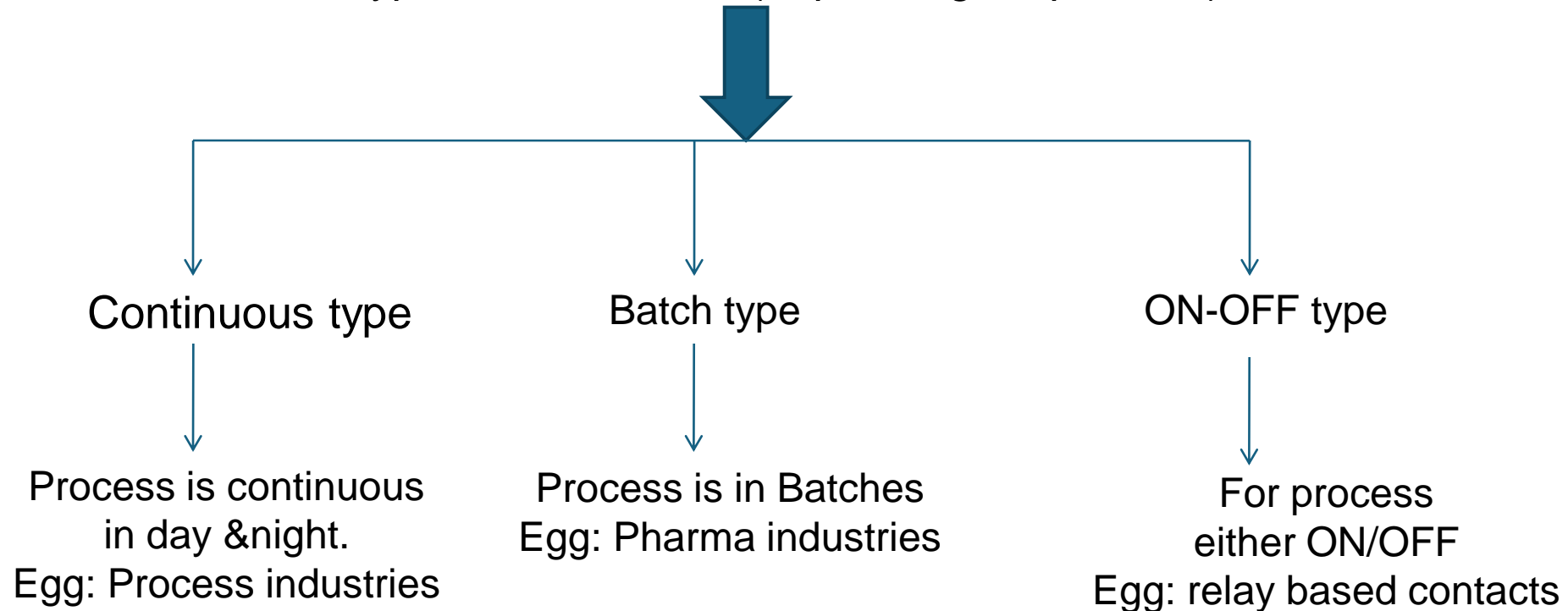


Unit-1

-

Types of controllers (depending on process)

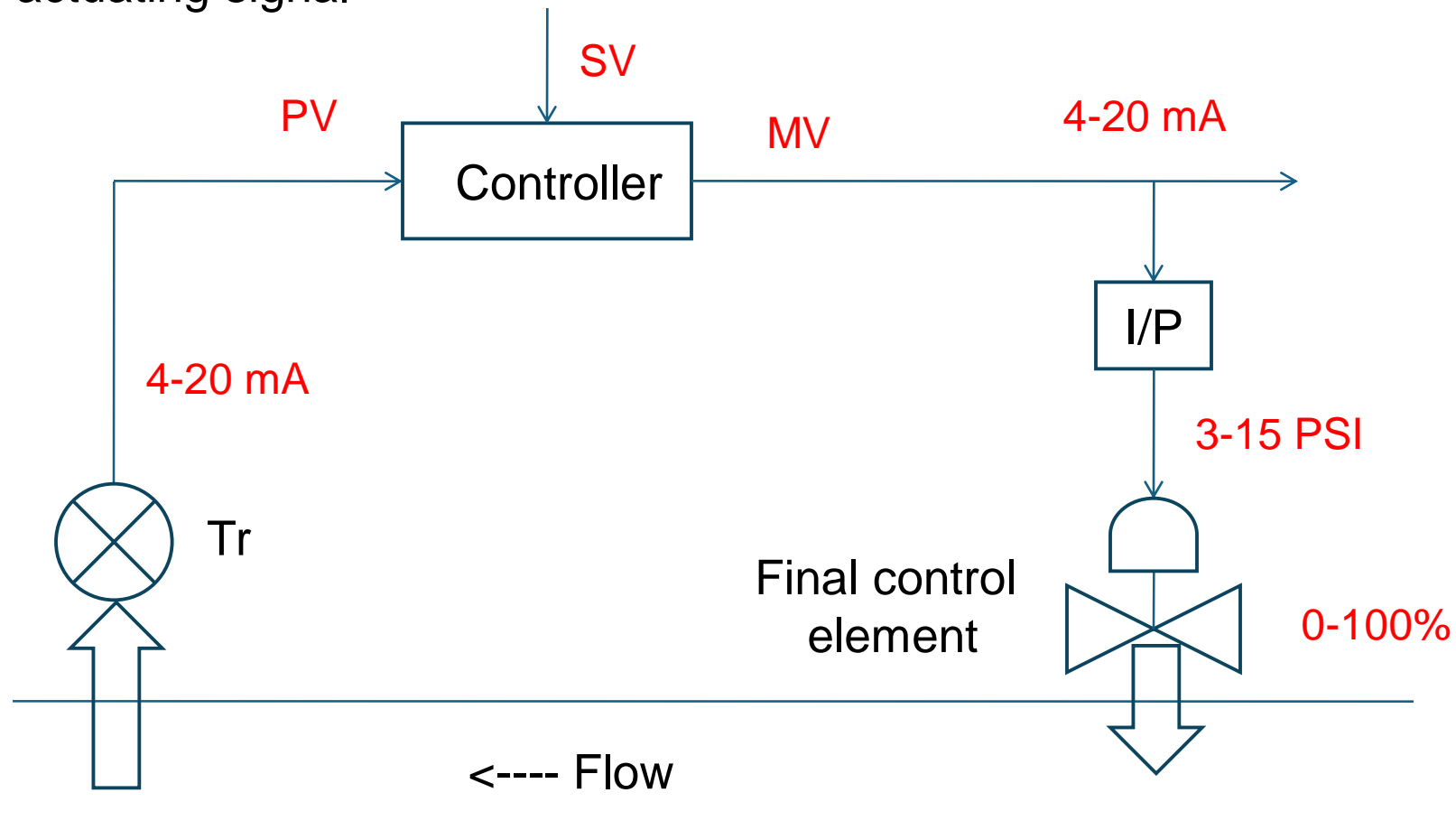


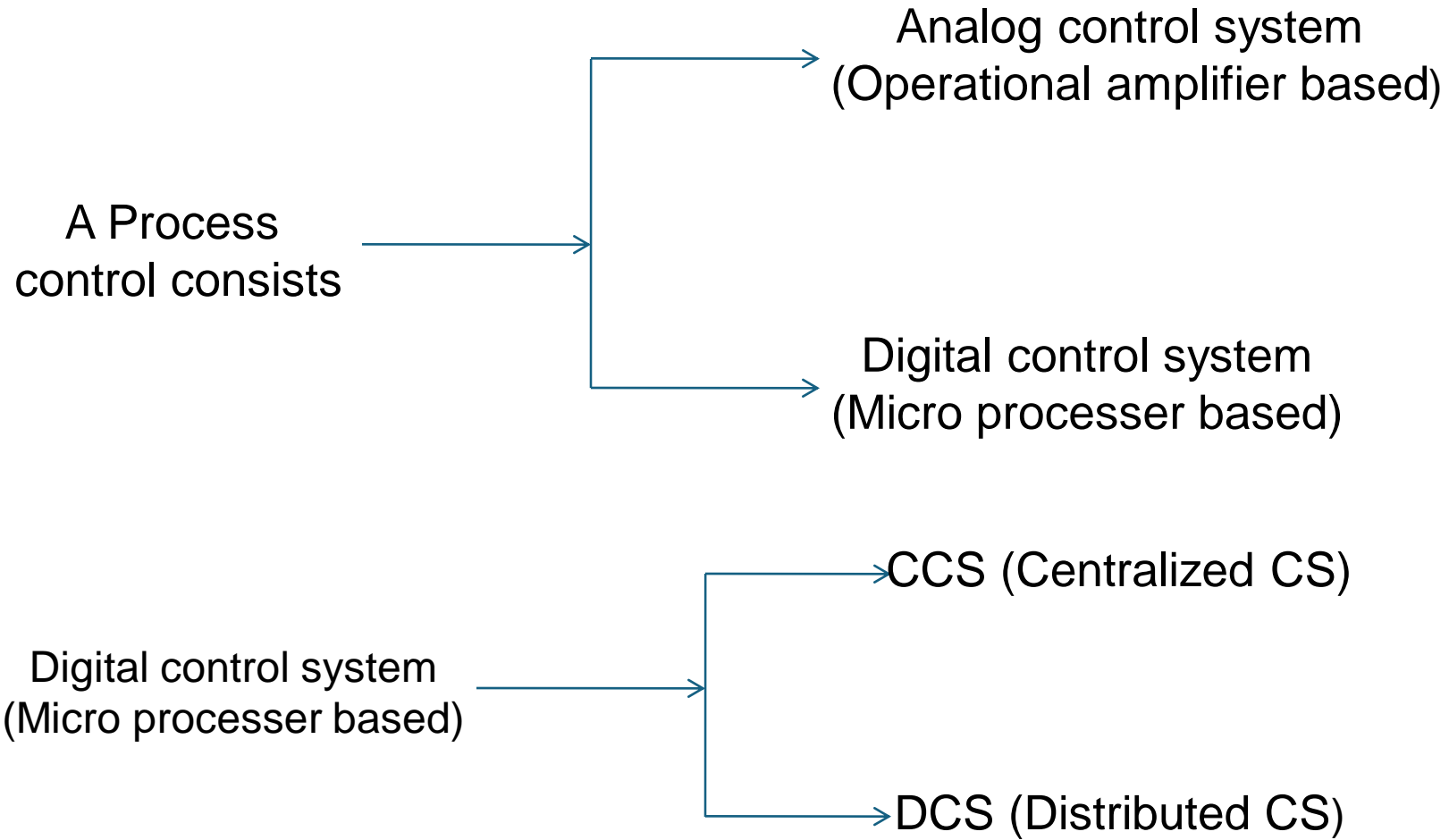
SIMPLE PROCESS

PV = Process variable value (or) controlled variable.

SV = Set point value (or) desired value (or) reference value.

MV = Manipulated value (or) output value (or) controlling variable (or) actuating signal





CONTROLLERS

- Controller is a device, which is used to control transient and steady state response as per the requirements.
- The best system demands smallest rise time, smallest settling time, smallest steady state error and smallest peak overshoot.
- Depending on the control actions provided the controllers can be classified as follows.
 1. Two position or on-off controllers
 2. proportional controllers
 3. integral controllers
 4. Derivative controllers

5. PI Controllers

6. PD Controllers

7. PID Controllers

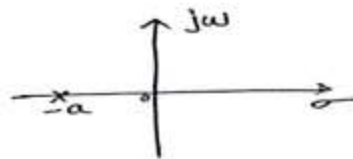
- The on-off control action may be provided by a relay. It is a device which has NO(Normally open) and NC(Normally closed) contacts, whose opening and closing are controlled by the relay coil. When the relay coil is excited, the relay operates and the contacts change their position (ie., NO to NC and NC to NO).

BIBO stability: A linear relaxed system is said have BIBO stability if every bounded (finite) input results in a bounded (finite) output.

Location of roots on the s-plane for stability.

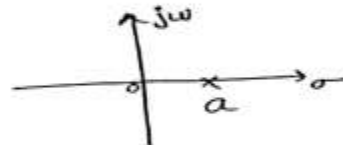
Transfer function and location of roots

① $M(s) = \frac{A}{s+a}$



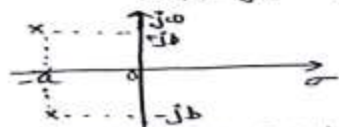
Root on -ve real axis

② $M(s) = \frac{A}{s-a}$



Root on +ve real axis

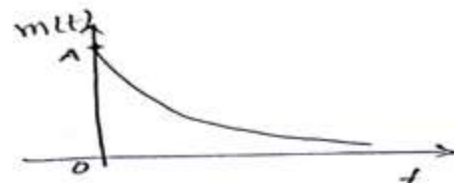
③ $M(s) = \frac{A}{s+a+jb} + \frac{A}{s+a-jb}$



Complex conjugate roots

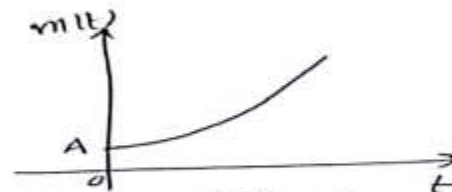
Sketch of time domain function.

$$m(t) = \mathcal{L}^{-1} \left[\frac{A}{s+a} \right] = A e^{-at}$$



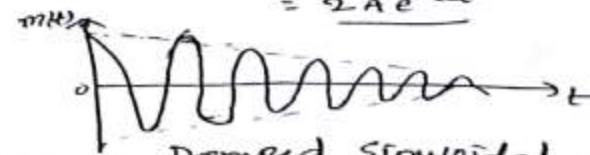
Exponentially decaying.

$$m(t) = \mathcal{L}^{-1} \left[\frac{A}{s-a} \right] = A e^{at}$$



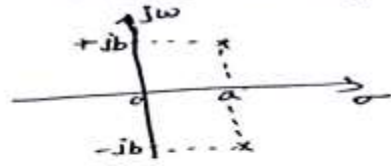
Exponentially Increasing.

$$m(t) = A e^{-(a+jb)t} + A e^{-(a-jb)t} = 2A e^{-at}$$



Damped Sinusoidal.

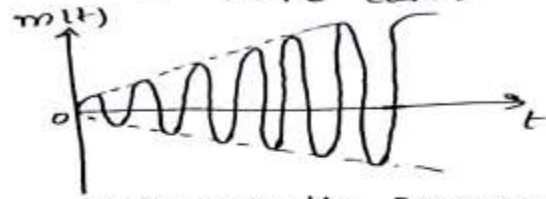
$$④ \quad M(s) = \frac{A}{s-a+jb} + \frac{A}{s-a-jb}$$



complex conjugate roots
on R.H.S.P.

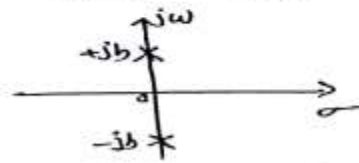
$$m(t) = Ae^{-(a+jb)t} + Ae^{-(a-jb)t}$$

$$= 2Ae^{-at} \cos bt$$



Exponentially increasing sinusoidal.

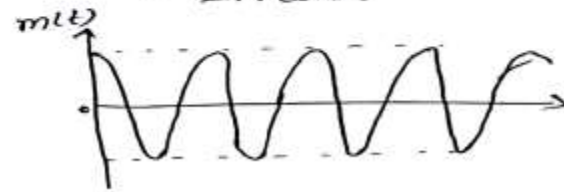
$$⑤ \quad M(s) = \frac{A}{s+jb} + \frac{A}{s-jb}$$



single pair of roots on imaginary axis.

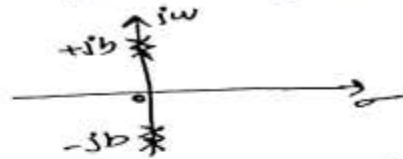
$$m(t) = Ae^{-jb} + Ae^{+jb}$$

$$= 2A \cos bt$$



Oscillatory.

$$⑥ \quad M(s) = \frac{A}{(s+jb)^2} + \frac{A}{(s-jb)^2}$$



Double pair of roots on I.Axial.

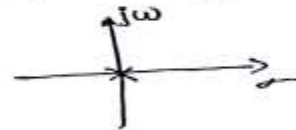
$$m(t) = At e^{-jb} + At e^{+jb}$$

$$= 2At \cos bt$$



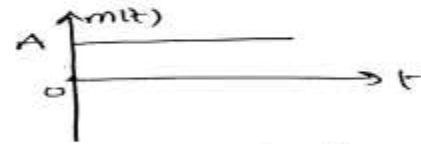
Linearly increasing sinusoidal.

$$⑦ \quad M(s) = \frac{A}{s}$$



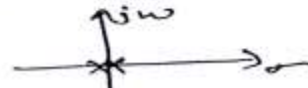
single root at origin

$$m(t) = A$$



constant.

$$⑧ \quad M(s) = \frac{A}{s^2}$$

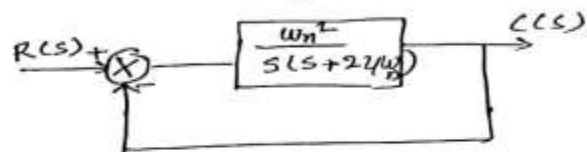


Double root at origin



Linearly increasing.

Standard form of closed loop transfer function of second order system.



$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

ω_n = Undamped natural frequency rad/sec
 ζ = Damping ratio.

Depending on ' ζ ' the system can be classified as

Case 1: Undamped system; $\zeta = 0$.

$$CE = s^2 + 2\zeta\omega_n s + \omega_n^2 = 0.$$

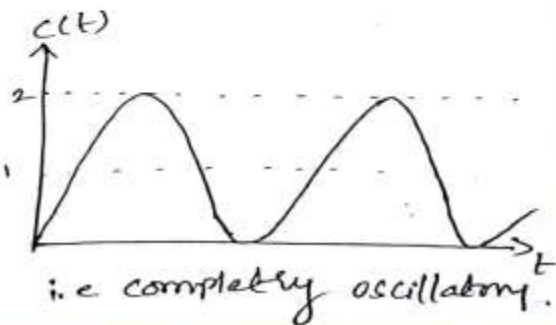
$$\text{roots} \Rightarrow s_1, s_2 = \pm j\omega_n$$

i.e. roots are purely imaginary, system undamped.

→ Response of undamped 2nd order system for unit step input.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + \omega_n^2}$$

$$c(t) = 1 - \cos\omega_n t$$



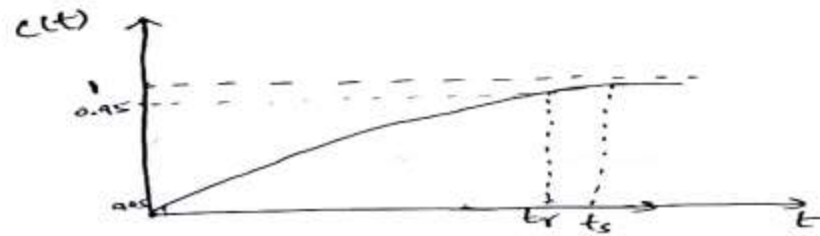
Case 2: Critically damped system. ($\alpha=1$)

$$\text{roots} \Rightarrow s_1, s_2 = -\omega_n.$$

i.e; roots are real and equal.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s + \omega_n)^2}$$

$$\Rightarrow \boxed{c(t) = 1 - e^{-\omega_n t} (1 + \omega_n t)}$$

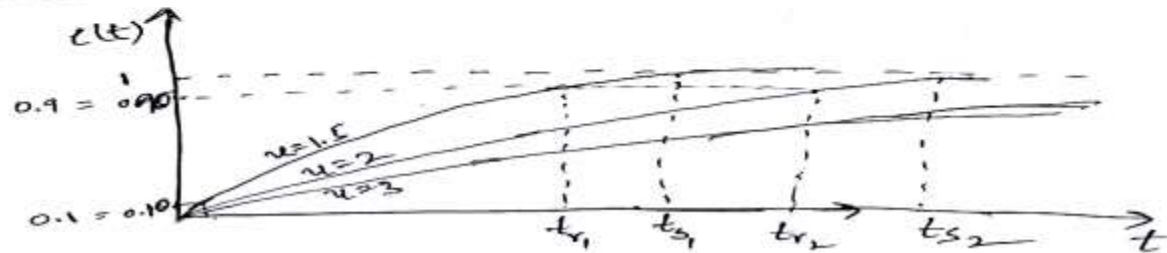


Case 3: overdamped system ($\alpha > 1$)

$$\text{roots} \Rightarrow s_1, s_2 = -\alpha\omega_n \pm \omega_n \sqrt{\alpha^2 - 1}$$

i.e; roots are real and unequal.

$$\boxed{c(t) = 1 - \frac{\omega_n}{2\sqrt{\alpha^2 - 1}} \left(\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2} \right)}$$



Case 4: under damped system. $0 < \zeta < 1$

$$\text{roots} \Rightarrow s_1, s_2 = -\zeta \omega_n \pm j \omega_n \sqrt{1 - \zeta^2}$$

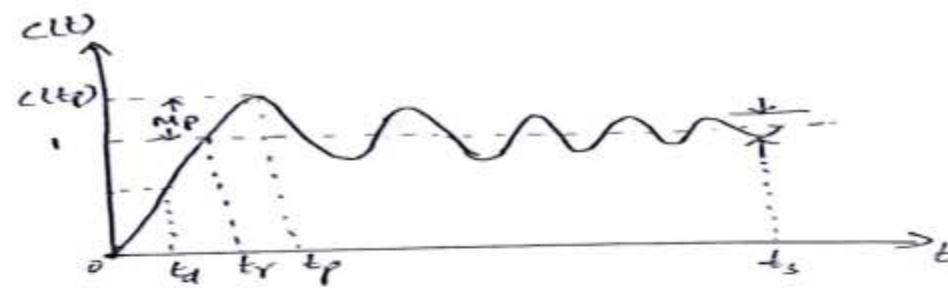
$$= -\zeta \omega_n \pm j \omega_d$$

i.e; roots are complex conjugate.

where, $\omega_d = \omega_n \sqrt{1 - \zeta^2}$, ω_d is called damped frequency of oscillations.

→ Response for unit step input

$$c(t) = 1 - \frac{e^{-\zeta \omega_n t}}{\sqrt{1 - \zeta^2}} \sin(\omega_d t + \theta), \quad \theta = \tan^{-1} \frac{\sqrt{1 - \zeta^2}}{\zeta}$$



Damped oscillatory response.

→ The transient response of a practical control system often exhibits damped oscillation before reaching steady state.