UNIT IV

Scattering matrix

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UNIT-IV

Scattering matrix

Scattering matrix: Scattering Matrix Properties, Directional Couplers – 2 Hole, Bethe Hole, [s] matrix of Magic Tee and Circulator. Microwave Measurements: Description of Microwave Bench – Different Blocks and their Features, Errors and Precautions, Measurement of Attenuation, Frequency. Standing Wave Measurements, measurement of Low and High VSWR, Cavity Q, Impedance Measurements.

S (Scattering) - Parameters

- Low frequency circuits can be described by two port networks and their parameters such as Z, Y, H, ABCD etc. as per network theory related to total voltages and total currents.
- In a similar way at microwave frequencies, travelling waves with associated powers instead of Voltages and Currents and microwave junction can be defined by what are called as S-parameters or Scattering parameters.

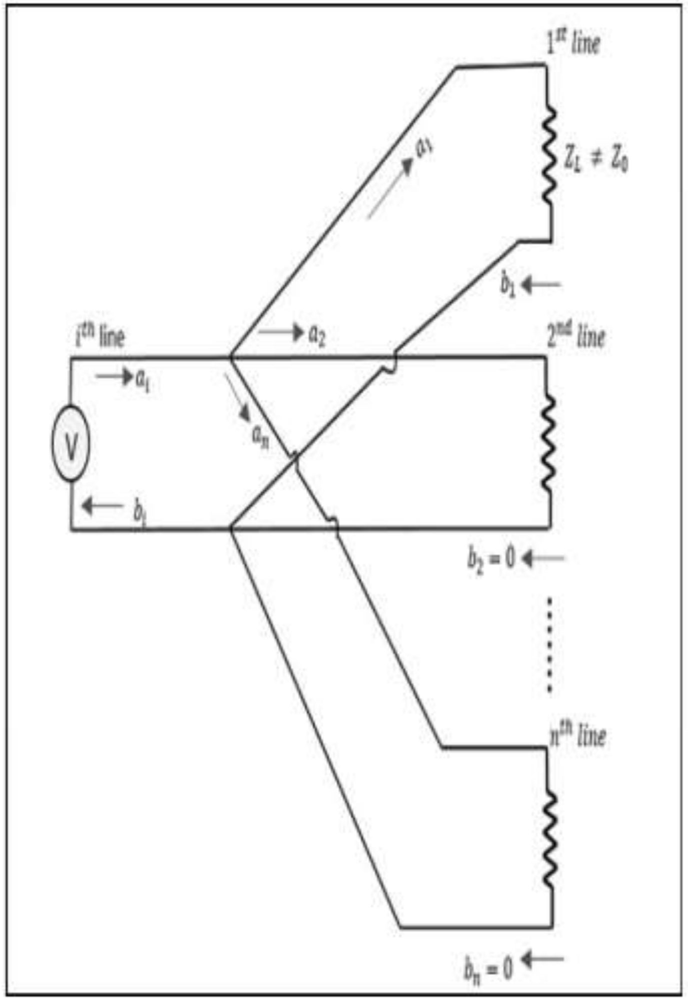
S (Scattering) - Parameters

- The matrix used to represent S- parameters is called as *Scattering matrix or S-matrix*.
- It is a square matrix which gives all the combinations of power relationships between the various input and output ports of microwave junction.
- The elements in this matrix are also called as *scattering coefficient*.

S (Scattering) - Parameters

To obtained the relation ship between scattering matrix and scattering coefficients, consider a junction of ‘n’ number of transmission lines where in the ith line is terminated in a source.

Case I: Let the first line be terminated in an impedance other than the characteristic impedance $Z_L \neq Z_o$ and all the remaining lines in an impedance equal to Z_o



S (Scattering) - Parameters

- If a_i be the incident wave at the junction due to the source at the ith line, then it divides itself among (n-1) number of lines as a_1, a_2, a_3…..a_n.
- There will be no reflections from 2nd to nth line and the incident waves are absorbed since their impedances are equal to characteristic impedance.
- But there is a mismatch at the 1st line and hence there will be a reflected wave b_1 going back in to the junction.

Therefore $$b_1 = (reflection\ coefficient)a_1 = S_{i1}a_1$$

$$S_{i1} = Reflection\ coefficient\ of\ 1st\ line$$

S (Scattering) - Parameters

Case II: Let all the (n-1) lines be terminated in an impedance other than characteristic impedance Z_o

there will be reflections into the junction from every line

Therefore $b_i = S_{i1}a_1 + S_{i2}a_2 + S_{i3}a_3 + \cdots + S_{in}a_n$

$$b_1 = S_{11}a_1 + S_{12}a_2 + S_{13}a_3 + \ldots\ldots\ldots\ldots\ldots + S_{1n}a_n$$

$$b_2 = S_{21}a_1 + S_{22}a_2 + S_{23}a_3 + \ldots\ldots\ldots\ldots\ldots + S_{2n}a_n$$

$$b_n = S_{n1}a_1 + S_{n2}a_2 + S_{n3}a_3 + \ldots\ldots\ldots\ldots\ldots + S_{nn}a_n$$

S (Scattering) - Parameters

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ . \\ . \\ . \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & \dots & S_{1n} \\ S_{21} & S_{22} & S_{23} & \dots & S_{2n} \\ . & . & . & \dots & . \\ . & . & . & \dots & . \\ . & . & . & \dots & . \\ S_{n1} & S_{n2} & S_{n3} & \dots & S_{nn} \end{bmatrix} \times \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ . \\ . \\ . \\ a_n \end{bmatrix}$$

Column matrix $[b]$ **Scattering matrix** $[S]$ **Matrix** $[a]$

$$[b] = [S]\,[a]$$

Properties of S (Scattering)-Matrix

1. $[S]$ is always a square matrix of order (n x n)
2. $[S]$ is symmetric matrix i.e., $S_{ij} = S_{ji}$
3. $[S]$ is a unitary matrix i.e., $[S][S]^* = [I]$
4. The sum of the products of each term of any row or column multiplied by the complex conjugate of the corresponding terms of any other row or column is zero.

$$\sum_{i=1}^{n} S_{ik} S_{ij}^* = 0 \quad \text{for } k \neq j \qquad (k = 1, 2, 3, \dots n) \quad (j = 1, 2, 3, \dots n)$$

5. If any of the terminal or reference planes are moved away from the junction by an electrical distance $\beta_k l_k$, each of the coefficients S_{ij} involving k will be multiplied by the factor $e^{-j\beta_k l_k}$.

Microwave T-Junctions

- A T- Junction is an interconnection of three waveguides in the form of English alphabet “T”.
- There are several types of Tee junctions
  - H-plane Tee junction
  - E-plane Tee junction
  - E-H plane Tee junction (Hybrid T junction)
  - Magic-T junction

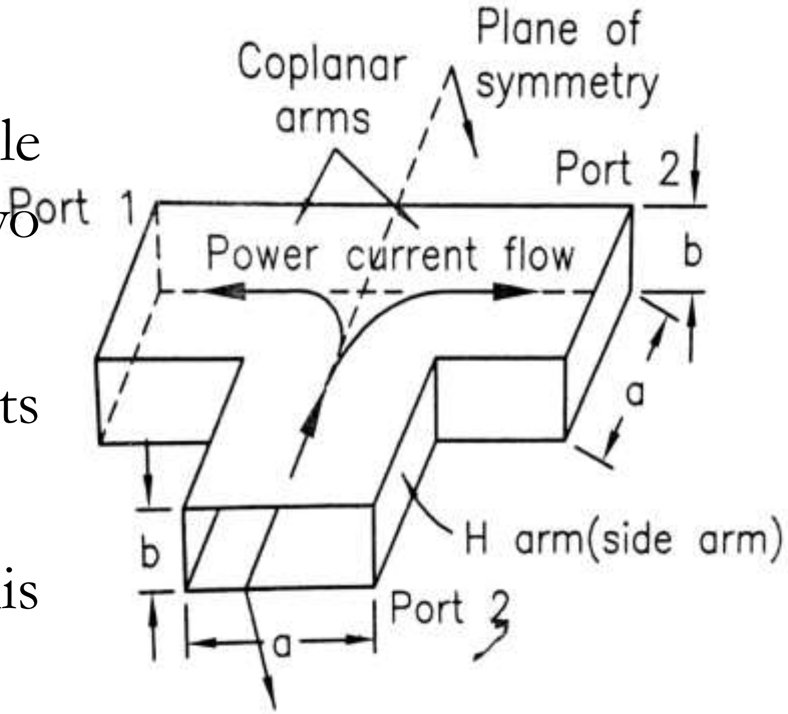
H-plane Tee Junction

An H-Plane Tee junction is formed by attaching a simple waveguide to a rectangular waveguide which already has two ports.

The arms of rectangular waveguides make two ports called **collinear ports** i.e., Port1 and Port2,

while the new one, Port3 is called as Side arm or **H-arm**. This H-plane Tee is also called as **Shunt Tee**.

As the axis of the side arm is parallel to the magnetic field, this junction is called H-Plane Tee junction.



H-plane Tee Junction

This is also called as Current junction, as the magnetic field divides itself into arms.

- The properties of H-Plane Tee can be defined by its [S] matrix.
- It is a 3×3 matrix as there are 3 possible inputs and 3 possible outputs.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

- Now we determine the S parameters S_{ij}, i→ 1, 2, 3, j → 1,2,3, by applying the properties of $[S]$

1. Because of plane of symmetry of the junction scattering coefficients $S_{13}$and S_{23} must be equal

$$S_{13} = S_{23}$$

H-plane Tee Junction

2. From the symmetric property $S_{ij} = S_{ji}$

$S_{12} = S_{21},\ S_{13} = S_{31},\ S_{23} = S_{32} = S_{13}$

3. Since port is perfectly matched to the junction $S_{33} = 0$

With these parameters $[S]$ matrix becomes

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix}$$

4. From the Unitary property

$$[S]\ [S]* = [I]$$

$$i.e., \quad \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

H-plane Tee Junction

Multiplying we get

$$R_1C_1: \quad S_{11}S_{11}^* + S_{12}S_{12}^* + S_{13}S_{13}^* = 1$$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1$$

$$\text{Similarly } R_2C_2: \quad |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1$$

$$R_3C_3: \quad |S_{13}|^2 + |S_{13}|^2 = 1$$

$$R_3C_1: \quad S_{13}S_{11}^* + S_{13}S_{12}^* = 0$$

From equation

$$2|S_{13}|^2 = 1 \quad \text{or} \quad S_{13} = \frac{1}{\sqrt{2}}$$

By comparing eqns and

$$|S_{11}|^2 = |S_{22}|^2$$

$$S_{11} = S_{22}$$

From eqn

$$S_{13}(S_{11}^* + S_{12}^*) = 0$$

$$S_{13} \neq 0, S_{11}^* + S_{12}^* = 0, \quad \text{or} \quad S_{11}^* = -S_{12}^*$$

$$S_{11} = -S_{12} \quad \text{or} \quad S_{12} = -S_{11}$$

H-plane Tee Junction

Using above relation in eq $|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} = 1 \text{ or } 2\,|S_{11}|^2 = \frac{1}{2} \text{ or } S_{11} = \frac{1}{2}$

From eqns and $S_{12} = -\frac{1}{2} \qquad S_{22} = \frac{1}{2}$

By substituting above eq

$$[S] = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ & & 0 \end{bmatrix}$$

We know that [b] = [S] [a]

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

H-plane Tee Junction

$$b_1 = \frac{1}{2}a_1 - \frac{1}{2}a_2 + \frac{1}{\sqrt{2}}a_3$$

$$b_2 = -\frac{1}{2}a_1 - \frac{1}{2}a_2 + \frac{1}{\sqrt{2}}a_3$$

$$b_3 = \frac{1}{\sqrt{2}}a_1 + \frac{1}{\sqrt{2}}a_2$$

Case I: $a_3 \neq 0, a_1 = 0, a_2 = 0,$

Input is given at port no inputs at port and port . By substituting these values in above equations

$$b_1 = \frac{a_3}{\sqrt{2}}, b_2 = \frac{a_3}{\sqrt{2}} \text{ and } b_3 = 0$$

H-plane Tee Junction

Let P_3 be the power input at port 3. then this power divides equally between ports and in phase

i.e., $P_1 = P_2$

$$P_3 = P_1 + P_2 = 2\,P_1 = 2\,P_2$$

H-plane Tee Junction

The amount of power coming out of port or port 2 due to input at port

$$= 10\log_{10}\frac{P_1}{P_3} = 10\log_{10}\frac{P_1}{2P_1} = 10\log_{10}\left(\frac{1}{2}\right)$$

$$= -10\log_{10}^{2} = -10(0.3010) \cong -3\text{ dB}$$

Hence the power coming out of port or port is 3dB down w.r.t input power at port 3, hence the H-plane is called as *3-dB splitter*.

Case 2 :

$$a_1 = a_2 = a,\ a_3 = 0$$

$$b_1 = \frac{a}{2} - \frac{a}{2} + \frac{1}{\sqrt{2}}a_3 = \frac{a_3}{\sqrt{2}} = 0$$

$$b_2 = -\frac{a}{2} + \frac{a}{2} + \frac{1}{\sqrt{2}}a_3 = \frac{a_3}{\sqrt{2}} = 0$$

$$b_3 = \frac{a_1}{\sqrt{2}} + \frac{a_2}{\sqrt{2}} = \frac{a}{\sqrt{2}} + \frac{a}{\sqrt{2}}$$

The output at port is addition of the two inputs at port and port and these are added in phase.

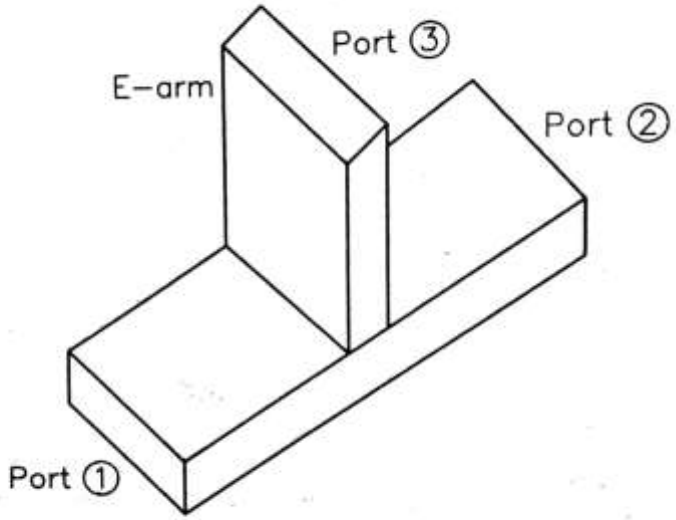
E-plane Tee Junction

An E-Plane Tee junction is formed by attaching a simple waveguide to the broader dimension of a rectangular waveguide, which already has two ports.

The arms of rectangular waveguides make two ports called collinear ports i.e., Port1 and Port2, while the new one, Port3 is called as Side arm or E-arm.

This is also called as **Voltage** or **Series junction**. The ports 1 and 2 are 180° out of phase with each other.

As the axis of the side arm is parallel to the electric field, this junction is called E-Plane Tee junction.



E-plane Tee Junction

- The properties of E-Plane Tee can be defined by its [S] matrix.

1. It is a 3×3 matrix as there are 3 possible inputs and 3 possible outputs.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

2. The scattering coefficient $S_{23} = -S_{13}$

   Since output at ports 1 and 2 are out of phase by 180 degrees with an input at port 3.

3. If port 3 is perfectly matched to the junction

$$S_{33} = 0$$

4. From the symmetric property

$$S_{12} = S_{21}$$

$$S_{13} = S_{31}$$

$$S_{23} = S_{32}$$

E-plane Tee Junction

With the above properties [S] becomes

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix}$$

5. From unitary property

$$[S].[S]* = [I]$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} . \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1 C_1$: $|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1$

$R_2 C_2$: $|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1$

$R_3 C_3$: $|S_{13}|^2 + |S_{13}|^2 = 1$

$R_3 C_1$: $S_{13} \cdot S_{11}^* - S_{13} S_{12}^* = 0$

E-plane Tee Junction

From eq and $S_{11} = S_{22}$

From eq $S_{13} = \frac{1}{\sqrt{2}}$

From eq $S_{13}(S_{11}^* - S_{12}^*) = 0$ or $S_{11} = S_{12} = S_{22}$

Substitute eqn in eq

$$|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} = 1$$

$$2\,|S_{11}|^2 = \frac{1}{2} \text{ or } S_{11} = \frac{1}{2}$$

By using above values

$$[S] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \end{bmatrix}$$

E-plane Tee Junction

$$[b] = [S]\ [a]$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$b_1 = \frac{1}{2} a_1 + \frac{1}{2} a_2 + \frac{1}{\sqrt{2}} a_3$$

$$b_2 = \frac{1}{2} a_1 + \frac{1}{2} a_2 - \frac{1}{\sqrt{2}} a_3$$

$$b_3 = \frac{1}{\sqrt{2}} a_1 - \frac{1}{\sqrt{2}} a_2$$

E-plane Tee Junction

Case I: $a_1 = a_2 = 0,\ a_3 \neq 0$

$$b_1 = \frac{1}{\sqrt{2}} a_3 \, ; \, b_2 = -\frac{1}{\sqrt{2}} a_3 \, ; \, b_3 = 0$$

i.e., An input at port 3 equally divides between 1 and 2 but introduces a phase shift of 180 degrees between the two outputs. Hence E-plane Tee also acts as a 3-dB splitter

Case II: $a_1 = a_2 = a, a_3 = 0$ then $b_1 = \frac{a}{2} + \frac{a}{2} \, ; \, b_2 = \frac{a}{2} + \frac{a}{2} \, ; \, b_3 = \frac{1}{\sqrt{2}} a - \frac{1}{\sqrt{2}} a = 0$

i.e., equal inputs at port1 and port 2 result in no output at port 3.

Case III: $a_1 \neq 0, a_2 = 0, a_3 = 0$

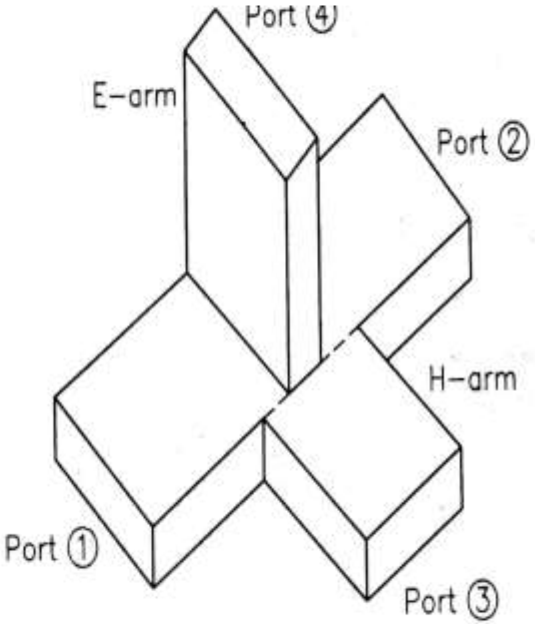
$$b_1 = \frac{a_1}{2} \, ; \, b_2 = \frac{a_1}{2} \, ; \, b_3 = \frac{-a_1}{\sqrt{2}},$$

Similarly we can have all combinations of inputs and outputs.

E-H plane (Hybrid or Magic) Tee Junction

An E-H plane T junction is formed by attaching two simple waveguides one parallel and all the other series, to rectangular waveguide which already has two ports. This is also called as Magic Tee or Hybrid Tee or 3 dB coupler.

The arms of rectangular waveguides make two ports called collinear ports i.e., Port 1 and Port 2, while the Port 3 is called as H-Arm or Sum port or Parallel port. Port 4 is called as E-Arm or Difference port or Series port.



E-H plane (Hybrid or Magic) Tee Junction

Using the properties of E-H plane Tee, its scattering matrix can be obtained

1. [S] matrix is a 4x4 matrix since there are 4 ports.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

2. Because of H-Plane Tee $S_{23} = S_{13}$
3. Because of E-Plane Tee $S_{24} = -S_{14}$
4. Because of geometry of the junction an input at port 3 cannot come out of port 4 since they are isolated ports and vice versa. $S_{34} = S_{43} = 0$
5. From the symmetric Property

$$S_{12} = S_{21}\,;\, S_{13} = S_{31}\,;\, S_{23} = S_{32}\,;$$

$$S_{34} = S_{43}\,;\, S_{24} = S_{42}\,;\, S_{41} = S_{14}$$

E-H plane (Hybrid or Magic) Tee Junction

6. If ports 3 and 4 are perfectly matched to the junction $S_{33} = S_{44} = 0$

7. Substituting above properties in [S] matrix

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix}$$

8. From Unitary property $[S][S]^* = [I]$

$$i.e., \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

E-H plane (Hybrid or Magic) Tee Junction

R_1C_1 : $|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$

R_2C_2 : $|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$

R_3C_3 : $|S_{13}|^2 + |S_{13}|^2 = 1$

R_4C_4 : $|S_{14}|^2 + |S_{14}|^2 = 1$

$$S_{13} = \frac{1}{\sqrt{2}} \qquad S_{14} = \frac{1}{\sqrt{2}}$$

$$S_{11} = S_{22}$$

$$|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} + \frac{1}{2} = 1$$

$$|S_{11}|^2 + |S_{12}|^2 = 0$$

$$S_{11} = S_{12} = 0$$

$$S_{22} = 0$$

E-H plane (Hybrid or Magic) Tee Junction

This means ports 1 and 2 are also perfectly matched to the junction. Hence in any four port junction, if any two ports are perfectly matched to the junction then the remaining two ports are automatically matched to the junction. Such a junction where in all the four ports are perfectly matched to the junction is called a Magic Tee.

The [S] of Magic Tee is obtained by substituting the scattering parameters

$$[S] = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

E-H plane (Hybrid or Magic) Tee Junction

We know that, $[b] = [S]\,[a]$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$$\therefore \quad b_1 = \frac{1}{\sqrt{2}}(a_3 + a_4)\,; \quad b_3 = \frac{1}{\sqrt{2}}(a_1 + a_2)$$

$$b_2 = \frac{1}{\sqrt{2}}(a_3 - a_4)\,; \quad b_4 = \frac{1}{\sqrt{2}}(a_1 - a_2)$$

Case I: $a_3 \neq 0, a_1 = a_2 = a_4 = 0$ then $b_1 = \frac{a_3}{\sqrt{2}}; b_2 = \frac{a_3}{\sqrt{2}}; b_3 = b_4 = 0$

This is the property of H-Plane Tee.

E-H plane (Hybrid or Magic) Tee Junction

Case II: $a_4 \neq 0, a_1 = a_2 = a_3 = 0$ then $b_1 = \frac{a_4}{\sqrt{2}}; b_2 = -\frac{a_4}{\sqrt{2}}; b_3 = b_4 = 0$

This is the property of E-Plane Tee.

Case III: $a_1 \neq 0, a_2 = a_3 = a_4 = 0$ then $b_1 = 0; b_2 = 0; b_3 = \frac{a_1}{\sqrt{2}}; b_4 = \frac{a_1}{\sqrt{2}}$

i.e., When power is fed into port 1, nothing comes out of port 2 even though they are collinear ports (Magic). Hence port 1 and port 2 are called isolated ports. Similarly an input at port 2 cannot come out at port 1. similarly E and H ports are isolated ports.

Case IV: $a_3 = a_4, a_1 = a_2 = 0$ then $b_1 = \frac{1}{\sqrt{2}}(2a_3); b_2 = 0; b_3 = b_4 = 0$

This is nothing but the additive property. Equal inputs at ports 3 and 4 result in an output at port 1.

E-H plane (Hybrid or Magic) Tee Junction

Case V: $a_1 = a_2,\ a_3 = a_4 = 0$

then $b_1 = 0 = b_2 = b_4;\ b_3 = \frac{1}{\sqrt{2}}(2\,a_1)$

i.e., Equal inputs at ports 1 and 2 results in an output at port 3, no output ports at 1, 2, 4.

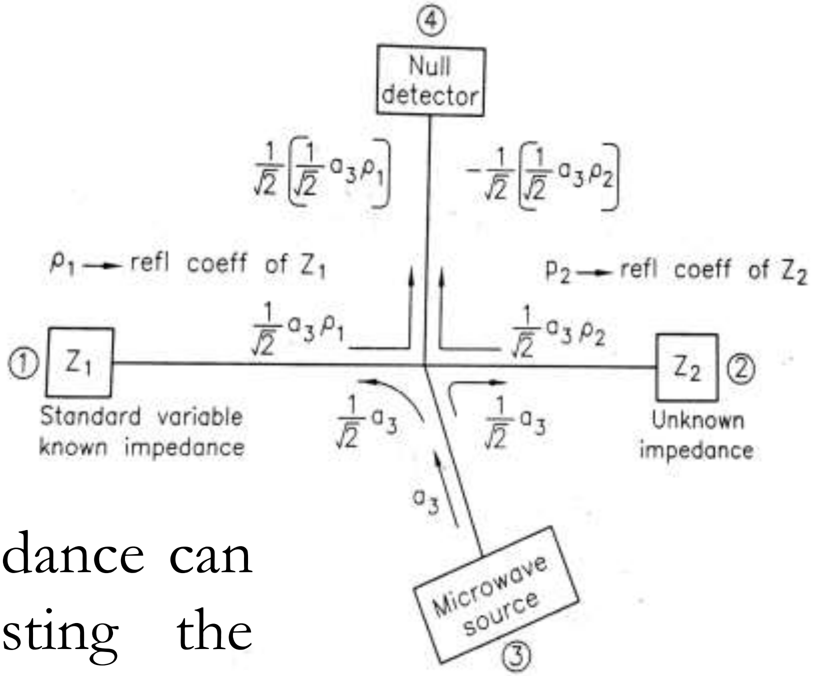
Applications of Magic Tee

1. **Measurement of Impedance:**
   - A Magic Tee has been used in the form of bridge for measuring impedance.
   - Microwave source is connected in the arm 3 and a null detector in arm 4. The unknown impedance is connected in arm 2 and a standard variable known impedance in arm 1.
   - Using the properties of Magic Tee the power from microwave source (a_3) gets divided equally between arms 1 and 2.
   - These impedances are not equal to characteristic impedances Z_o and hence there will be reflections from arms 1 and 2.
   - If ρ_1 and ρ_2 are the reflection coefficients, powers $\frac{1}{\sqrt{2}} a_3\rho_1$ and $\frac{1}{\sqrt{2}} a_3\rho_2$ enter the Magic Tee junction from arms 1 and 2.
   - The resultant wave into arm 4 i.e., the null detector can be calculated as follows

Applications of Magic Tee

The net wave reaching the null detector

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}a_3\rho_1\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}a_3\rho_2\right) = \frac{1}{2}a_3(\rho_1 - \rho_2)$$



For perfect balancing of the bridge above eq is

$$\frac{1}{2}a_3(\rho_1 - \rho_2) = 0$$

$$\rho_1 - \rho_2 = 0 \quad \text{or} \quad \rho_1 = \rho_2$$

$$\frac{Z_1 - Z_z}{Z_1 + Z_z} = \frac{Z_2 - Z_z}{Z_2 + Z_z}$$

$$Z_1 = Z_2$$

$$R_1 + jX_1 = R_2 + jX_2$$

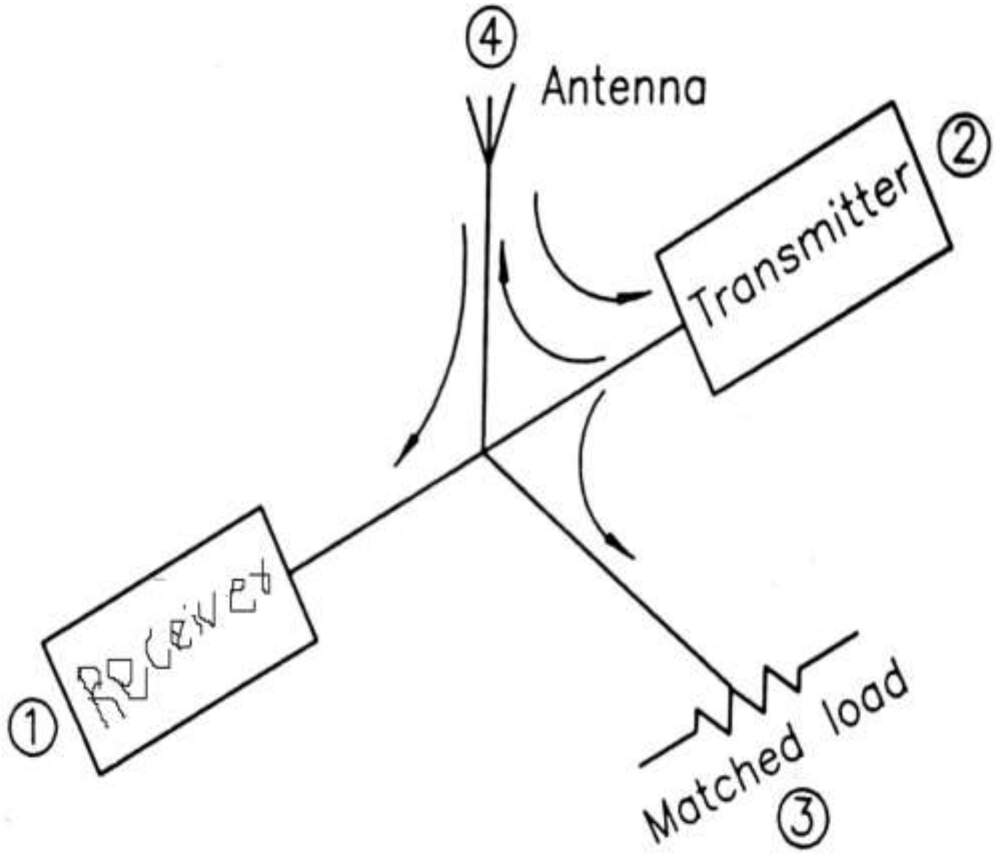
$$R_1 = R_2 \quad \text{and} \quad X_1 = X_2.$$

Thus the unknown impedance can be measured by adjusting the standard variable impedance till the bridge is balanced and both impedances become equal

Applications of Magic Tee

2. **Magic Tee as a Duplexer:**

- The transmitter and receiver are connected in ports 2 and 1 respectively, antenna in the E-arm or port 4 and port 3 of Magic Tee is terminated in a matched load.
- During transmission half of the power reaches the antenna from where it is radiated into space. Other half reaches the matched load where it is absorbed without reflections.
- No transmitter power reaches the receiver since port 1 and 3 are isolated ports.
- During the reception, half of the received power goes to the receiver and the other half to the transmitter are isolated as well as during transmission.

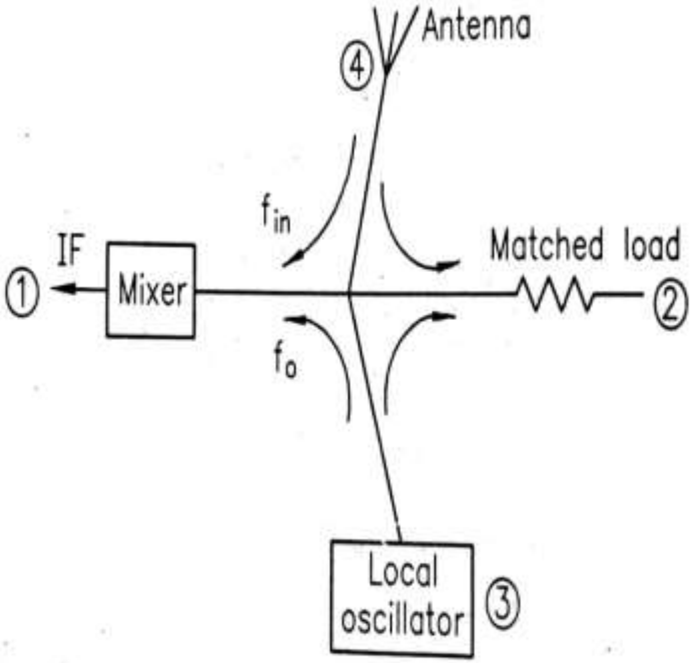


Applications of Magic Tee

3. Magic Tee as a Mixer:

- The Magic Tee can also be used in microwave receivers as a mixer where the signal and local oscillator are fed into the E and H arms.

- Half of the local oscillator power and half of the received power from antenna goes to the mixer where they are mixed to generate the IF frequency.



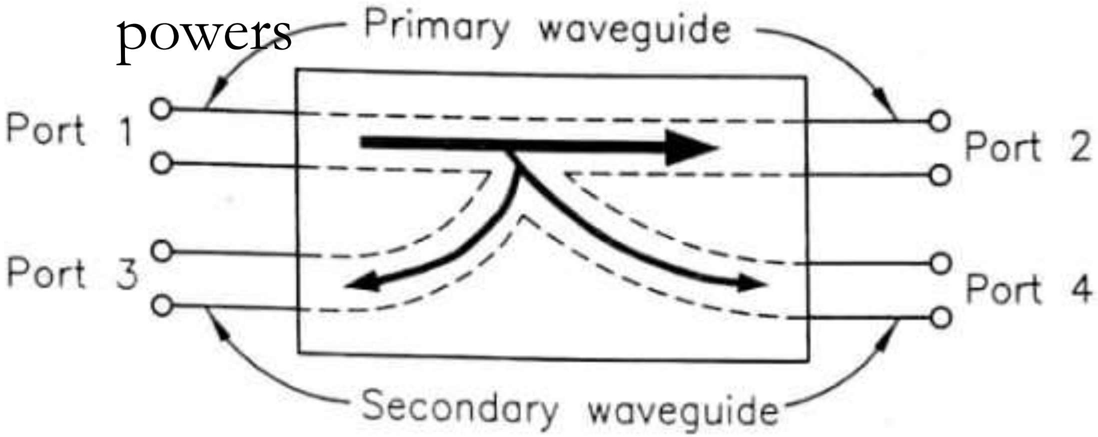
Directional Couplers

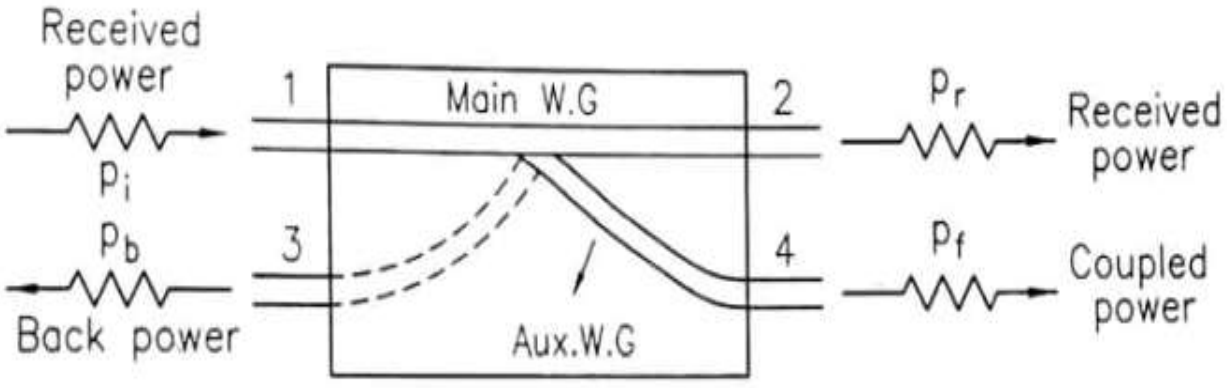
A Directional coupler is a device that samples a small amount of microwave power for measurement purposes.

Measurements include Incident power, Reflected power and VSWR etc..,

Directional Coupler is a four port waveguide junction consisting of primary main waveguide and a secondary auxiliary waveguide.

A Schematic of a directional coupler Directional coupler indicating powers





Directional Couplers

- Directional coupler is used to couple the microwave power which may be unidirectional or bi-directional.

Properties of Directional Couplers

- All the terminations are matched to the ports.
- When the power travels from Port 1 to Port 2, some portion of it gets coupled to Port 4 but not to Port 3.
- As it is also a bi-directional coupler, when the power travels from Port 2 to Port 1, some portion of it gets coupled to Port 3 but not to Port 4.
- If the power is incident through Port 3, a portion of it is coupled to Port 2, but not to Port 1.
- If the power is incident through Port 4, a portion of it is coupled to Port 1, but not to Port 2. Port 1 and 3 are decoupled as are Port 2 and Port 4.

Ideally, the output of Port 3 should be zero. However, practically, a small amount of power called **back power** is observed at Port 3.

Directional Couplers

Coupling Factor C: The coupling factor of a directional coupler is defined as the ratio of the incident power to the forward power measured in dB.

$$C = 10\, log_{10} \frac{P_i}{P_f} dB$$

Directivity: The directivity of the directional coupler as the ration of the froward power to the back power measured in dB.

$$D = 10\, log_{10} \frac{P_f}{P_b} dB$$

Isolation: It is defined to describe the directive properties of a directional coupler. It is the ratio of incident power to the back power measured in dB

$$I = 10\, log_{10} \frac{P_i}{P_b} dB$$

Directional Couplers

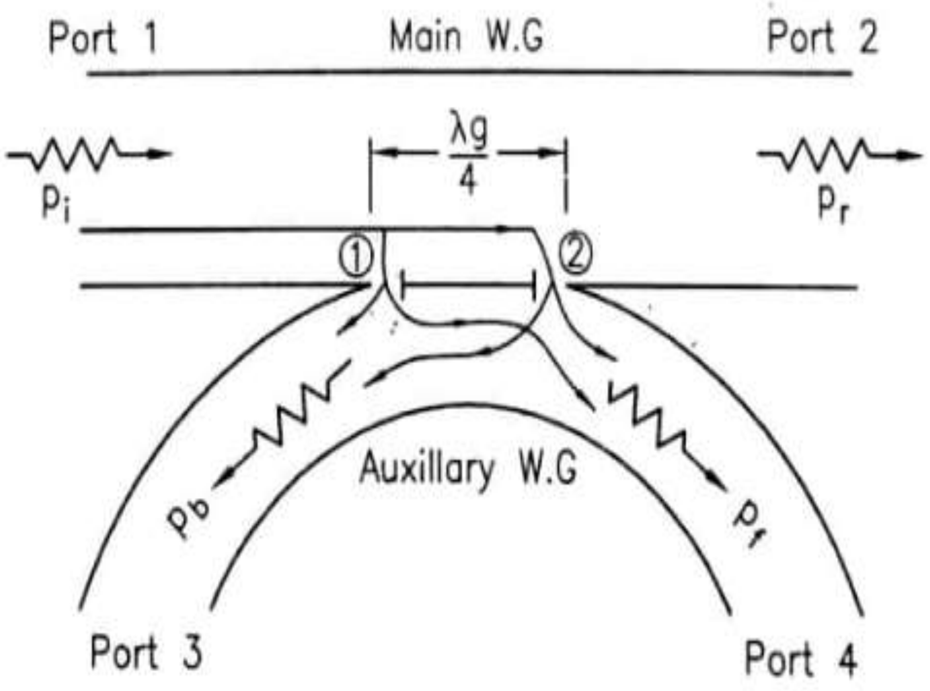
Two Hole Directional Coupler:

Two hole directional coupler consists of two guides the main and auxiliary with two tiny holes common between them.

The two holes are at a distance of $\frac{\lambda_g}{4}$

A two hole directional coupler is designed to meet the requirement of directional coupler, which is to avoid back power.

Some of the power while travelling between port 1 and port 2 escapes through the holes 1 and 2.



Directional Couplers

- The magnitude of the power depends upon the dimensions of the holes.
- This leakage power at both holes are in phase at hole 2, adding up the power contributing to the forward power P_f.
- The leakage power at both holes are out of phase at hole1, cancelling each other and making back power to zero $P_b = 0$.
- The degree of coupling is determined by size and location of the holes in the waveguide walls.
- High degree of directivity can be achieved.

Directional Couplers

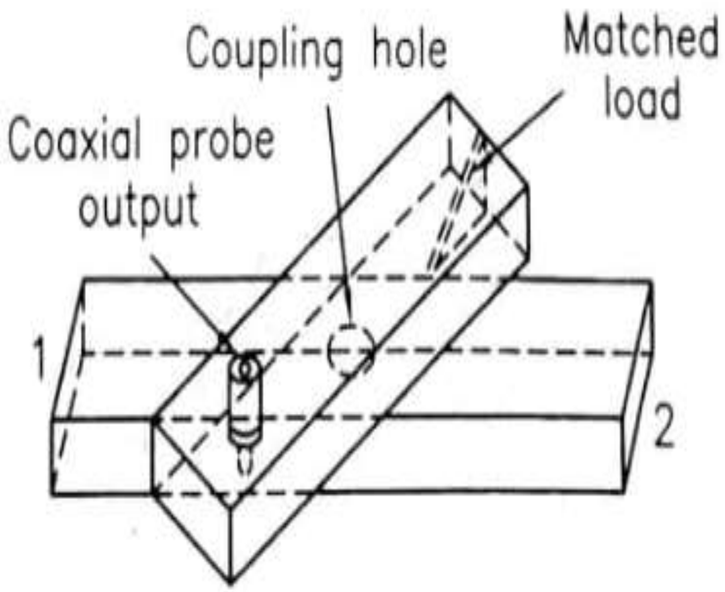
Bethe or Single hole coupler

The power entering to the port 1 is coupled to the coaxial probe output and the power entering port 2 is absorbed by the matched load.

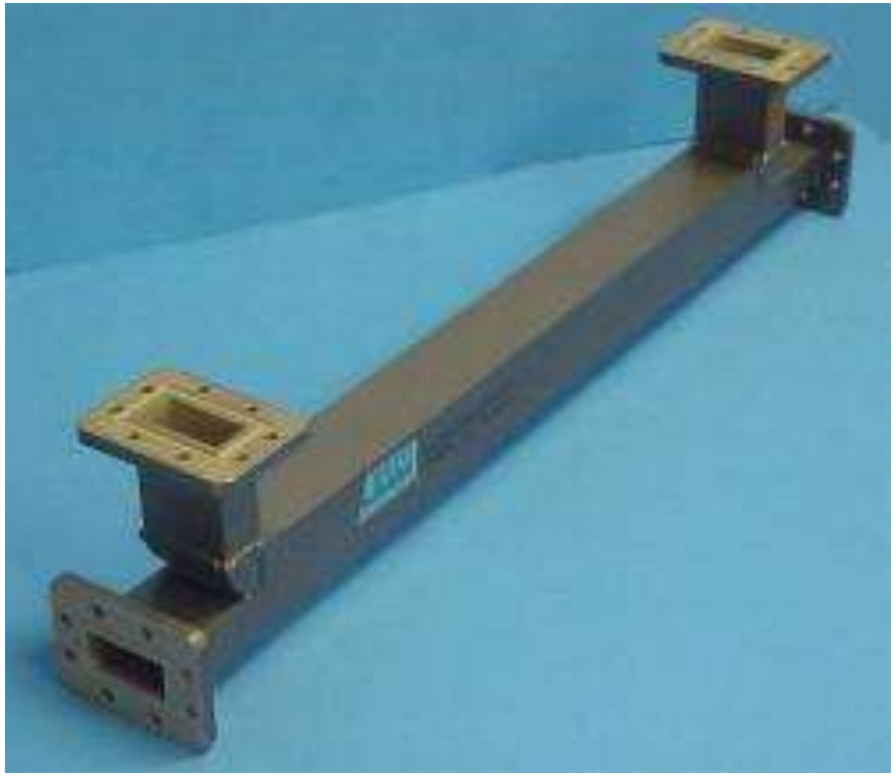
The waves in the auxiliary guide are generated through a single hole which include both electrical and magnetic fields.

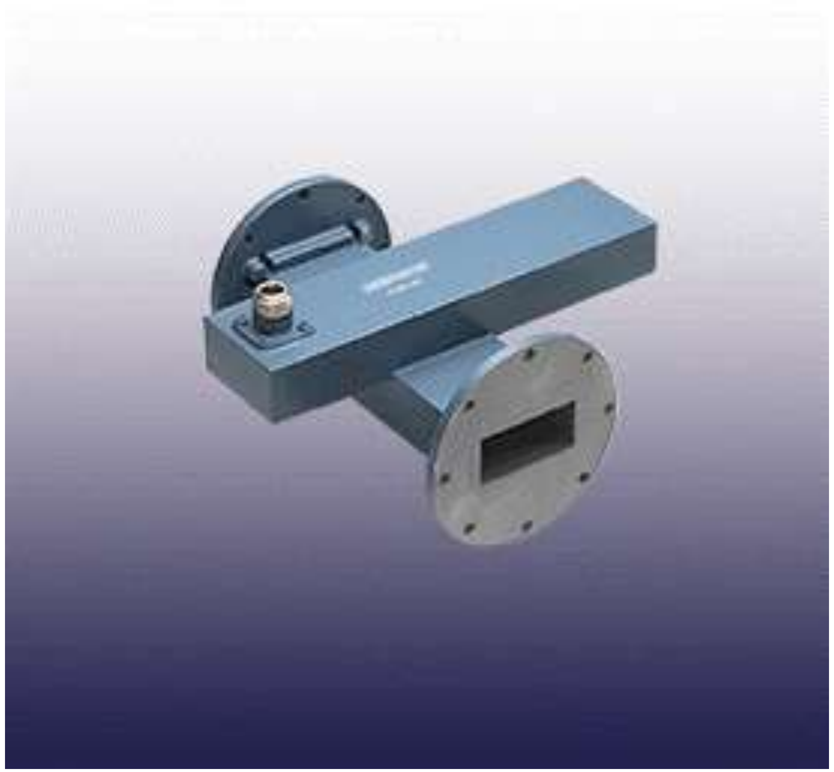
Because of the phase involved in the coupling process, the signals generated by the two types of coupling cancel in the forward direction and reinforce in the reverse direction.

Bethe coupler relies on single hole for coupling process rather than the separation between the holes and improves the directivity.



Directional Couplers





Directional Couplers

Scattering Matrix of Directional Coupler

- Directional coupler is a four port network and hence **[S]** is a 4x4 matrix

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

- In Directional coupler all four ports are perfectly matched to the junction.

$$S_{11} = S_{22} = S_{33} = S_{44} = 0$$

- From the symmetric Property

$$S_{12} = S_{21} \, ; S_{13} = S_{31} \, ; S_{23} = S_{32} \, ;$$
$$S_{34} = S_{43} \, ; S_{24} = S_{42} \, ; S_{41} = S_{14}$$

$$S_{13} = S_{31} = 0$$

- Ideally the back power is zero $P_b = 0$. i.e., There is no coupling between port 1 and port 3

Directional Couplers

- Also there is no coupling between port 2 and port 4. $S_{24} = S_{42} = 0$

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix}$$

- From the Unitary Property

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Directional Couplers

$$R_1C_1: \quad |S_{12}|^2 + |S_{14}|^2 = 1$$

$$R_2C_2: \quad |S_{12}|^2 + |S_{23}|^2 = 1$$

$$R_3C_3: \quad |S_{23}|^2 + |S_{34}|^2 = 1$$

$$R_1C_3: \quad S_{12}\, S_{23}^* + S_{14}\, S_{34}^* = 0$$

$$S_{14} = S_{23}$$
$$S_{12} = S_{34}$$

$$S_{12} = S_{34} = P = S_{34}^*$$

$$\begin{aligned} P \quad S_{23}^* + S_{23}\, P &= 0 \\ P \quad [S_{23} + S_{23}^*] &= 0 \end{aligned}$$

$$P \neq 0,\ S_{23} + S_{23}^* = 0$$

$$\begin{aligned} S_{23} &= jy \\ S_{23}^* &= -jy \end{aligned}$$

Directional Couplers

Let $$S_{23} = jq = S_{14}$$

$$S_{12} = S_{34} = P \quad \text{and} \quad S_{23} = S_{14} = jq$$

Substituting above values in **[S]** matrix, the **[S]** matrix of a directional coupler is reduced to

$$[S] = \begin{bmatrix} 0 & P & 0 & jq \\ P & 0 & jq & 0 \\ 0 & jq & 0 & P \\ jq & 0 & P & 0 \end{bmatrix}$$

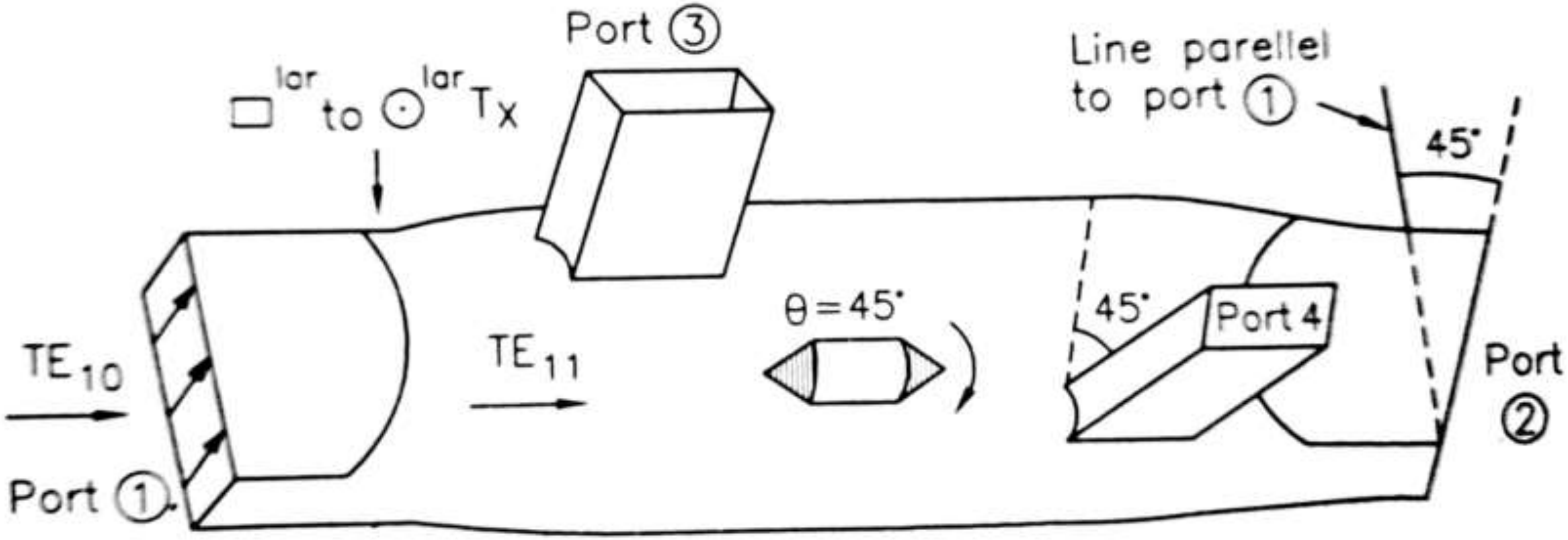
Circulator

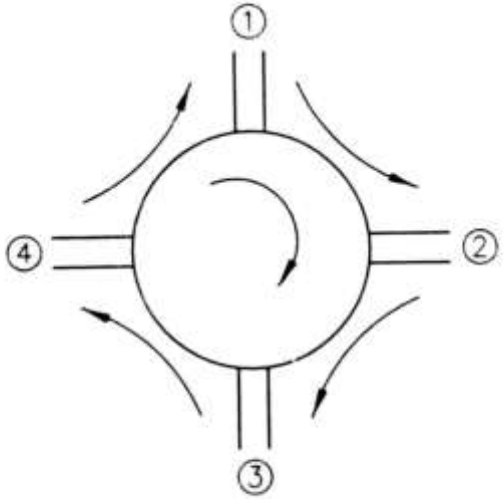
- A circulator is a four port microwave device which has a peculiar property that each terminal is connected only to the next clockwise terminal i.e., port 1 is connected to port 2 only and not to port 3 and port 4.
- Although there is no restriction on the number of ports, four ports are most commonly used.

Operation

- The power entering port 1 is TE_{10} mode and is converted to TE_{11} mode because of gradual rectangular to circular transition.
- This power passes port 3 unaffected since the electric filed is not significantly cut and is rotated through 45ºdue to the ferrite, passes port 4 unaffected for the same reason and finally emerges out of port 2.
- Power from port 2 will have plane of polarization already titled by 45º with respect to port 1.

Circulator


Port ③
□ lar to ⊙ lar Tx
Line parellel to port ①
45°
TE10
TE11
θ=45°
45°
Port 4
Port ②
Port ①


①
②
③
④

Circulator

- This power passes port 4 unaffected and gets rotated by 45º due to ferrite rod in the clockwise direction. And now totally plane of polarization is tilted through 90º finds port 3 suitably aligned and emerges out of it.
- Similarly port 3 in coupled only to port 4 and port 4 to port 1.

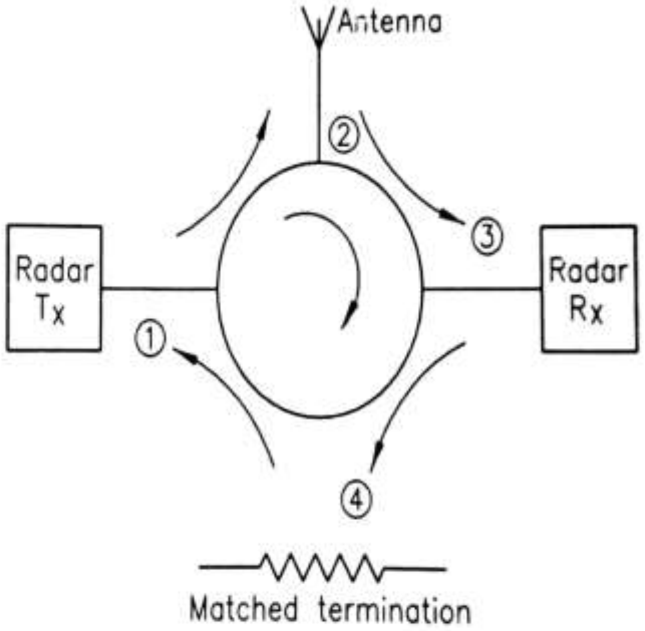
Applications of Circulator

Circulator as a Duplexer

Circulator can be used as a duplexer for a radar antenna system.

Transmitter feeds the antenna while the received energy is directed to the receiver.

The powerful radar transmitter is isolated from the receiver and also the same antenna can be used for both transmission and reception.



Introduction

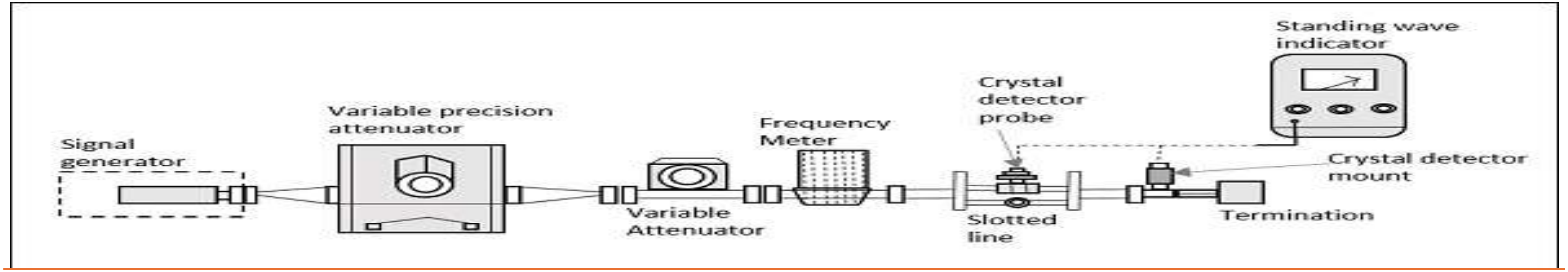
- Measurement of voltage and current is easy at Low frequencies therefore power calculation is easy where as at microwave frequencies it is difficult to measure voltage and current as they are vary with position in a transmission line.
- So at microwave frequencies it is convenient to measure power directly instead of voltage and current.
- At low frequencies, circuits use lumped elements which can be identified and measured. At microwave frequencies, circuit elements are distributed and hence it is not important to know what element make up a line.
- Unlike low frequency measurements, many quantities measured at microwave frequencies are relative and it is not necessary to know their absolute values.
- Further for power measurement, it is usually sufficient to know the ratio of two powers rather than exact input or output powers

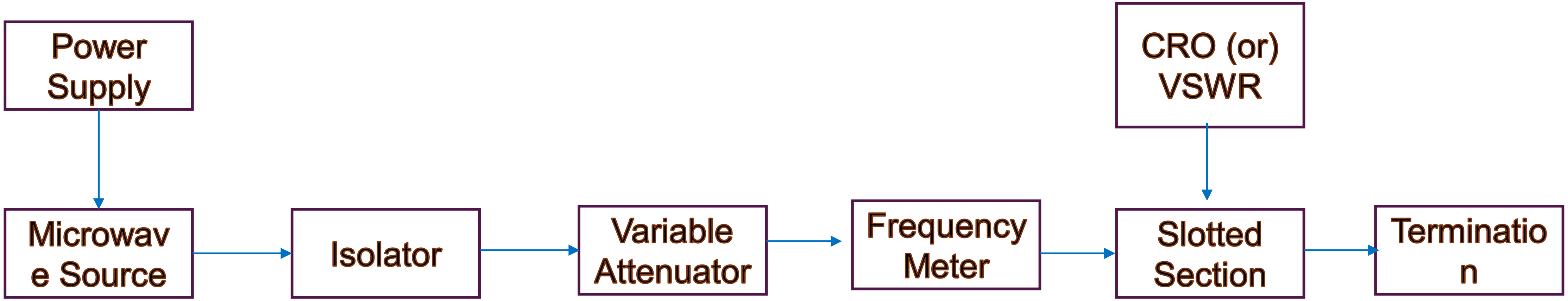
Microwave Measurements

The following parameters can be conveniently measured at microwave frequencies

1. Frequency
2. Power
3. Attenuation
4. Voltage Standing Wave Ratio (VSWR)
5. Phase
6. Impedance
7. Insertion Loss
8. Dielectric Constant
9. Noise factor

Microwave Bench





Power Supply

CRO (or) VSWR

Microwave Source

Isolator

Variable Attenuator

Frequency Meter

Slotted Section

Termination

Microwave Bench

The general set up for measurement of any parameter in microwaves is normally don by a microwave bench.

Power Supply:

- The power supply gives necessary beam voltage and beam current to the circuit. Also repeller voltage delivered by this unit.
- In lab typically we use 300V beam voltage , 24 mA beam current and take output readings by varying repeller voltage from -50V to 270V.

Microwave Source:

- The source of microwave may be Gunn diode oscillator, Reflex Klystron or BWO.
- Microwave source can provide either a continuous wave (CW) or square wave modulated at an audio rate which is normally 1KHz

Microwave Bench

Isolator:

- Isolator is used to protect the source from the reflected power due to mismatch of the load.
- Power flows in only one direction from source to load.

Precision Attenuator or Variable Attenuator:

- The precision attenuator can provide 0 to 50 dB attenuation above insertion loss.
- The variable flat attenuator is also used in addition, whose calibration can be checked against readings of the precision attenuator.

Frequency Meter:

- This is the device which measures the frequency of the signal. With this frequency meter, the signal can be adjusted to its resonance frequency. It also gives provision to couple the signal to waveguide.

Microwave Bench

Crystal Detector:

- A crystal detector probe and crystal detector mount are indicated in the above figure, where the detector is connected through a probe to the mount. This is used to demodulate the signals.

Slotted Line:

- In a microwave transmission line or waveguide, the electromagnetic field is considered as the sum of incident wave from the generator and the reflected wave to the generator. The reflections indicate a mismatch or a discontinuity. The magnitude and phase of the reflected wave depends upon the amplitude and phase of the reflecting impedance.
- The standing waves obtained are measured to know the transmission line imperfections which is necessary to have a knowledge on impedance mismatch for effective transmission. This slotted line helps in measuring the standing wave ratio of a microwave device.

Microwave Bench

Standing Wave Indicator

- The standing wave voltmeter provides the reading of standing wave ratio in dB. The waveguide is slotted by some gap to adjust the clock cycles of the signal. Signals transmitted by waveguide are forwarded through BNC cable to VSWR or CRO to measure its characteristics.

Terminations:

- This can be a matched load. If the load is perfectly matched maximum power is transferred to the load and there are no reflections.
- If there is any mismatch the power is reflected and reflection coefficient is given by

$$\sigma = \frac{Power\ Reflected}{Power\ Incident}$$

Waveguide Attenuators

- Attenuator is an electronic device that reduces the power of the signal without effecting or reducing the waveform of the signal.
- A device used to control the amount of microwave power transferred from one point to another on a microwave transmission systems is called microwave attenuator.
- Microwave attenuators control the flow of microwave power either by reflecting it or absorbing it.
- Attenuators are commonly used for
  - Measuring power gain or loss in dB
  - Providing isolation between instruments
  - Reducing the power I/P to a particular stage to prevent overloading.

Attenuators can be classified as fixed or variable type

Waveguide Attenuators

Fixed Attenuators:

- Fixed attenuators in circuits are used to lower voltage, dissipate power and to impedance matching.
- These are used where fixed amount of attenuation is to provided. If such a fixed attenuator absorbs all the energy entering into it, we call it as a waveguide terminator.
- This normally consists of a short section of waveguide with a tapered plug of absorbing material at the end.
- The tapering is done for providing a gradual transition from the wave guide medium to the absorbing medium thus reducing the reflection occurring at the media interface.

Waveguide Attenuators

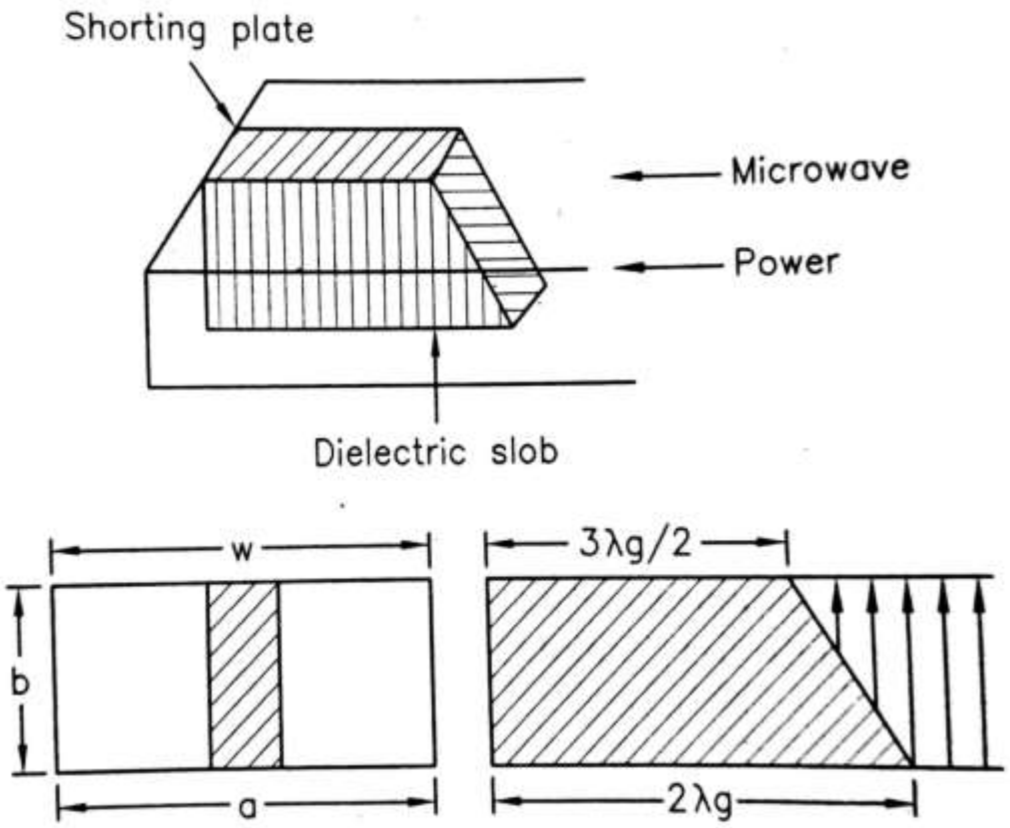


Figure shows fixed attenuator where a dielectric slab consisting of glass slab coated with aquadag or carbon film has been used as a plug

Waveguide Attenuators

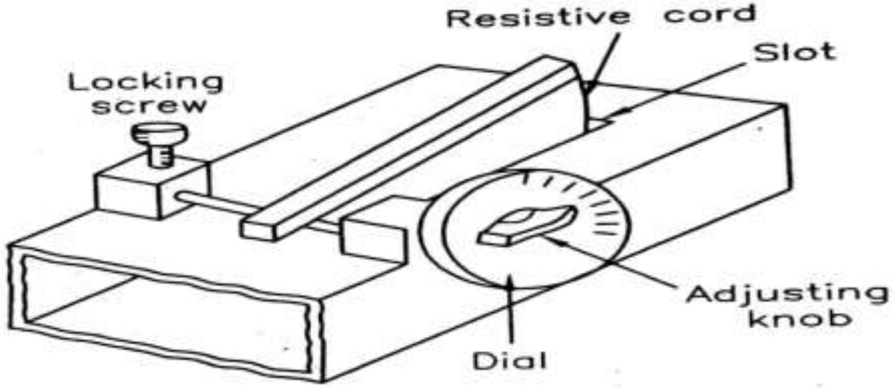
Variable Attenuators:

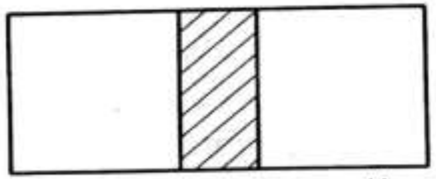
- Variable attenuators provide continuous or step wise variable attenuation.
- For rectangular waveguides, these attenuators can be flap type or vane type.
- For circular waveguide rotary type is used.

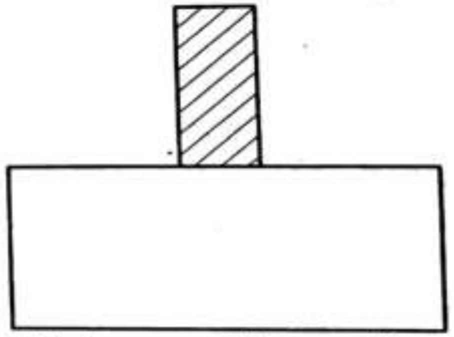
Waveguide Attenuators

Resistive or Flap type Attenuators:

- Flap type attenuator consists of a resistive element or disc inserted into a longitudinal slot cut along the center of the wider dimension of the guide.
- Flap is mounted on the hinged arm allowing it to descent into the centre of waveguide.
- Degree of attenuation can be determined by depth of insertion of the flap.




Maximum attenuation

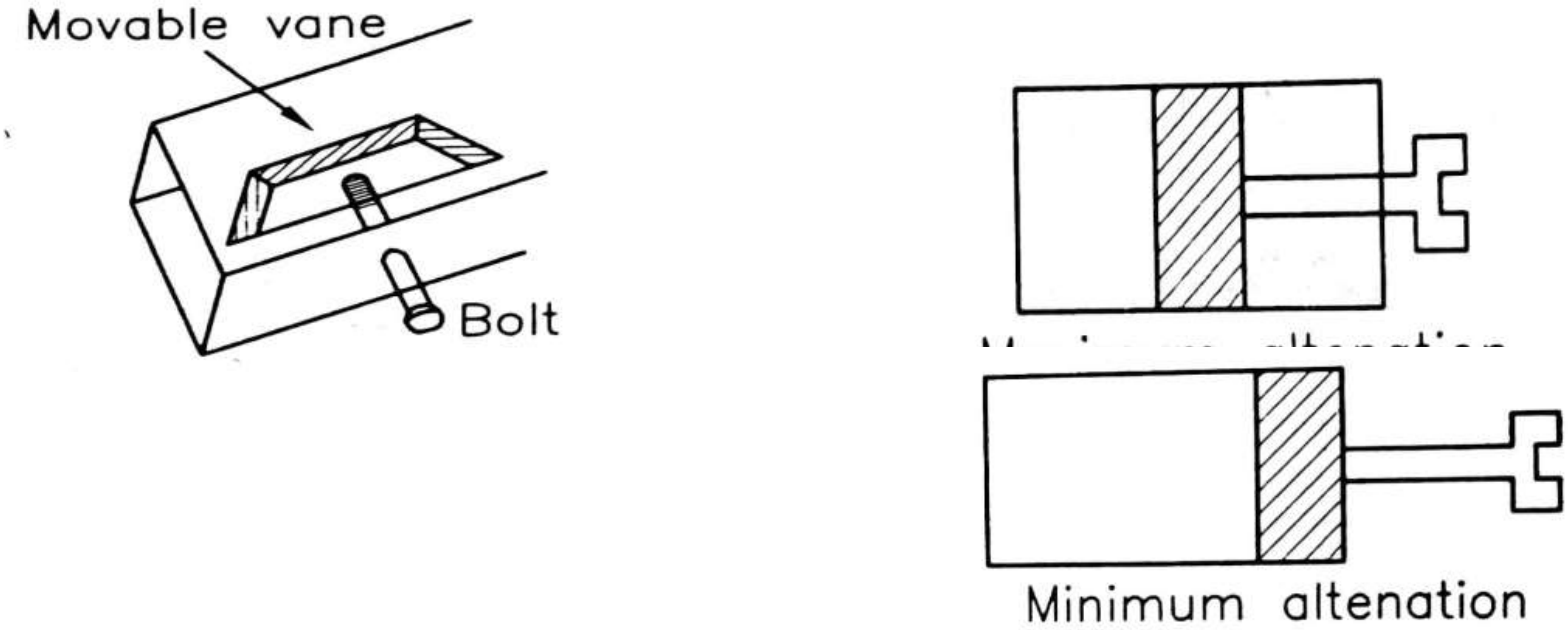

Minimum attenuation

Waveguide Attenuators

Vane type Attenuators:

- The vane type attenuator, basically consists of a glass vane with a coating of aquadag or carbon similar to a fixed vane attenuator.
- If this vane used at the center is made movable, it can be used as a variable attenuator.
- The vane positioned at center of waveguide can be moved laterally from center, where it provides maximum attenuation to the edges where the attenuation is considerably reduced since E lines are always concentrated at the center of waveguide.
- The vane is tapered at both ends for matching the attenuator to the waveguide.
- An adequate match is obtained if the taper length is made equal to $\lambda_g/2$

Waveguide Attenuators

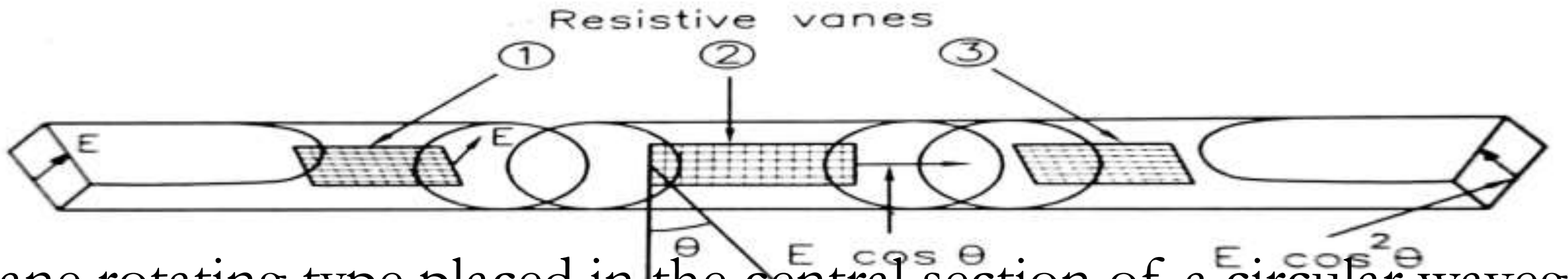


Movable vane attenuator

Waveguide Attenuators

Rotatory vane Attenuator:

A resistive rotary vane attenuator consists of three vanes.



The central vane rotating type placed in the central section of a circular waveguide arrangement tapered at both ends.

The other two vanes are rectangular sections.

Waveguide Attenuators

- When all the three vanes are aligned their planes are at 90° to the direction of electric field. Hence there is no attenuation.
- Vane 1 prevents any horizontal polarization and hence output of vane 1 is vertically polarized.
- Center vane 2 is rotating type and if it is rotated by an angle θ, the E sinθ component is attenuated and E cosθ component is present at the output of vane 2 and the final output of attenuator becomes $E\,cos^2\theta$, which has the same polarization as the input wave.
- The attenuation due to this rotary vane is then equal to

$$20\log cos^2\theta = 40\log cos\theta$$

Which is independent of frequency and is precise.

Attenuation Measurement

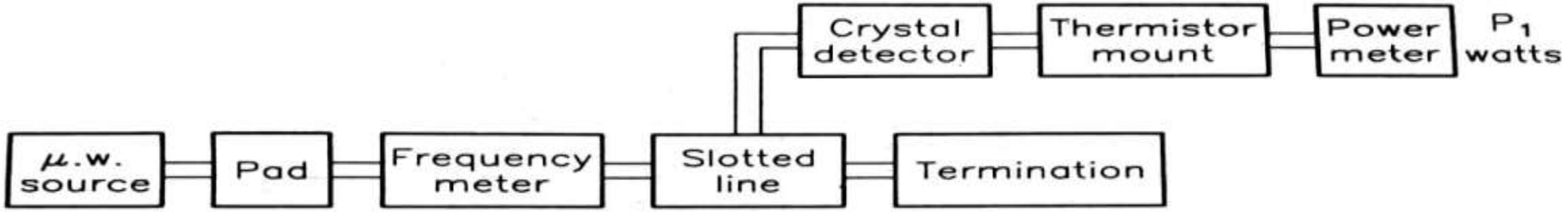
- Attenuation is the ratio of input power to the output power and is normally expressed in decibels.

$$Attenuation\ (in\ dBs) = 10 \log \frac{P_{in}}{P_{out}}$$

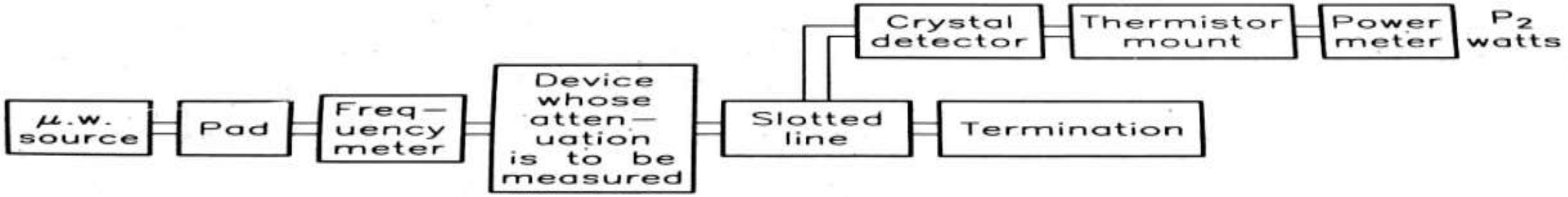
- The amount of attenuation can be measured by two methods
  - Power Ratio Method
  - RF Substitution Method

Attenuation Measurement

- This method involves measuring the input power and output power with and without the device whose attenuation is to be measured as shown in set up 1 and set up 2.



Set up 1, power ratio method.



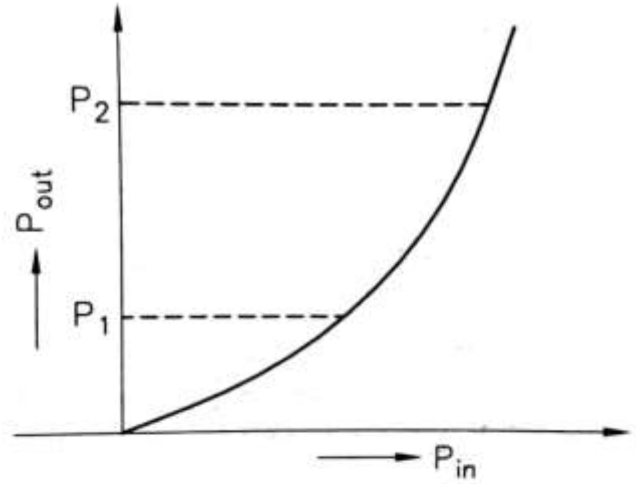
Set up 2, power ratio method.

Attenuation Measurement (Power Ratio Method)

The ratio of powers P_1/P_2 expressed in decibels gives the attenuation.

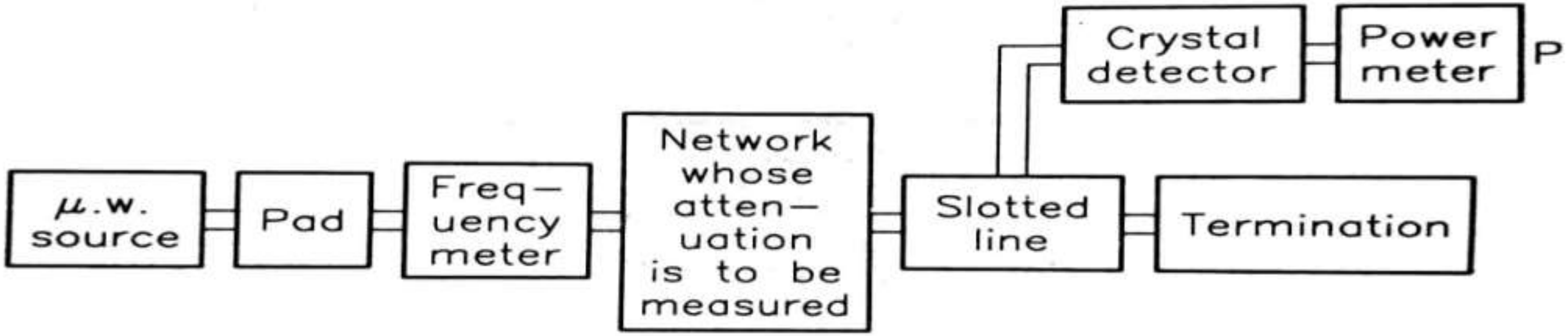
The drawback of this method is that the attenuation measured corresponds to two positions on the power meter with a square law crystal detector characteristics.

Due to non-linear characteristics the two powers measured and the attenuation calculated will not be accurate particularly if the attenuation of the network is large and if the input power is low.



Attenuation Measurement (RF Substitution Method)

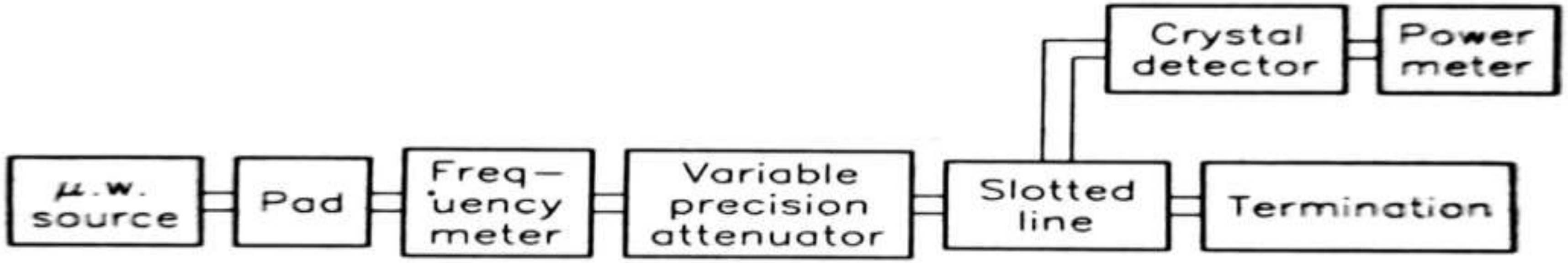
- This method overcomes the drawback of power ratio method since here we measure attenuation at a single power position.
- This method consists of measuring the output power ‘P’ by including the network whose attenuation is to be measured in set up 1.



Set up 1, RF substitution method.

Attenuation Measurement (RF Substitution Method)

- In set up 2 this network is replaced by a precision calibrated attenuator which can be adjusted to obtain the same power 'P' as measured in set up 1.
- Under this condition the attenuation read on the precision attenuator would give attenuation of the network directly.



Set up 2, RF substitution method.

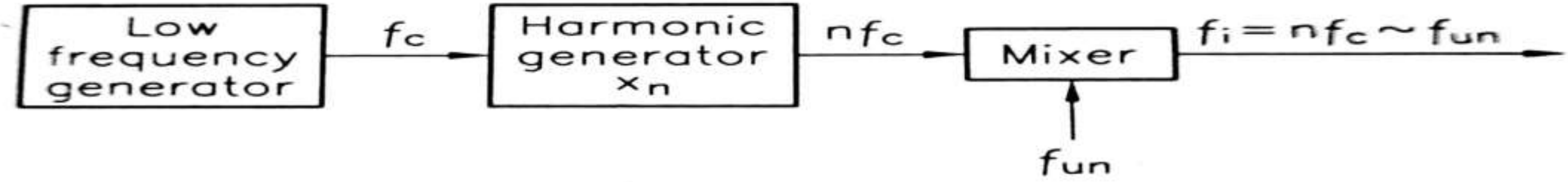
Frequency Measurement

- Microwave frequency can be measured by either electronic or mechanical techniques

Electronic techniques

- These techniques generally are more accurate but expensive.
- Here the unknown frequency is compared with harmonics of a known lower frequency by use of a low frequency generator, a harmonic generator and a mixer.

This method is 99.99% accurate.



Frequency Measurement

Mechanical techniques

Slotted line technique

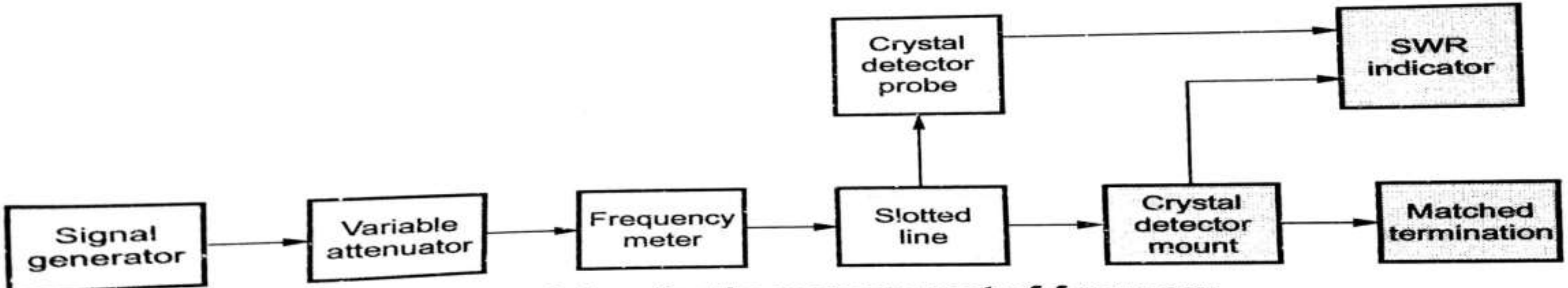
the above techniques operation and accuracy depends upon the physical dimensions of the mechanical devices.

Slotted Line Technique

- A slotted line is a piece of transmission line and it is constructed in such a way that the voltage and current along it can be measured continuously over its length.
- The general set up for the measurement of microwave frequency is shown

Frequency Measurement

- When a waveguide is mismatched by a load, a standing wave is created in the waveguide.
- The distance between the two adjacent maxima or minima is one half of the wavelength.
- Standing waves are set up in a slotted line producing minima every half wavelength apart.
- The distance between minima can be measured and guide wavelength hence frequency can be measured.



Set-up for the measurement of frequency

Frequency Measurement

$$\frac{\lambda_g}{2} = (D_2 - D_1) = \Delta D$$

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

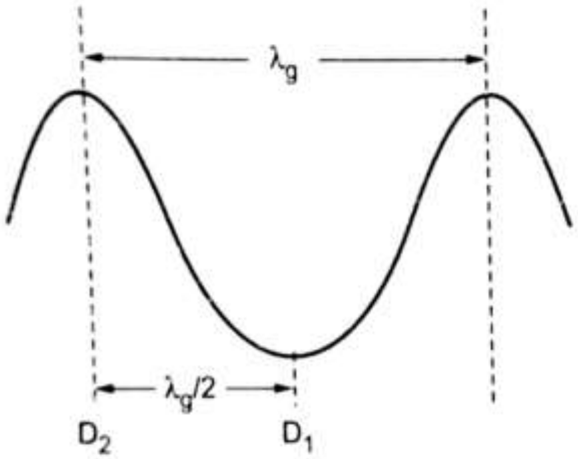
But for TE$_{10}$ mode

$$\lambda_c = 2a$$

and

$$\lambda_0 = \frac{c}{f}$$

$$\therefore \qquad f = \frac{c}{\lambda_0}$$



Maxima and minima of a wave

Accuracies obtained by this technique are limited to 1%. Since λ_g is dependent on guide dimensions.

Measurement of VSWR

Any mismatched load leads to reflected waves resulting in standing waves along the length of the line.

The ratio of maximum to minimum voltages gives the VSWR

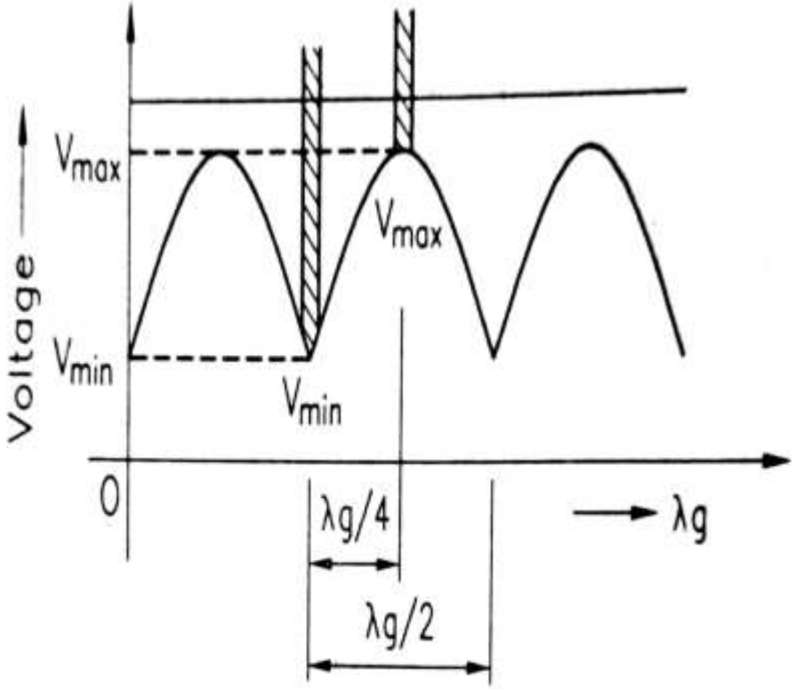
$$S = \frac{V_{max}}{V_{min}} = \frac{1+\rho}{1-\rho}$$

where, ρ = reflection coefficient = $\frac{P_{reflected}}{P_{incident}}$

S varies from 1 to ∞

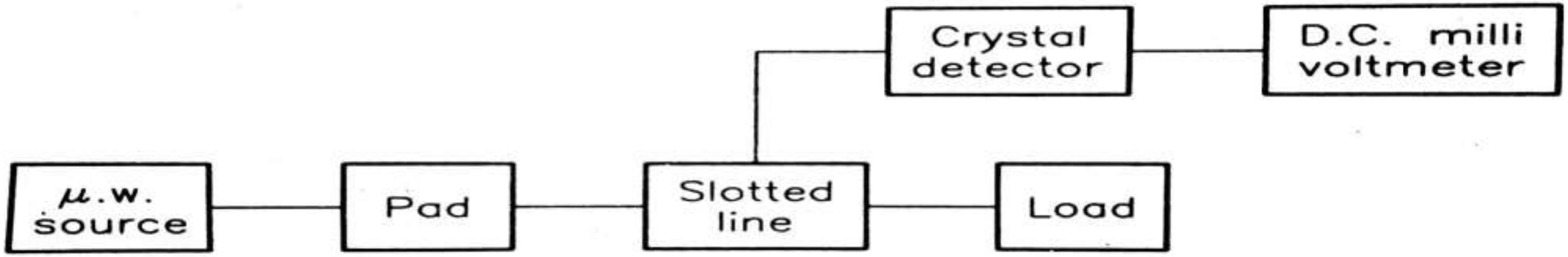
as ρ varies from 0 to 1

Hence minimum value of S is unity



Measurement of Low VSWR (S<10)

- Values of VSWR not exceeding 10 are very easily measured with the set up shown



Measurement of Low VSWR (S<10)

- The measurement basically consists of simply adjusting the attenuator to give an adequate reading on the DC mullivolt meter.
- The probe on the slotted waveguide is moved to get maximum reading on the meter V_{max}.

Measurement of Low VSWR (S<10)

- Next the probe on the slotted line is adjusted to get minimum reading on the meter V_{min}. The ratio of first reading to the second (V_{max}/V_{min}) gives the VSWR.
- The meter itself can be calibrated in terms of VSWR.
- The meter will be congested and the measurement will not be accurate for VSWR>10.
- This method is not useful for VSWR>10

Measurement of HIGH VSWR (S>10)

- For VSWR>10, we use double minimum method
- In this method the probe is inserted to a depth where the minimum can be read without difficulty.
- The probe is then moved to a point where the power is twice the minimum.
- Let this position be denoted by d_1. The probe is then moved to twice the power point on the other side of the minimum (say d_2) as shown

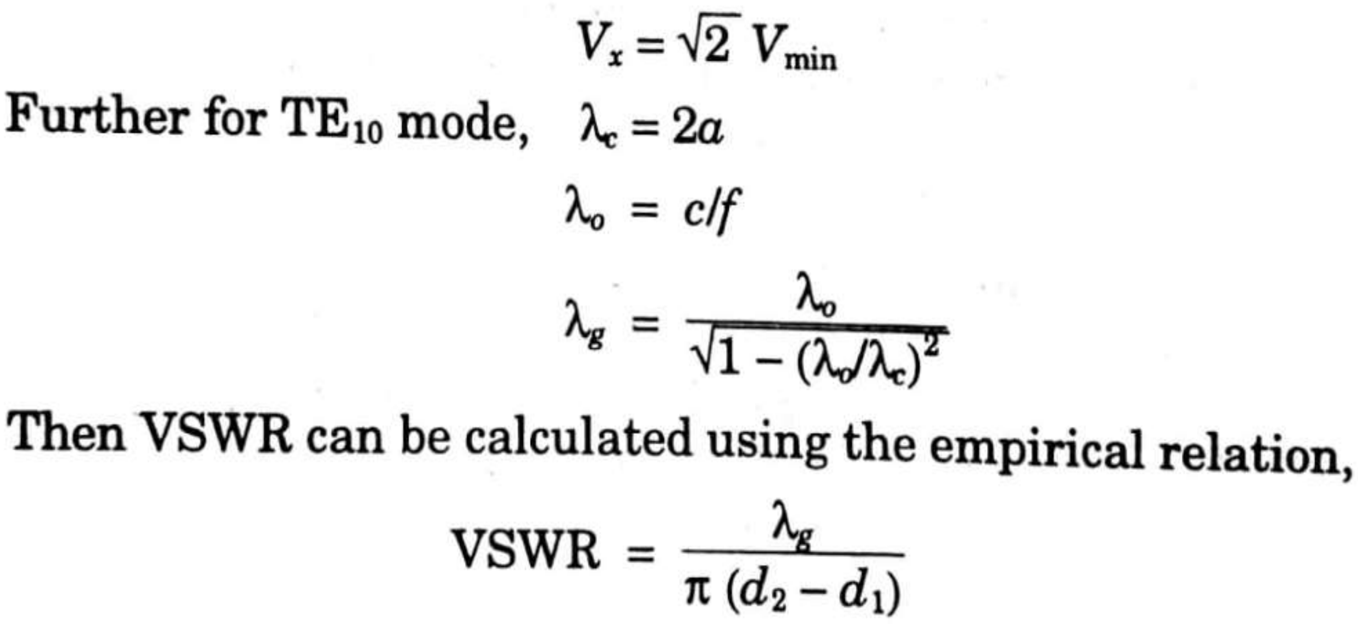
$$P_{\min} \propto V_{\min}^2$$

$$2\,P_{\min} \propto V_x^2$$

$$\frac{1}{2} = \frac{V_{\min}^2}{V_x^2}$$

$$V_x^2 = 2\,(V_{\min})^2$$

Measurement of HIGH VSWR (S>10)



$$V_x = \sqrt{2}\, V_{min}$$

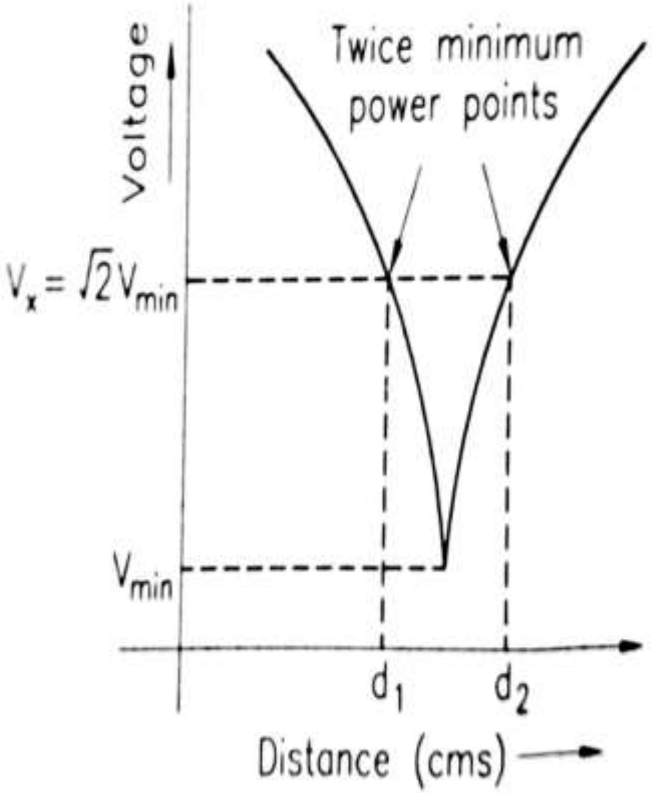
Further for TE$_{10}$ mode, $\lambda_c = 2a$

$$\lambda_o = c/f$$

$$\lambda_g = \frac{\lambda_o}{\sqrt{1-(\lambda_o/\lambda_c)^2}}$$

Then VSWR can be calculated using the empirical relation,

$$\text{VSWR} = \frac{\lambda_g}{\pi\,(d_2 - d_1)}$$



Measurement of Impedance

- Impedance at microwave frequencies can be measured by using following 3 methods
    - Using Magic T
    - Using Slotted line
    - Using Reflectometer

Measurement of Impedance

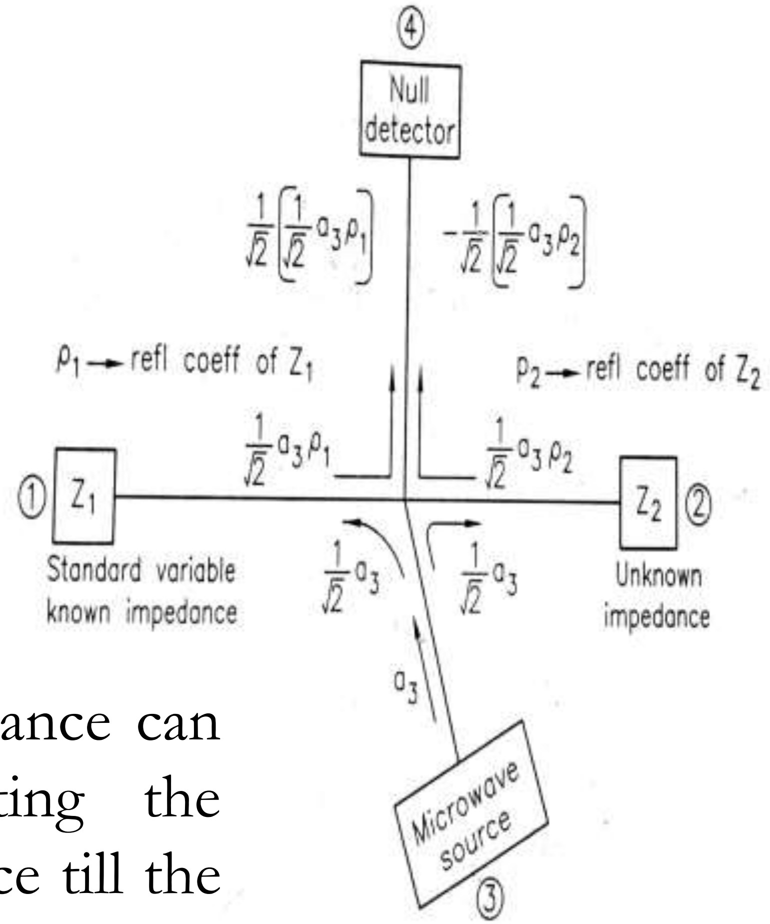
Measurement of Impedance using Magic T:

- A Magic Tee has been used in the form of bridge for measuring impedance.
- Microwave source is connected in the arm 3 and a null detector in arm 4. The unknown impedance is connected in arm 2 and a standard variable known impedance in arm 1.
- Using the properties of Magic Tee the power from microwave source (a_3) gets divided equally between arms 1 and 2.
- These impedances are not equal to characteristic impedances Z_o and hence there will be reflections from arms 1 and 2.
- If $\rho1$ and $\rho2$ are the reflection coefficients, powers $\frac{1}{\sqrt{2}} a_3\rho_1$ and $\frac{1}{\sqrt{2}} a_3\rho_2$ ıter the Magic Tee junction from arms 1 and 2.
- The resultant wave into arm 4 i.e., the null detector can be calculated as follows

Measurement of Impedance

The net wave reaching the null detector

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} a_3 \rho_1\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} a_3 \rho_2\right) = \frac{1}{2} a_3 (\rho_1 - \rho_2)$$



For perfect balancing of the bridge above eq is equated to zero.

$$\frac{1}{2} a_3 (\rho_1 - \rho_2) = 0$$

$$\rho_1 - \rho_2 = 0 \quad \text{or} \quad \rho_1 = \rho_2$$

$$\frac{Z_1 - Z_z}{Z_1 + Z_z} = \frac{Z_2 - Z_z}{Z_2 + Z_z}$$

$$Z_1 = Z_2$$

$$R_1 + jX_1 = R_2 + jX_2$$

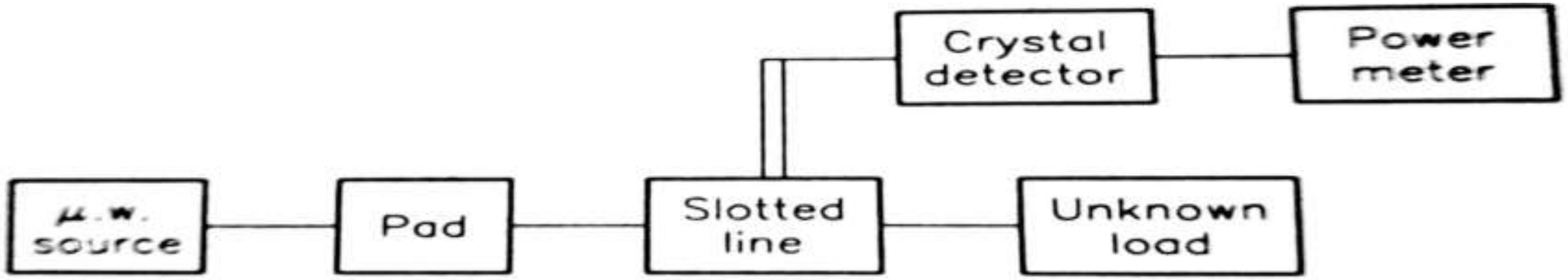
$$R_1 = R_2 \quad \text{and} \quad X_1 = X_2.$$

Thus the unknown impedance can be measured by adjusting the standard variable impedance till the bridge is balanced and both impedances become equal

Measurement of Impedance

Measurement of Impedance using Slotted line

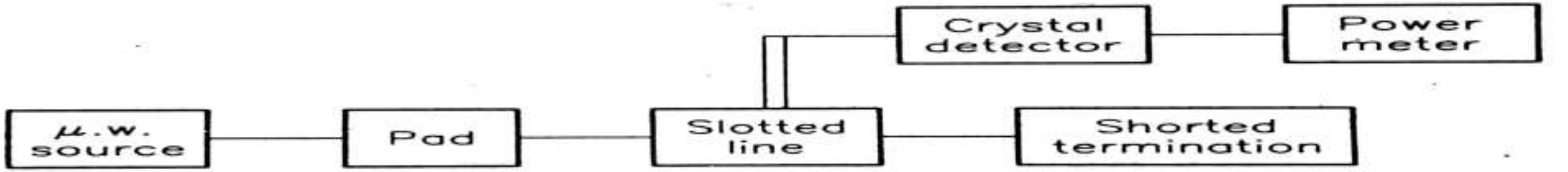
- Incident and reflected waves will be present proportional to the mismatch of the load under test resulting in standing waves.
- Using slotted waveguide and the load Z_L in the circuit shown, the position of V_{max} and V_{min} can be accurately determined.



Set up 1, Impedance measurement using slotted line.

Measurement of Impedance

- Now the load Z_L is replaced by a short circuit as shown and shift in minimum is measured.

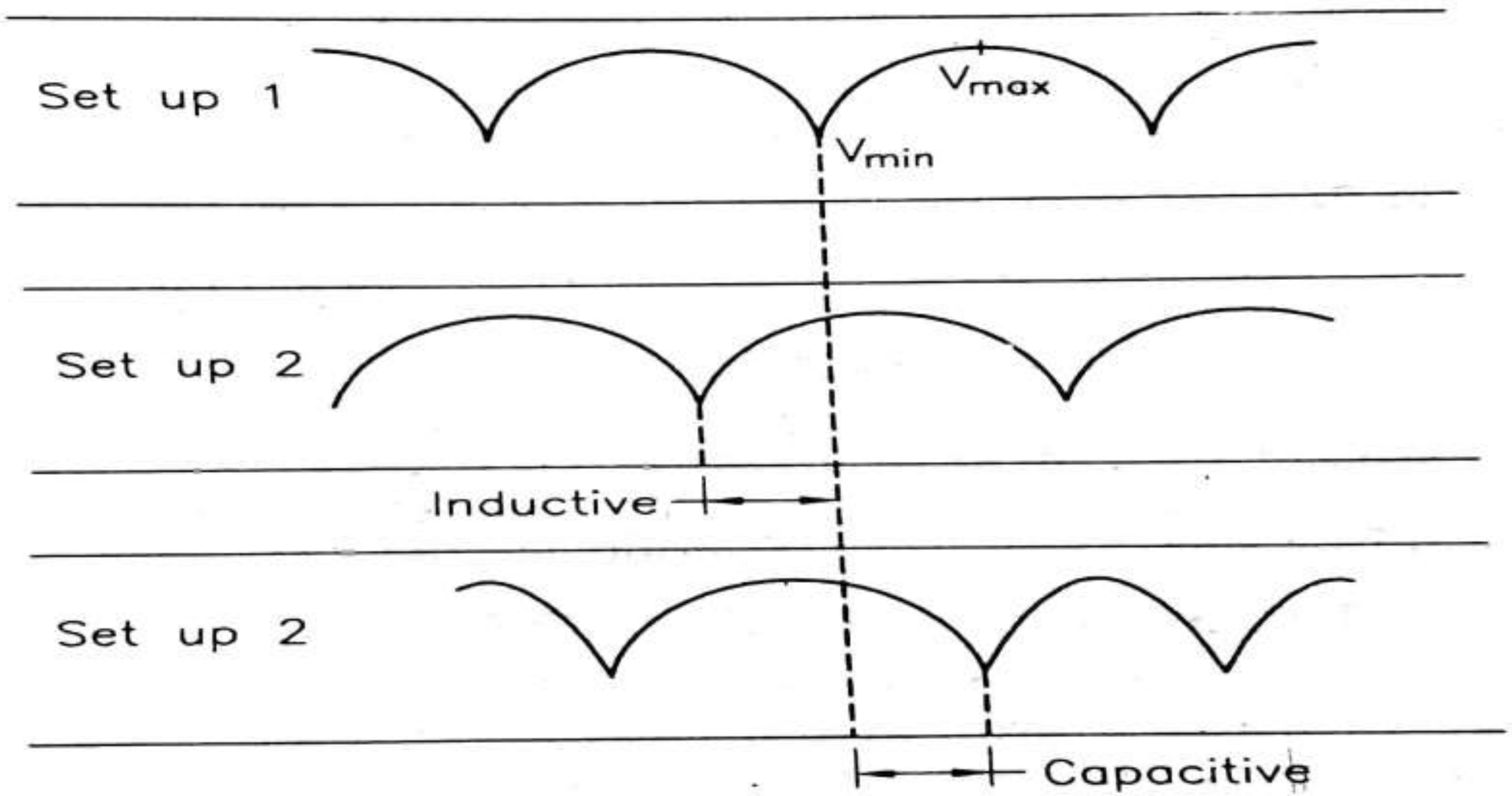


Set up 2, Impedance measurement using slotted line.

Measurement of Impedance

- If minimum is shifted to the left, then the impedance is inductive and if it shifts to the right then the impedance id capacitive.
- Using the smith chart the normalized or unknown impedance can be obtained in terms of magnitude and phase.

Measurement of Impedance



Ouput standing waves of set up 1 and 2.

Measurement of Impedance

Measurement of Impedance using Reflectometer

- The reflectometer indicates magnitude of impedance but not the phase angle, whereas using slotted line waveguide measurement gives both.
- In reflectometer technique, two directional couplers are used to sample the incident power P_i and reflected power P_r from the load.
- The magnitude of the reflection coefficient ρ, can be directly obtained on the reflectometer

$$\rho = \sqrt{\frac{P_r}{P_i}}$$

$$S = \frac{1+\rho}{1-\rho} \quad \text{and} \quad \frac{z - z_g}{z + z_g} = \rho$$

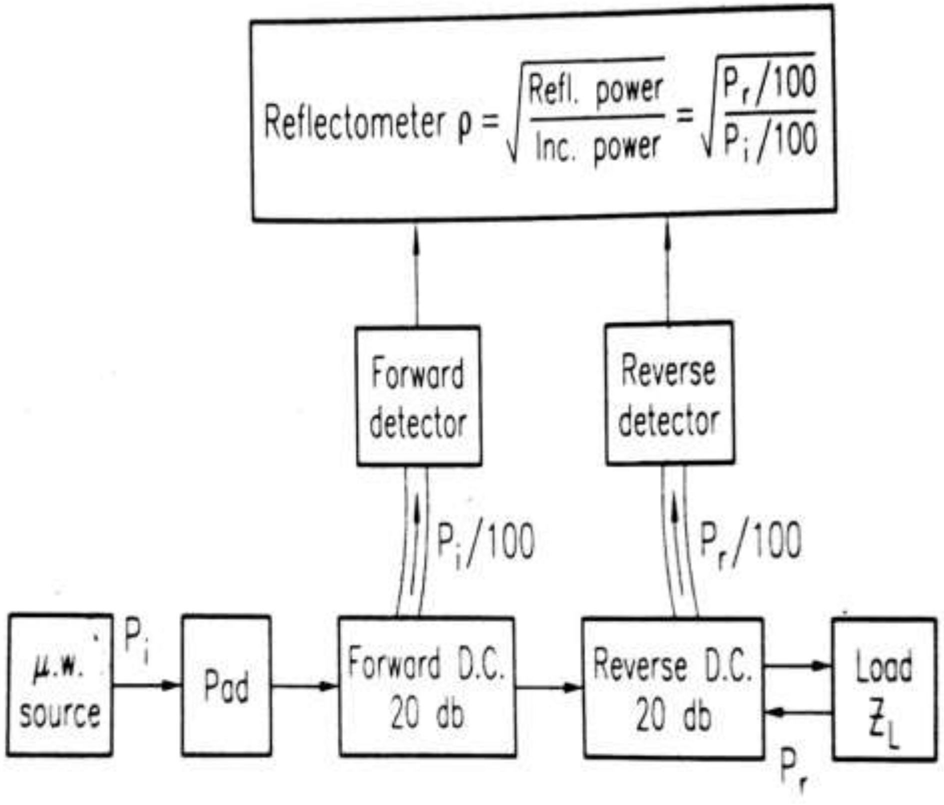
- By using ρ we can calculate VSWR and impedance

where z = unknown impedance z_g = known wave impedance

Measurement of Impedance

Due to directional property of the couplers, there will be no interference between forward and reverse waves.

The reflectometer accuracy is greatest at low VSWR



Set up for measuring impedance using reflectometer.