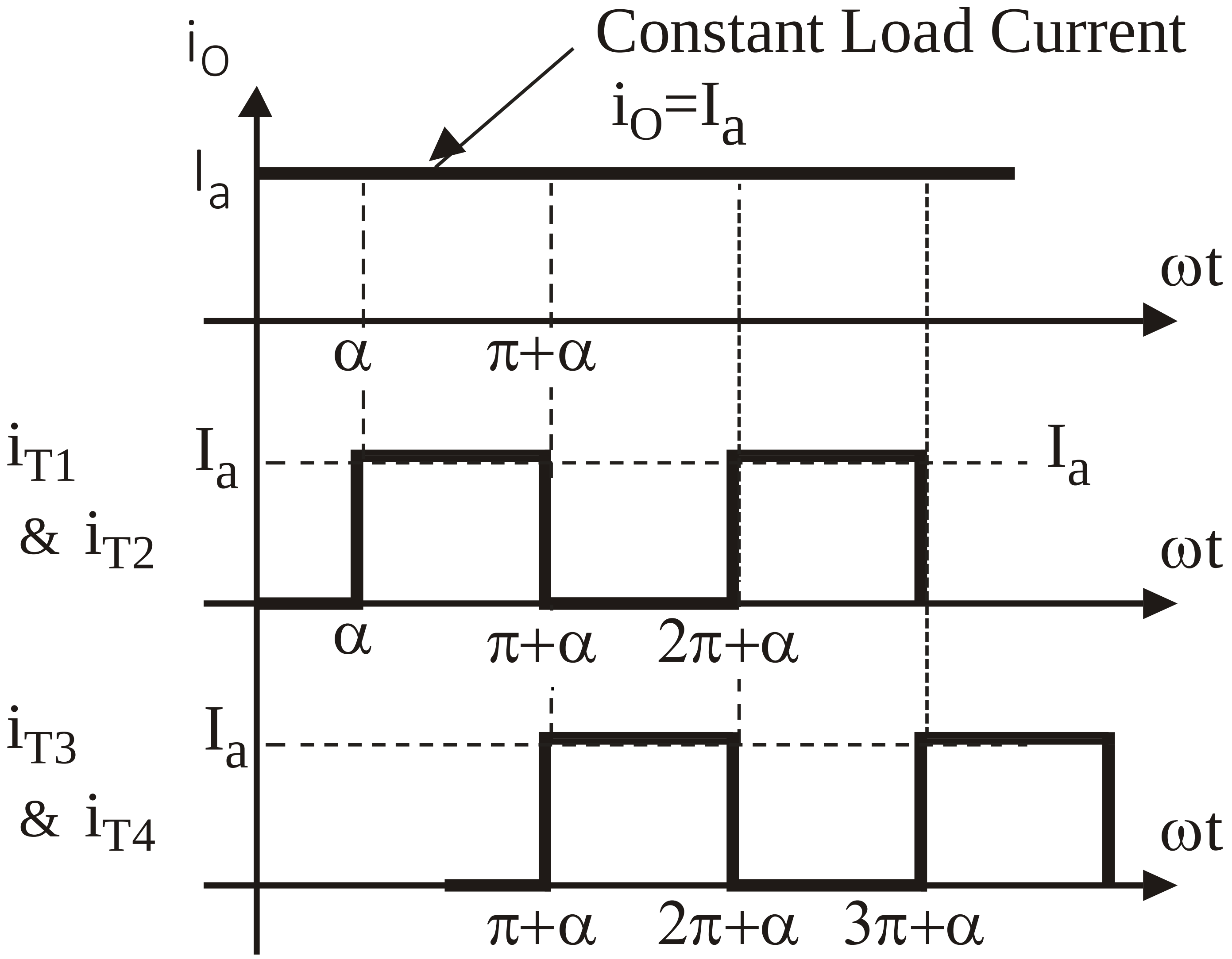
Waveforms of
Single Phase Full Converter
Assuming Continuous (Constant Load Current)
&
Ripple Free Load Current



T3, T4
T1, T2
T3, T4
On
vs
Vm
v = Vm sin ωt
0
ωt
α
π
π + α
2π
vo
2π
0
ωt
α
π
π + α



i_o
I_a
Load current
0
ωt
π
2π
i_s
I_a
π + α
2π
0
ωt
α
π
−I_a


Constant Load Current
iO=Ia
iO
Ia
ωt
α
π+α
iT1
& iT2
Ia
Ia
ωt
α
π+α
2π+α
iT3
& iT4
Ia
ωt
π+α
2π+α
3π+α

To Derive An Expression For The Average DC Output Voltage of a Single Phase Full Converter assuming Continuous & Constant Load Current

The average dc output voltage

can be determined by using the expression

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi}\left[\int_{0}^{2\pi} v_O.d\left(\omega t\right)\right];$$

The o/p voltage waveform consists of two o/p pulses during the input supply time period of 0 to 2π radians. Hence the Average or dc o/p voltage can be calculated as

$$V_{O(dc)} = V_{dc} = \frac{2}{2\pi}\left[\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t . d(\omega t)\right]$$

$$V_{O(dc)} = V_{dc} = \frac{2V_m}{2\pi}\left[-\cos\omega t\right]_{\alpha}^{\pi+\alpha}$$

$$V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi}\cos\alpha$$

Maximum average dc output voltage is calculated for a trigger angle $\alpha = 0^0$ and is obtained as

$$V_{dc(\max)} = V_{dm} = \frac{2V_m}{\pi} \times \cos(0) = \frac{2V_m}{\pi}$$

$$\therefore V_{dc(\max)} = V_{dm} = \frac{2V_m}{\pi}$$

The normalized average output voltage is given by

$$V_{dcn} = V_n = \frac{V_{O(dc)}}{V_{dc(\max)}} = \frac{V_{dc}}{V_{dm}}$$

$$\therefore V_{dcn} = V_n = \frac{\frac{2V_m}{\pi}\cos\alpha}{\frac{2V_m}{\pi}} = \cos\alpha$$

By plotting $V_{O(dc)}$ *versus* α,

we obtain the control characteristic of a single phase full wave fully controlled bridge converter

(single phase full converter)

for constant & continuous

load current operation.

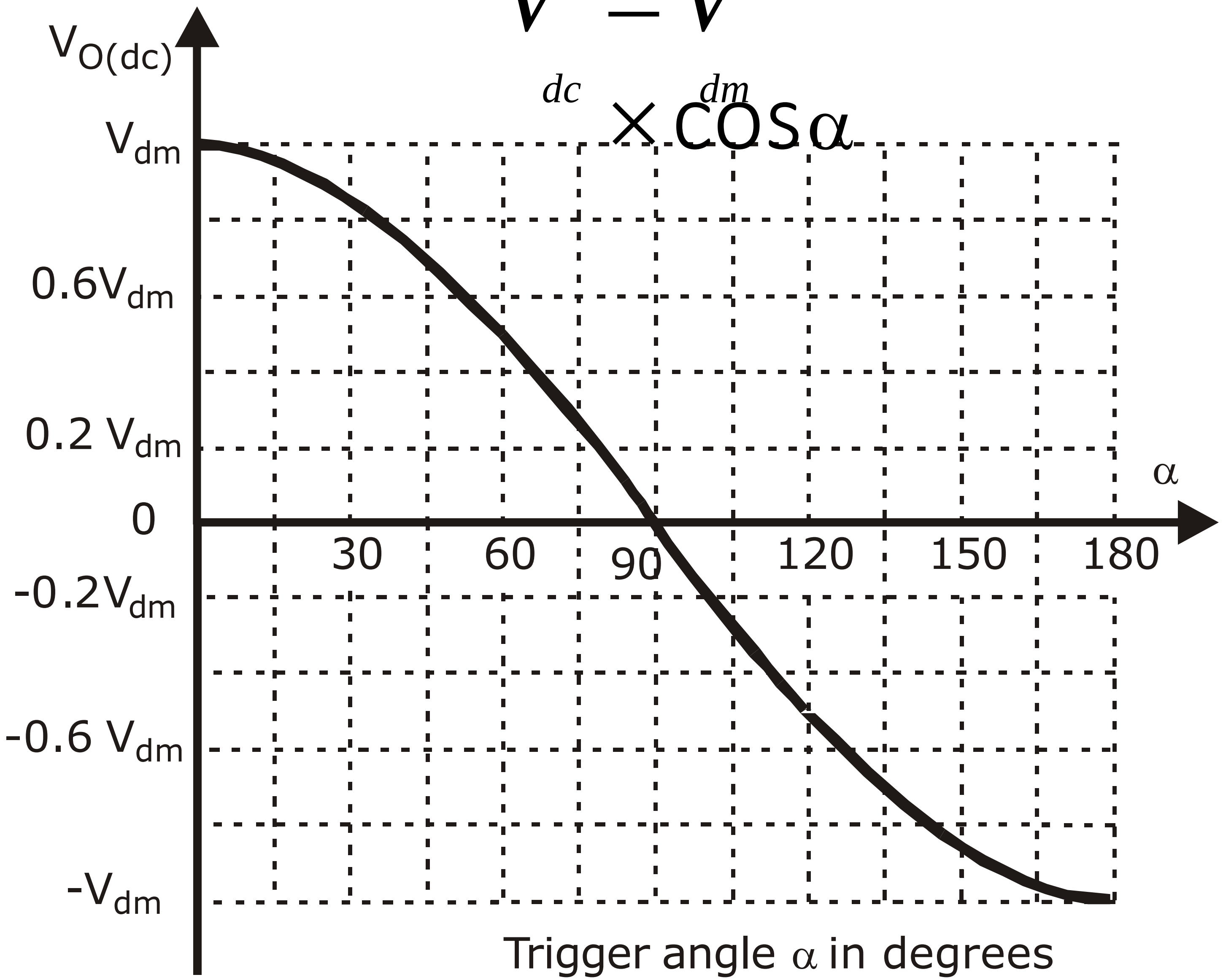
To plot the control characteristic of a Single Phase Full Converter for constant & continuous load current operation.

- We use the equation for the average/ dc output voltage

$$V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi}\cos\alpha$$

Trigger angle α in degrees	$V_{O(dc)}$	Remarks
0	$V_{dm} = \left(\dfrac{2V_m}{\pi}\right)$	Maximum dc output voltage $V_{dc(\max)} = V_{dm} = \left(\dfrac{2V_m}{\pi}\right)$
30^0	$0.866\ V_{dm}$	
60^0	$0.5\ V_{dm}$	
90^0	$0\ V_{dm}$	
120^0	$-0.5\ V_{dm}$	
150^0	$-0.866\ V_{dm}$	
180^0	$-V_{dm} = -\left(\dfrac{2V_m}{\pi}\right)$	

$$V_{dc} = V_{dm} \times \cos\alpha$$



- During the period from $\omega t = \alpha$ to π the input voltage v_S and the input current i_S are both positive and the power flows from the supply to the load.
- The converter is said to be operated in the rectification mode

Controlled Rectifier Operation

for $0 < \alpha < 90^0$

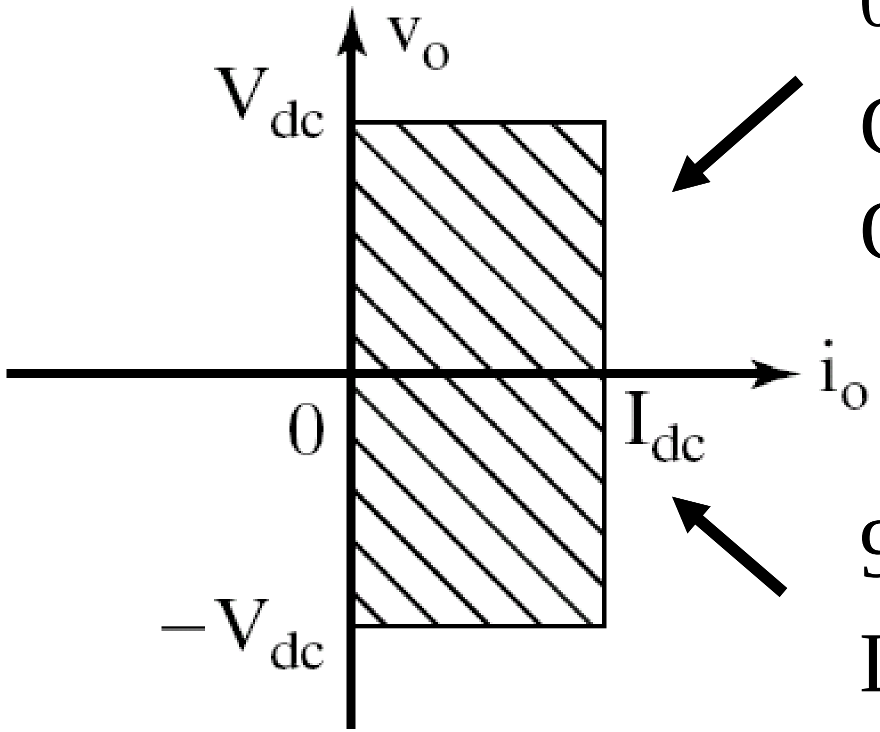
- During the period from $\omega t = \pi$ to $(\pi+\alpha)$, the input voltage v_S is negative and the input current i_S is positive and the output power becomes negative and there will be reverse power flow from the load circuit to the supply.
- The converter is said to be operated in the inversion mode.

Line Commutated Inverter Operation

for $90^0 < \alpha < 180^0$

Two Quadrant Operation of a Single Phase Full Converter

Two Quadrant Operation of a Single Phase Full Converter



$0 < \alpha < 90^0$

Controlled Rectifier Operation

$90^0 < \alpha < 180^0$

Line Commutated Inverter Operation

To Derive An Expression For The RMS Value Of The Output Voltage

The rms value of the output voltage is calculated as

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_{0}^{2\pi} v_O^2 . d(\omega t)\right]}$$

The single phase full converter gives two output voltage pulses during the input supply time period and hence the single phase full converter is referred to as a two pulse converter. The rms output voltage can be calculated as

$$V_{O(RMS)} = \sqrt{\frac{2}{2\pi}\left[\int_{\alpha}^{\pi+\alpha} v_O^2 . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{1}{\pi}\left[\int_{\alpha}^{\pi+\alpha} V_m^2 \sin^2 \omega t . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\pi}\left[\int_{\alpha}^{\pi+\alpha} \sin^2 \omega t . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\pi}\left[\int_{\alpha}^{\pi+\alpha} \frac{(1-\cos 2\omega t)}{2} . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[\int_{\alpha}^{\pi+\alpha} d(\omega t) - \int_{\alpha}^{\pi+\alpha} \cos 2\omega t . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[\left(\omega t\right)\Big/_{\alpha}^{\pi+\alpha} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{\pi+\alpha} \right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[\left(\pi+\alpha-\alpha\right) - \left(\frac{\sin 2\left(\pi+\alpha\right) - \sin 2\alpha}{2}\right) \right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[\left(\pi\right) - \left(\frac{\sin\left(2\pi+2\alpha\right) - \sin 2\alpha}{2}\right) \right]};$$

$$\sin\left(2\pi+2\alpha\right) = \sin 2\alpha$$

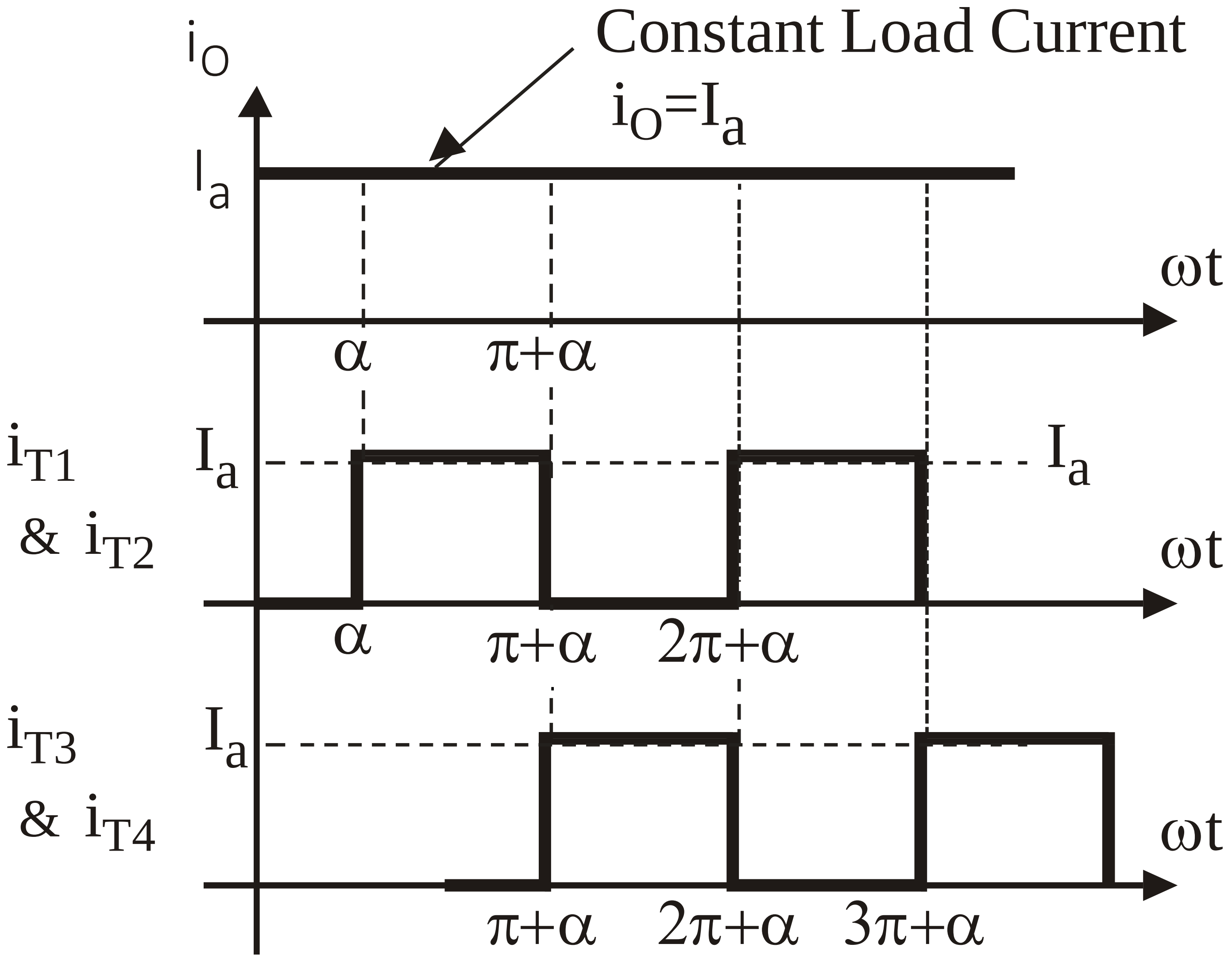
$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[(\pi) - \left(\frac{\sin 2\alpha - \sin 2\alpha}{2}\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}(\pi) - 0} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}}$$

$$\therefore V_{O(RMS)} = \frac{V_m}{\sqrt{2}} = V_S$$

Hence the rms output voltage is same as the rms input supply voltage

Thyristor Current Waveforms


Constant Load Current
iO=Ia
iO
Ia
ωt
α
π+α
iT1
& iT2
Ia
Ia
ωt
α
π+α
2π+α
iT3
& iT4
Ia
ωt
π+α
2π+α
3π+α

The rms thyristor current can be calculated as

$$I_{T(RMS)} = \frac{I_{O(RMS)}}{\sqrt{2}}$$

The average thyristor current can be calculated as

$$I_{T(Avg)} = \frac{I_{O(dc)}}{2}$$

THREE PHASE LINE COMMUTATED CONVERTERS

Introduction to Three phase converters

3 Phase Controlled Rectifiers

- Three phase converters are 3-phase controlled rectifiers which are used to convert ac input power supply into dc output power across the load

Features of 3-phase controlled rectifiers

- Operate from 3 phase ac supply voltage.
- They provide higher dc output voltage.
- Higher dc output power.
- Higher output voltage ripple frequency.
- Filtering requirements are simplified for smoothing out load voltage and load current.

- Extensively used in high power variable speed industrial dc drives.
- Three single phase half-wave converters can be connected together to form a three phase half-wave converter.

Classification of 3-phase converters

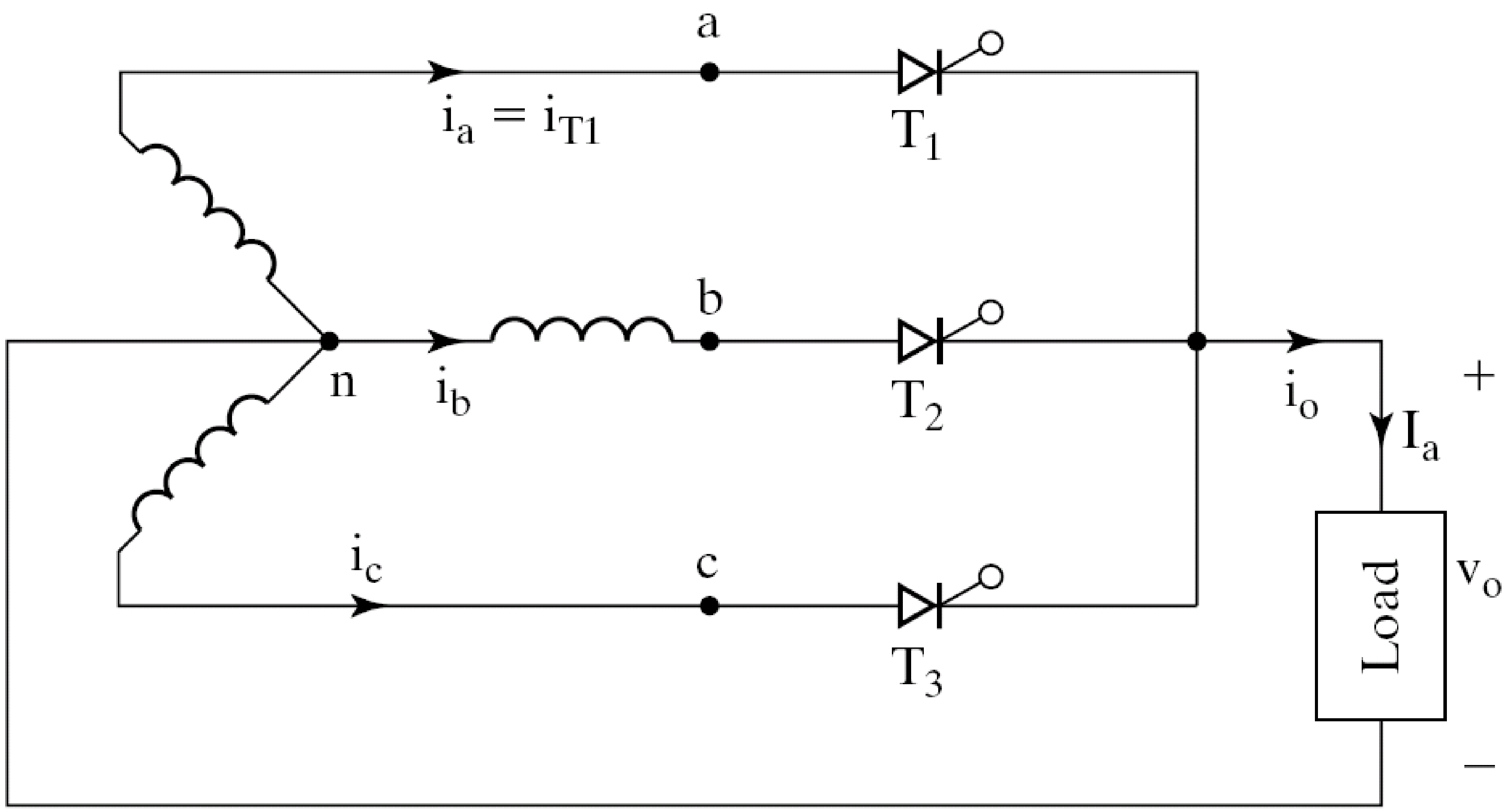
- 3-phase half wave converter
- 3-phase semi converter
- 3-phase full converter
- 3- phase dual converter

Classification according to no of pulses in the output wave

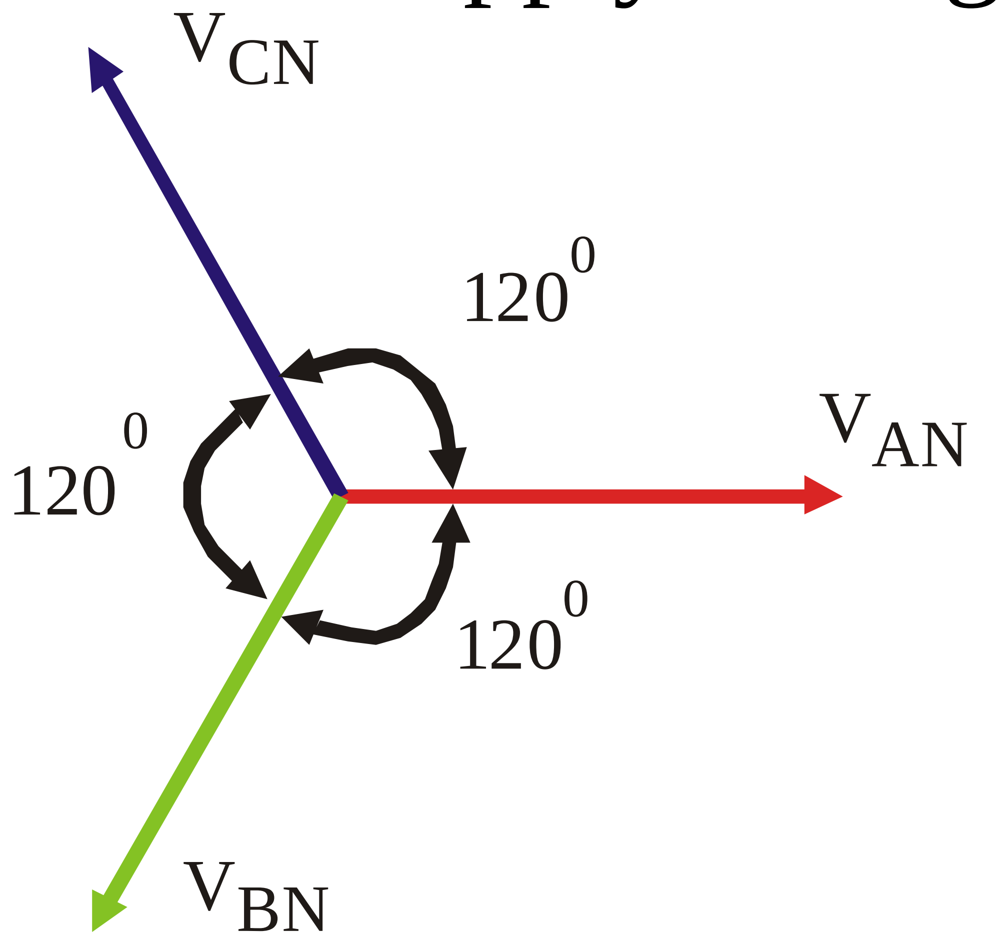
- 3- pulse converter
- 6-pulse converter
- 12- pulse converter

3-Phase
Half Wave Converter
(3-Pulse Converter)
with
R-L Load
Continuous & Constant
Load Current Operation

Circuit Diagram of 3- pulse converter



Vector Diagram of
3 Phase Supply Voltages



$$v_{RN} = v_{AN}$$

$$v_{YN} = v_{BN}$$

$$v_{BN} = v_{CN}$$

3 Phase Supply Voltage Equations

We deifine three line to neutral voltages (3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t;$$

$V_m =$ Max. Phase Voltage

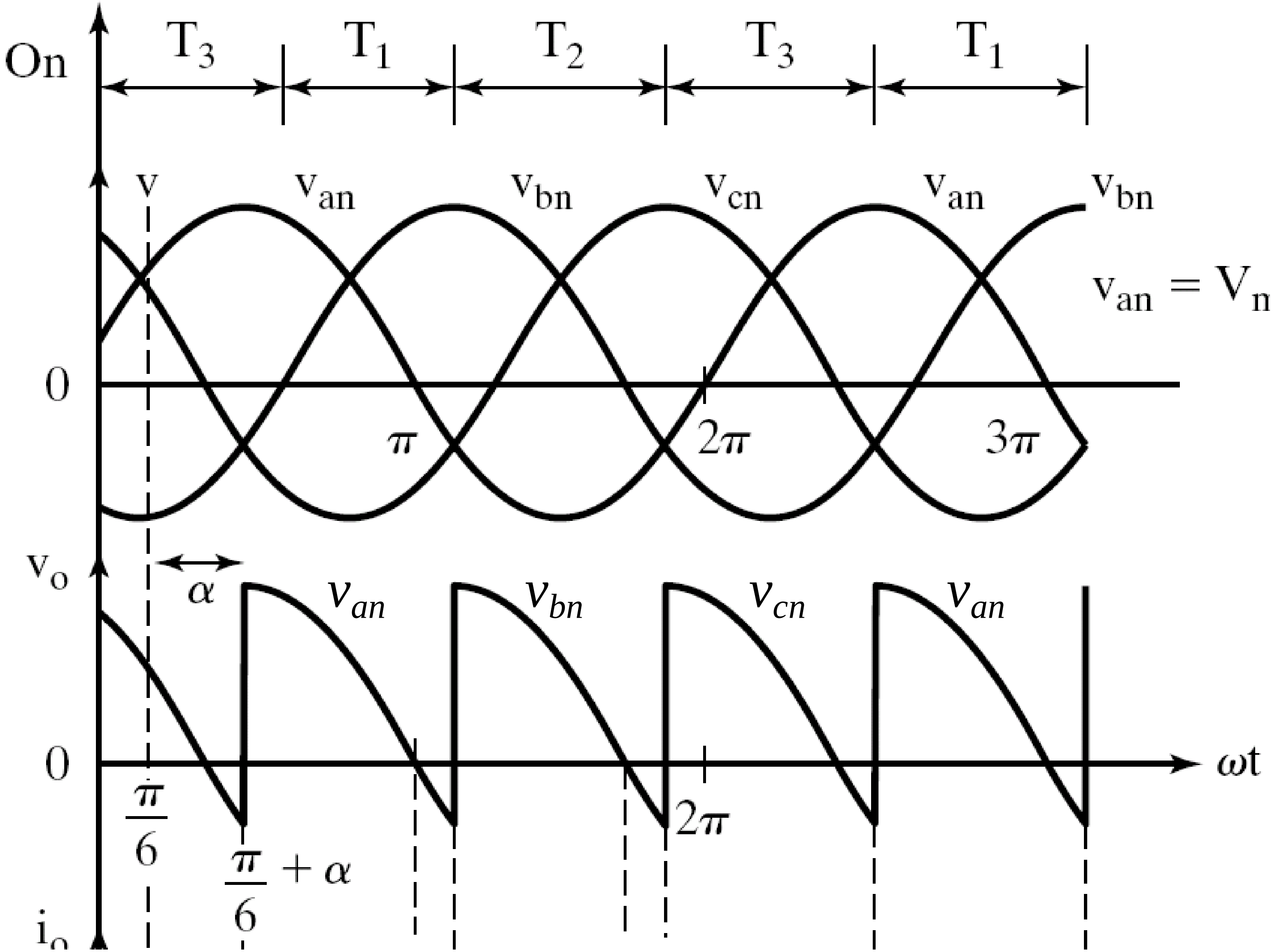
$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$= V_m \sin\left(\omega t - 120^0\right)$$

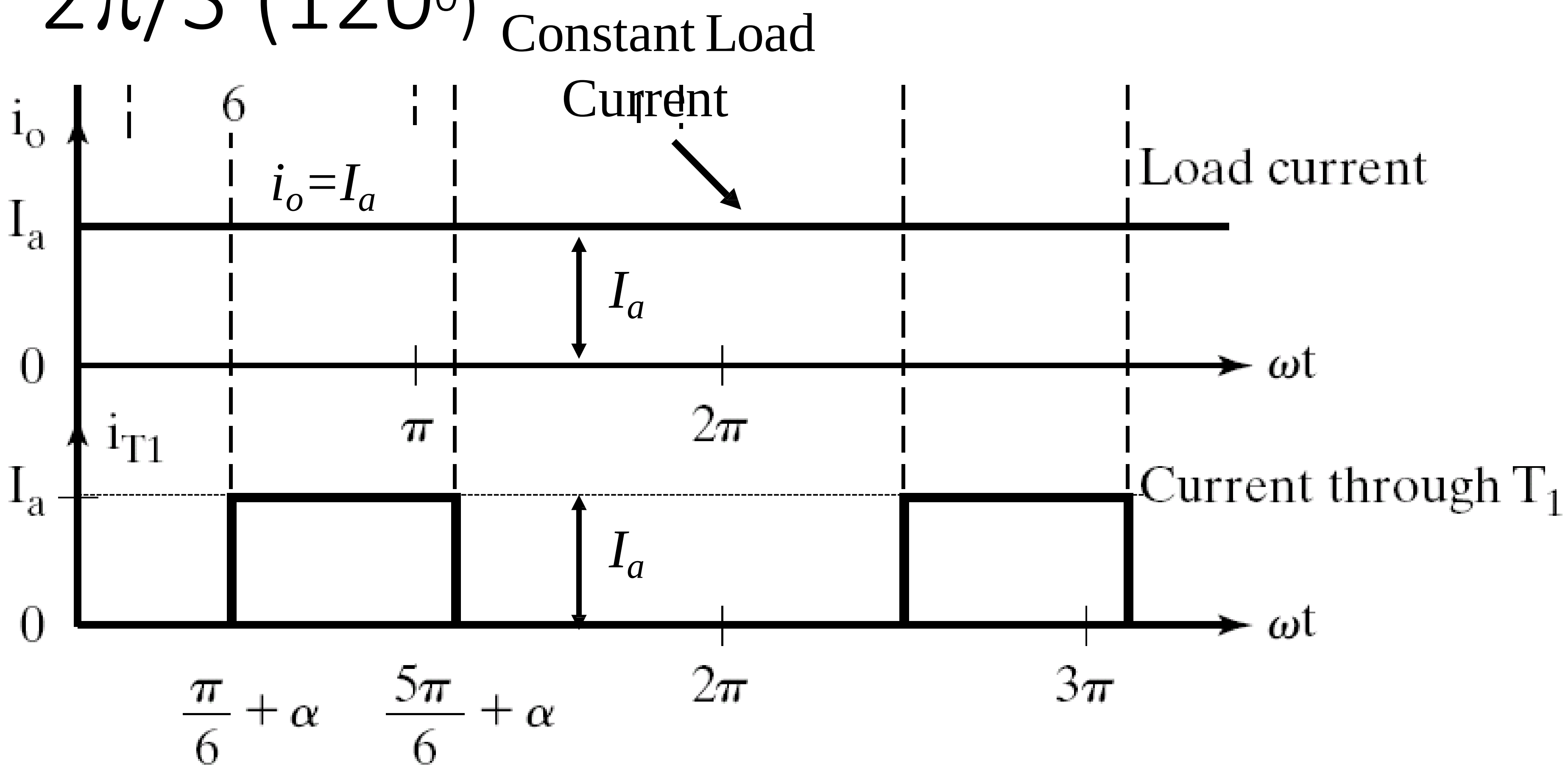
$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$= V_m \sin\left(\omega t + 120^0\right)$$

$$= V_m \sin\left(\omega t - 240^0\right)$$


On
T3
T1
T2
T3
T1
v
van
vbn
vcn
van
vbn
van = Vn
0
π
2π
3π
vo
α
van
vbn
vcn
van
0
ωt
π/6
2π
π/6 + α
io

Each thyristor conducts for $2\pi/3$ (120^0)



To Derive an Expression for the Average Output Voltage of a 3-Phase Half Wave Converter with RL Load for Continuous Load Current

$$T_1 \ is \ triggered \ \ at \ \omega t = \left(\frac{\pi}{6} + \alpha \right) = \left(30^0 + \alpha \right)$$

$$T_2 \ is \ triggered \ \ at \ \omega t = \left(\frac{5\pi}{6} + \alpha \right) = \left(150^0 + \alpha \right)$$

$$T_3 \ is \ triggered \ \ at \ \omega t = \left(\frac{7\pi}{6} + \alpha \right) = \left(270^0 + \alpha \right)$$

Each thytistor conducts for 120^0 or $\frac{2\pi}{3}$ radians

If the reference phase voltage is

$v_{RN} = v_{an} = V_m \sin \omega t$, the average or dc output voltage for continuous load current is calculated using the equation

$$V_{dc} = \frac{3}{2\pi} \left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin \omega t . d(\omega t) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} \sin\omega t.d(\omega t)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[(-\cos\omega t)\Big/_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha}\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos\left(\frac{5\pi}{6}+\alpha\right)+\cos\left(\frac{\pi}{6}+\alpha\right)\right]$$

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Note from the trigonometric relationship

$$\cos(A+B) = (\cos A.\cos B - \sin A.\sin B)$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\begin{array}{l} -\cos\left(\frac{5\pi}{6}\right)\cos(\alpha) + \sin\left(\frac{5\pi}{6}\right)\sin(\alpha) \\ \qquad + \cos\left(\frac{\pi}{6}\right).\cos(\alpha) - \sin\left(\frac{\pi}{6}\right)\sin(\alpha) \end{array}\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\begin{array}{l} -\cos(150^0)\cos(\alpha) + \sin(150^0)\sin(\alpha) \\ \qquad + \cos(30^0).\cos(\alpha) - \sin(30^0)\sin(\alpha) \end{array}\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\begin{array}{l} -\cos\left(180^0 - 30^0\right)\cos(\alpha) + \sin\left(180^0 - 30^0\right)\sin(\alpha) \\ \qquad + \cos\left(30^0\right).\cos(\alpha) - \sin\left(30^0\right)\sin(\alpha) \end{array}\right]$$

Note: $\cos\left(180^0 - 30^0\right) = -\cos\left(30^0\right)$

$$\sin\left(180^0 - 30^0\right) = \sin\left(30^0\right)$$

$$\therefore V_{dc} = \frac{3V_m}{2\pi}\left[\begin{array}{l} +\cos\left(30^0\right)\cos(\alpha) + \sin\left(30^0\right)\sin(\alpha) \\ \qquad + \cos\left(30^0\right).\cos(\alpha) - \sin\left(30^0\right)\sin(\alpha) \end{array}\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[2\cos\left(30^0\right)\cos(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[2\times\frac{\sqrt{3}}{2}\cos(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\sqrt{3}\cos(\alpha)\right] = \frac{3\sqrt{3}V_m}{2\pi}\cos(\alpha)$$

$$V_{dc} = \frac{3V_{Lm}}{2\pi}\cos(\alpha)$$

Where $V_{Lm} = \sqrt{3}V_m =$ Max. line to line supply voltage

The maximum average or dc output voltage is obtained at a delay angle $\alpha = 0$ and is given by

$$V_{dc(\max)} = V_{dm} = \frac{3\sqrt{3}\,V_m}{2\pi}$$

Where V_m is the peak phase voltage.

And the normalized average output voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = \cos\alpha$$

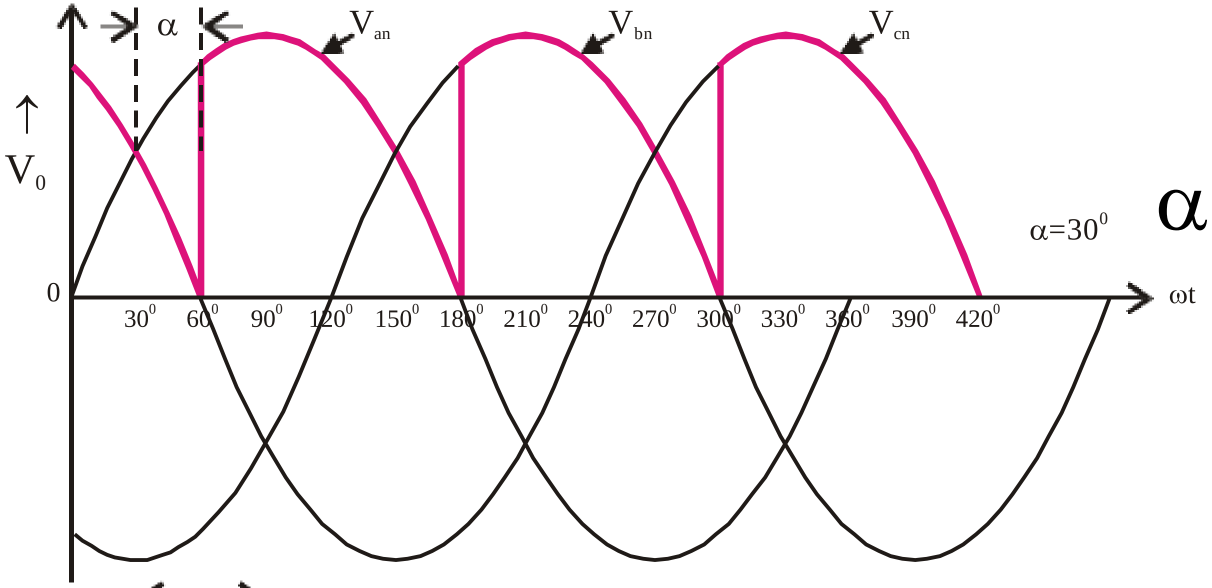
The rms value of output voltage is found by using the equation

$$V_{O(RMS)} = \left[\frac{3}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m^2 \sin^2 \omega t . d(\omega t) \right]^{\frac{1}{2}}$$

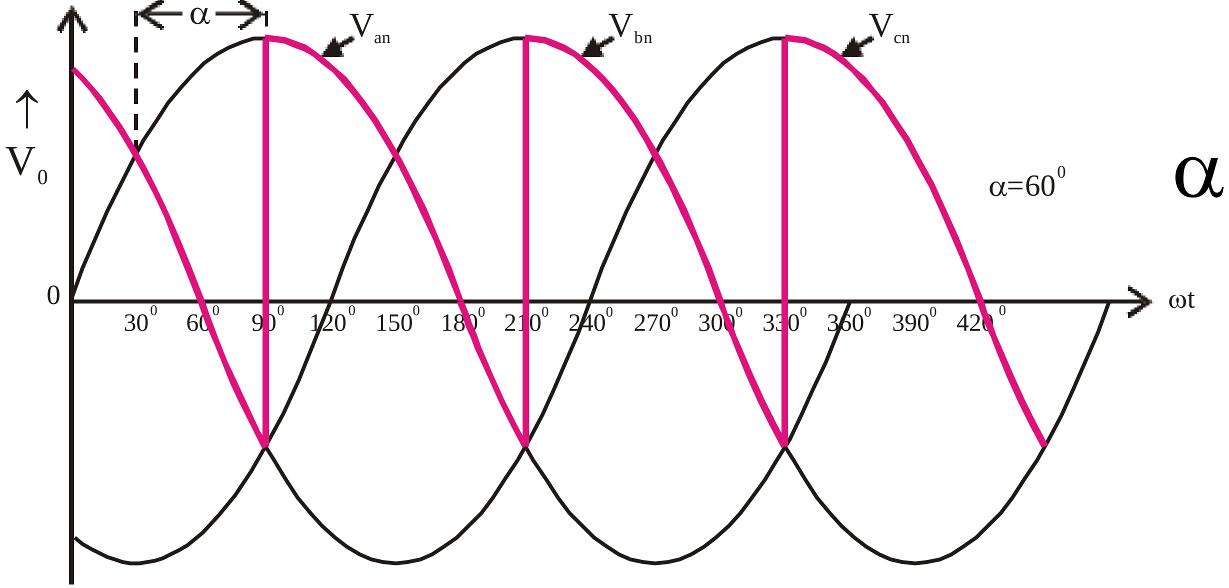
and we obtain

$$V_{O(RMS)} = \sqrt{3} V_m \left[\frac{1}{6} + \frac{\sqrt{3}}{8\pi} \cos 2\alpha \right]^{\frac{1}{2}}$$

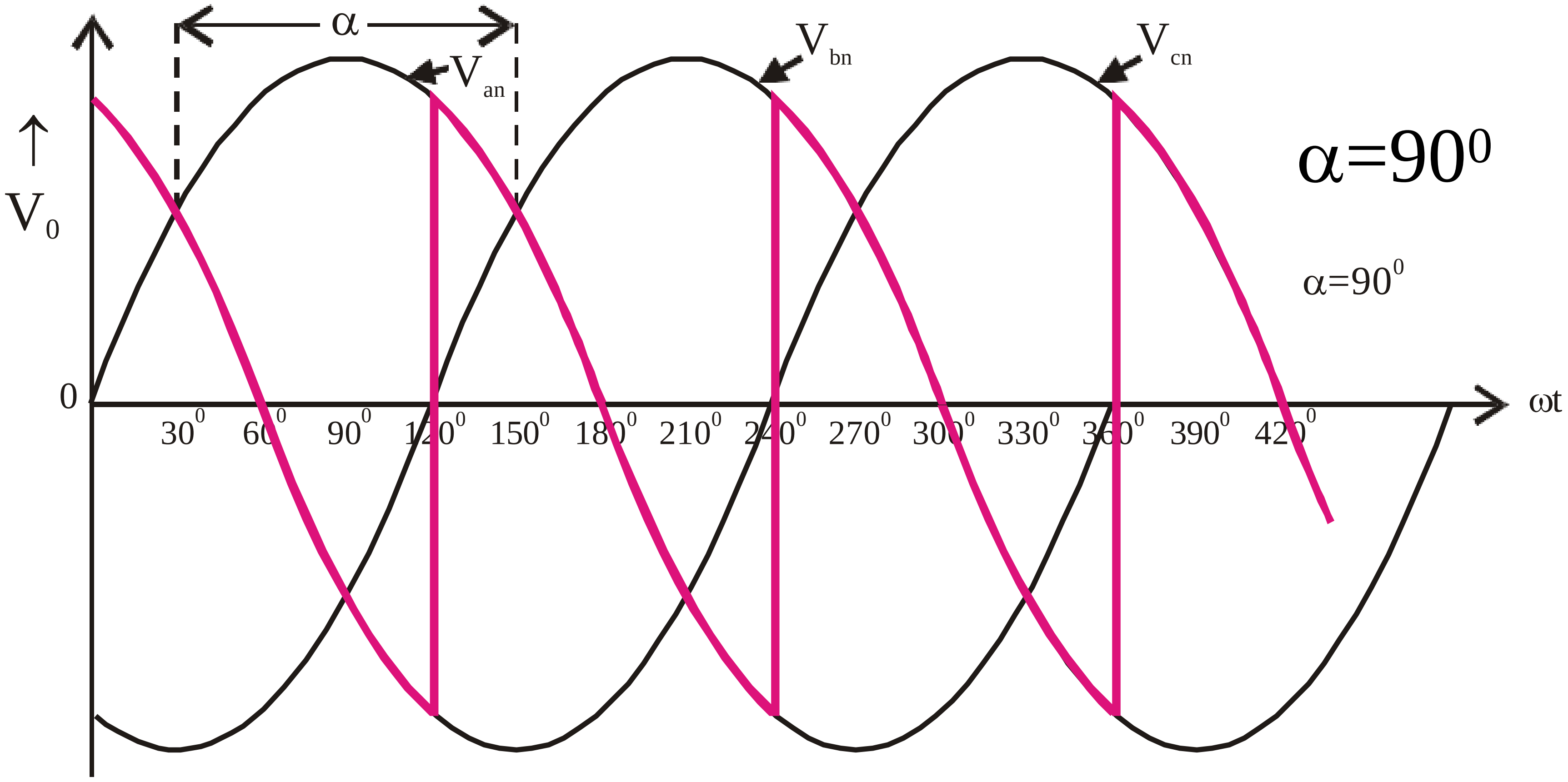
3 Phase Half Wave Controlled Rectifier Output Voltage Waveforms For RL Load at Different Trigger Angles



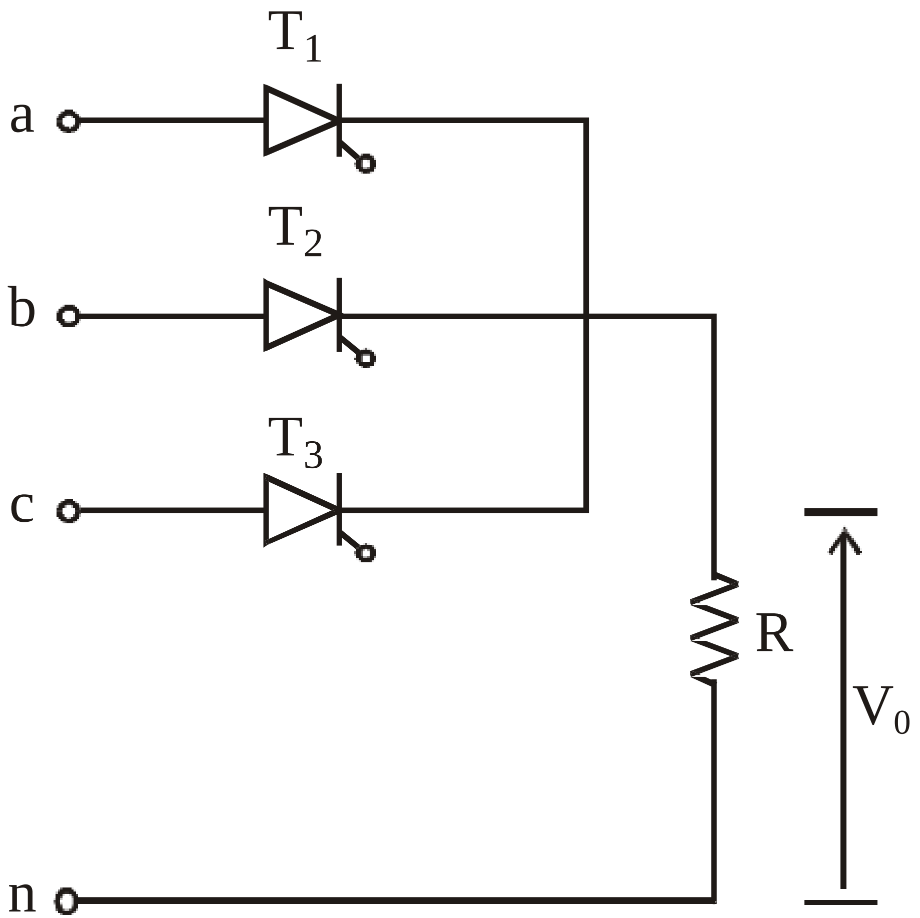
$\alpha=30^0$

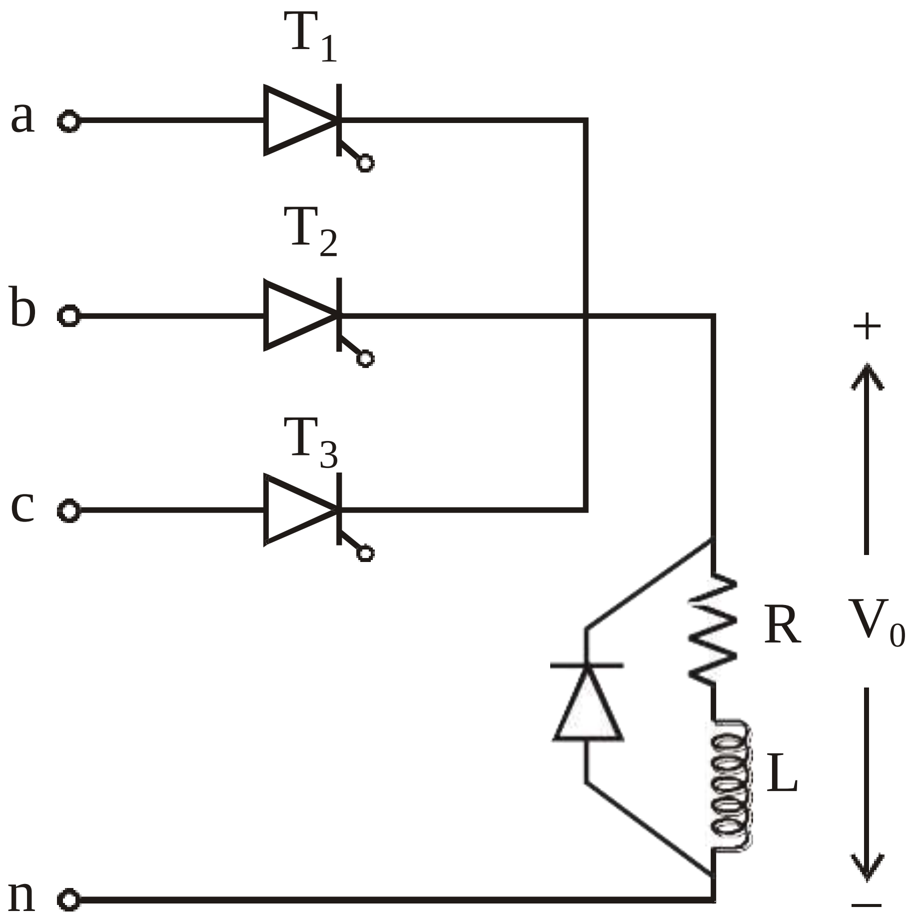


$\alpha=60^0$

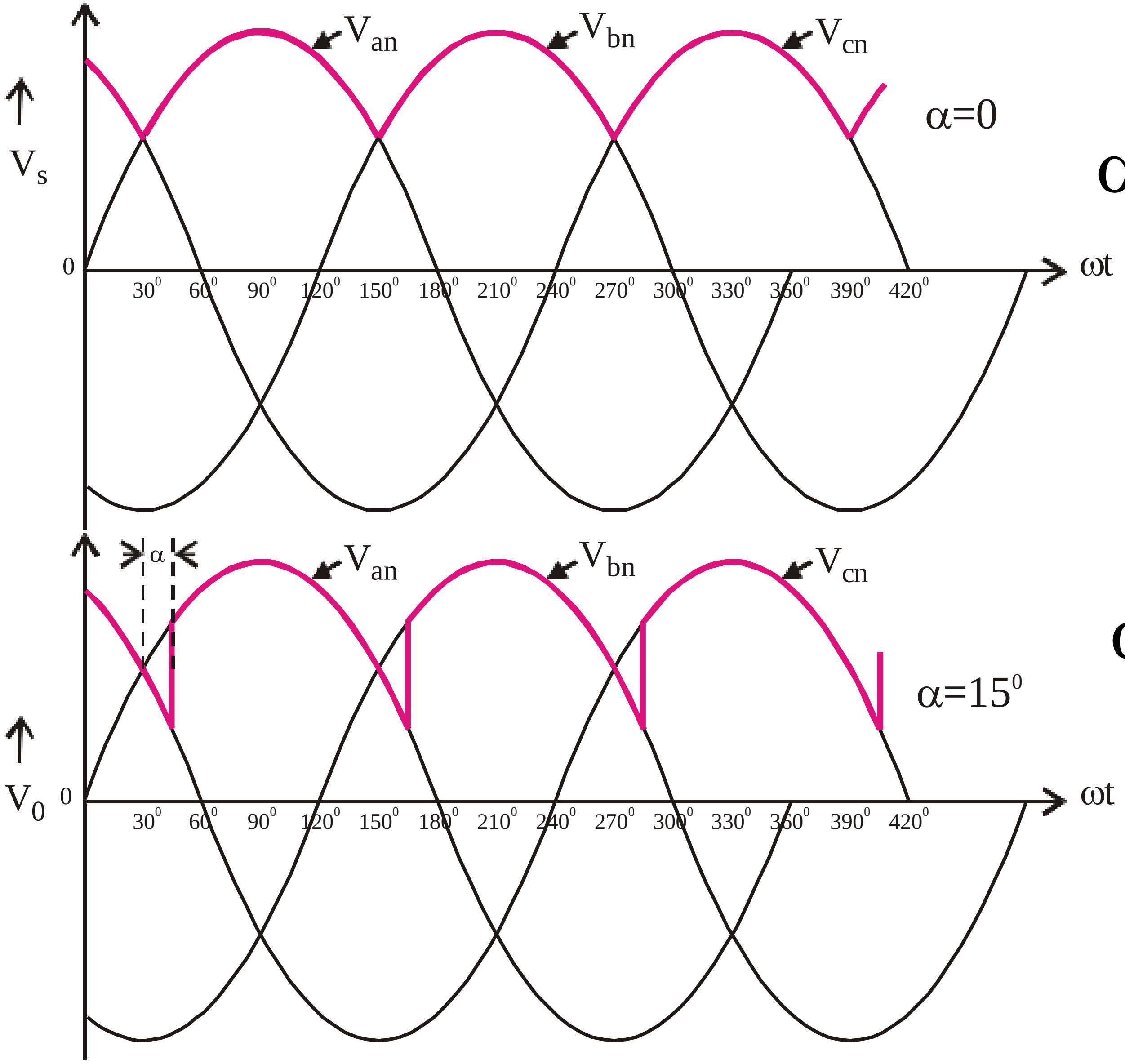

α
V an
V bn
V cn
V 0
0
ωt
30⁰ 60⁰ 90⁰ 120⁰ 150⁰ 180⁰ 210⁰ 240⁰ 270⁰ 300⁰ 330⁰ 360⁰ 390⁰ 420⁰
α=90⁰
α=90⁰

3 Phase Half Wave Controlled Rectifier With R Load and RL Load with FWD


T1
T2
T3
a
b
c
n
R
V0

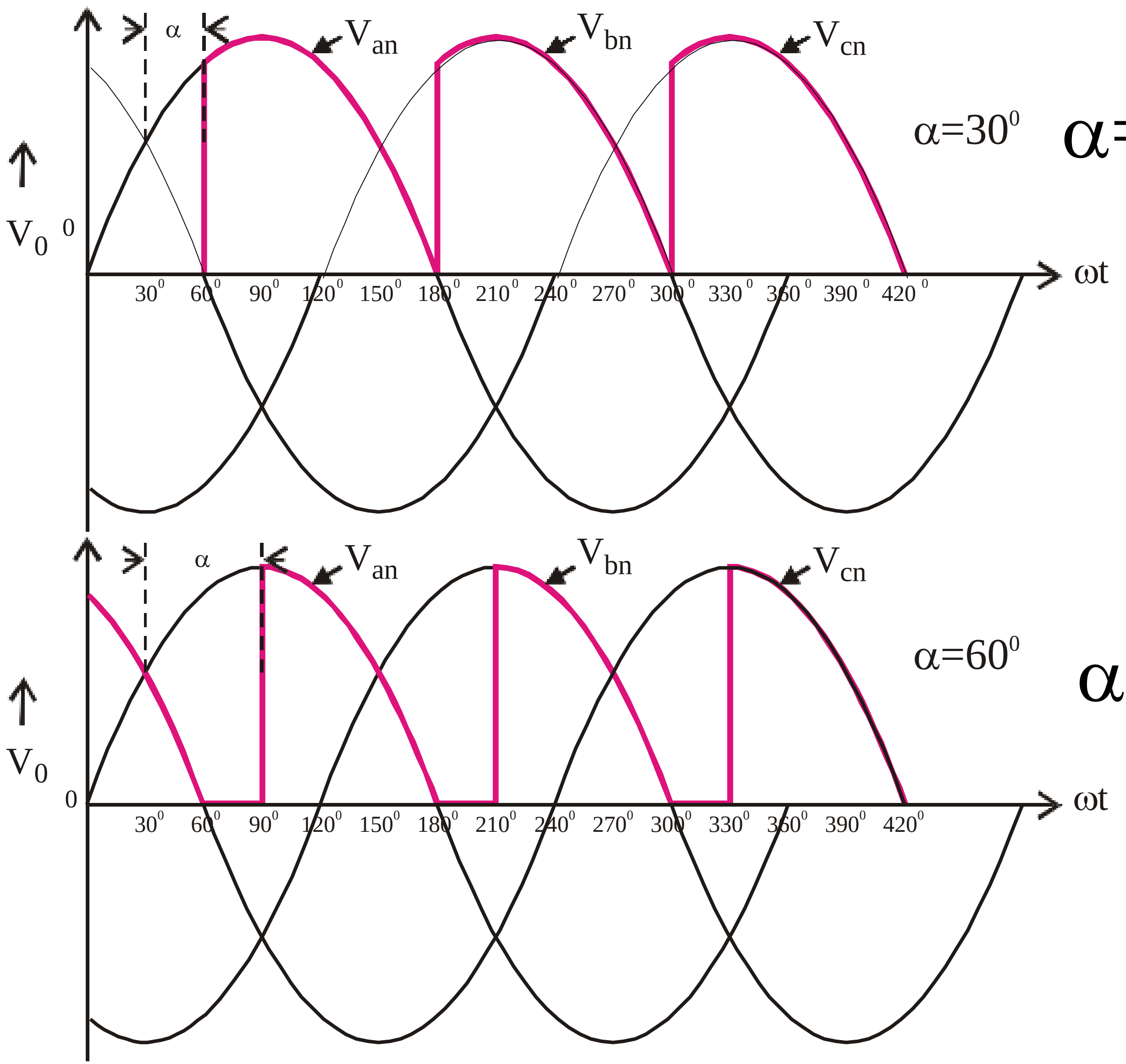

T1
T2
T3
a
b
c
n
R
L
+
V0
–

3 Phase Half Wave Controlled Rectifier Output Voltage Waveforms For R Load or RL Load with FWD at Different Trigger Angles



$\alpha = 0^0$

$\alpha = 15^0$



$\alpha=30^0$

$\alpha=60^0$

To Derive An Expression For The Average Or Dc Output Voltage Of A 3 Phase Half Wave Converter With Resistive Load Or RL Load With FWD

$$T_1 \ \textit{is triggered at}\ \omega t = \left(\frac{\pi}{6} + \alpha\right) = \left(30^0 + \alpha\right)$$

$$T_1 \ \textit{conducts from}\ \left(30^0 + \alpha\right) \textit{to}\ 180^0 \quad ;$$

$$v_O = v_{an} = V_m \sin \omega t$$

$$T_2 \ \textit{is triggered at}\ \omega t = \left(\frac{5\pi}{6} + \alpha\right) = \left(150^0 + \alpha\right)$$

$$T_2 \ \textit{conducts from}\ \left(150^0 + \alpha\right) \textit{to}\ 300^0 \quad ;$$

$$v_O = v_{bn} = V_m \sin\left(\omega t - 120^0\right)$$

$$T_3 \text{ is triggered at } \omega t = \left(\frac{7\pi}{6} + \alpha \right) = \left(270^0 + \alpha \right)$$

$$T_3 \text{ conducts from } \left(270^0 + \alpha \right) \text{ to } 420^0 \;\; ;$$

$$v_O = v_{cn} = V_m \sin\left(\omega t - 240^0 \right)$$

$$= V_m \sin\left(\omega t + 120^0 \right)$$

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\alpha+30^0}^{180^0} v_O.d(\omega t)\right]$$

$$v_O = v_{an} = V_m \sin\omega t;\ \text{for } \omega t = \left(\alpha + 30^0\right) \text{ to } \left(180^0\right)$$

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\alpha+30^0}^{180^0} V_m \sin\omega t.d(\omega t)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\int_{\alpha+30^0}^{180^0} \sin\omega t.d(\omega t)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos\omega t \Big/_{\alpha+30^0}^{180^0}\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos 180^0 + \cos\left(\alpha + 30^0\right)\right]$$

$\because$ $\cos 180^0 = -1$, we get

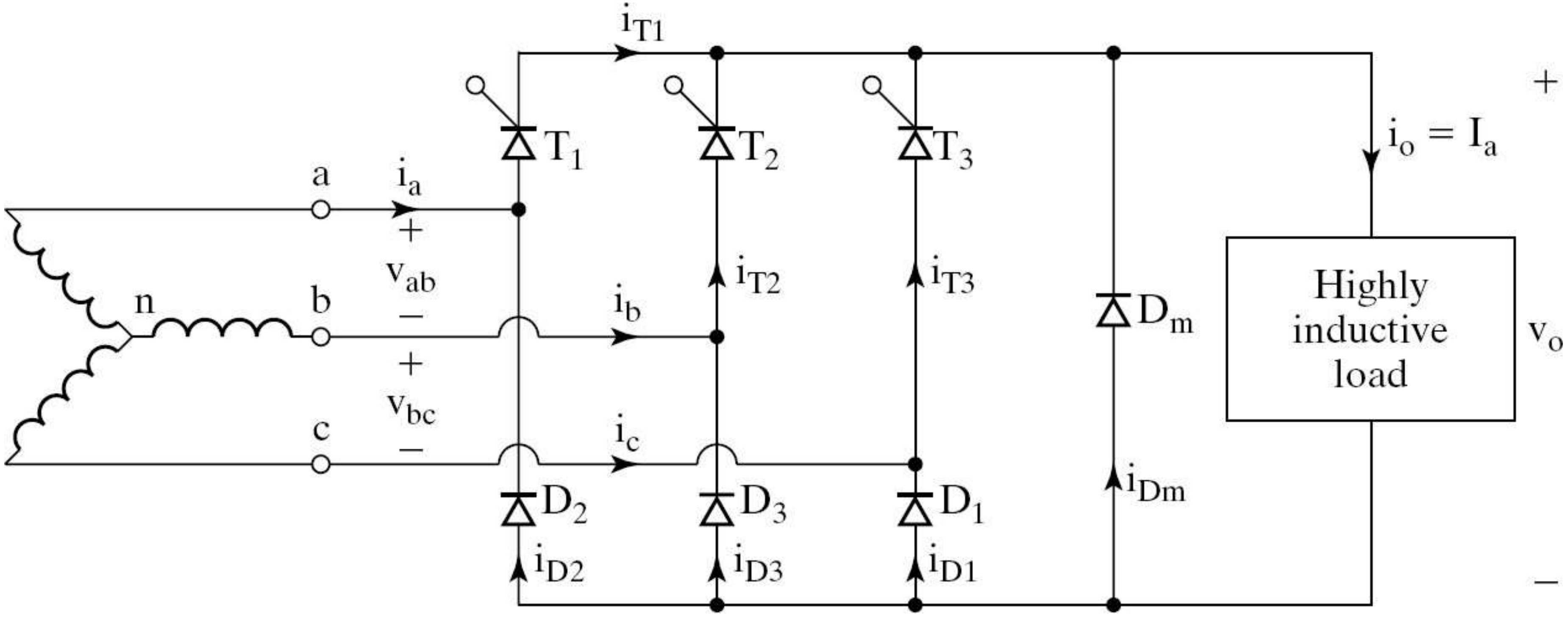
$$V_{dc} = \frac{3V_m}{2\pi}\left[1 + \cos\left(\alpha + 30^0\right)\right]$$

Three Phase Semi-converters

Three Phase Semi-converters

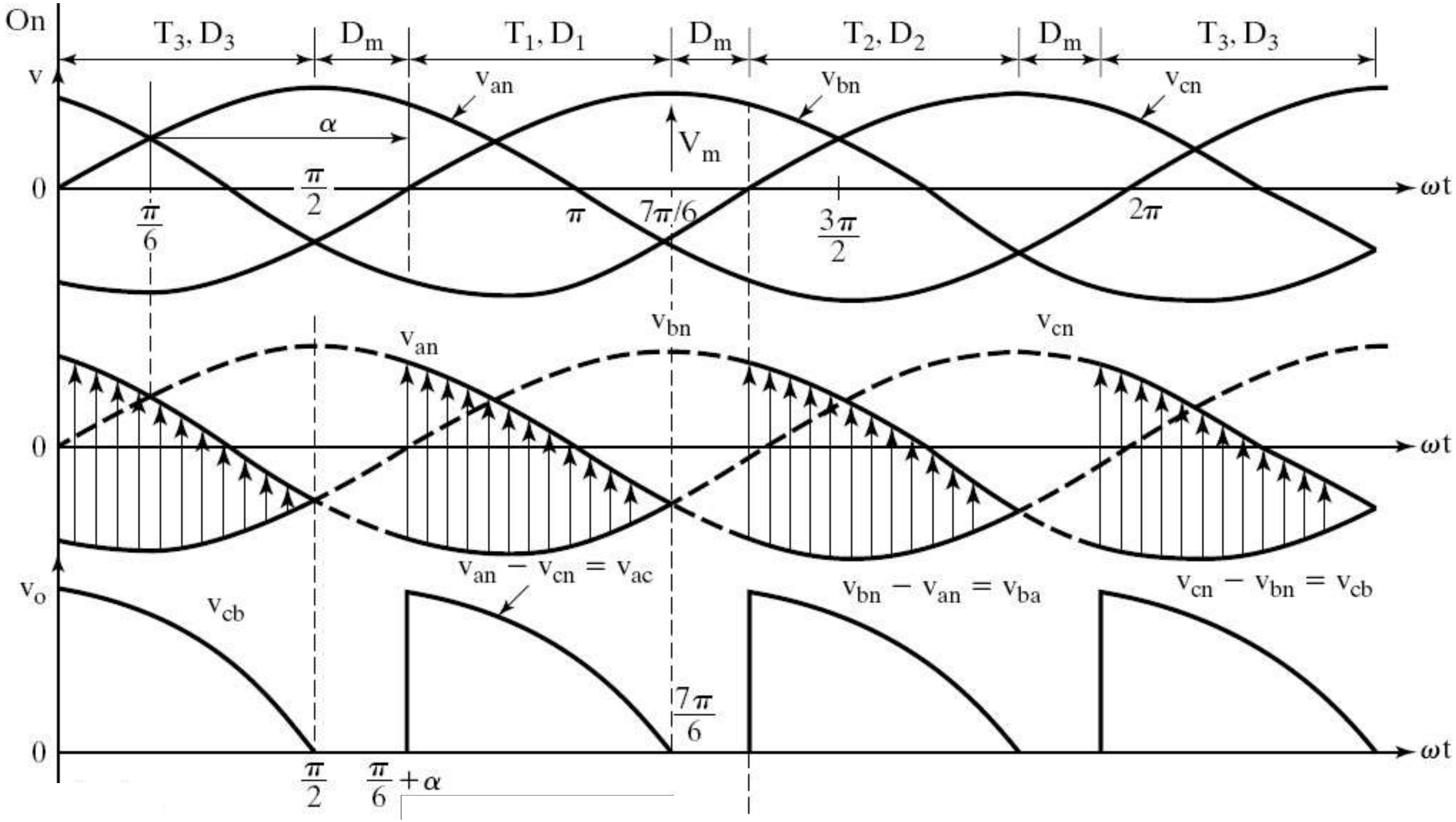
- 3 Phase semi-converters are used in Industrial dc drive applications upto 120kW power output.
- Single quadrant operation is possible.
- Power factor decreases as the delay angle increases.
- Power factor is better than that of 3 phase half wave converter.

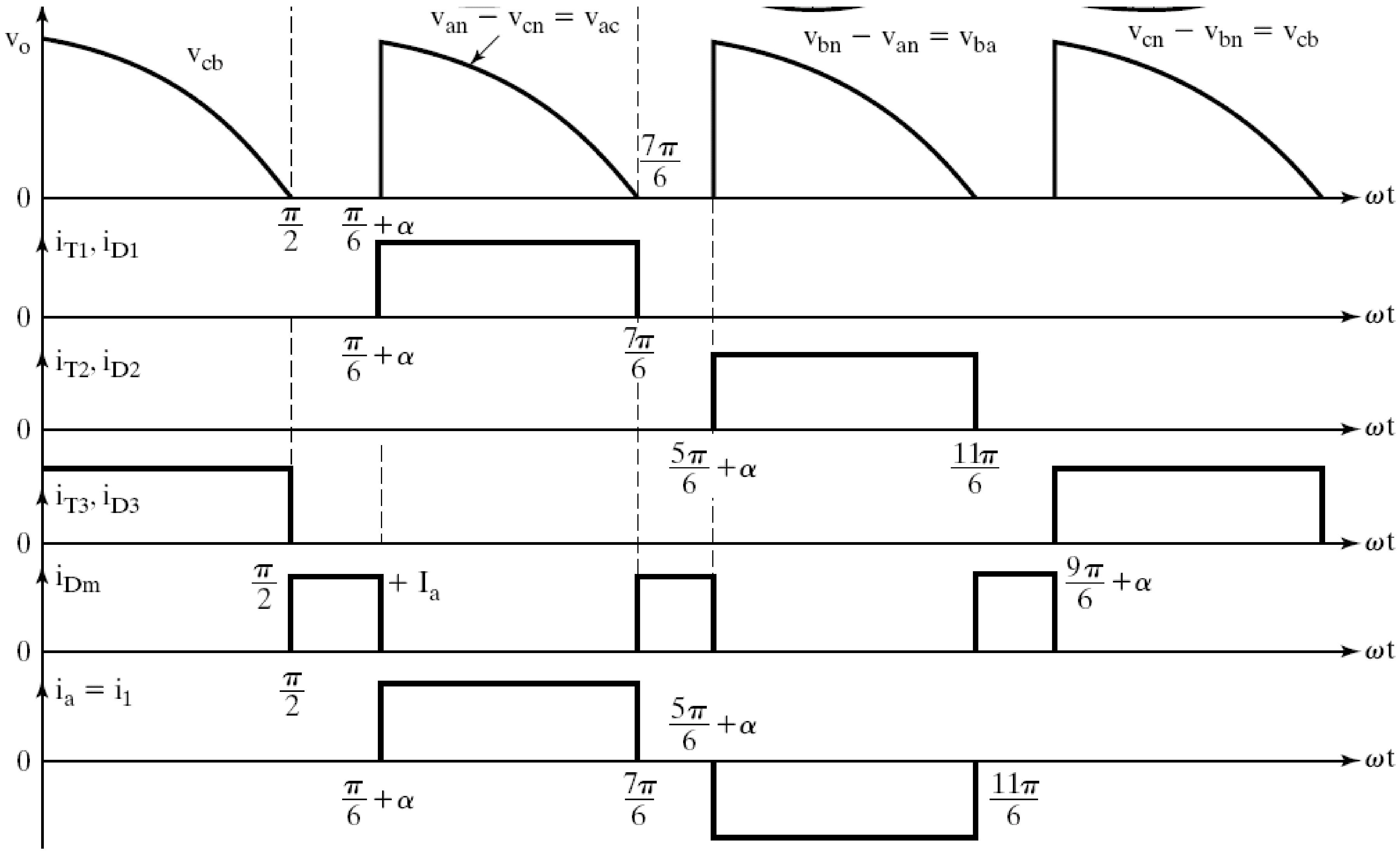
3 Phase
Half Controlled Bridge Converter
(Semi Converter)
with Highly Inductive Load &
Continuous Ripple free Load Current



i_{T1}
T_1
T_2
T_3
+
$i_o = I_a$
a
i_a
+
v_{ab}
n
b
−
i_{T2}
i_{T3}
i_b
D_m
Highly inductive load
v_o
+
v_{bc}
c
i_c
−
D_2
D_3
D_1
i_{Dm}
i_{D2}
i_{D3}
i_{D1}
−

Wave forms of 3 Phase Semiconverter for $\alpha > 60^0$


On
T_3, D_3
D_m
T_1, D_1
D_m
T_2, D_2
D_m
T_3, D_3
v
v_{an}
v_{bn}
v_{cn}
α
V_m
0
$\frac{\pi}{6}$
$\frac{\pi}{2}$
π
$7\pi/6$
$\frac{3\pi}{2}$
2π
ωt
v_{an}
v_{bn}
v_{cn}
0
ωt
v_o
v_{cb}
$v_{an} - v_{cn} = v_{ac}$
$v_{bn} - v_{an} = v_{ba}$
$v_{cn} - v_{bn} = v_{cb}$
$\frac{7\pi}{6}$
0
$\frac{\pi}{2}$
$\frac{\pi}{6} + \alpha$
ωt



v_o
v_{cb}
$v_{an} - v_{cn} = v_{ac}$
$v_{bn} - v_{an} = v_{ba}$
$v_{cn} - v_{bn} = v_{cb}$
$\frac{7\pi}{6}$
$\frac{\pi}{2}$
$\frac{\pi}{6} + \alpha$
i_{T1}, i_{D1}
i_{T2}, i_{D2}
i_{T3}, i_{D3}
i_{Dm}
$+ I_a$
$i_a = i_1$
$\frac{5\pi}{6} + \alpha$
$\frac{11\pi}{6}$
$\frac{9\pi}{6} + \alpha$
0
ωt

3 phase semiconverter output ripple frequency of output voltage is $3 f_S$

The delay angle α can be varied from 0 to π

During the period

$$30^0 \leq \omega t < 210^0$$

$$\frac{\pi}{6} \leq \omega t < \frac{7\pi}{6}$$, thyristor T_1 is forward biased

If thyristor T_1 is triggered at $\omega t = \left(\frac{\pi}{6} + \alpha\right)$,

T_1 & D_1 conduct together and the line to line voltage v_{ac} appears across the load.

At $\omega t = \frac{7\pi}{6}$, v_{ac} becomes negative & FWD D_m conducts.

The load current continues to flow through FWD D_m;

T_1 and D_1 are turned off.

If FWD D_m is not used the T_1 would continue to conduct until the thyristor T_2 is triggered at $\omega t = \left(\frac{5\pi}{6} + \alpha \right)$, and Free wheeling action would be accomplished through T_1 & D_2.

If the delay angle $\alpha \leq \frac{\pi}{3}$, each thyristor conducts for $\frac{2\pi}{3}$ and the FWD D_m does not conduct.

We deifine three line neutral voltages

(3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin\omega t \quad ; \quad V_m = \text{ Max. Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right) = V_m \sin\left(\omega t - 120^0\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right) = V_m \sin\left(\omega t + 120^0\right)$$

$$= V_m \sin\left(\omega t - 240^0\right)$$

V_m is the peak phase voltage of a wye-connected source

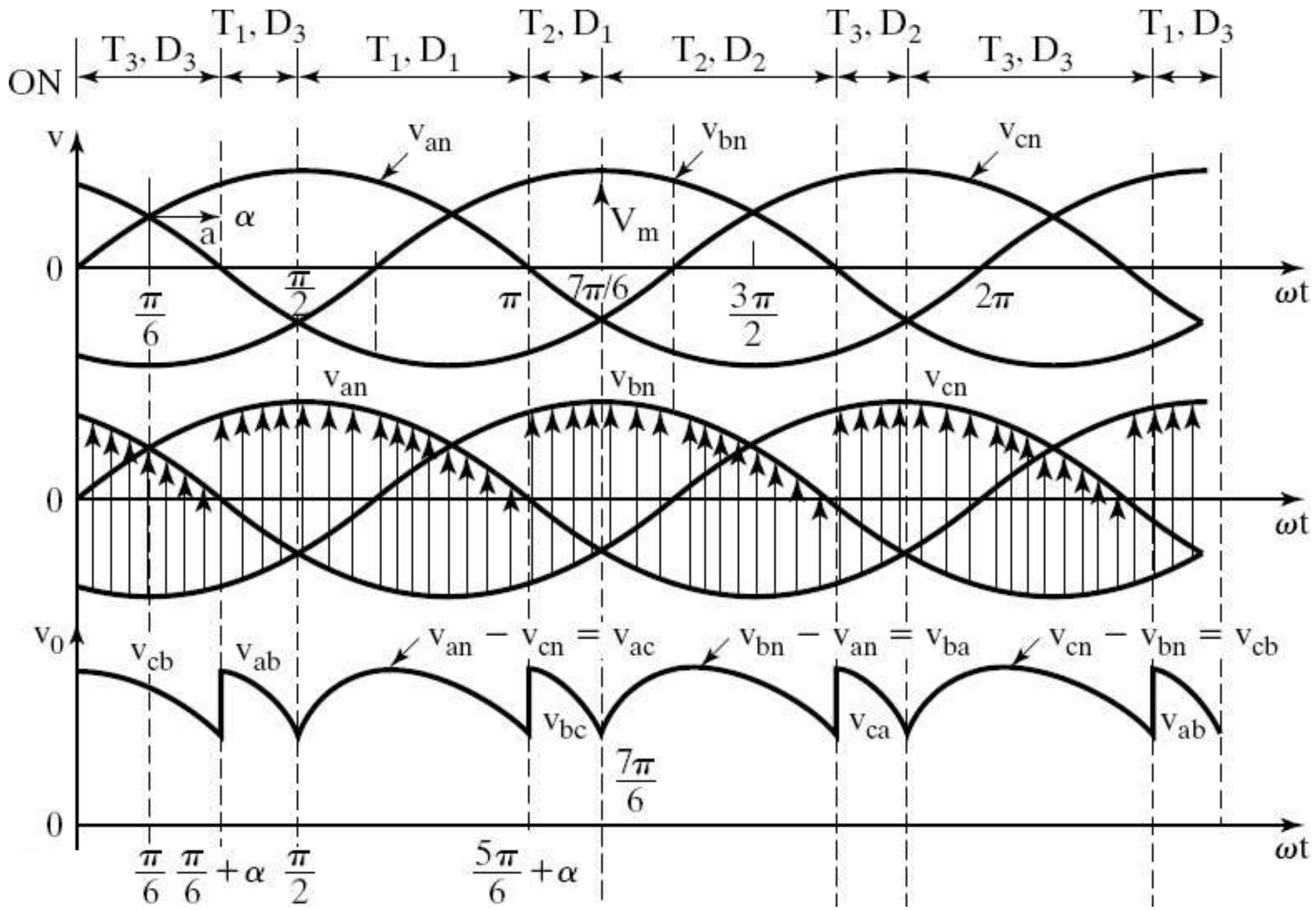
$$v_{RB} = v_{ac} = \left(v_{an} - v_{cn}\right) = \sqrt{3} V_m \sin\left(\omega t - \frac{\pi}{6}\right)$$

$$v_{YR} = v_{ba} = \left(v_{bn} - v_{an}\right) = \sqrt{3} V_m \sin\left(\omega t - \frac{5\pi}{6}\right)$$

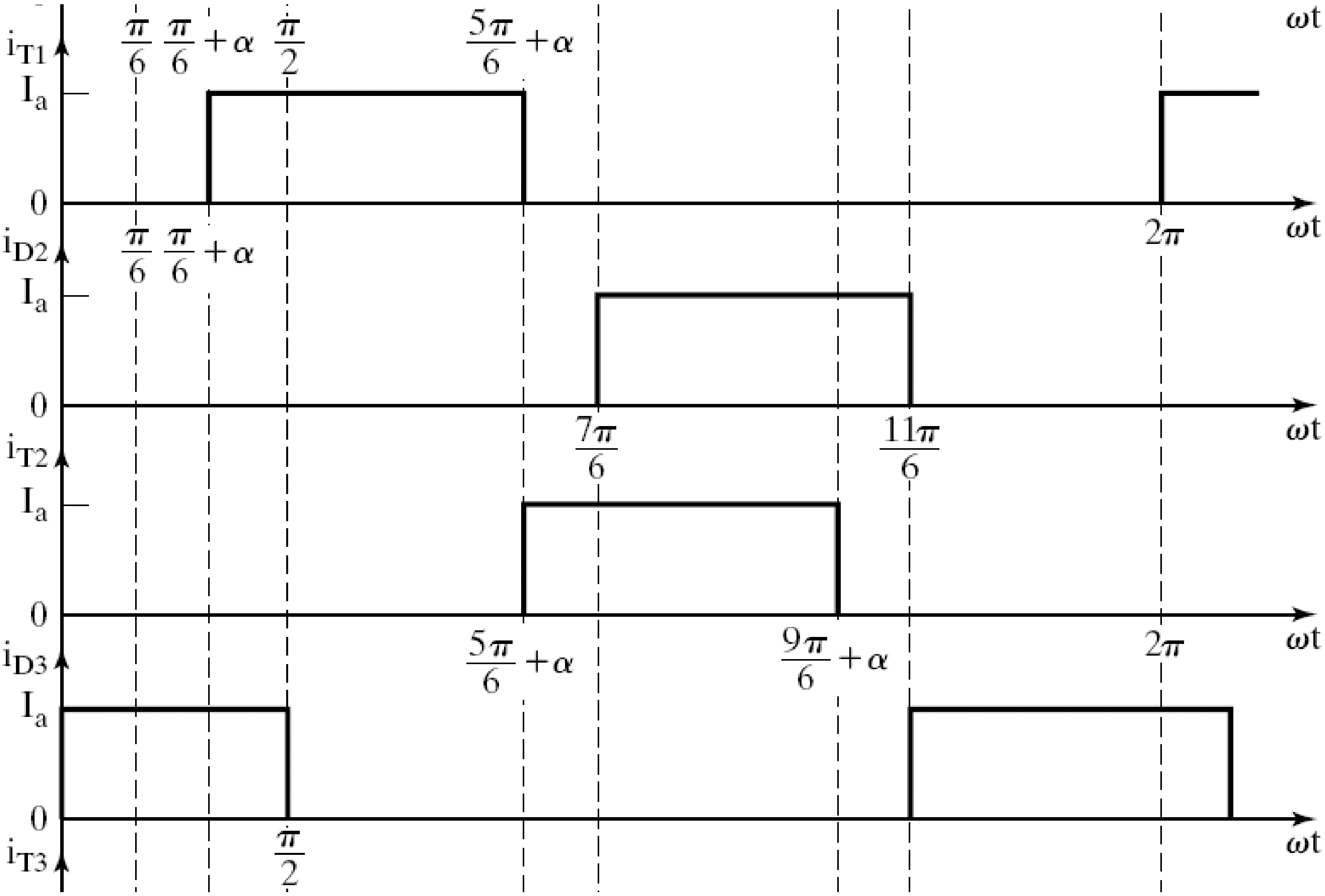
$$v_{BY} = v_{cb} = \left(v_{cn} - v_{bn}\right) = \sqrt{3} V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$v_{RY} = v_{ab} = \left(v_{an} - v_{bn}\right) = \sqrt{3} V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

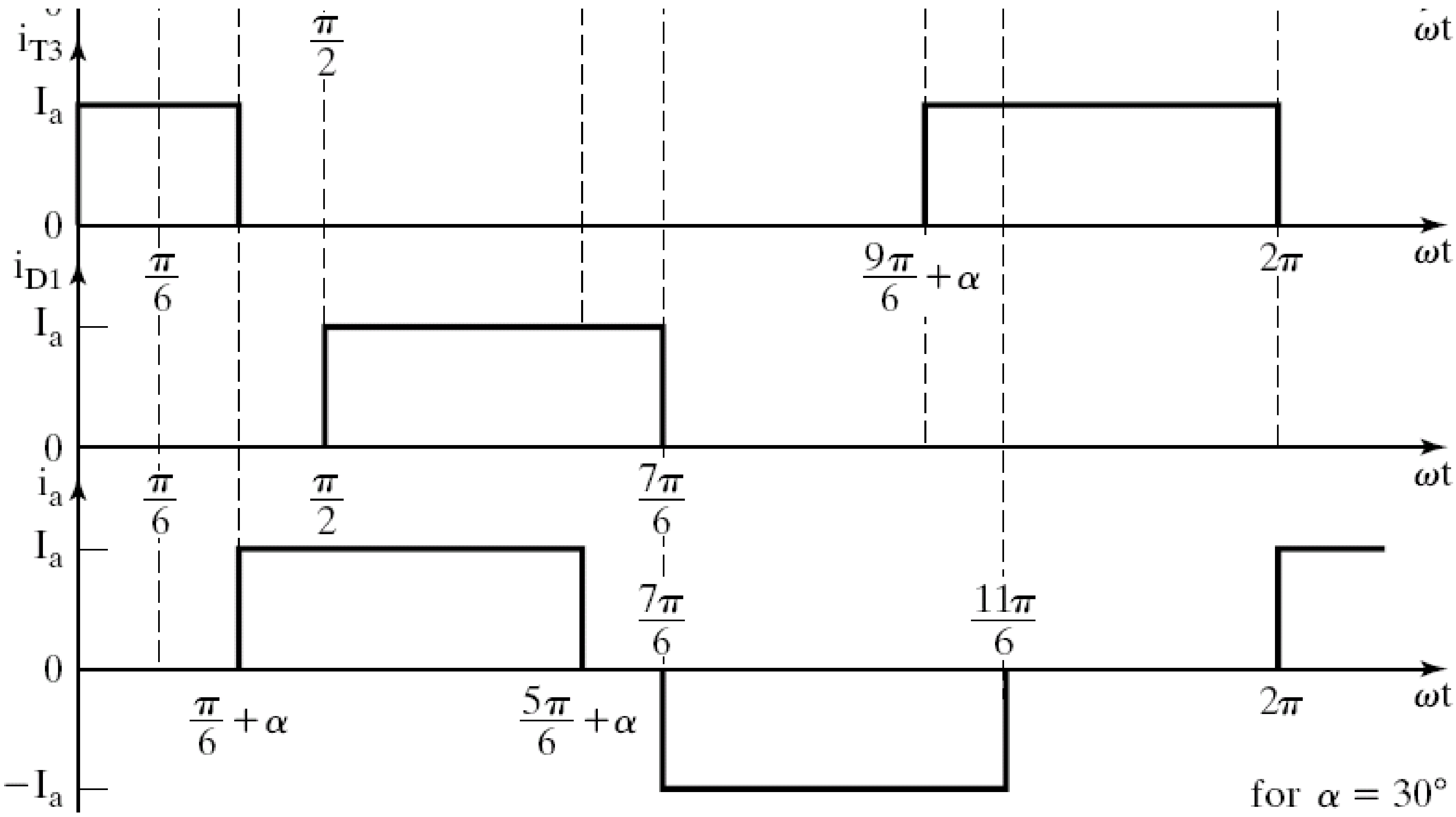
Wave forms of 3 Phase Semiconverter for $\alpha \leq 60^0$



ON
T3, D3
T1, D3
T1, D1
T2, D1
T2, D2
T3, D2
T3, D3
T1, D3
v
van
vbn
vcn
α
a
Vm
0
π/6
π/2
π
7π/6
3π/2
2π
ωt
van
vbn
vcn
0
ωt
v0
vcb
vab
van − vcn = vac
vbn − van = vba
vcn − vbn = vcb
vbc
vca
vab
7π/6
0
π/6
π/6 + α
π/2
5π/6 + α
ωt



i_{T1}
I_a
0
$\frac{\pi}{6}$ $\frac{\pi}{6}+\alpha$ $\frac{\pi}{2}$
$\frac{5\pi}{6}+\alpha$
ωt
2π
i_{D2}
I_a
0
$\frac{\pi}{6}$ $\frac{\pi}{6}+\alpha$
ωt
$\frac{7\pi}{6}$
$\frac{11\pi}{6}$
i_{T2}
I_a
0
ωt
$\frac{5\pi}{6}+\alpha$
$\frac{9\pi}{6}+\alpha$
2π
i_{D3}
I_a
0
ωt
$\frac{\pi}{2}$
i_{T3}
ωt



i_{T3}
ωt
I_a
0
$\frac{\pi}{2}$
i_{D1}
$\frac{\pi}{6}$
$\frac{9\pi}{6}+\alpha$
2π
ωt
I_a
0
ωt
i_a
$\frac{\pi}{6}$
$\frac{\pi}{2}$
$\frac{7\pi}{6}$
ωt
I_a
$\frac{7\pi}{6}$
$\frac{11\pi}{6}$
0
$\frac{\pi}{6}+\alpha$
$\frac{5\pi}{6}+\alpha$
2π
ωt
$-I_a$
for $\alpha = 30°$

To derive an Expression for the Average Output Voltage of 3 Phase Semi-converter for $\alpha > \pi / 3$ and Discontinuous Output Voltage

For $\alpha \geq \frac{\pi}{3}$ and discontinuous output voltage:

the Average output voltage is found from

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\pi/6+\alpha}^{7\pi/6} v_{ac}.d\left(\omega t\right)\right]$$

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\pi/6+\alpha}^{7\pi/6} \sqrt{3}\; V_m \sin\left(\omega t - \frac{\pi}{6}\right) d\left(\omega t\right)\right]$$

$$V_{dc} = \frac{3\sqrt{3}V_m}{2\pi}\left(1+\cos\alpha\right)$$

$$V_{dc} = \frac{3V_{mL}}{2\pi}\left(1+\cos\alpha\right)$$

$V_{mL} = \sqrt{3}V_m =$ Max. value of line-to-line supply voltage

The maximum average output voltage that occurs at a delay angle of $\alpha = 0$ is

$$V_{dc(\max)} = V_{dm} = \frac{3\sqrt{3}V_m}{\pi}$$

The normalized average output voltage is

$$V_n = \frac{V_{dc}}{V_{dm}} = 0.5\left(1+\cos\alpha\right)$$

The rms output voltage is found from

$$V_{O(rms)} = \left[\frac{3}{2\pi}\int_{\pi/6+\alpha}^{7\pi/6} v_{ac}^2 . d\left(\omega t\right)\right]^{\frac{1}{2}}$$

$$V_{O(rms)} = \left[\frac{3}{2\pi} \int_{\pi/6+\alpha}^{7\pi/6} 3V_m^2 \sin^2\left(\omega t - \frac{\pi}{6}\right) d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(rms)} = \sqrt{3} V_m \left[\frac{3}{4\pi} \left(\pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{\frac{1}{2}}$$

Average or DC Output Voltage of a 3-Phase Semi-converter for $\alpha \leq \pi / 3$, and Continuous Output Voltage

For $\alpha \leq \dfrac{\pi}{3}$, and continuous output voltage

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\pi/6+\alpha}^{\pi/2} v_{ab}.d(\omega t) + \int_{\pi/2}^{5\pi/6+\alpha} v_{ac}.d(\omega t)\right]$$

$$V_{dc} = \frac{3\sqrt{3}V_m}{2\pi}\left(1+\cos\alpha\right)$$

$$V_n = \frac{V_{dc}}{V_{dm}} = 0.5\left(1 + \cos\alpha\right)$$

RMS value of o/p voltage is calculated by using the equation

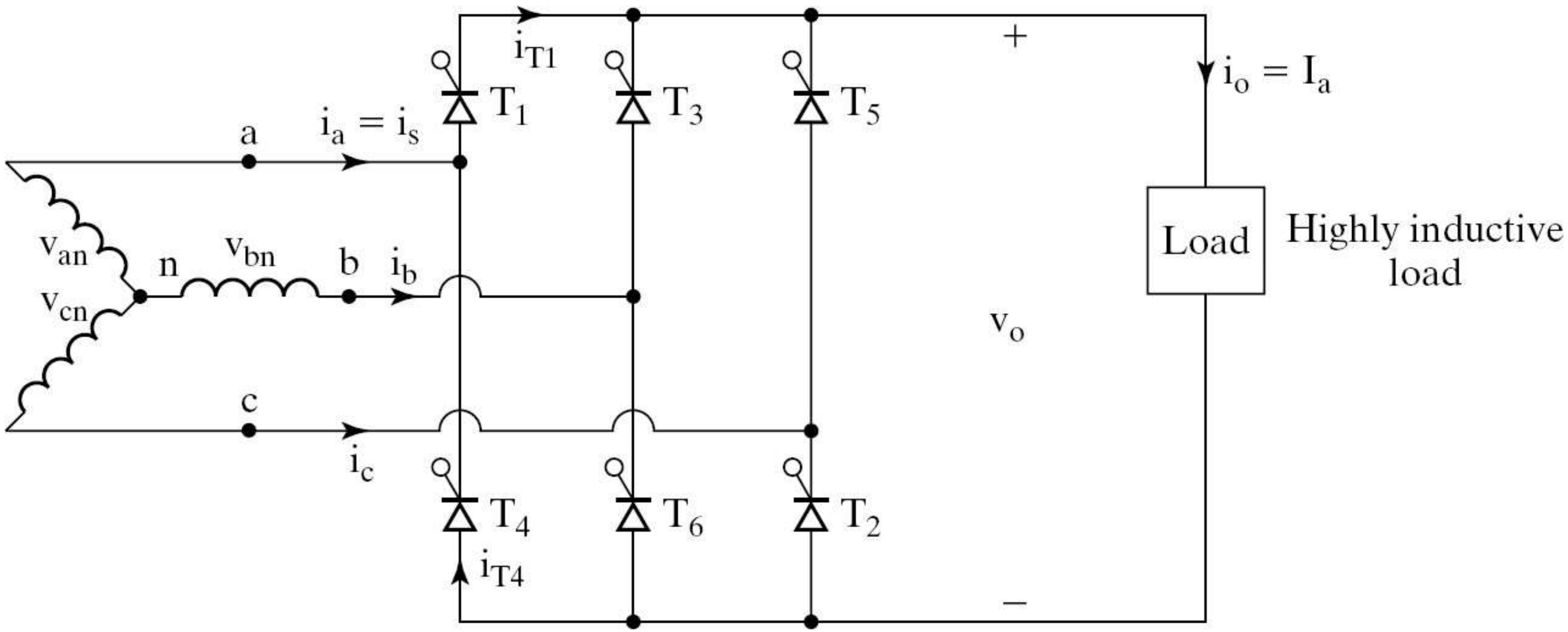
$$V_{O(rms)} = \left[\frac{3}{2\pi} \int_{\pi/6+\alpha}^{\pi/2} v_{ab}^{2}.d\left(\omega t\right) + \int_{\pi/2}^{5\pi/6+\alpha} v_{ac}^{2}.d\left(\omega t\right) \right]^{\frac{1}{2}}$$

$$V_{O(rms)} = \sqrt{3}V_m \left[\frac{3}{4\pi} \left(\frac{2\pi}{3} + \sqrt{3}\cos^2\alpha \right) \right]^{\frac{1}{2}}$$

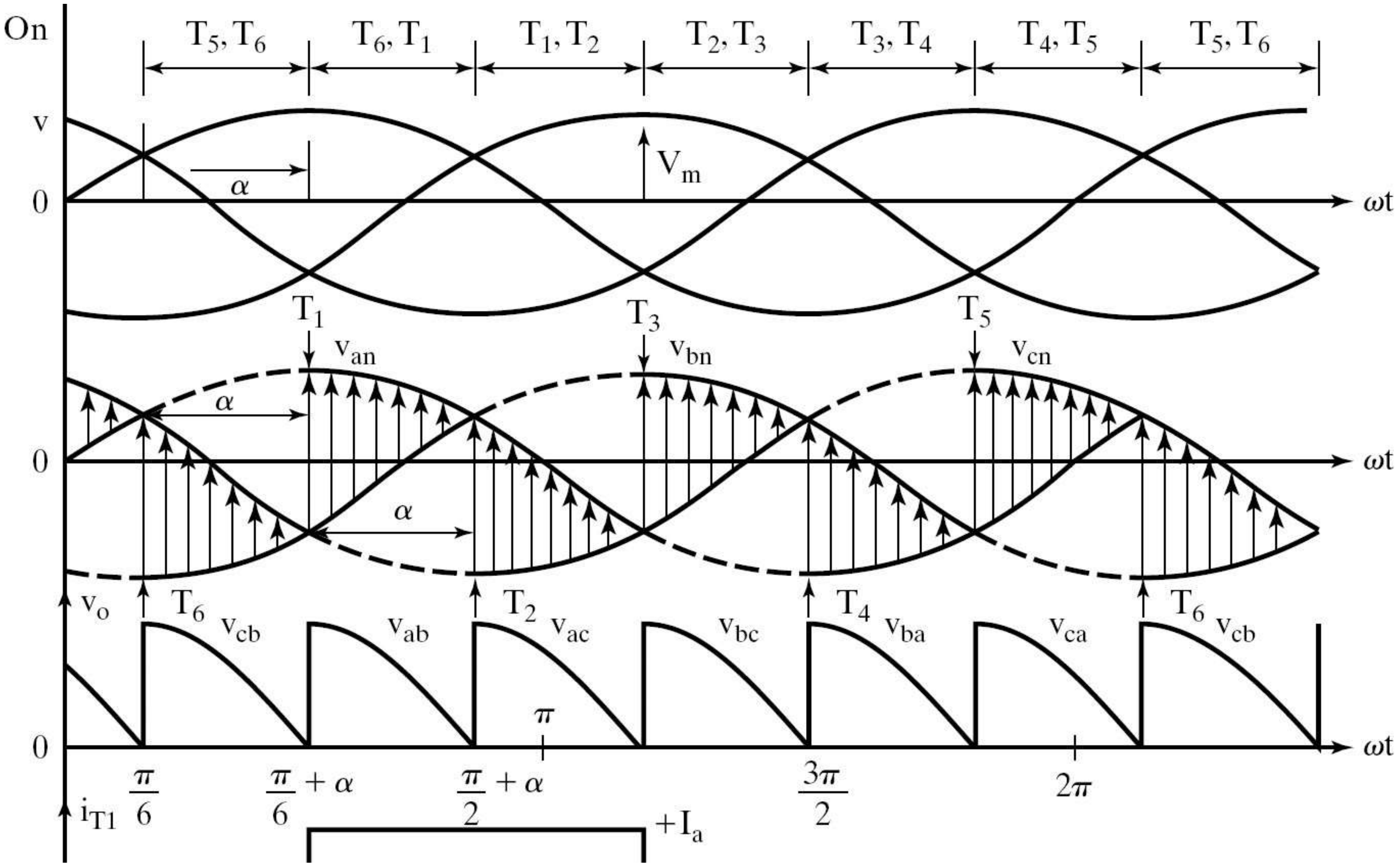
Three Phase Full Converter

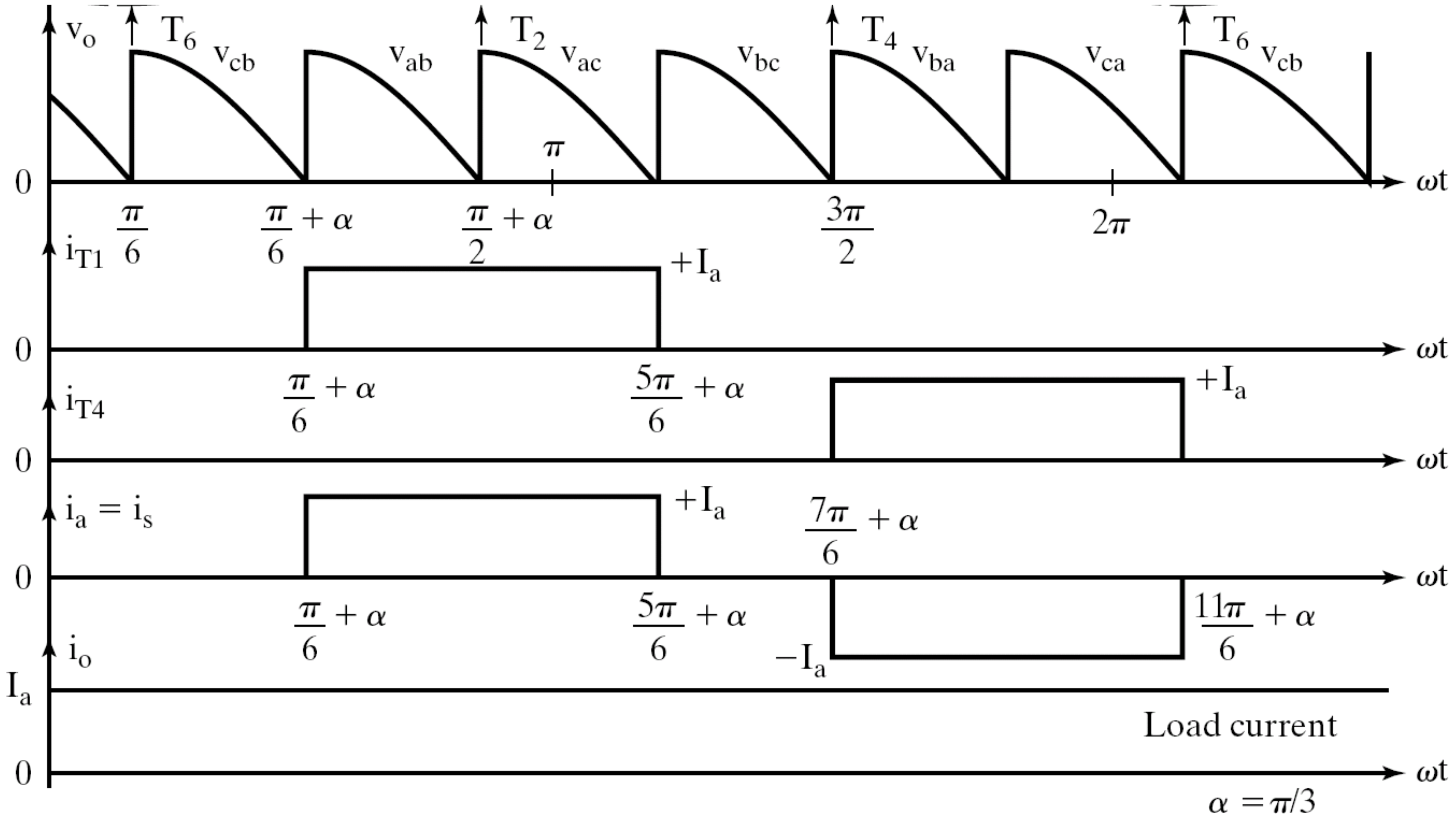
Three Phase Full Converter

- 3 Phase Fully Controlled Full Wave Bridge Converter.
- Known as a 6-pulse converter.
- Used in industrial applications up to 120kW output power.
- Two quadrant operation is possible.



i_{T1}
+
$i_o = I_a$
T_1
T_3
T_5
a
$i_a = i_s$
Load
Highly inductive load
v_{an}
n
v_{bn}
b
i_b
v_{cn}
v_o
c
i_c
T_4
T_6
T_2
i_{T4}
–


On
T5, T6
T6, T1
T1, T2
T2, T3
T3, T4
T4, T5
T5, T6
v
Vm
α
0
ωt
T1
T3
T5
van
vbn
vcn
α
0
ωt
α
T6
T2
T4
T6
vo
vcb
vab
vac
vbc
vba
vca
vcb
0
ωt
π
π/6
π/6 + α
π/2 + α
3π/2
2π
iT1
+Ia



v_o
T_6 v_{cb}
v_{ab}
T_2 v_{ac}
v_{bc}
T_4 v_{ba}
v_{ca}
T_6 v_{cb}
π
0
ωt
$\frac{\pi}{6}$
$\frac{\pi}{6} + \alpha$
$\frac{\pi}{2} + \alpha$
$\frac{3\pi}{2}$
2π
i_{T1}
$+I_a$
0
ωt
i_{T4}
$\frac{\pi}{6} + \alpha$
$\frac{5\pi}{6} + \alpha$
$+I_a$
0
ωt
$i_a = i_s$
$+I_a$
$\frac{7\pi}{6} + \alpha$
0
ωt
$\frac{\pi}{6} + \alpha$
$\frac{5\pi}{6} + \alpha$
$\frac{11\pi}{6} + \alpha$
$-I_a$
i_o
I_a
Load current
0
ωt
$\alpha = \pi/3$

- The thyristors are triggered at an interval of $\pi/3$.
- The frequency of output ripple voltage is $6f_S$.
- T_1 is triggered at $\omega t = (\pi/6 + \alpha)$, T_6 is already conducting when T_1 is turned ON.
- During the interval $(\pi/6 + \alpha)$ to $(\pi/2 + \alpha)$, T_1 and T_6 conduct together & the output load voltage is equal to $v_{ab} = (v_{an} - v_{bn})$

- T_2 is triggered at $\omega t = (\pi/2 + \alpha)$, T_6 turns off naturally as it is reverse biased as soon as T_2 is triggered.
- During the interval $(\pi/2 + \alpha)$ to $(5\pi/6 + \alpha)$, T_1 and T_2 conduct together & the output load voltage $v_O = v_{ac} = (v_{an} - v_{cn})$
- Thyristors are numbered in the order in which they are triggered.
- The thyristor triggering sequence is 12, 23, 34, 45, 56, 61, 12, 23, 34, ………

We deifine three line neutral voltages

(3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t \quad ; \quad V_m = \text{ Max. Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right) = V_m \sin\left(\omega t - 120^0\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right) = V_m \sin\left(\omega t + 120^0\right)$$

$$= V_m \sin\left(\omega t - 240^0\right)$$

V_m is the peak phase voltage of a wye-connected source.

The corresponding line-to-line supply voltages are

$$v_{RY} = v_{ab} = \left(v_{an} - v_{bn}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$v_{YB} = v_{bc} = \left(v_{bn} - v_{cn}\right) = \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$v_{BR} = v_{ca} = \left(v_{cn} - v_{an}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

To Derive An Expression For The Average Output Voltage Of 3-phase Full Converter With Highly Inductive Load Assuming Continuous And Constant Load Current

The output load voltage consists of 6 voltage pulses over a period of 2π radians, Hence the average output voltage is calculated as

$$V_{O(dc)} = V_{dc} = \frac{6}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} v_O . d\omega t \quad ;$$

$$v_O = v_{ab} = \sqrt{3} V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$V_{dc} = \frac{3}{\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} \sqrt{3} V_m \sin\left(\omega t + \frac{\pi}{6}\right).d\omega t$$

$$V_{dc} = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha = \frac{3V_{mL}}{\pi}\cos\alpha$$

Where $V_{mL} = \sqrt{3}V_m =$ Max. line-to-line supply voltage

The maximum average dc output voltage is

obtained for a delay angle $\alpha = 0$,

$$V_{dc(\max)} = V_{dm} = \frac{3\sqrt{3}V_m}{\pi} = \frac{3V_{mL}}{\pi}$$

The normalized average dc output voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = \cos\alpha$$

The rms value of the output voltage is found from

$$V_{O(rms)} = \left[\frac{6}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} v_O^2 . d\left(\omega t\right) \right]^{\frac{1}{2}}$$

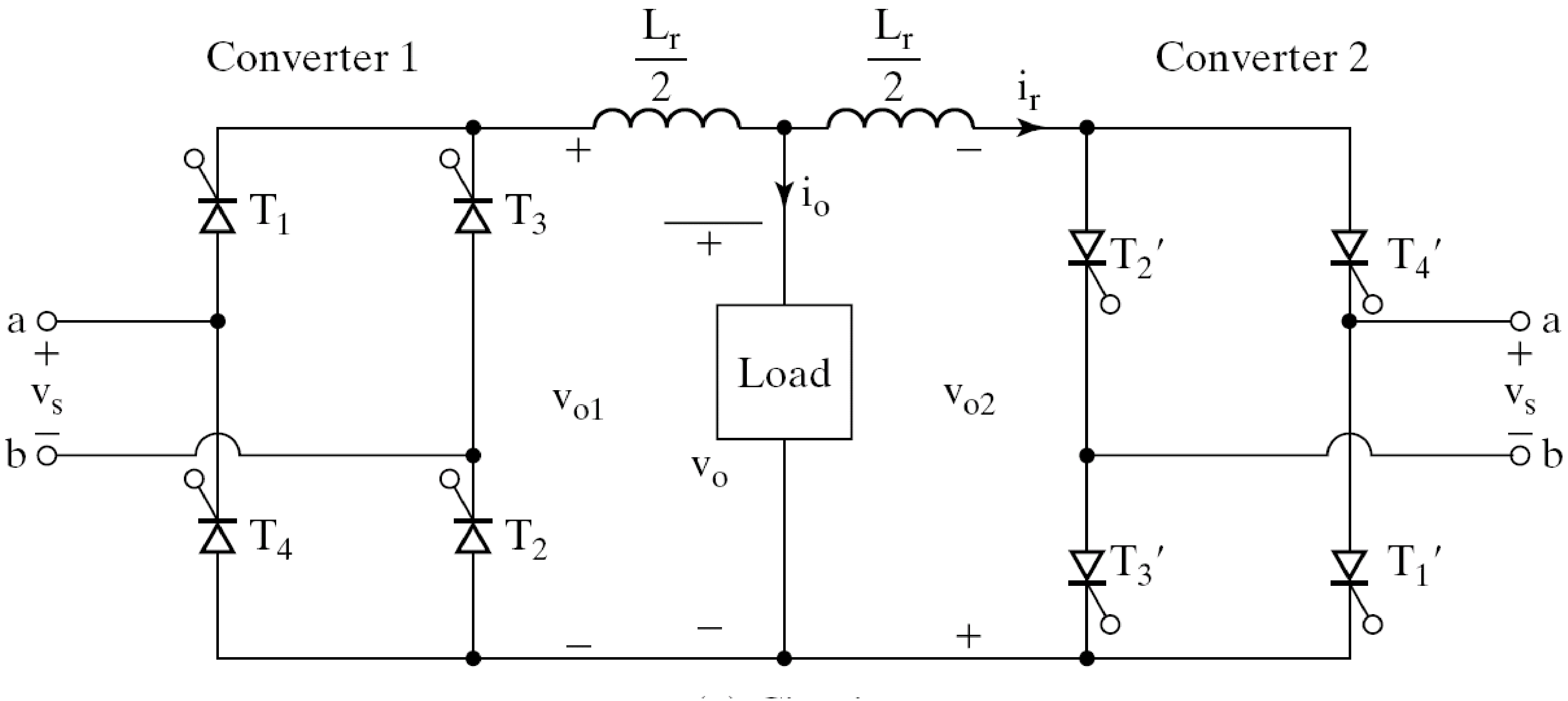
$$V_{O(rms)} = \left[\frac{6}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} v_{ab}^2 . d(\omega t) \right]^{\frac{1}{2}}$$

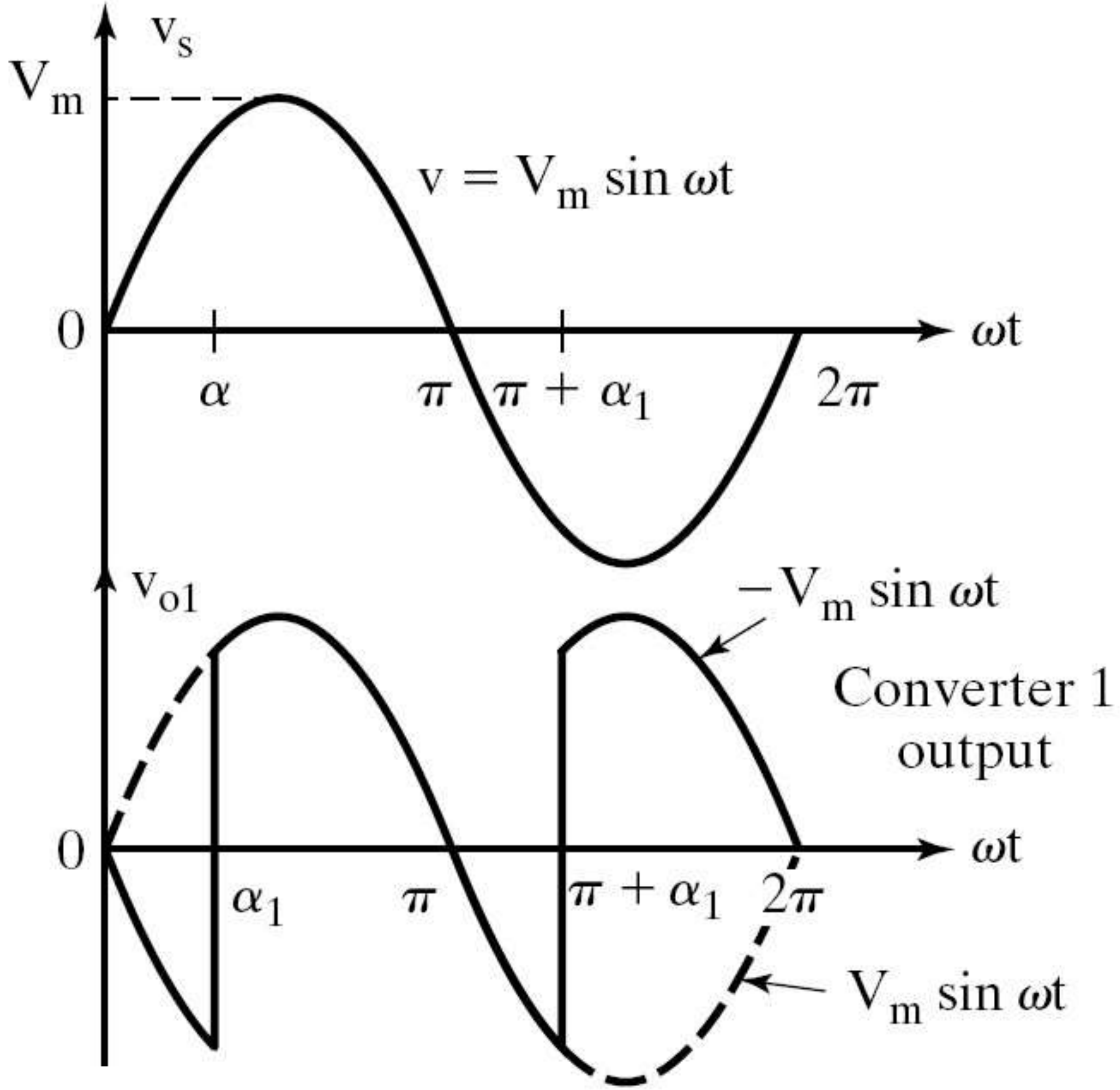
$$V_{O(rms)} = \left[\frac{3}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} 3V_m^2 \sin^2\left(\omega t + \frac{\pi}{6}\right) . d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(rms)} = \sqrt{3} V_m \left(\frac{1}{2} + \frac{3\sqrt{3}}{4\pi} \cos 2\alpha \right)^{\frac{1}{2}}$$

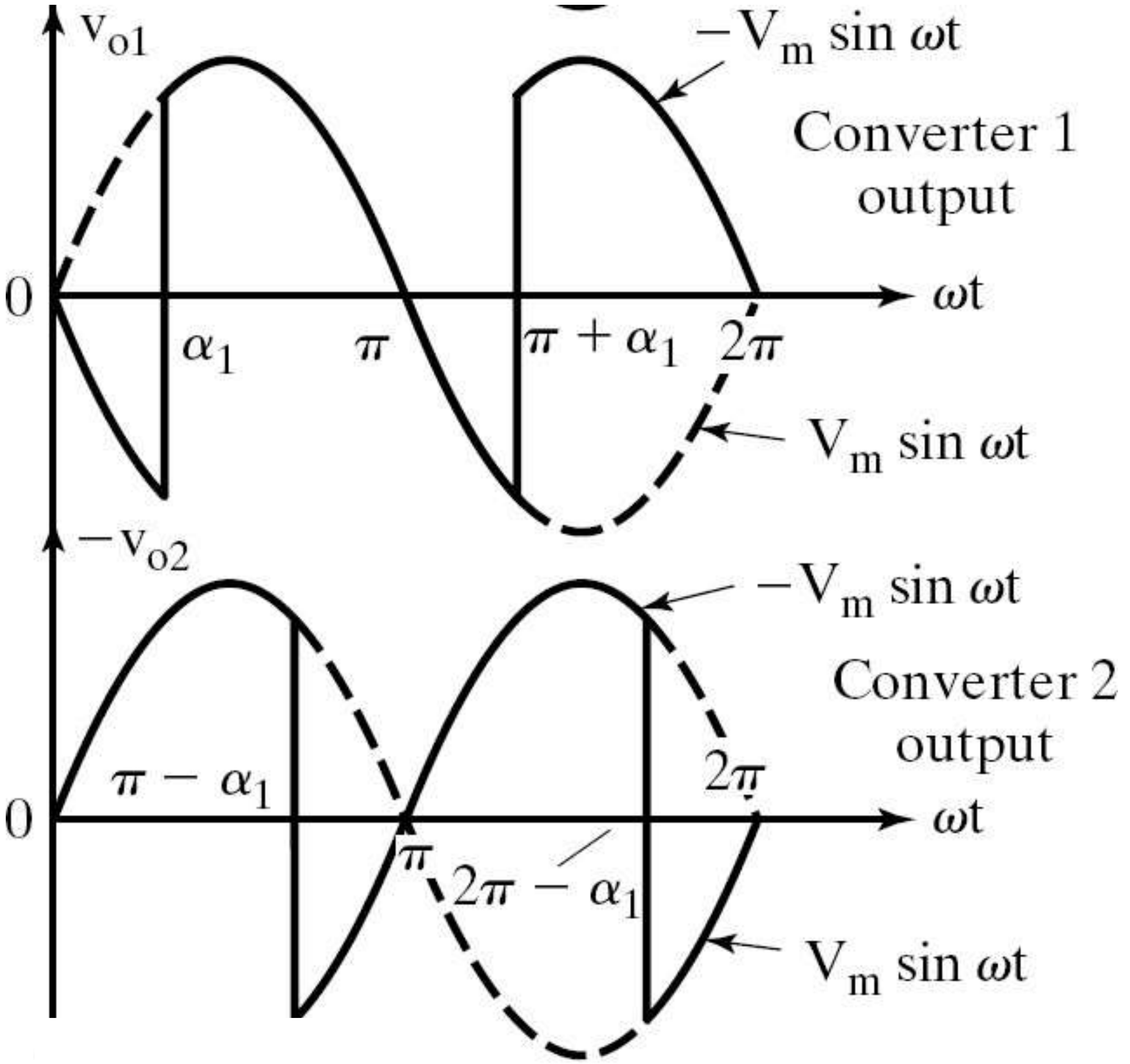
Single Phase Dual Converter

Single Phase Dual Converter

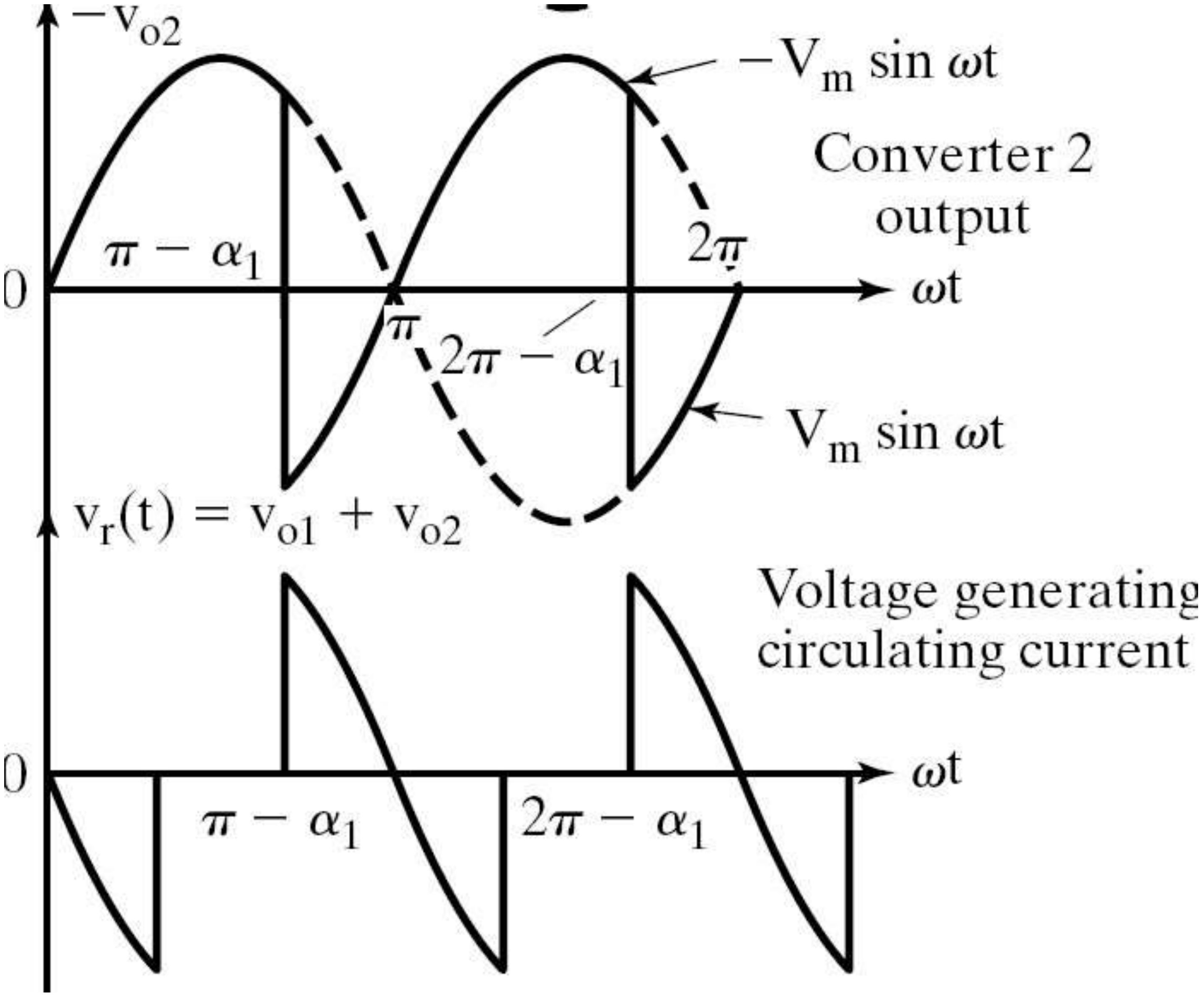




vs
Vm
v = Vm sin ωt
0
ωt
α
π
π + α1
2π
vo1
−Vm sin ωt
Converter 1
output
0
ωt
α1
π
π + α1
2π
Vm sin ωt



v_{o1}
$-V_m \sin \omega t$
Converter 1 output
0
ωt
α_1
π
$\pi + \alpha_1$
2π
$V_m \sin \omega t$
$-v_{o2}$
$-V_m \sin \omega t$
Converter 2 output
$\pi - \alpha_1$
2π
0
ωt
π
$2\pi - \alpha_1$
$V_m \sin \omega t$


$-v_{o2}$
$-V_m \sin \omega t$
Converter 2
output
2π
0
$\pi - \alpha_1$
ωt
π
$2\pi - \alpha_1$
$V_m \sin \omega t$
$v_r(t) = v_{o1} + v_{o2}$
Voltage generating
circulating current
0
ωt
$\pi - \alpha_1$
$2\pi - \alpha_1$

The average dc output voltage of converter 1 is

$$V_{dc1} = \frac{2V_m}{\pi}\cos\alpha_1$$

The average dc output voltage of converter 2 is

$$V_{dc2} = \frac{2V_m}{\pi}\cos\alpha_2$$

In the dual converter operation one converter is operated as a controlled rectifier

with $\alpha < 90^0$ & the second converter is operated as a line commutated inverter in the inversion mode with $\alpha > 90^0$

$$\therefore \qquad V_{dc1} = -V_{dc2}$$

$$\frac{2V_m}{\pi}\cos\alpha_1 = \frac{-2V_m}{\pi}\cos\alpha_2 = \frac{2V_m}{\pi}\left(-\cos\alpha_2\right)$$

$$\therefore \qquad \cos\alpha_1 = -\cos\alpha_2$$

or

$$\cos\alpha_2 = -\cos\alpha_1 = \cos\left(\pi - \alpha_1\right)$$

$$\therefore \qquad \alpha_2 = \left(\pi - \alpha_1\right) \text{ or}$$

$$\left(\alpha_1 + \alpha_2\right) = \pi \text{ radians}$$

Which gives

$$\alpha_2 = \left(\pi - \alpha_1\right)$$

To Obtain an Expression for the Instantaneous Circulating Current

- v_{O1} = Instantaneous o/p voltage of converter 1.
- v_{O2} = Instantaneous o/p voltage of converter 2.
- The circulating current i_r can be determined by integrating the instantaneous voltage difference (which is the voltage drop across the circulating current reactor L_r), starting from $\omega t = (2\pi - \alpha_1)$.
- As the two average output voltages during the interval $\omega t = (\pi+\alpha_1)$ to $(2\pi - \alpha_1)$ are equal and opposite their contribution to the instantaneous circulating current i_r is zero.

$$i_r = \frac{1}{\omega L_r}\left[\int_{(2\pi-\alpha_1)}^{\omega t} v_r . d(\omega t)\right] ; \quad v_r = (v_{O1} - v_{O2})$$

As the o/p voltage v_{O2} is negative

$$v_r = (v_{O1} + v_{O2})$$

$$\therefore \quad i_r = \frac{1}{\omega L_r}\left[\int_{(2\pi-\alpha_1)}^{\omega t} (v_{O1} + v_{O2}) . d(\omega t)\right];$$

$$v_{O1} = -V_m \sin\omega t \text{ for } (2\pi - \alpha_1) \text{ to } \omega t$$

$$i_r = \frac{V_m}{\omega L_r}\left[\int_{(2\pi-\alpha_1)}^{\omega t} -\sin\omega t.d(\omega t) - \int_{(2\pi-\alpha_1)}^{\omega t} \sin\omega t.d(\omega t)\right]$$

$$i_r = \frac{2V_m}{\omega L_r}\left(\cos\omega t - \cos\alpha_1\right)$$

The instantaneous value of the circulating current depends on the delay angle.

For trigger angle (delay angle) $\alpha_1 = 0$,

the magnitude of circulating current becomes min. when $\omega t = n\pi,\ n = 0, 2, 4, ...$ & magnitude becomes max. when $\omega t = n\pi,\ n = 1, 3, 5, ...$

If the peak load current is I_p, one of the converters that controls the power flow may carry a peak current of

$$\left(I_p + \frac{4V_m}{\omega L_r} \right),$$

wher
e

$$I_p = I_{L(\max)} = \frac{V_m}{R_L},$$

&

$$i_{r(\max)} = \frac{4V_m}{\omega L_r} = \text{ max. circulating current}$$

Different Modes Of Operation of Dual converter

- Non-circulating current (circulating current free) mode of operation.

- Circulating current mode of operation.

Non-Circulating Current Mode of Operation

- In this mode only one converter is operated at a time.
- When converter 1 is ON, $0 < \alpha_1 < 90^0$
- V_{dc} is positive and I_{dc} is positive.
- When converter 2 is ON, $0 < \alpha_2 < 90^0$
- V_{dc} is negative and I_{dc} is negative.

Circulating Current Mode Of Operation

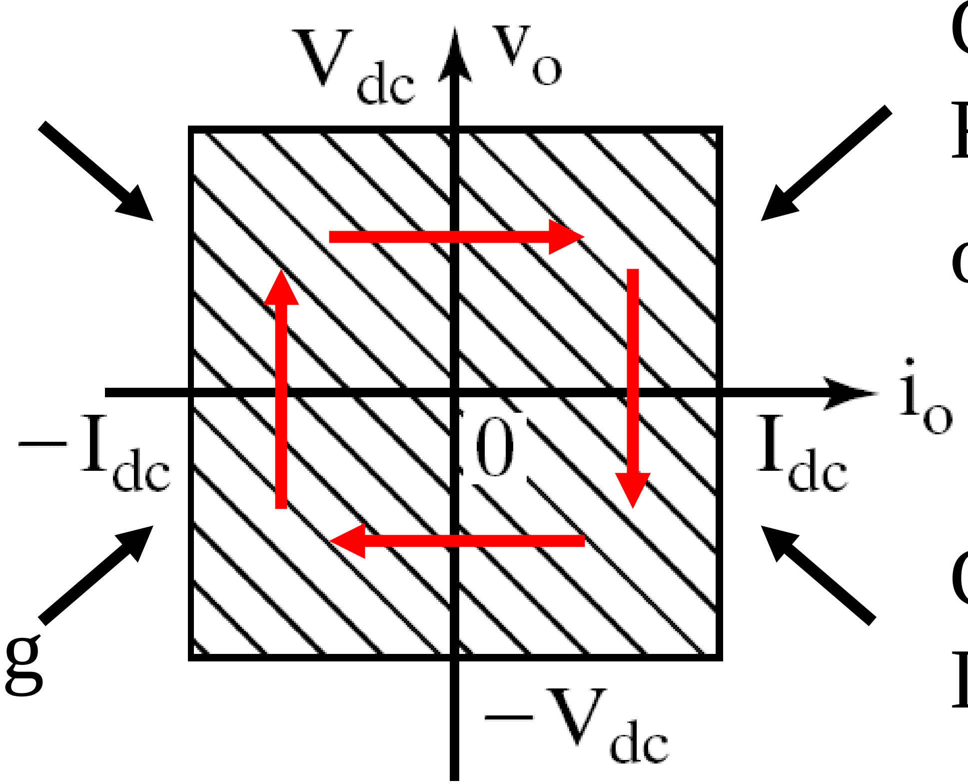
- In this mode, both the converters are switched ON and operated at the same time.
- The trigger angles α_1 and α_2 are adjusted such that $(\alpha_1 + \alpha_2) = 180^0$; $\alpha_2 = (180^0 - \alpha_1)$.

- When $0 < \alpha_1 < 90^0$, converter 1 operates as a controlled rectifier and converter 2 operates as an inverter with $90^0 < \alpha_2 < 180^0$.
- In this case V_{dc} and I_{dc}, both are positive.
- When $90^0 < \alpha 1 < 180^0$, converter 1 operates as an Inverter and converter 2 operated as a controlled rectifier by adjusting its trigger angle α_2 such that $0 < \alpha_2 < 90^0$.
- In this case V_{dc} and I_{dc}, both are negative.

Four Quadrant Operation

Conv. 2
Inverting
$\alpha_2 > 90^0$

Conv. 1
Rectifying
$\alpha_1 < 90^0$



Conv. 2
Rectifying
$\alpha_2 < 90^0$

Conv. 1
Inverting
$\alpha_1 > 90^0$

Advantages of Circulating Current Mode Of Operation

- The circulating current maintains continuous conduction of both the converters over the complete control range, independent of the load.
- One converter always operates as a rectifier and the other converter operates as an inverter, the power flow in either direction at any time is possible.

- As both the converters are in continuous conduction we obtain faster dynamic response. i.e., the time response for changing from one quadrant operation to another is faster.

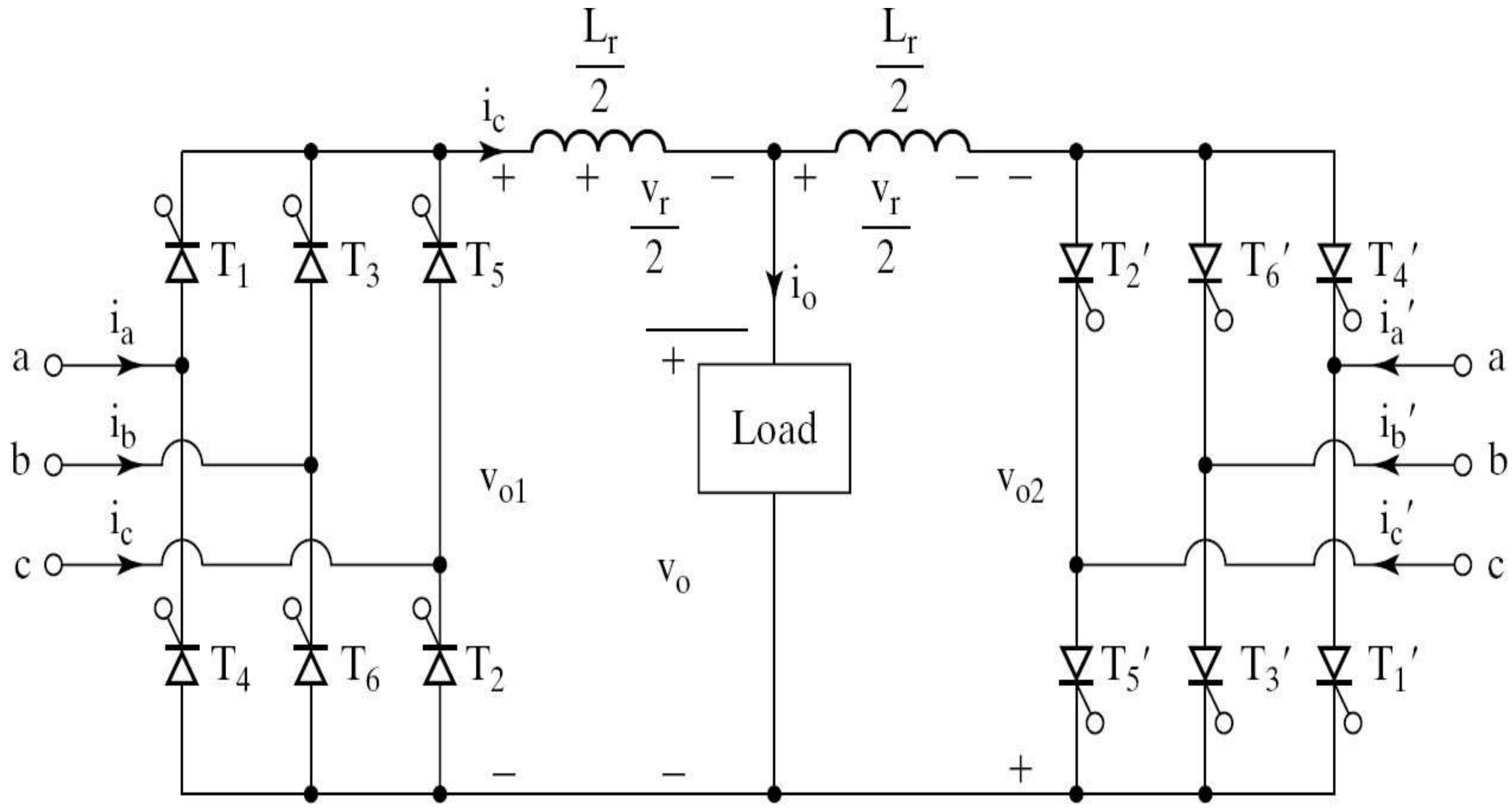
Disadvantages of Circulating Current Mode Of Operation

- There is always a circulating current flowing between the converters.
- When the load current falls to zero, there will be a circulating current flowing between the converters so we need to connect circulating current reactors in order to limit the peak circulating current to safe level.
- The converter thyristors should be rated to carry a peak current much greater than the peak load current.

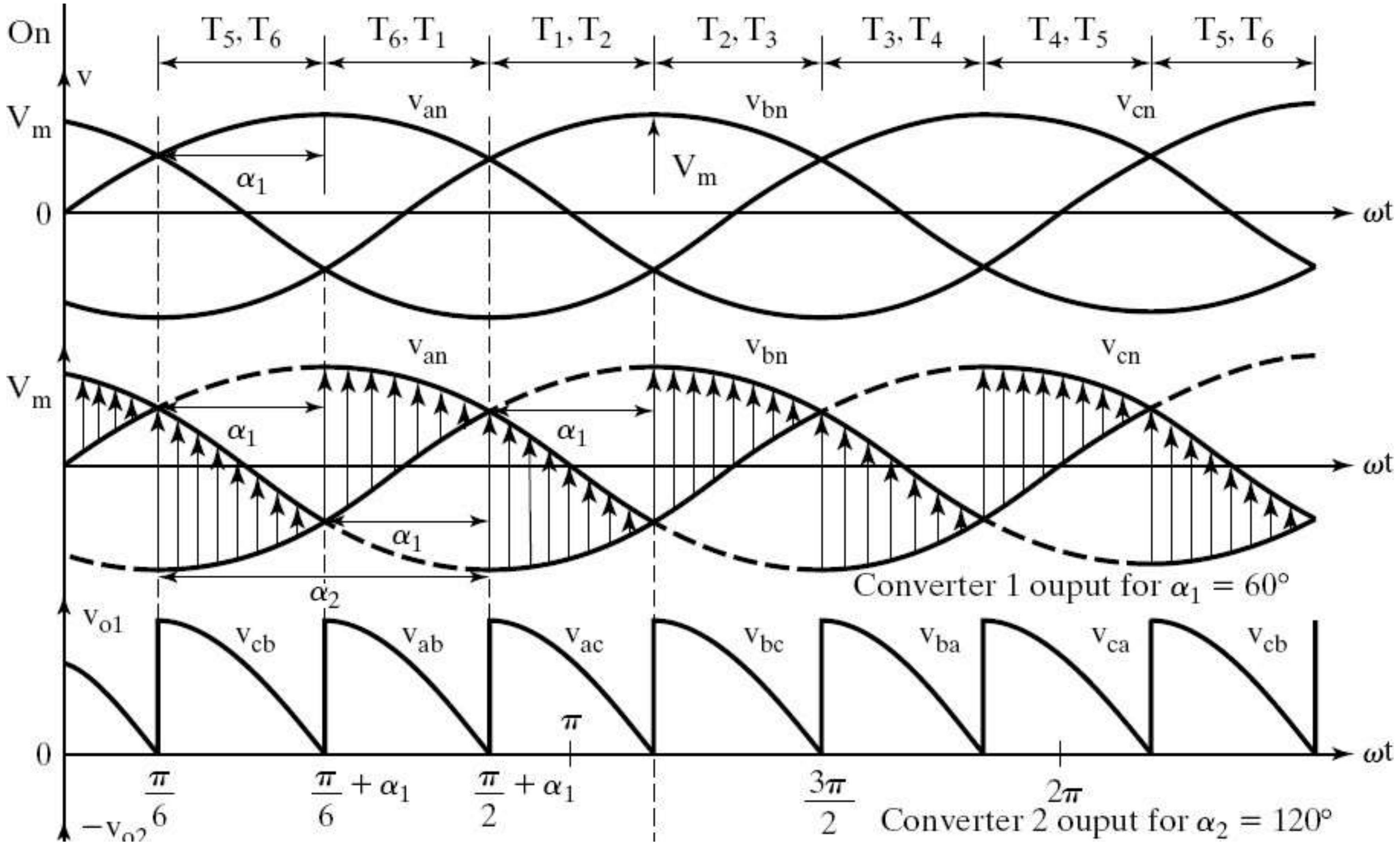
Three Phase Dual Converters

Three Phase Dual Converters

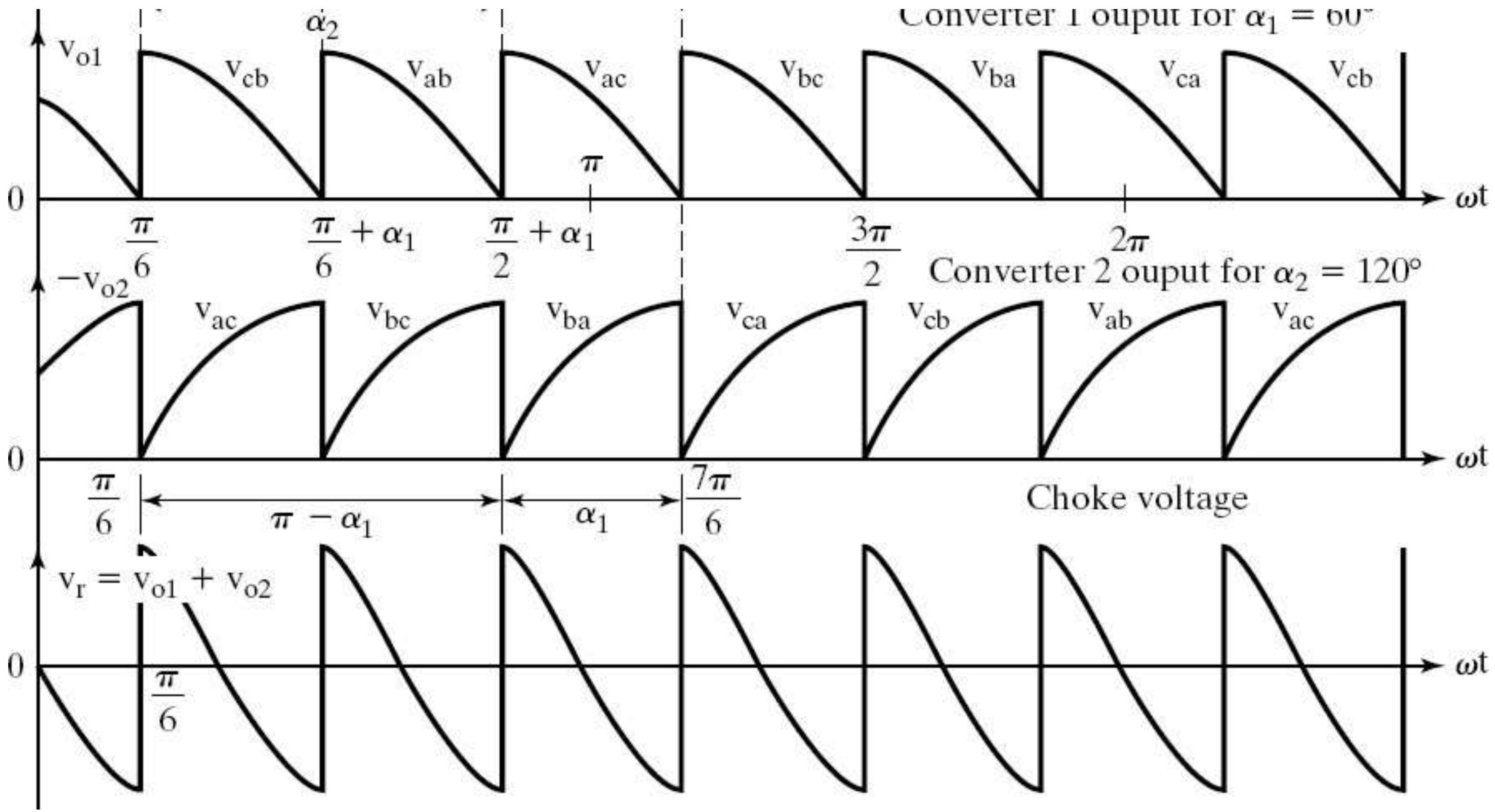
- For four quadrant operation in many industrial variable speed dc drives , 3 phase dual converters are used.
- Used for applications up to 2 mega watt output power level.
- Dual converter consists of two 3 phase full converters which are connected in parallel & in opposite directions across a common load.



i_c
L_r / 2
L_r / 2
+
+
v_r / 2
−
+
v_r / 2
− −
T_1
T_3
T_5
T_2′
T_6′
T_4′
i_o
i_a
i_a′
a
a
+
Load
i_b
i_b′
b
b
v_o1
v_o2
i_c
i_c′
c
c
v_o
T_4
T_6
T_2
T_5′
T_3′
T_1′
−
−
+



On
T_5, T_6
T_6, T_1
T_1, T_2
T_2, T_3
T_3, T_4
T_4, T_5
T_5, T_6
v
V_m
v_{an}
v_{bn}
v_{cn}
α_1
V_m
0
ωt
v_{an}
v_{bn}
v_{cn}
V_m
α_1
α_1
α_1
ωt
α_2
Converter 1 ouput for $\alpha_1 = 60°$
v_{o1}
v_{cb}
v_{ab}
v_{ac}
v_{bc}
v_{ba}
v_{ca}
v_{cb}
π
0
ωt
$\frac{\pi}{6}$
$\frac{\pi}{6} + \alpha_1$
$\frac{\pi}{2} + \alpha_1$
$\frac{3\pi}{2}$
2π
$-v_{o2}$
Converter 2 ouput for $\alpha_2 = 120°$



Converter 1 ouput for $\alpha_1 = 60°$
v_{o1}
v_{cb} v_{ab} v_{ac} v_{bc} v_{ba} v_{ca} v_{cb}
α_2
π
0
ωt
$\frac{\pi}{6}$
$\frac{\pi}{6} + \alpha_1$
$\frac{\pi}{2} + \alpha_1$
$\frac{3\pi}{2}$
2π
Converter 2 ouput for $\alpha_2 = 120°$
$-v_{o2}$
v_{ac} v_{bc} v_{ba} v_{ca} v_{cb} v_{ab} v_{ac}
0
ωt
$\frac{\pi}{6}$
$\pi - \alpha_1$
α_1
$\frac{7\pi}{6}$
Choke voltage
$v_r = v_{o1} + v_{o2}$
0
ωt
$\frac{\pi}{6}$

Outputs of Converters 1 & 2

- During the interval ($\pi/6 + \alpha_1$) to ($\pi/2 + \alpha_1$), the line to line voltage v_{ab} appears across the output of converter 1 and v_{bc} appears across the output of converter 2

We deifine three line neutral voltages

(3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t \qquad ;$$

$$V_m = \text{ Max. Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right) = V_m \sin\left(\omega t - 120^0\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right) = V_m \sin\left(\omega t + 120^0\right)$$

$$= V_m \sin\left(\omega t - 240^0\right)$$

The corresponding line-to-line supply voltages are

$$v_{RY} = v_{ab} = \left(v_{an} - v_{bn}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$v_{YB} = v_{bc} = \left(v_{bn} - v_{cn}\right) = \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$v_{BR} = v_{ca} = \left(v_{cn} - v_{an}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

To obtain an Expression for the Circulating Current

- If v_{O1} and v_{O2} are the output voltages of converters 1 and 2 respectively, the instantaneous voltage across the current limiting inductor during the interval $(\pi/6 + \alpha_1) \leq \omega t \leq (\pi/2 + \alpha_1)$ is given by

$$v_r = v_{O1} + v_{O2} = v_{ab} - v_{bc}$$

$$v_r = \sqrt{3}V_m \left[\sin\left(\omega t + \frac{\pi}{6}\right) - \sin\left(\omega t - \frac{\pi}{2}\right) \right]$$

$$v_r = 3V_m \cos\left(\omega t - \frac{\pi}{6}\right)$$

The circulating current can be calculated by using the equation

$$i_r(t) = \frac{1}{\omega L_r} \int_{\frac{\pi}{6}+\alpha_1}^{\omega t} v_r . d(\omega t)$$

$$i_r(t) = \frac{1}{\omega L_r} \int_{\frac{\pi}{6}+\alpha_1}^{\omega t} 3V_m \cos\left(\omega t - \frac{\pi}{6}\right) . d(\omega t)$$

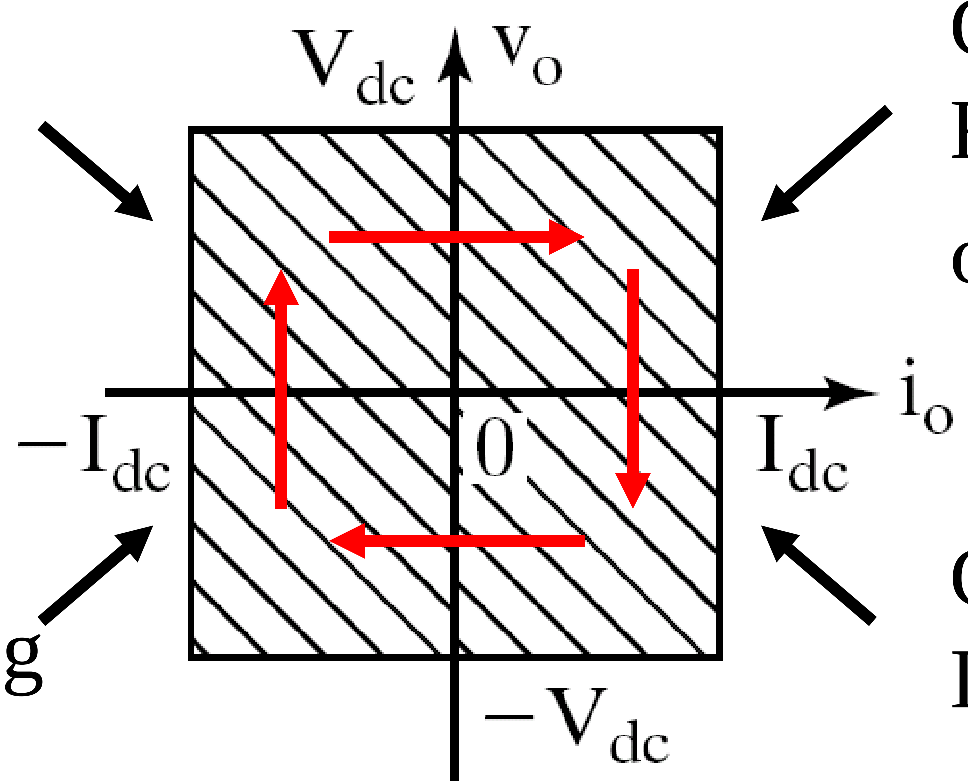
$$i_r(t) = \frac{3V_m}{\omega L_r}\left[\sin\left(\omega t - \frac{\pi}{6}\right) - \sin\alpha_1\right]$$

$$i_{r(\max)} = \frac{3V_m}{\omega L_r}$$

Four Quadrant Operation

Conv. 2
Inverting
$\alpha_2 > 90^0$

Conv. 1
Rectifying
$\alpha_1 < 90^0$



Conv. 2
Rectifying
$\alpha_2 < 90^0$

Conv. 1
Inverting
$\alpha_1 > 90^0$

Contd…

- There are two different modes of operation.
  - Circulating current free (non circulating) mode of operation
  - Circulating current mode of operation

Non Circulating
Current Mode Of Operation

- In this mode of operation only one converter is switched on at a time
- When the converter 1 is switched on,

  For $\alpha_1 < 90^0$ the converter 1 operates in the Rectification mode

  V_{dc} is positive, I_{dc} is positive and hence the average load power P_{dc} is positive.
- Power flows from ac source to the load

- When the converter 1 is on,

  For $\alpha_1 > 90^0$ the converter 1 operates in the Inversion mode

  V_{dc} is negative, I_{dc} is positive and the average load power P_{dc} is negative.
- Power flows from load circuit to ac source.

- When the converter 2 is switched on,

  For $\alpha_2 < 90^0$ the converter 2 operates in the Rectification mode

  V_{dc} is negative, I_{dc} is negative and the average load power P_{dc} is positive.

- The output load voltage & load current reverse when converter 2 is on.
- Power flows from ac source to the load

- When the converter 2 is switched on,

  For $\alpha_2 > 90^0$ the converter 2 operates in the Inversion mode

  V_{dc} is positive, I_{dc} is negative and the average load power P_{dc} is negative.
- Power flows from load to the ac source.
- Energy is supplied from the load circuit to the ac supply.

Circulating Current Mode Of Operation

- Both the converters are switched on at the same time.
- One converter operates in the rectification mode while the other operates in the inversion mode.
- Trigger angles α_1 & α_2 are adjusted such that $(\alpha_1 + \alpha_2) = 180^0$

- When $\alpha_1 < 90^0$, converter 1 operates as a controlled rectifier. α_2 is made greater than 90^0 and converter 2 operates as an Inverter.
- V_{dc} is positive & I_{dc} is positive and P_{dc} is positive.