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DEPARTMENT OF CIVIL ENGINEERING

Lecture Notes on 23CE703 - Prestressed Concrete

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UNIT III

Analysis of Members under Flexure

1.1 Introduction

Similar to members under axial load, the analysis of members under flexure refers to the evaluation of the following.

1) Permissible prestress based on allowable stresses at transfer.
2) Stresses under service loads. These are compared with allowable stresses under service conditions.
3) Ultimate strength. This is compared with the demand under factored loads.
4) The entire load versus deformation behaviour.

The analyses at transfer and under service loads are presented in this section.

The evaluation of the load versus deformation behaviour is required in special type of analysis.

Assumptions

The analysis of members under flexure considers the following.

1) Plane sections remain plane till failure (known as Bernoulli's hypothesis).
2) Perfect bond between concrete and prestressing steel for bonded tendons.

Principles of Mechanics

The analysis involves three principles of mechanics.

1) **Equilibrium** of internal forces with the external loads. The compression in concrete (*C*) is equal to the tension in the tendon (*T*). The couple of C and T are equal to the moment due to external loads.
2) **Compatibility** of the strains in concrete and in steel for bonded tendons. The

formulation also involves the first assumption of plane section remaining plane after bending. For unbonded tendons, the compatibility is in terms of deformation.

3) **Constitutive** relationships relating the stresses and the strains in the materials.

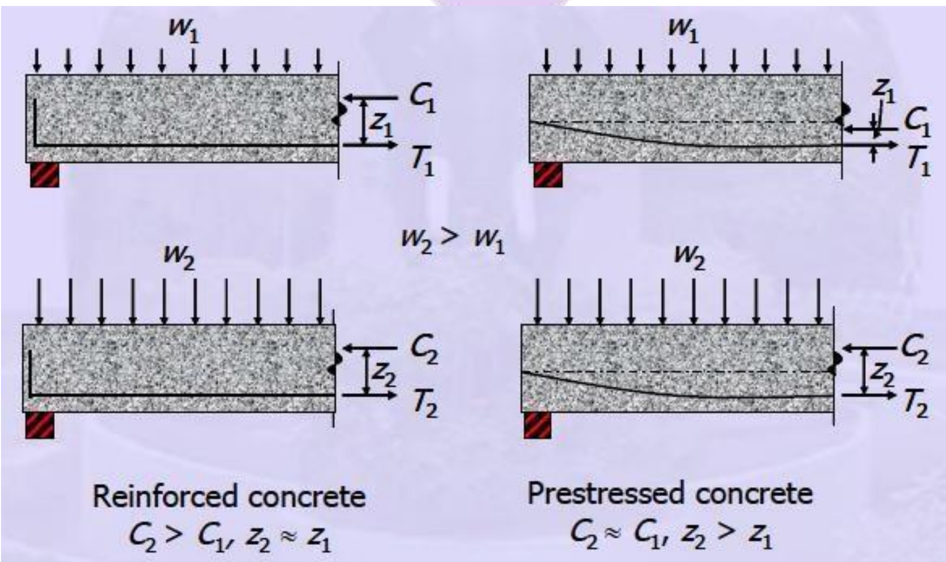
1.2 Variation of Internal Forces

In reinforced concrete members under flexure, the values of compression in concrete (C) and tension in the steel (*T*) increase with increasing external load.

The change in the lever arm (*z*) is not large.

In prestressed concrete members under flexure, at transfer of prestress *C* is located close to *T*. The couple of *C* and *T* balance only the self-weight. At service loads, *C* shifts up and the lever arm (*z*) gets large. The variation of *C* or *T* is not appreciable.

The following figure explains this difference schematically for a simply supported beam under uniform load.



C_1 , T_1 = compression and tension at transfer due to self-weight

C_2 , T_2 = compression and tension under service loads

w_1 = self weight

w_2 = service loads

z_1 = lever arm at transfer

z_2 = lever arm under service loads.

1.3 Analyses at Transfer and at Service

The analyses at transfer and under service loads are similar. Hence, they are presented together. A prestressed member usually remains uncracked under service loads. The concrete and steel are treated as elastic materials. The principle of superposition is applied. The increase in stress in the prestressing steel due to bending is neglected.

There are three approaches to analyse a prestressed member at transfer and under service loads. These approaches are based on the following concepts.

a) Based on stress concept.

b) Based on force concept.

c) Based on load balancing concept.

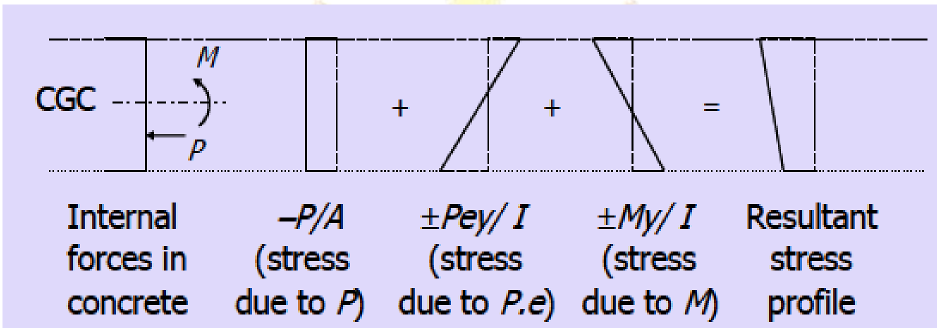
The following material explains the three concepts.

- **Based on Stress Concept**

In the approach based on stress concept, the stresses at the edges of the section under the internal forces in concrete are calculated. The stress concept is used to compare the calculated stresses with the allowable stresses.

The following figure shows a simply supported beam under a uniformly distributed load (UDL) and prestressed with constant eccentricity (e) along its length.

The first stress profile is due to the compression *P*. The second profile is due to the eccentricity of the compression. The third profile is due to the moment. At transfer, the moment is due to self-weight. At service the moment is due to service loads.



Stress profiles at a section due to internal forces

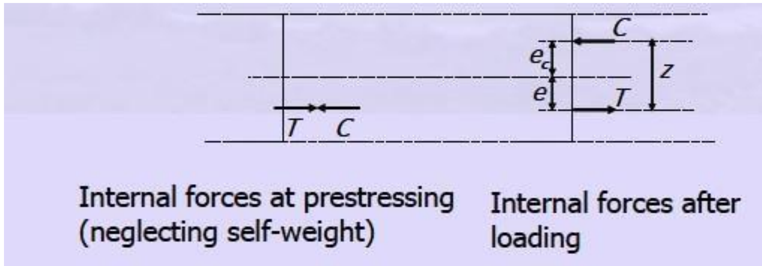
The resultant stress at a distance *y* from the CGC is given by the principle of superposition as follows.

$$f = -\frac{P}{A} \pm \frac{Pey}{I} \pm \frac{My}{I}$$

- ## <u>Based on Force Concept</u>

The approach based on force concept is analogous to the study of reinforced concrete. The tension in prestressing steel (*T*) and the resultant compression in concrete (*C*) are considered to balance the external loads. This approach is used to determine the dimensions of a section and to check the service load capacity. Of course, the stresses in concrete calculated by this approach are same as those calculated based on stress concept. The stresses at the extreme edges are compared with the allowable stresses.

The following figures show the internal forces in the section.



The equilibrium equations are as follows.

$C=T$

$M=C.z$

$M = C(e + e)$

The resultant stress in concrete at distance y from the CGC is given as follows.

$$f = -\frac{C}{A} \pm \frac{Ce_c y}{I}$$

Substituting $C = P$ and $Ce = M\text{-}Pec$

The expression of stress becomes same as that given by the stress concept.

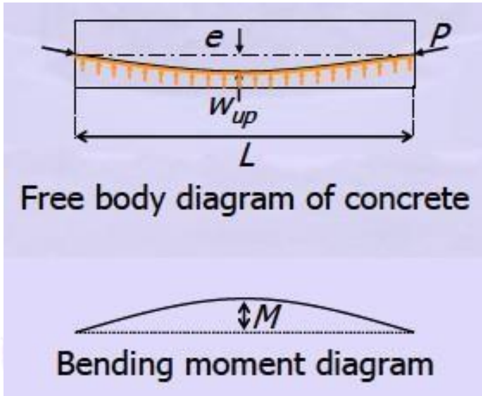
$$f = -\frac{P}{A} \pm \frac{Pey}{I} \pm \frac{My}{I}$$

Based on Load Balancing Concept

The approach based on load balancing concept is used for a member with curved or harped tendons and in the analysis of indeterminate continuous beams. The moment, upward thrust and upward deflection (camber) due to the prestress in the tendons are calculated. The upward thrust balances part of the superimposed load.

The expressions for three profiles of tendons in simply supported beams are given.

a) For a Parabolic Tendon



Free body diagram of concrete

Bending moment diagram

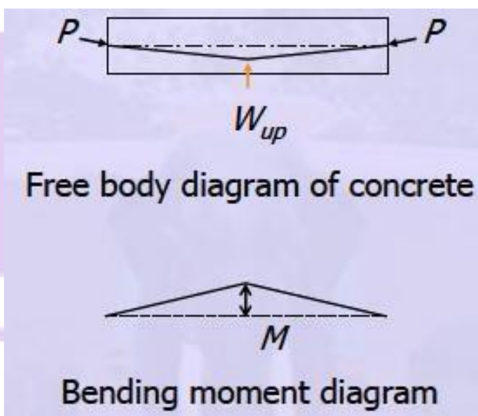
Simply supported beam with parabolic tendon

The moment at the centre due to the uniform upward thrust (w_{up}) is given by the following equation.

$$M = \frac{w_{up}L^2}{8}$$

The moment at the centre from the prestressing force is given as $M = Pe$. The expression of wup is calculated by equating the two expressions of M. The upward deflection (Δ) can be calculated from w_{up} based on elastic analysis.

b) For Singly Harped Tendon



Free body diagram of concrete

Bending moment diagram

Simply supported beam with singly harped tendon

The moment at the centre due to the upward thrust (Wup) is given by the following equation. It is equated to the moment due to the eccentricity of the tendon. As before, the upward thrust and the deflection can be calculated.

$$M = W_{up} aL = Pe$$

$$W_{up} = \frac{Pe}{aL}$$

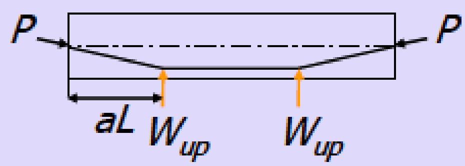
$$\Delta = \frac{a\left(3-4a^2\right) W_{up} L^3}{24EI}$$

$$M = \frac{W_{up} L}{4} = Pe$$

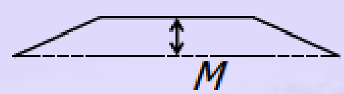
$$W_{up} = \frac{4Pe}{L}$$

$$\Delta = \frac{W_{up} L^3}{48EI}$$

c) For Doubly Harped Tendon



Free body diagram of concrete



Bending moment diagram

Simply supported beam with doubly harped tendon

The moment at the centre due to the upward thrusts (W_{up}) is given by the following equation.

It is equated to the moment due to the eccentricity of the tendon. As before, the upward thrust and the deflection can be calculated.

Analysis for Shear

Introduction

The analysis of reinforced concrete and prestressed concrete members for shear is more difficult compared to the analyses for axial load or flexure.

The analysis for axial load and flexure are based on the following principles of mechanics.

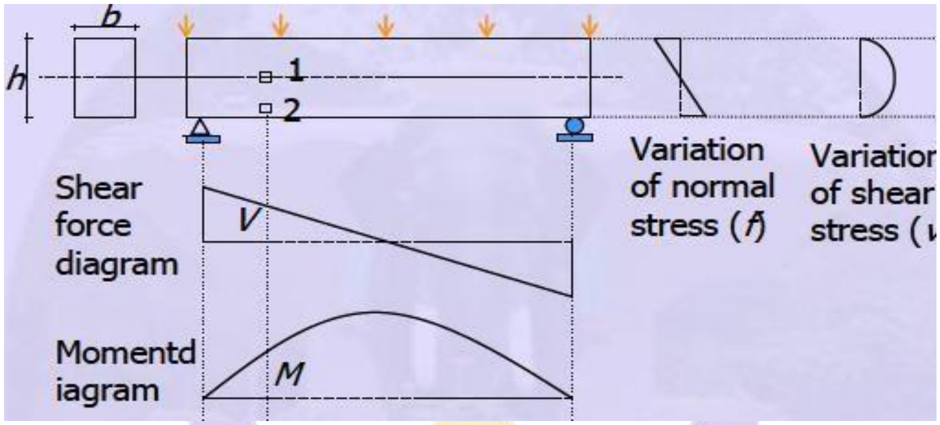
1) **Equilibrium** of internal and external forces
2) **Compatibility** of strains in concrete and steel
3) **Constitutive relationships** of materials.

The conventional analysis for shear is based on equilibrium of forces by a simple equation. The compatibility of strains is not considered. The constitutive relationships (relating stress and strain) of the materials, concrete or steel, are not used. The strength of each material corresponds to the ultimate strength. The strength of concrete under shear although based on test results, is empirical in nature.

Shear stresses generate in beams due to bending or twisting. The two types of shear stress are called flexural shear stress and torsional shear stress, respectively.

1.4 Stresses in an Uncracked Beam

The following figure shows the variations of shear and moment along the span of a simply supported beam under a uniformly distributed load. The variations of normal stress and shear stress along the depth of a section of the beam are also shown.



Variations of forces and stresses in a simply supported beam

Under a general loading, the shear force and the moment vary along the length. The normal stress and the shear stress vary along the length, as well as along the depth. The combination of the normal and shear stresses generate a two-dimensional stress field at a point. At any point in the beam, the state of two-dimensional stresses can be expressed in terms of the principal stresses. The Mohr's circle of stress is helpful to understand the state of stress.