UNIT – IV

Matrix Methods - Stiffness Matrix Method

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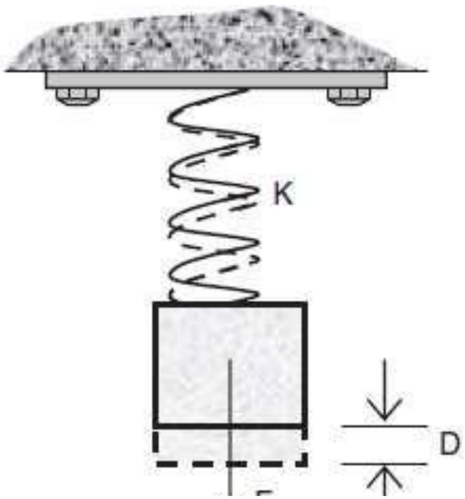
MATRIX STIFFNESS METHOD OF ANALYSIS

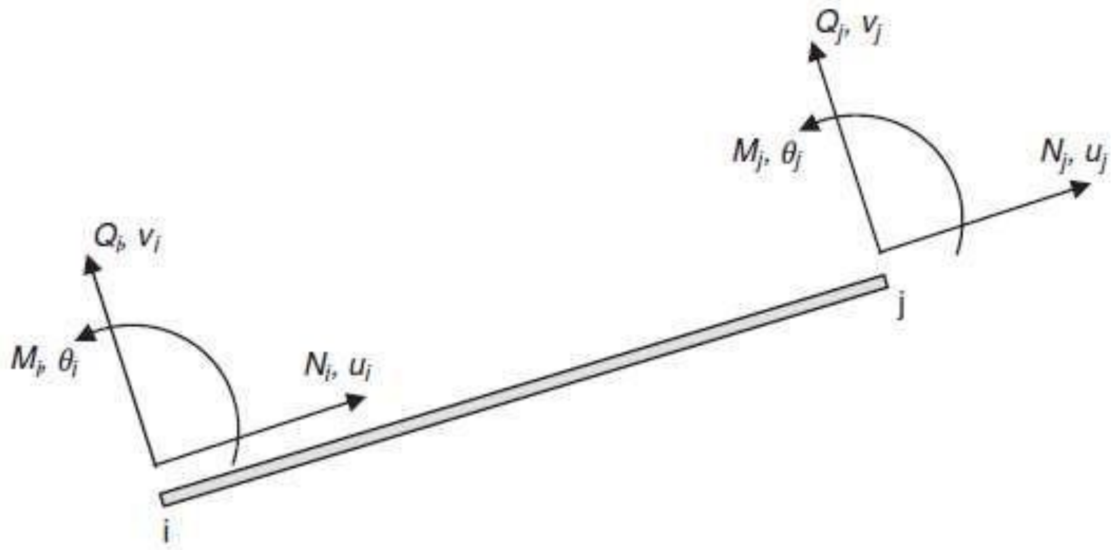


$$\{F\}_{n\times 1} = [K]_{n\times n}\{D\}_{n\times 1}$$

$$F = KD \qquad \{F\}_{n\times 1} = [K]_{n\times n}\{D\}_{n\times 1}$$

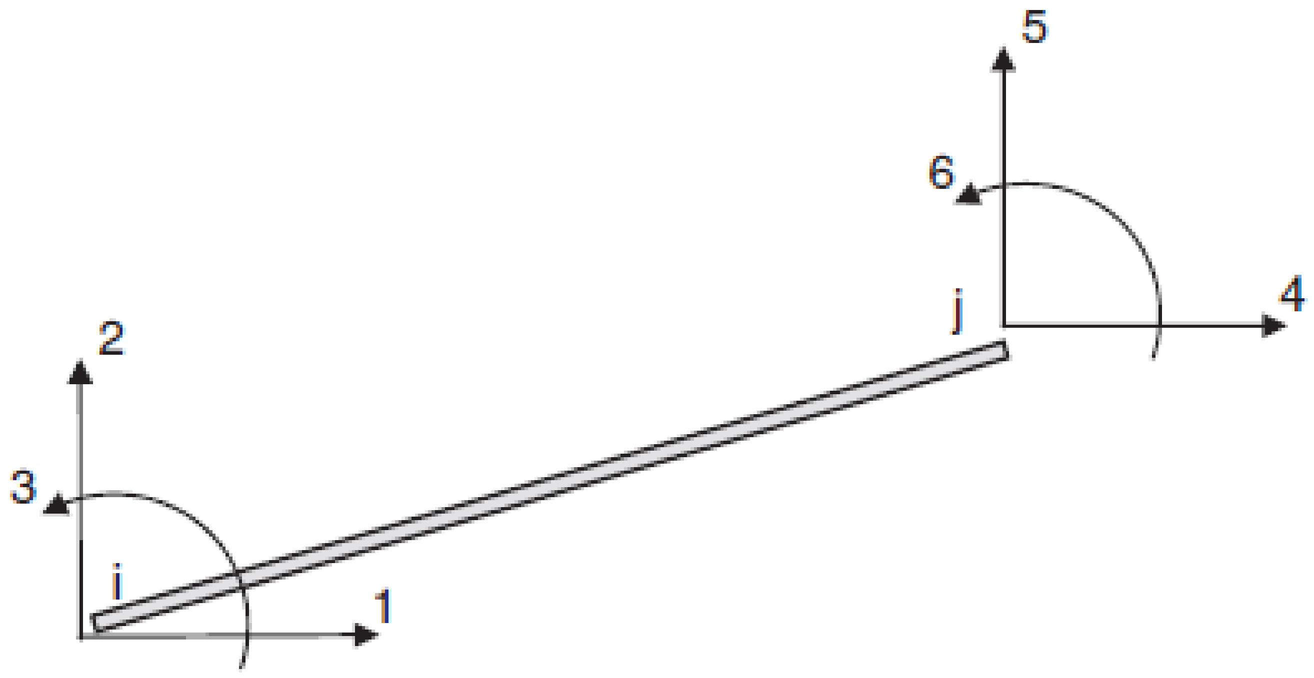
$$D = F/K \qquad \{D\} = [K]^{-1}\{F\}$$

LOCAL COORDINATE SYSTEM



$M_{i,j}$, $\theta_{i,j}$ = bending moments and corresponding rotations at ends i, j, respectively; $N_{i,j}$, $u_{i,j}$ are axial forces and corresponding axial deformations at ends i, j, respectively; and $Q_{i,j}$, $v_{i,j}$ are shear forces and corresponding transverse displacements at ends i, j, respectively. The directions of the actions and movements are positive when using the stiffness method.

DEGREES OF FREEDOM



MEMBER STIFFNESS MATRIX

The structure stiffness matrix $[K]$ is assembled on the basis of the equilibrium and compatibility conditions between the members. For a general frame, the equilibrium matrix equation of a member is

$$\{P\} = [K_e]\{d\} \tag{1.9}$$

where $\{P\}$ is the member force vector, $[K_e]$ is the member stiffness matrix, and $\{d\}$ is the member displacement vector, all in the member's local coordinate system. The elements of the matrices in Equation (1.9) are given as

$$\{P\} = \begin{Bmatrix} N_i \\ Q_i \\ M_i \\ N_j \\ Q_j \\ M_j \end{Bmatrix}; [K_e] = \begin{bmatrix} K_{11} & 0 & 0 & K_{14} & 0 & 0 \\ 0 & K_{22} & K_{23} & 0 & K_{25} & K_{26} \\ 0 & K_{32} & K_{33} & 0 & K_{35} & K_{36} \\ K_{41} & 0 & 0 & K_{44} & 0 & 0 \\ 0 & K_{52} & K_{53} & 0 & K_{55} & K_{56} \\ 0 & K_{62} & K_{63} & 0 & K_{65} & K_{66} \end{bmatrix}; \{d\} = \begin{Bmatrix} u_i \\ v_i \\ \theta_i \\ u_j \\ v_j \\ \theta_j \end{Bmatrix}$$

ELEMENS OF MEMBER STIFFNESS MATRIX

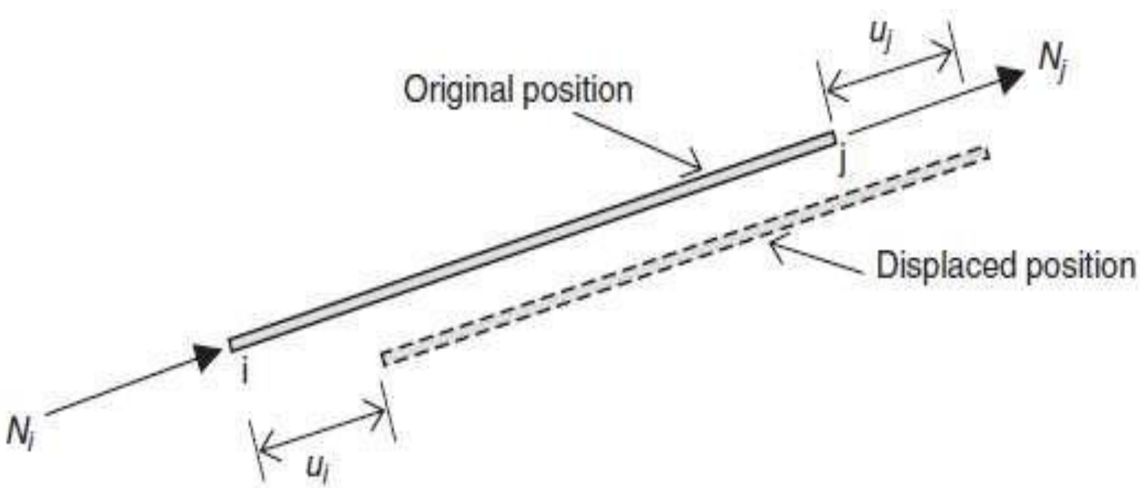
AXIAL LOADING

A member under axial forces N_i and N_j acting at its ends produces axial displacements u_i and u_j as shown in Figure 1.10. From the stress-strain relation, it can be shown that

$$N_i = \frac{EA}{L}(u_i - u_j) \tag{1.10a}$$

$$N_j = \frac{EA}{L}(u_j - u_i) \tag{1.10b}$$

where E is Young's modulus, A is cross-sectional area, and L is length of the member. Hence, $K_{11} = -K_{14} = -K_{41} = K_{44} = \dfrac{EA}{L}$.



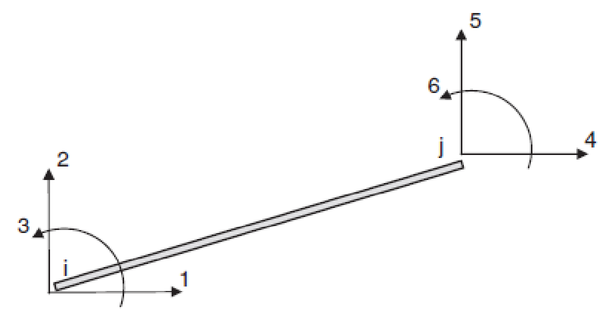


FIGURE 1.10. Member under axial forces.

BENDING MOMENTS AND SHEAR

For a member with shear forces Q_i, Q_j and bending moments M_i, M_j acting at its ends as shown in Figure 1.11, the end displacements and rotations are related to the bending moments by the slope-deflection equations as

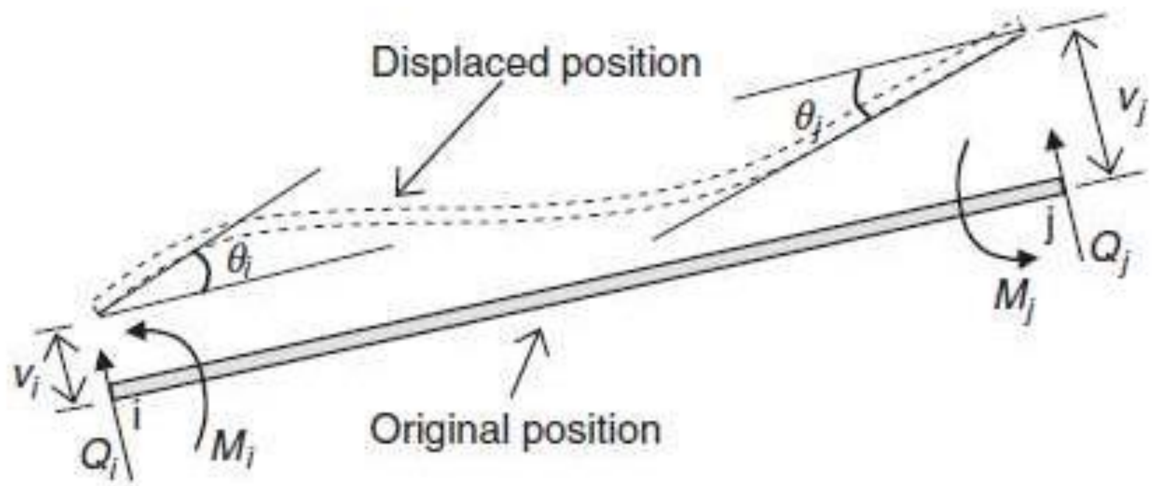


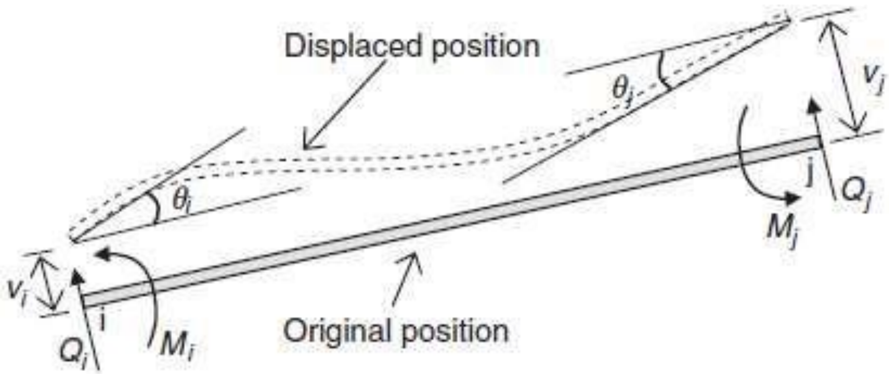
FIGURE 1.11. Member under shear forces and bending moments.

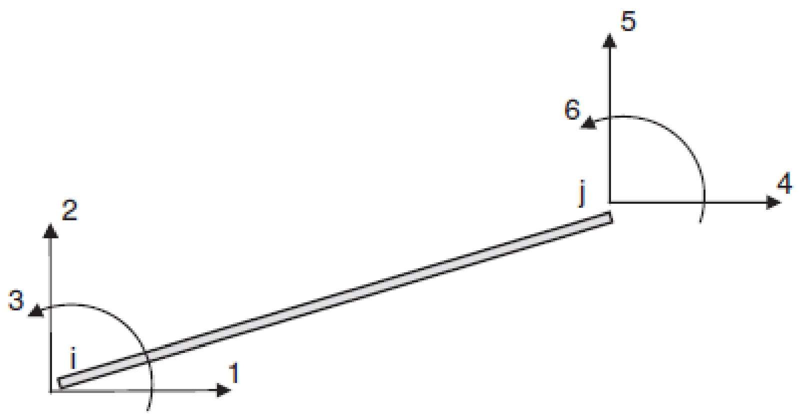
BENDING MOMENTS AND SHEAR

$$M_i = \frac{2EI}{L}\left[2\theta_i + \theta_j - \frac{3(v_j - v_i)}{L}\right]$$

$$M_j = \frac{2EI}{L}\left[2\theta_j + \theta_i - \frac{3(v_j - v_i)}{L}\right]$$

$$\text{Hence, } K_{62} = -K_{65} = \frac{6EI}{L^2},\ K_{63} = \frac{2EI}{L},\ \text{and } K_{66} = \frac{4EI}{L}.$$

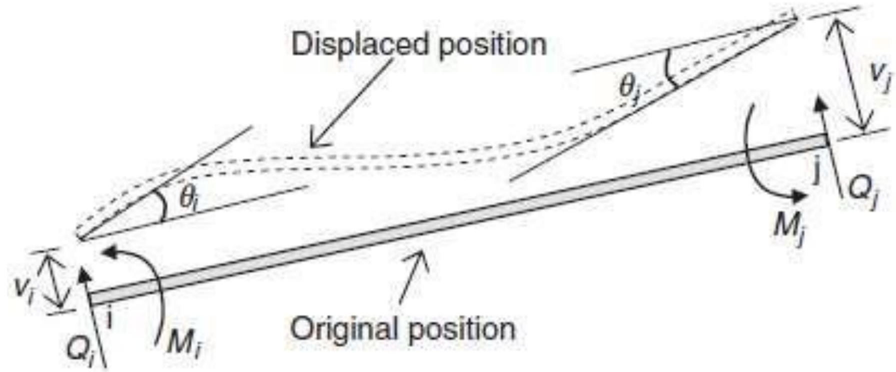




BENDING MOMENTS AND SHEAR

By taking the moment about end j of the member in Figure 1.11, we obtain

$$Q_i = \frac{M_i + M_j}{L} = \frac{2EI}{L^2}\left[3\theta_i + 3\theta_j - \frac{6(v_j - v_i)}{L}\right] \qquad (1.12a)$$



Also, by taking the moment about end i of the member, we obtain

$$Q_j = -\left(\frac{M_i + M_j}{L}\right) = -Q_i \qquad (1.12b)$$

$$K_{22} = K_{55} = -K_{25} = -K_{52} = \frac{12EI}{L^3} \text{ and } K_{23} = K_{26} = -K_{53} = -K_{66} = \frac{6EI}{L^2}.$$

STIFFNESS MATRIX

In summary, the resulting member stiffness matrix is symmetric about the diagonal:

$$[K_e] = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \tag{1.13}$$

COORDINATES TRANSFORMATION

In order to establish the equilibrium conditions between the member forces in the local coordinate system and the externally applied loads in the global coordinate system, the member forces are transformed into the global coordinate system by force resolution. Figure 1.12 shows a member inclined at an angle α to the horizontal.

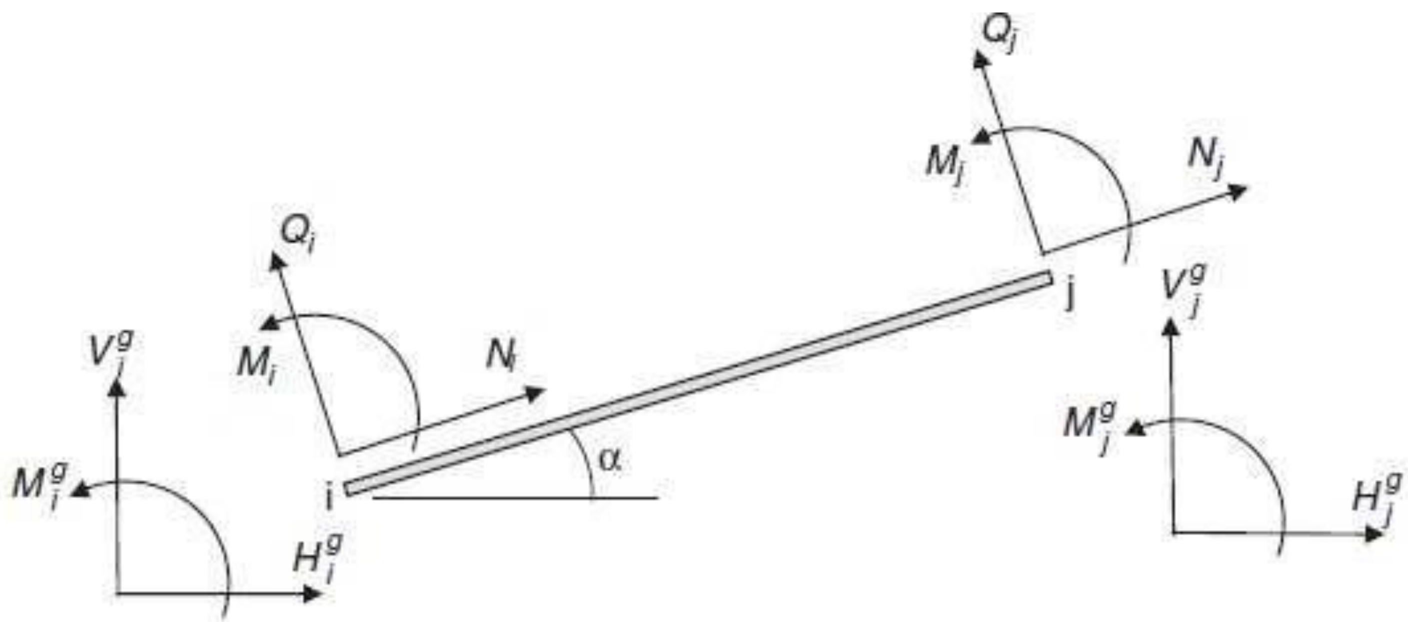
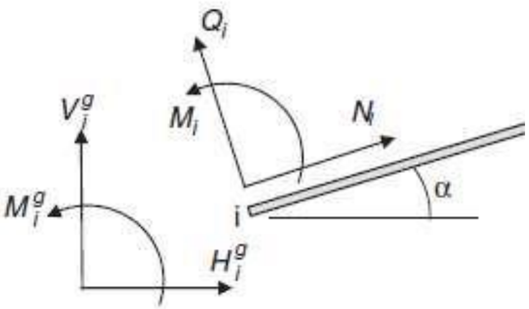


FIGURE 1.12. Forces in the local and global coordinate systems.

LOAD TRANSFORMATION

The forces in the global coordinate system shown with superscript "g" in Figure 1.12 are related to those in the local coordinate system by

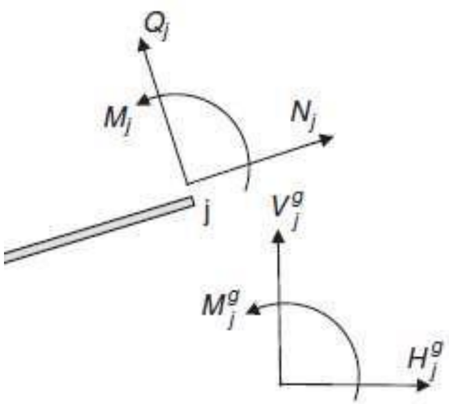


$$H_i^g = N_i \cos\alpha - Q_i \sin\alpha \qquad (1.14a)$$

$$V_i^g = N_i \sin\alpha + Q_i \cos\alpha \qquad (1.14b)$$

$$M_i^g = M_i \qquad (1.14c)$$

Similarly,



$$H_j^g = N_j \cos\alpha - Q_j \sin\alpha \qquad (1.14d)$$

$$V_j^g = N_j \sin\alpha + Q_j \cos\alpha \qquad (1.14e)$$

$$M_j^g = M_j \qquad (1.14f)$$

In matrix form, Equations (1.14a) to (1.14f) can be expressed as

$$\{F_e^g\} = [T]\{P\} \qquad (1.15)$$

LOAD TRANSFORMATION

In matrix form, Equations (1.14a) to (1.14f) can be expressed as

$$\{F_e^g\} = [T]\{P\} \tag{1.15}$$

where $\{F_e^g\}$ is the member force vector in the global coordinate system and $[T]$ is the transformation matrix, both given as

$$\{F_e^g\} = \begin{Bmatrix} H_i^g \\ V_i^g \\ M_i^g \\ H_j^g \\ V_j^g \\ M_j^g \end{Bmatrix} \text{ and } [T] = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & 0 & 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

DISPLACEMENT TRANSFORMATION

The displacements in the global coordinate system can be related to those in the local coordinate system by following the procedure similar to the force transformation. The displacements in both coordinate systems are shown in Figure 1.13.

From Figure 1.13,

$$u_i = u_i^g \cos\alpha + v_i^g \sin\alpha \tag{1.16a}$$

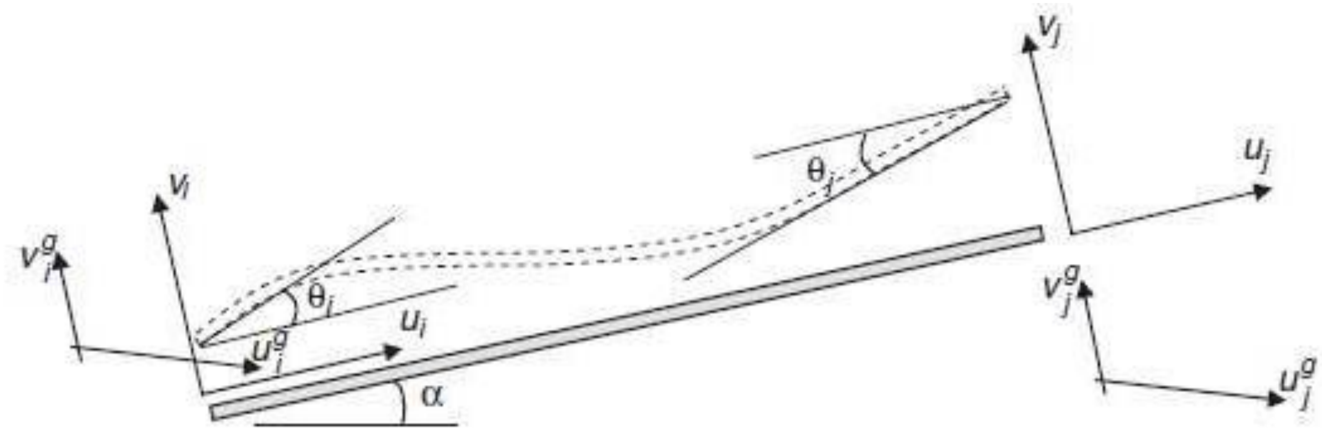


FIGURE 1.13. Displacements in the local and global coordinate systems.

DISPLACEMENT TRANSFORMATION

$$v_i = -u_i^g \sin\alpha + v_i^g \cos\alpha \tag{1.16b}$$

$$\theta_i = \theta_i^g \tag{1.16c}$$

$$u_j = u_j^g \cos\alpha + v_j^g \sin\alpha \tag{1.16d}$$

$$v_j = -u_j^g \sin\alpha + v_j^g \cos\alpha \tag{1.16e}$$

$$\theta_j = \theta_j^g \tag{1.16f}$$

DISPLACEMENT TRANSFORMATION

In matrix form, Equations (1.16a) to (1.16f) can be expressed as

$$\{d\} = [T]^t \{D_e^g\} \tag{1.17}$$

where $\{D_e^g\}$ is the member displacement vector in the global coordinate system corresponding to the directions in which the freedom codes are specified and is given as

$$\{D_e^g\} = \begin{Bmatrix} u_i^g \\ v_i^g \\ \theta_i^g \\ u_j^g \\ v_j^g \\ \theta_j^g \end{Bmatrix}$$

and $[T]^t$ is the transpose of $[T]$.

Member Stiffness in Global Coordinate System

From Equation (1.15),

$$
\begin{aligned}
\{F_e^g\} &= [T]\{P\} \\
&= [T][K_e]\{d\} && \text{from Equation (1.9)} \\
&= [T][K_e][T]^t\{D_e^g\} && \text{from Equation (1.17)} \\
&= [K_e^g]\{D_e^g\}
\end{aligned}
$$

where $[K_e^g] = [T][K_e][T]^t$ = member stiffness matrix in the global coordinate system.

Member Stiffness in Global Coordinate System

An explicit expression for $[K_e^g]$ is

$$[K_e^g]=\begin{bmatrix} C^2\frac{EA}{L}+S^2\frac{12EI}{L^3} & SC\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & -S\frac{6EI}{L^2} & -\left(C^2\frac{EA}{L}+S^2\frac{12EI}{L^3}\right) & -SC\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & -S\frac{6EI}{L^2} \\ & S^2\frac{EA}{L}+C^2\frac{12EI}{L^3} & C\frac{6EI}{L^2} & -SC\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & -\left(S^2\frac{EA}{L}+C^2\frac{12EI}{L^3}\right) & C\frac{6EI}{L^2} \\ & & \frac{4EI}{L} & S\frac{6EI}{L^2} & -C\frac{6EI}{L^2} & \frac{2EI}{L} \\ & & & C^2\frac{EA}{L}+S^2\frac{12EI}{L^3} & SC\left(\frac{EA}{L}-\frac{12EI}{L^3}\right) & S\frac{6EI}{L^2} \\ & \textit{Symmetric} & & & S^2\frac{EA}{L}+C^2\frac{12EI}{L^3} & -C\frac{6EI}{L^2} \\ & & & & & \frac{4EI}{L} \end{bmatrix} \quad (1.19)$$

where C = cos α; S = sin α.

Assembly of Structure Stiffness Matrix

Consider part of a structure with four externally applied forces, F1, F2, F4, and F5, and two applied moments, M3 and M6, acting at the two joints *p* and *q* connecting three members A, B, and C as shown in Figure 1.14. The freedom codes at joint *p* are {1, 2, 3} and at joint *q* are {4, 5, 6}. The structure stiffness matrix [K] is assembled on the basis of two conditions: compatibility and equilibrium conditions at the joints.

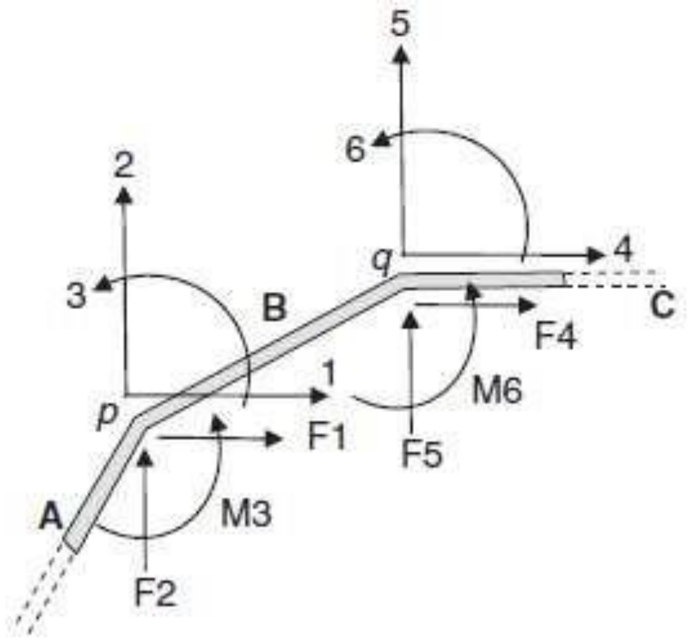


FIGURE 1.14. Assembly of structure stiffness matrix [*K*].

Compatibility Condition

At joint p, the global displacements are $D1$ (horizontal), $D2$ (vertical), and $D3$ (rotational). Similarly, at joint q, the global displacements are $D4$ (horizontal), $D5$ (vertical), and $D6$ (rotational). The compatibility condition is that the displacements ($D1$, $D2$, and $D3$) at end p of member A are the same as those at end p of member B. Thus, $(u_j^g)_A = (u_i^g)_B = D1$, $(v_j^g)_A = (v_i^g)_B = D2$, and $(\theta_j^g)_A = (\theta_i^g)_B = D3$. The same condition applies to displacements ($D4$, $D5$, and $D6$) at end q of both members B and C.

The member stiffness matrix in the global coordinate system given in Equation (1.19) can be written as

$$[K_e^g] = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} \end{bmatrix} \tag{1.20}$$

where $k_{11} = C^2 \dfrac{EA}{L} + S^2 \dfrac{12EI}{L^3}$, etc.

Compatibility Condition

For member A, from Equation (1.18),

$$\left(H_j^g\right)_A = ... + ... + ... + (k_{44})_A D_1 + (k_{45})_A D_2 + (k_{46})_A D_3 \qquad (1.21a)$$

$$\left(V_j^g\right)_A = ... + ... + ... + (k_{54})_A D_1 + (k_{55})_A D_2 + (k_{56})_A D_3 \qquad (1.21b)$$

$$\left(M_j^g\right)_A = ... + ... + ... + (k_{64})_A D_1 + (k_{65})_A D_2 + (k_{66})_A D_3 \qquad (1.21c)$$

Similarly, for member B,

$$(H_i^g)_B = (k_{11})_B D_1 + (k_{12})_B D_2 + (k_{13})_B D_3 + (k_{14})_B D_4 + (k_{15})_B D_5 + (k_{16})_B D_6 \qquad (1.21d)$$

$$(V_i^g)_B = (k_{21})_B D_1 + (k_{22})_B D_2 + (k_{23})_B D_3 + (k_{24})_B D_4 + (k_{25})_B D_5 + (k_{26})_B D_6 \qquad (1.21e)$$

$$(M_i^g)_B = (k_{31})_B D_1 + (k_{32})_B D_2 + (k_{33})_B D_3 + (k_{34})_B D_4 + (k_{35})_B D_5 + (k_{36})_B D_6 \qquad (1.21f)$$

Compatibility Condition

$$\left(H_j^g\right)_B = (k_{41})_B D_1 + (k_{42})_B D_2 + (k_{43})_B D_3 + (k_{44})_B D_4 + (k_{45})_B D_5 + (k_{46})_B D_6 \tag{1.21g}$$

$$\left(V_j^g\right)_B = (k_{51})_B D_1 + (k_{52})_B D_2 + (k_{53})_B D_3 + (k_{54})_B D_4 + (k_{55})_B D_5 + (k_{56})_B D_6 \tag{1.21h}$$

$$\left(M_j^g\right)_B = (k_{61})_B D_1 + (k_{62})_B D_2 + (k_{63})_B D_3 + (k_{64})_B D_4 + (k_{65})_B D_5 + (k_{66})_B D_6 \tag{1.21i}$$

Similarly, for member C,

$$(H_i^g)_C = (k_{11})_C D_1 + (k_{12})_C D_2 + (k_{13})_C D_3 + \ldots\ldots + \ldots\ldots + \ldots\ldots \tag{1.21j}$$

$$(V_i^g)_C = (k_{21})_C D_1 + (k_{22})_C D_2 + (k_{23})_C D_3 + \ldots\ldots + \ldots\ldots + \ldots\ldots \tag{1.21k}$$

$$(M_i^g)_C = (k_{31})_C D_1 + (k_{32})_C D_2 + (k_{33})_C D_3 + \ldots\ldots + \ldots\ldots + \ldots\ldots \tag{1.21l}$$

Equilibrium Condition

Any of the externally applied forces or moments applied in a certain direction at a joint of a structure is equal to the sum of the member forces acting in the same direction for members connected at that joint in the global coordinate system. Therefore, at joint p,

$$F1 = \left(H_j^g\right)_A + \left(H_i^g\right)_B \tag{1.22a}$$

$$F2 = \left(V_j^g\right)_A + \left(V_i^g\right)_B \tag{1.22b}$$

$$M3 = \left(M_j^g\right)_A + \left(M_i^g\right)_B \tag{1.22c}$$

Also, at joint q,

$$F4 = \left(H_j^g\right)_B + \left(H_i^g\right)_C \tag{1.22d}$$

$$F5 = \left(V_j^g\right)_B + \left(V_i^g\right)_C \tag{1.22e}$$

$$M6 = \left(M_j^g\right)_B + \left(M_i^g\right)_C \tag{1.22f}$$

Equilibrium Condition

$$\begin{Bmatrix} \bullet \\ F1 \\ F2 \\ M3 \\ F4 \\ F5 \\ M6 \\ \bullet \end{Bmatrix} = \begin{bmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & (k_{44})_A + (k_{11})_B & (k_{45})_A + (k_{12})_B & (k_{46})_A + (k_{13})_B & (k_{14})_B & (k_{15})_B & (k_{16})_B & \bullet \\ \bullet & (k_{54})_A + (k_{21})_B & (k_{55})_A + (k_{22})_B & (k_{56})_A + (k_{23})_B & (k_{24})_B & (k_{25})_B & (k_{26})_B & \bullet \\ \bullet & (k_{64})_A + (k_{31})_B & (k_{65})_A + (k_{32})_B & (k_{66})_A + (k_{33})_B & (k_{34})_B & (k_{35})_B & (k_{36})_B & \bullet \\ \bullet & (k_{41})_B & (k_{42})_B & (k_{43})_B & (k_{44})_B + (k_{11})_C & (k_{45})_B + (k_{12})_C & (k_{46})_B + (k_{13})_C & \bullet \\ \bullet & (k_{51})_B & (k_{52})_B & (k_{53})_B & (k_{54})_B + (k_{21})_C & (k_{55})_B + (k_{22})_C & (k_{56})_B + (k_{23})_C & \bullet \\ \bullet & (k_{61})_B & (k_{62})_B & (k_{63})_B & (k_{64})_B + (k_{31})_C & (k_{65})_B + (k_{32})_C & (k_{66})_B + (k_{33})_C & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{bmatrix} \begin{Bmatrix} \bullet \\ \bullet \\ D1 \\ D2 \\ D3 \\ D4 \\ D5 \\ D4 \\ \bullet \\ \bullet \end{Bmatrix} \tag{1.23}$$

where the "•" stands for matrix coefficients contributed from the other parts of the structure. In simple form, Equation (1.23) can be written as

$$\{F\} = [K]\{D\}$$

which is identical to Equation (1.7). Equation (1.23) shows how the structure equilibrium equation is set up in terms of the load vector $\{F\}$, structure stiffness matrix $[K]$, and the displacement vector $\{D\}$.

Assembly of Structure Stiffness Matrix

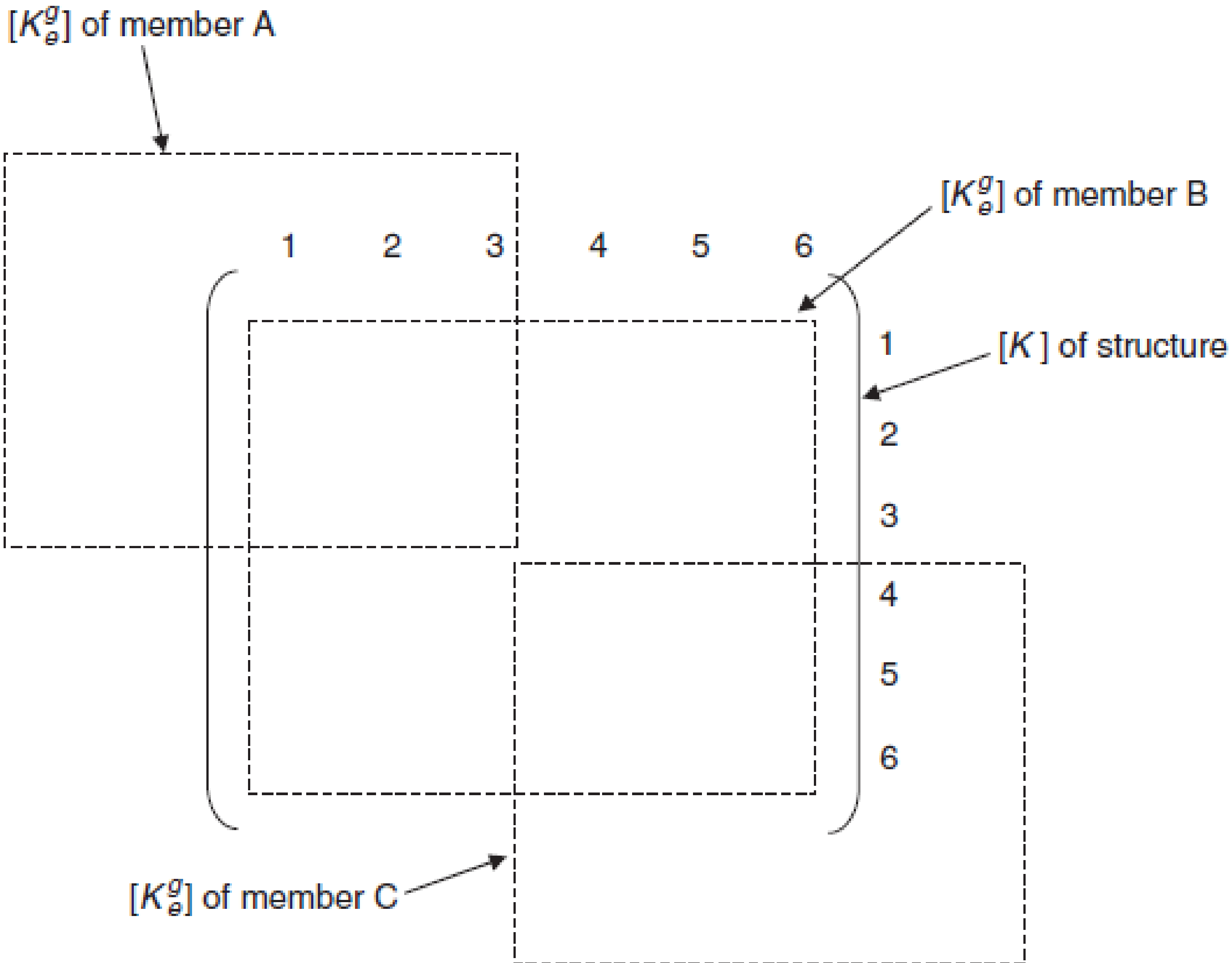


FIGURE 1.16. Assembly of structure stiffness matrix.

ASSEMBLY OF LOAD VECTOR

$$\{F\} = \begin{array}{cc} & \boxed{\text{Freedom codes}} \\ \begin{Bmatrix} \bullet \\ F1 \\ F2 \\ F3 \\ F4 \\ F5 \\ F6 \\ \bullet \end{Bmatrix} & \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ \\ \end{array} \end{array}$$

FIGURE 1.17. Assembly of load vector.

METHODS OF SOLUTION

The displacements of the structure can be found by solving Equation (1.23). Because of the huge size of the matrix equation usually encountered in practice, Equation (1.23) is solved routinely by numerical methods such as the Gaussian elimination method and the iterative Gauss–Seidel method. It should be noted that in using these

numerical methods, the procedure is analogous to inverting the structure stiffness matrix, which is subsequently multiplied by the load vector as in Equation (1.8):

$$\{D\} = [K]^{-1}\{F\} \tag{1.8}$$

METHODS OF SOLUTION

The numerical procedure fails only if an inverted $[K]$ cannot be found. This situation occurs when the determinant of $[K]$ is zero, implying an unstable structure. Unstable structures with a degree of statically indeterminacy, f_r, greater than zero (see Section 1.2) will have a zero determinant of $[K]$. In numerical manipulation by computers, an exact zero is sometimes difficult to obtain. In such cases, a good indication of an unstable structure is to examine the displacement vector $\{D\}$, which would include some exceptionally large values.

CALCULATION OF MEMBER FORCES

Member forces are calculated according to Equation (1.9). Hence,

$$\begin{aligned}\{P\} &= [K_e]\{d\} \\ &= [K_e]\,[T]^t\{D_e^g\}\end{aligned} \qquad (1.24)$$

where $\{D_e^g\}$ is extracted from $\{D\}$ for each member according to its freedom codes and

$$[K_e][T]^t = \begin{bmatrix} C\frac{EA}{L} & S\frac{EA}{L} & 0 & -C\frac{EA}{L} & -S\frac{EA}{L} & 0 \\ -S\frac{12EI}{L^3} & C\frac{12EI}{L^3} & \frac{6EI}{L^2} & S\frac{12EI}{L^3} & -C\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ -S\frac{6EI}{L^2} & C\frac{6EI}{L^2} & \frac{4EI}{L} & S\frac{6EI}{L^2} & -C\frac{6EI}{L^2} & \frac{2EI}{L} \\ -C\frac{EA}{L} & -S\frac{EA}{L} & 0 & C\frac{EA}{L} & S\frac{EA}{L} & 0 \\ S\frac{12EI}{L^3} & -C\frac{12EI}{L^3} & -\frac{6EI}{L^2} & -S\frac{12EI}{L^3} & C\frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ -S\frac{6EI}{L^2} & C\frac{6EI}{L^2} & \frac{2EI}{L} & S\frac{6EI}{L^2} & -C\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

MEMBER FORCES

$$\{P\} = \begin{Bmatrix} N_i \\ Q_i \\ M_i \\ N_j \\ Q_j \\ M_j \end{Bmatrix} = \begin{bmatrix} C\dfrac{EA}{L} & S\dfrac{EA}{L} & 0 & -C\dfrac{EA}{L} & -S\dfrac{EA}{L} & 0 \\ -S\dfrac{12EI}{L^3} & C\dfrac{12EI}{L^3} & \dfrac{6EI}{L^2} & S\dfrac{12EI}{L^3} & -C\dfrac{12EI}{L^3} & \dfrac{6EI}{L^2} \\ -S\dfrac{6EI}{L^2} & C\dfrac{6EI}{L^2} & \dfrac{4EI}{L} & S\dfrac{6EI}{L^2} & -C\dfrac{6EI}{L^2} & \dfrac{2EI}{L} \\ -C\dfrac{EA}{L} & -S\dfrac{EA}{L} & 0 & C\dfrac{EA}{L} & S\dfrac{EA}{L} & 0 \\ S\dfrac{12EI}{L^3} & -C\dfrac{12EI}{L^3} & -\dfrac{6EI}{L^2} & -S\dfrac{12EI}{L^3} & C\dfrac{12EI}{L^3} & -\dfrac{6EI}{L^2} \\ -S\dfrac{6EI}{L^2} & C\dfrac{6EI}{L^2} & \dfrac{2EI}{L} & S\dfrac{6EI}{L^2} & -C\dfrac{6EI}{L^2} & \dfrac{4EI}{L} \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{Bmatrix}$$

SUMMARY

1. Assign freedom codes to each joint indicating the displacement freedom at the ends of the members connected at that joint. Assign a freedom code of "zero" to any restrained displacement.

2. Assign an arrow to each member so that ends *i* and *j* are defined. Also, the angle of orientation α for the member is defined in Figure 1.18 as:

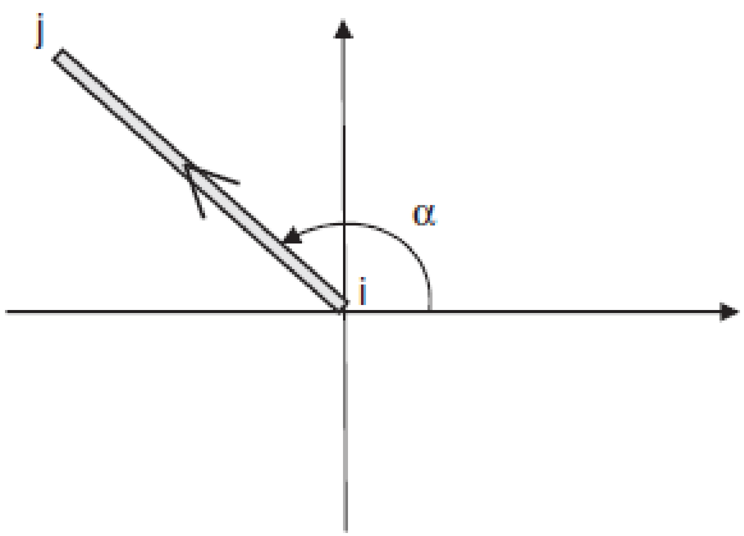


FIGURE 1.18. Definition of angle of orientation for member.

SUMMARY

3. Assemble the structure stiffness matrix $[K]$ from each of the member stiffness matrices.

4. Form the load vector $\{F\}$ of the structure.

5. Calculate the displacement vector $\{D\}$ by solving for $\{D\} = [K]^{-1}\{F\}$.

6. Extract the local displacement vector $\{D_e^g\}$ from $\{D\}$ and calculate the member force vector $\{P\}$ using $\{P\} = [K_e][T]^t\{D_e^g\}$.

Sign Convention for Member Force

Positive member forces and displacements obtained from the stiffness method of analysis are shown in Figure 1.19. To plot the forces in conventional axial force, shear force, and bending moment diagrams, it is necessary to translate them into a system commonly adopted for plotting.

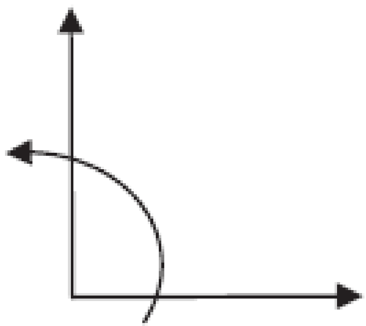


FIGURE 1.19. Direction of positive forces and displacements using stiffness method.

Axial Force

For a member under compression, the axial force at end *i* is positive (from analysis) and at end *j* is negative (from analysis), as shown in Figure 1.20.

FIGURE 1.20. Member under compression.

Shear Force

A shear force plotted positive in diagram is acting upward (positive from analysis) at end *i* and downward (negative from analysis) at end *j* as shown in Figure 1.21. Positive shear force is usually plotted in the space above the member.

FIGURE 1.21. Positive shear forces.

Bending Moment

A member under sagging moment is positive in diagram (clockwise and negative from analysis) at end *i* and positive (anticlockwise and positive from analysis) at end *j* as shown in Figure 1.22. Positive bending moment is usually plotted in the space beneath the member. In doing so, a bending moment is plotted on the tension face of the member.

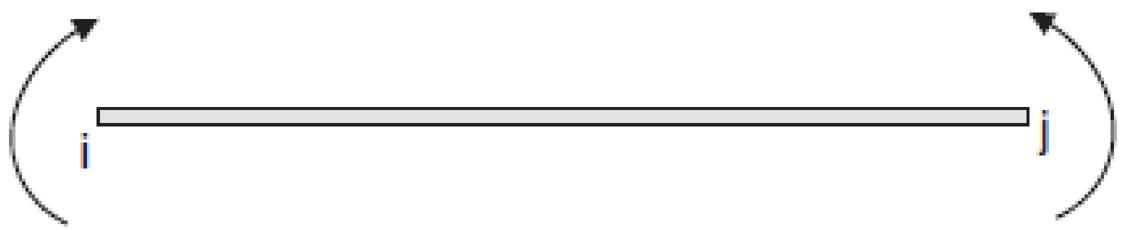


FIGURE 1.22. Sagging moment of a member.

THANK YOU