UNIT-II

LIMIT STATE DESIGN

Introduction:

- identify the regions where the beam shall be designed as a flanged and where it will be rectangular in normal slab beam construction,
- define the effective and actual widths of flanged beams,
- state the requirements so that the slab part is effectively coupled with the flanged beam,
- write the expressions of effective widths of T and L-beams both for continuous and isolated cases,
- derive the expressions of C, T and M_u for four different cases depending on the location of the neutral axis and depth of the flange.

Reinforced concrete slabs used in floors, roofs and decks are mostly cast monolithic from the bottom of the beam to the top of the slab. Such rectangular beams having slab on top are different from others having either no slab (bracings of elevated tanks, lintels etc.) or having disconnected slabs as in some pre-cast systems (Figs. 5.10.1 a, b and c). Due to monolithic casting, beams and a part of the slab act together. Under the action of positive bending moment, i.e., between the supports of a continuous beam, the slab, up to a certain width greater than the width of the beam, forms the top part of the beam. Such beams having slab on top of the rectangular rib are designated as the flanged beams - either T or L type depending on whether the slab is on both sides or on one side of the beam (Figs. 5.10.2 a to e) . Over the supports of a continuous beam, the bending moment is negative and the slab, therefore, is in tension while a part of the rectangular beam (rib) is in compression. The continuous beam at support is thus equivalent to a rectangular beam (Figs. 5.10.2 a, c, f and g).

The actual width of the flange is the spacing of the beam, which is the same as the distance between the middle points of the adjacent spans of the slab, as shown in Fig. 5.10.2 b. However, in a flanged beam, a part of the width less than the actual width, is effective to be considered as a part of the beam. This width of the slab is designated as the effective width of the flange.

Effective Width

IS code requirements?

The following requirements (cl. 23.1.1 of IS 456) are to be satisfied to ensure the combined action of the part of the slab and the rib (rectangular part of the beam).

The slab and the rectangular beam shall be cast integrally or they shall be effectively bonded in any other manner.

Slabs must be provided with the transverse reinforcement of at least 60 per cent of the main reinforcement at the mid span of the slab if the main reinforcement of the slab isparallel to the transverse beam (Figs. 5.10.3 a)

The variation of compressive stress (Fig. 5.10.4) along the actual width of the flange shows that the compressive stress is more in the flange just above the rib than the same at some distance away from it. The nature of variation is complex and, therefore, the concept of effective width has been introduced. The effective width is a convenient hypothetical width of the flange over which the compressive stress is assumed to be uniform to give the same compressive force as it would have been in case of the actual width with the true variation of compressive stress.

5.10.2.2 IS code specifications

Clause 23.1.2 of IS 456 specifies the following effective widths of T and L-beams:

(a) For T-beams, the lesser of

(i) $b_f = l_o/6 + b_w + 6D_f$

(iv) b_f=Actual width of the flange

4.8.3 For isolated T-beams, the lesser of

(i) $b_f = \dfrac{l_o}{(l_o/b) + 4} + b_w$

b_f=Actual width of the flange

(ii) For L-beams, the lesser of

(i) $b_f = l_o/12 + b_w + 3D_f$

b_f=Actual width of the flange

(i) For isolated L-beams, the lesser of

(i) $b_f = \dfrac{0.5\, l_o}{(l_o/b) + 4} + b_w$

(ii) b_f= Actual width of the flange

where b_f = effective width of the flange,

l_o= distance between points of zero moments in the beam, which is the effective span for simply supported beams and 0.7 times the effective span for continuous beams and frames,

b_w= beadth of the web,

D_f= thickness of the flange,

b = actual width of theflange

Four Different Cases

The neutral axis of a flanged beam may be either in the flange or in the web depending on the physical dimensions of the effective width of flange b_f, effective width of web b_w, thickness of flange $D_f$and effective depth of flanged beam d (Fig. 5.10.4). The flanged beam may be considered as a rectangular beam of width $b_f$and effective depth d if the neutral axis is in the flange as the concrete in tension is ignored. However, if the neutral axis is in the web, the compression is taken by the flange and a part of the web.

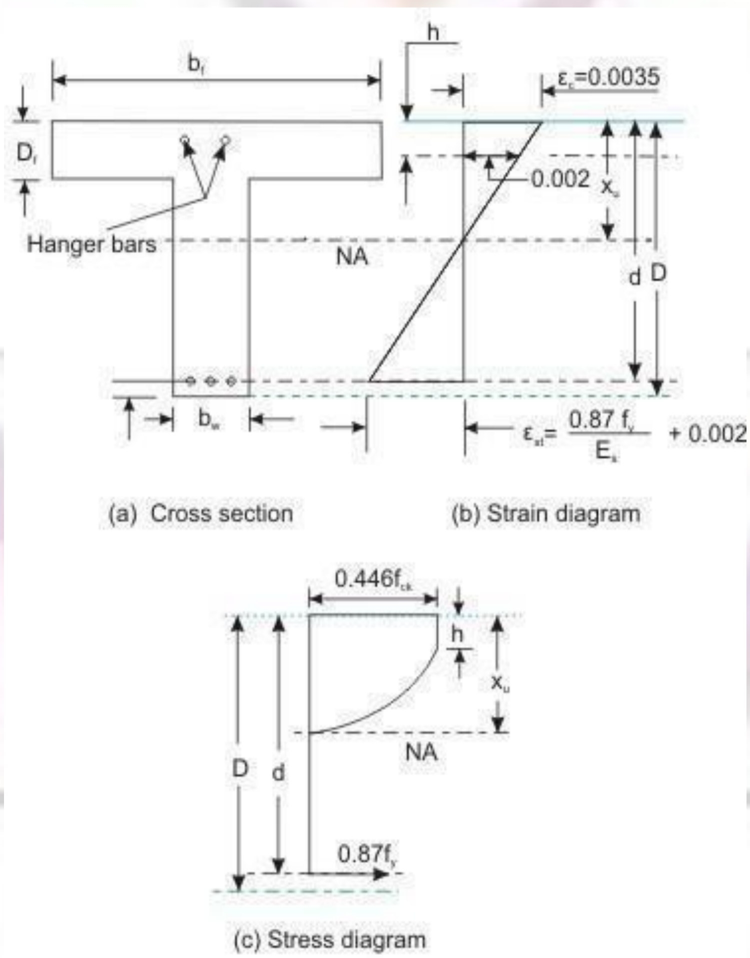


Fig. 5.10.5: A typical T-beam section

All the assumptions made in sec. 3.4.2 of Lesson 4 are also applicable for the flanged beams. As explained in Lesson 4, the compressive stress remains constant between the strains of 0.002and0.0035.Itisimportanttofindthedepthhofthebeamwherethestrainis0.002(Fig.5.10.5b).Ifit is locate din the web, the whole of flange willbe under the constantstresslevelof 0.446 f_{ck}.

The following gives the relation of $D_f$and d to facilitate the determination of the depth h where the strain will be 0.002.

$$h/d = 0.2 \qquad (5.2)$$

It is now clear that the three values of h are around 0.2 d for the three grades of steel. The maximum value of h may be D_f, at the bottom of the flange where the strain will be 0.002, if D_f/d = 0.2. This reveals that the thickness of the flange may be considered small if D_f/d does not exceed 0.2 and in that case, the position of the fibre of 0.002 strain will be in the web and the entire flange will be under a constant compressive stress of 0.446 f_{ck}.

On the other hand, if $D_f$is > 0.2 d, the position of the fibre of 0.002 strain will be in the flange. In that case, a part of the slab will have the constant stress of 0.446 $f_{ck}$where the strain will be more than0.002.

Thus, in the balanced and over-reinforced flanged beams(when $x_u = x_{u,max}$), the ratioofDf /d is important to determine if the rectangular stress block is for the full depth of the flange (when D_f/d does not exceed 0.2) of for a part of the flange (when $D_f/d > 0.2$). Similarly, for the under-reinforced flanged beams, the ratio of $D_f/x_u$is considered in place of D_f/d. If D_f/x_udoes notexceed

0.43 (see Eq. 5.1), the constant stress block is for the full depth of the flange. If $D_f/x_u >$ 0.43, the constant stress block is for a part of the depth of theflange.

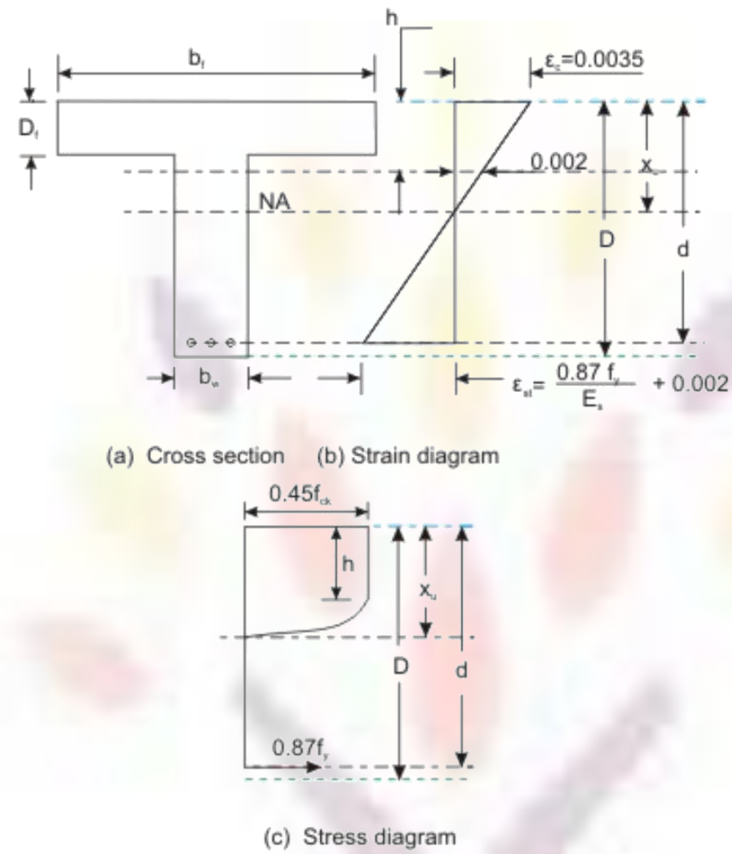


Fig. 5.10.9: T-beam, case (iii a), when $D_f / x_u <= 0.43$ and under-reinforced $x_u > D_f$

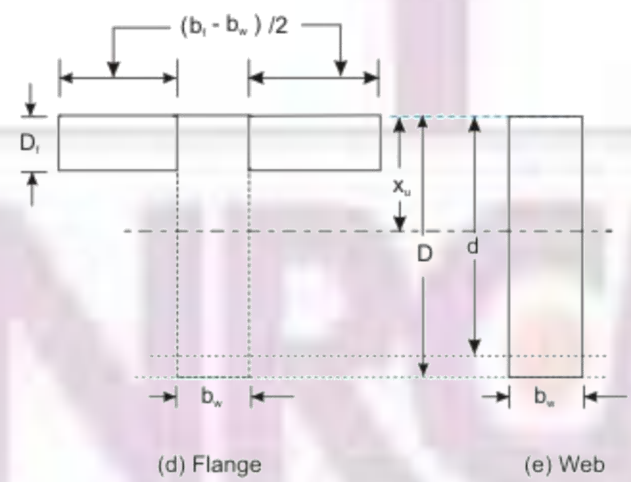


Fig. 5.10.9: T-beam, case (iii a), when $D_f / x_u <= 0.43$ and under-reinforced $x_u > D_f$

14. Neutralaxis is in the web and the section is under-reinforced ($x_{u,max} > x_u > D_f$), (Figs. 5.10.9 and 10 a to e)

This has two situations: (a) when D_f/x_u does not exceed 0.43, the full depth of flange is having the constant stress (Fig. 5.10.9), and (b) when $D_f/x_u > 0.43$, the constant stress is for a part of the depth of flange

(Fig. 5.10.10).

(i) Neutral axis is in the web and the section is over-reinforced ($x_u > x_{u,max} > D_f$), (Figs. 5.10.7 and 8 a toe)

As mentioned earlier, the value of x_u is then taken as $x_{u,max}$ when $x_u > x_{u,max}$. Therefore, this case also will have two situations depending on D_f/d not exceeding 0.2 or
> 0.2 as in (ii) above. The governing equations of the four different cases are now taken up.

Governing Equations

The following equations are only for the singly reinforced T-beams.

Additional terms involving $M_{u,lim}$, M_{u2}, A_{sc}, A_{st1} and A_{st2} are to be included from Eqs. 4.1 to 4.8 of sec. 4.8.3 of Lesson 8 depending on the particular case.

Applications of these terms are explained through the solutions of numerical problems of doubly reinforced T-beams in Lessons 11 and 12.

Case (i): When the neutral axis is in the flange ($x_u < D_f$), (Figs. 5.10.6 a to c)

Concrete below the neutral axis is in tension and is ignored. The steel reinforcement takes the tensile force (Fig. 5.10.6). Therefore, T and L-beams are considered as rectangular beams of width b_f and effective depth d. All the equations of singly and doubly reinforced rectangular beams derived in Lessons 4 to 5 and 8 respectively, are also applicable here.

Case (ii): When the neutral axis is in the web and the section is balanced ($x_{u,max} > D_f$), (Figs. 5.10.7 and 8 a to e)

(a) When D_f/d doesnotexceed 0.2, (Figs. 5.10.7 a toe)

As explained in sec. 5.10.3, the depth of the rectangular portion of the stress block (of constant stress = 0.446 f_{ck}) in this case is greater than D_f (Figs. 5.10.7 a, b and c). The section is split into two parts: (i) rectangular web of width b_w and effective depth d, and (ii) flange of width (b_f - b_w) and depth D_f (Figs.

d ande).

Total compressive force = Compressive force of rectangular beam of width $b_w$and depth d + Compressive force of rectangular flange of width (b_f- b_w) and depth D_f.

Thus, total compressive force

$$C=0.36\, f_{ck}b_w x_{u,max}+0.45f_{ck}(b_f- b_w)\, D_f \qquad (5.5)$$

(Assuming the constant stress of concrete in the flange as0.45 f_{ck}in place of 0.446 f_{ck},as per G-2.2 of IS 456), and the tensile force

$$T=0.87f_yA_{st} \qquad (5.6)$$

The lever arm of the rectangular beam (web part) is (d - 0.42 $x_{u,max}$) and the same for the flanged part is (d - 0.5 D_f).

So, the total moment = Moment due to rectangular web part + Moment due to rectangular flange part

$$\text{or} \quad M_u = 0.36f_{ck}\, b_w\, x_{u,max}(d - 0.42\, x_{u,max}) + 0.45f_{ck}(b_f - b_w)\, D_f(d - D_f/2)$$

$$\text{or} \quad M_u = 0.36(x_{u,max}/d)\{1-0.42(x_{u,max}/d)\}f_{ck}b_wd^2+0.45f_{ck}(b_f-b_w)D_f(d-D_f/2) \qquad (5.7)$$

Equation 5.7 is given in G-2.2 of IS 456.

(b) WhenD_f/d >0.2, (Figs. 5.10.8 a toe)

In this case, the depth of rectangular portion of stress block is within the flange (Figs. a, b and c). It is assumed that this depth of constantstress (0.45 f_{ck}) is y_f,where

$$y_f= 0.15\, x_{u,max}+0.65\, D_f, \text{ but notgreaterthan } D_f \qquad (5.8)$$

The above expressionofy_f is derived in sec.5.10.4.5.

As in the previous case (ii a), when D_f /d does not exceed 0.2, equations of C, T and M_u are obtained from Eqs. 5.5, 6 and 7 by changing $D_f$to y_f. Thus, we have (Figs. 5.10.8 d and e)

$T = 0.87 \quad f_y A_{st}$ (5.10)

The lever arm of the rectangular beam (web part) is (d - 0.42 $x_{u,max}$ same for the flange part is (d - 0.5 y_f). Accordingly, the expression of follows:

$$M_u = 0.36(x_{u,max}/d)\{1 - 0.42(x_{u,max}/d)\} f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w) y_f(d - y_f/2) \qquad (5.11)$$

Case (iii): When the neutral axis is in the web and the section is under-reinforced ($x_u > D_f$), (Figs. 5.10.9 and 10 a to e)

(a) When D_f/x_u doesnotexceed0.43, (Figs. 5.10.9 a toe)

Since D_f does not exceed 0.43 x_u and h (depth of fibre where the strain is 0.002) is at a depth of 0.43 x_u, the entire flange will be under a constant stress of 0.45 f_{ck} (Figs. 5.10.9 a, b and c). The equations of C, T and M_u can be written in the same manner as in sec. 5.10.4.2, case (ii a). The final forms of the equations are obtained from Eqs. 5.5, 6 and 7 by replacing $x_{u,max}$ by x_u. Thus, we have (Figs. 5.10.9 d and e)

$C = 0.36 f_{ck} \; b_w \; x_u + 0.45 \quad f_{ck}(b_f - b_w)D_f$ (5.12)

$T = 0.87 \quad f_y A_{st}$ (5.13)

$$M_u = 0.36(x_u/d)\{1 - 0.42(x_u/d)\} f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w) D_f(d - D_f/2) \qquad (5.14)$$

(b) When $D_f/x_u > \quad 0.43$, (Figs. 5.10.10 a toe)

Since $D_f > 0.43\, x_u$ and h (depth of fibre where the strain is 0.002) is at a depth of 0.43 x_u, the part of the flange having the constant stress of 0.45 f_{ck} is assumed as y_f (Fig. 5.10.10 a, b and c). The expressions of y_f, C, T and M_u can be written from Eqs. 5.8, 9, 10 and 11 of sec. 5.10.4.2, case (ii b), by replacing $x_{u,max}$ by x_u. Thus, we have (Fig. 5.10.10 d and e)

$y_f = 0.15 x_u + \quad 0.65 \; D_f$, but notgreaterthan D_f (5.15)

$C = 0.36 f_{ck} \; b_w \; x_u + 0.45 \quad f_{ck}(b_f - b_w) y_f$ (5.16)

$T = 0.87 \quad f_y A_{st}$ (5.17)

$$M_u = 0.36(x_u/d)\{1 - 0.42(x_u/d)\} f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w) y_f(d - y_f/2) \qquad (5.18)$$

Case (iv): When the neutral axis is in the web and the section is over-reinforced ($x_u > D_f$), (Figs. 5.10.7 and 8 a to e)

For the over-reinforced beam, the depth of neutral axis x_u is more than $x_{u,max}$ as in rectangular beams. However, x_u is restricted up to $x_{u,max}$. Therefore, the corresponding expressions of C, T and Mu for the two situations (a) when D_f/d does not exceed 0.2 and (b) when $D_f/d > 0.2$ are written from Eqs. 5.5 to 5.7 and 5.9 to 5.11, respectively of sec. 5.10.4.2 (Figs. 5.10.7 and 8). The expression of y_f for (b) is the same as that of Eq. 5.8.

(a) When D_f/d does not exceed 0.2 (Figs. 5.10.7 a to e)

The equations are:

$$C = 0.36 f_{ck} b_w x_{u,max} + 0.45 f_{ck}(b_f - b_w) D_f \quad (5.5)$$

$$T = 0.87 f_y A_{st} \quad (5.6)$$

$$M_u = 0.36(x_{u,max}/d)\{1 - 0.42(x_{u,max}/d)\} f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w) D_f(d - D_f/2) \quad (5.7)$$

(b) When $D_f/d > 0.2$ (Figs. 5.10.8 a to e)

$$Y_f = 0.15 x_{u,max} + 0.65 D_f, \text{ but not greater than } D_f \quad (5.8)$$

$$C = 0.36 f_{ck} b_w x_{u,max} + 0.45 f_{ck}(b_f - b_w) y_f \quad (5.9)$$

$$T = 0.87 f_y A_{st} \quad (5.10)$$

$$M_u = 0.36(x_{u,max}/d)\{1 - 0.42(x_{u,max}/d)\} f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w) y_f(d - y_f/2) \quad (5.11)$$

It is clear from the above that the over-reinforced beam will not have additional moment of resistance beyond that of the balanced one. Moreover, it will prevent steel failure. It is, therefore, recommended either to re-design or to go for doubly reinforced flanged beam than designing over-reinforced flanged beam.

Derivation of the equation to determine y_f, Eq. 5.8, Fig. 5.10.11

Whitney's stress block has been considered to derive Eq. 5.8. Figure 5.10.11 shows the two stress blocks of IS code and of Whitney.

y_f= Depth of constant portion of the stress block when $D_f/d > 0.2$. As y_f is a function of x_u and D_f and let us assume

$$y_f = A\, x_u + B\, D_f \qquad (5.19)$$

where A and B are to be determined from the following twoconditions:

Using the conditions of Eqs. 5.20 and 21 in Eq. 5.19, we get A = 0.15 and B = 0.65. Thus, we have

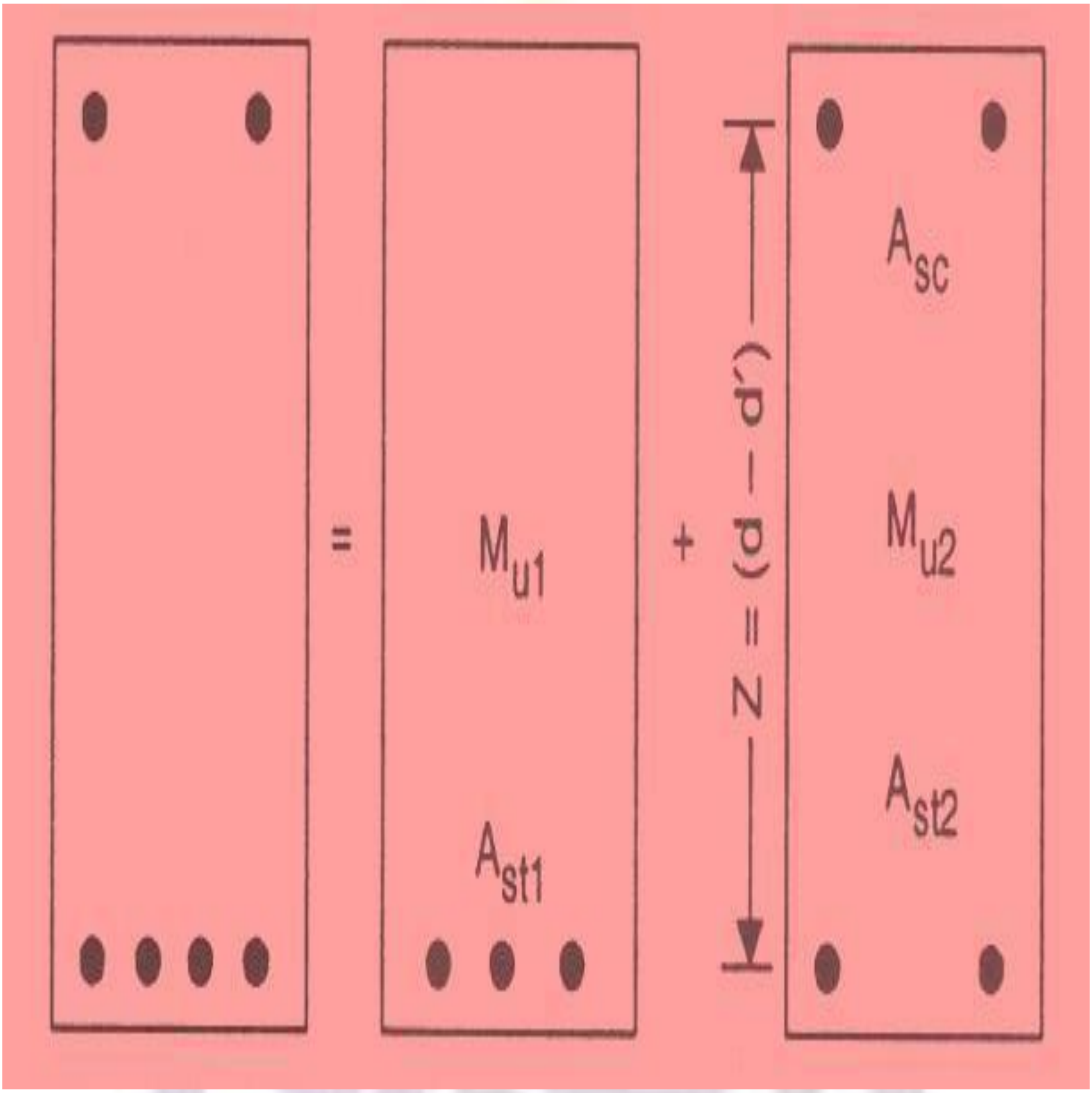
DESIGN OF DOUBLY REINFORCED BEAMS

Doubly Reinforced Beams

- **When beam depth is restricted and the moment the beam has to carry is greater than the moment capacity of the beam in concretefailure.**
- **When B.M at the section can changesign.**
- **When compression steel can substantially improve the ductility of beams and its use is therefore advisable in members when larger amount of tension steel becomes necessary for itsstrength.**
- **Compression steel is always used in structures in earthquake regions to increase theirductility.**
- **Compression reinforcement will also aid significantly in reducing the long-term deflections ofbeams.**

Doubly Reinforced Concrete Beam

Steel Beam Theory



(iv) A doubly reinforced concrete beam is reinforced in both compression and tension faces.

When depth of beam is restricted, strength available from a singly reinforced beam is inadequate.

At a support of a continuous beam, the bending moment changes sign, such a situation may also arise in design of a ringbeam.

2 Analysis of a doubly reinforced section involves determination of moment of resistance with given beam width, depth, area of tension and compression steels and theircovers.

3 In doubly reinforced concrete beams the compressive force consists of two parts; both in concrete and steel incompression.

4 Stress in steel at the limit state of collapse may be equal to yield stress or less depending on position of the neutralaxis.

Design Steps

Determine the limiting moment of resistance M_{um} for the given cross-section using the equation for a singly reinforced beam

$$M_{lim} = 0.87f_y.A_{st,1} [d - 0.42x_m] = 0.36\, f_{ck}.b.x_m [d - 0.42x_m]$$

(ii) If the factored moment M_u exceeds M_{lim}, a doubly reinforced section is required $(M_u - M_{lim}) = M_{u2}$

Additional area of tension steel A_{st2} is obtained by considering the equilibrium of force of compression in comp. steel and force of tension T_2in the additional tensionsteel

A_{sc} = compression steel.

σ_{cc} = Comp. stress in conc at the level of comp. steel = 0.446fck.

(iii) When beam section is shallow in depth, and the flexural strength obtained using balanced steel is insufficient i.e. the factored moment is more than the limiting ultimate moment of resistance of the beam section. Additional steel enhances the momentcapacity.

(iv) Steel bars in compression enhances ductility of beam at ultimatestrength.

(v) Compression steel reinforcement reduces deflection as moment of inertia of the beam section alsoincreases.

(vi) Long-term deflections of beam are reduced by compressionsteel.

(vii) Curvature due to shrinkage of concrete are alsoreduced.

(viii) Doubly reinforced beams are also used in reversal of externalload

Examples

(ii) A single reinforced rectangular beam is 400mm wide. The effective depth of the beam section is 560mm and its effective cover is 40mm. The steel reinforcement consists of 4 MS 18mm diameter bars in the beam section. The grade of concrete is M20. Locate the neutral axis of the beamsection.

(iii) In example 1, the bending moment at a transverse section of beam is 105 kN-m. Determine the strains at the extreme fibre of concrete in compression and steel bars provided as reinforcement in tension. Also determine the stress in steelbars.

(iv) In example 2, the strain in concrete at the extreme fibre in compression $\varepsilon_{cu}$is 0.00069 and the tensile stress in bending in steel is 199.55 N/mm^2. Determine the depth of neutral axis and the moment of resistance of the beamsection.

(v) Determine the moment of resistance of a section 300mm wide and 450mm deep up to the centre of reinforcement. If it is reinforced with (i) 4-12mm fe415 grade bars, (ii) 6-18mm fe415 gradebars.

(i) A rectangular beam section is 200mm wide and 400mm deep up to the centre of reinforcement. Determine the reinforcement required at the bottom if it has to resist a factored moment of 40kN-m. Use M20 grade concrete and fe415 gradesteel.

(ii) A rectangular beam section is 250mm wide and 500mm deep up to the centre of tension steel which consists of 4-22mm dia. bars. Find the position of the neutral axis, lever arm, forces of compression and tension and safe moment of resistance if concrete is M20 grade and steel is Fe500grade.

(iii) A rectangular beam is 200mm wide and 450 mm overall depth with an effective cover of 40mm. Find the reinforcement required if it has to resist a moment of 35 kN.m. Assume M20 concrete and Fe250 gradesteel.

Limit State of Serviceability

- explain the need to check for the limit state of serviceability after designing the structures by limit state ofcollapse,
- differentiate between short- and long-termdeflections,
- state the influencing factors to both short- and long-termdeflections,
- select the preliminary dimensions of structures to satisfy the requirements as per IS456,
- calculate the short- and long-term deflections of designedbeams.

Introduction

Structures designed by limit state of collapse are of comparatively smaller sections than those designed employing working stress method. They, therefore, must be checked for deflection and width of cracks. Excessive deflection of a structure or part thereof adversely affects the appearance and efficiency of the structure, finishes or partitions. Excessive cracking of concrete also seriously affects the appearance and durability of the structure. Accordingly, cl. 35.1.1 of IS 456 stipulates that the designer should consider all relevant limit states to ensure an adequate degree of safety and serviceability. Clause 35.3 of IS 456 refers to the limit state of serviceability comprising deflection in cl. 35.3.1 and cracking in cl. 35.3.2. Concrete is said to be durable when it performs satisfactorily in the working environment during its anticipated exposure conditions during service. Clause 8 of IS 456 refers to the durability aspects of concrete. Stability of the structure against overturning and sliding (cl. 20 ofIS

456), and fire resistance (cl. 21 of IS 456) are some of the other importance issues to be kept in mind while designing reinforced concretestructures.

This lesson discusses about the different aspects of deflection of beams and the requirements as per IS 456. In addition, lateral stability of beams is also taken up while selecting the preliminary dimensions of beams. Other requirements, however, are beyond the scope of this lesson.

Short- and Long-term Deflections

As evident from the names, short-term deflection refers to the immediate deflection after casting and application of partial or full service loads, while the long-term deflection occurs over a long period of time largely due toshrinkage and creep of the materials. The following factors influence the short-term deflection of structures:

(v) magnitude and distribution of liveloads,
(vi) span and type of endsupports,
(vii) cross-sectional area of themembers,
(viii) amount of steel reinforcement and the stress developed in thereinforcement,
(ix) characteristic strengths of concrete and steel,and
(x) amount and extent ofcracking.

The long-term deflection is almost two to three times of the short-term deflection. The following are the major factors influencing the long-term deflection of the structures.

humidity and temperature ranges duringcuring,

age of concrete at the time of loading,and

(c) type and size of aggregates, water-cement ratio, amount of compression reinforcement, size of members etc., which influence the creep and shrinkage of concrete.

Control of Deflection

Clause 23.2 of IS 456 stipulates the limiting deflections under two heads as given below:

5 The maximum final deflection should not normally exceed span/250 due to all loads including the effects of temperatures, creep and shrinkage and measured from the as-cast level of the supports of floors, roof and all other horizontalmembers.

6 The maximum deflection should not normally exceed the lesser of span/350 or 20 mm including the effects of temperature, creep and shrinkage occurring after erection of partitions and the application offinishes.

It is essential that both the requirements are to be fulfilled for every structure.

Selection of Preliminary Dimensions

The two requirements of the deflection are checked after designing the members. However, the structural design has to be revised if it fails to satisfy any one of the two or both the requirements. In order to avoid this, IS 456 recommends the guidelines to assume the initial dimensions of the members which will generally satisfy the deflection limits. Clause stipulates different span to effective depth ratios and cl. 23.3 recommends limiting slenderness ofbeams, a relation of b and d of the members, to ensure lateral stability. They are given below:

(A) For the deflectionrequirements

Different basic values of span to effective depth ratios for three different support conditions are prescribed for spans up to 10 m, which should be modified under any or all of the four different situations: (i) for spans above 10 m, (ii) depending on the amount and the stress of tension steel reinforcement, (iii) depending on the amount of compression reinforcement, and (iv) for flanged beams. These are furnished in Table 7.1.

(B) For lateralstability

The lateral stability of beams depends upon the slenderness ratio and the support conditions. Accordingly cl. 23.3 of IS code stipulates the following:

> For simply supported and continuous beams, the clear distance between the lateral restraints shall not exceed the lesser of 60b or $250b^2/d$, where d is the effective depth and b is the breadth of the compression face midway between the lateralrestraints.
>
> For cantilever beams, the clear distance from the free end of the cantilever tothe lateral restraint shall not exceed the lesser of 25b or$100b^2/d$.

Table7.1 Span/depth ratios and modificationfactors

Sl. No.	Items	Cantilever	Simply supported	Continuous
1	Basic values of span to effective depth ratio for spans up to 10 m	7	20	26
2	Modification factors for spans > 10 m	Not applicable as deflection calculations are to be done.	Multiply values of row 1 by 10/span in metres.	
3	Modification factors depending on area and stress of steel	Multiply values of row 1 or 2 with the modification factor from Fig.4 of IS 456.		
4	Modification factors depending as area of compression steel	Further multiply the earlier respective value with that obtained from Fig.5 of IS 456.		
5	Modification factors for flanged beams	(i)Modify values of row 1 or 2 as per Fig.6 of IS 456.		

		(ii)Further modify as per row 3 and/or 4 where reinforcement percentage to be used on area of section equal to b_f d.

Calculation of Short-Term Deflection

Clause C-2 of Annex C of IS 456 prescribes the steps of calculating the short-term deflection. The code recommends the usual methods for elastic deflections using the short-term modulus of elasticity of concrete E_c and effective moment of inertia I_{eff} given by the following equation:

$$I_{eff} = \frac{I_r}{1.2 - (M_r / M)(z / d)\left(1 - x / d\right)(b_w / b)} \text{; but } I_r \le I_{eff} \le I_{gr} \quad (7.1)$$

where I_r =moment of inertia of the cracked section,

M_r= cracking moment equal to (f_{cr} I_{gr})/y_t, where f_{cr} is the modulus of rupture of concrete, I_{gr} is the moment of inertia of the gross section about the centroidal axis neglecting the reinforcement, and y_t is the distance from centroidal axis of gross section, neglecting the reinforcement, to extreme fibre in tension,

M = maximum moment under service loads,

z = leverarm,

x = depth of neutral axis,

d = effective depth,

b_w= breadth of web, and

b = breadth of compression face.

For continuous beams, however, the values of I_r, I_{gr} and M_r are to be modified by the following equation:

$$X_e = k_1 \frac{X_1 + X_2}{2} + (1 - k_1) X_o$$

(7.2)

where X_e = modified value of X,

X_1, X_2 = values of X at the supports,

X_o = value of X at mid span,

k_1 = coefficient given in Table 25 of IS 456 and in Table 7.2 here, and

X = value of I_r, I_{gr} or M_r as appropriate.

Table 7.2 Values of coefficient k_1

k_1	0.5 or less	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
k_2	0	0.03	0.08	0.16	0.30	0.50	0.73	0.91	0.97	1.0

Note: k_2 is given by $(M_1 + M_2)/(M_{F1} + M_{F2})$, where M_1 and M_2 = support moments, and M_{F1} and M_{F2} = fixed end moments.

Deflection due to Shrinkage

Clause C-3 of Annex C of IS 456 prescribes the method of calculating the deflection due to shrinkage α_{cs} from the following equation:

$\alpha_{cs} = k_3 \psi_{cs} l^2$

(7.3)

where k_3 is a constant which is 0.5 for cantilevers, 0.125 for simply supported

members, 0.086 for members continuous at one end, and 0.063 for fully continuous members; ψ_{cs} is shrinkage curvature equal to $k_4\, \varepsilon_{cs}/D$ where ε_{cs} is the ultimate shrinkage strain of concrete. For ε_{cs}, cl. 6.2.4.1 of IS 456 recommends an approximate value of 0.0003 in the absence of test data.

)

$k_4 = 0.72(p_t - p_c)$ /

$= 0.65(p_t - p_c)$ /

(7.4

0,for 0.25≤ ≤1.0,for $p_t - p_c \geq 1.0$
≤1. $p_t - p_c < 1.0$

where $p_t = 100A_{st}/bd$ and $p_c = 100A_{sc}/bd$, D is the total depth of the section, and l is the length of span.

Deflection Due to Creep

Clause C-4 of Annex C of IS 456 stipulates the following method of calculating deflection due to creep. The creep deflection due to permanentloads

$\alpha_{cc(perm)}$ is obtained from the following equation:

$$\alpha_{cc(perm)} = \alpha_{1cc(perm)} - \alpha_{1(perm)} \tag{7.5}$$

where $\alpha_{1cc(perm)}$ =initial plus creep deflection due to permanent loads obtained usinganelasticanalysiswithaneffectivemodulusof elasticity,

$E_{ce} = E_c/(1 + \theta)$, θbeing the creep coefficient, and

$\alpha_{1(perm)}$ = short-term deflection due to permanent loads usingE_c.

(iii) NumericalProblems

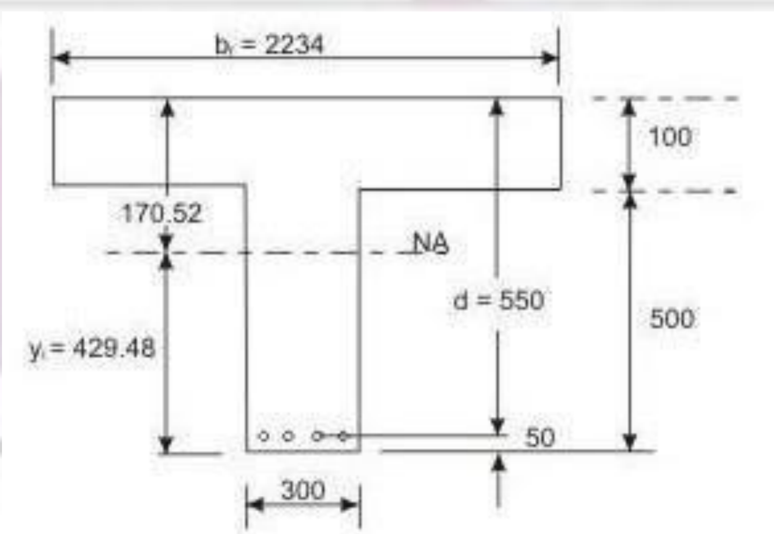


Fig. 7.17.1: Problem 1 (uncracked section)

$$+ 0.45 (20) (1000 - 300) (97.4) (450 - 97.4/2)$$

$$= 167.63+246.24 = 413.87 \text{ kNm}$$

It is seen that this over-reinforced beam has the same M_u as that of the balanced beam of Example 2.

Summary of Results of Examples 1-4

The results of four problems (Exs. 1-4) are given in Table 5.1 below. All the examples are having the common data except A_{st}.

Table5.1 Results of Examples 1-4 (Figs. 5.11.2 –5.11.5)

Ex. No.	Ast (mm^2)	Case	Section No.	M_u (kNm)	Remarks
1	1,963	(i)	5.10.4.1	290.06	x_u= 98.44 mm < $x_{u,max}$(= 216 mm), x_u < D_f(= 100 mm), Under-reinforced, (NA in the flange).
2	3,066	(ii b)	5.10.4.2 (b)	413.87	x_u=$x_{u,max}$= 216 mm, D_f/d = 0.222 > 0.2, Balanced, (NA in web).
3	2,591	(iii b)	5.10.4.3 (b)	369.18	x_u= 169.398 mm < $x_{u,max}$(= 216 mm), D_f /x_u= 0.59 > 0.43, Under-reinforced, (NA in the web).
4	4,825	(iv b)	5.10.4.4 (b)	413.87	x_u=241.95 mm > $x_{u,max}$(= 216 mm), D_f/d=0.222 > 0.2, Over-reinforced, (NA in web).

It is clear from the above table (Table 5.1), that Ex.4 is an over-reinforced flanged beam. The moment of resistance of this beam is the same as that of balanced beam of Ex.2. Additional reinforcement of 1,759 mm^2 (= 4,825 mm^2 – 3,066 mm^2) does not improve the M_u of the over- reinforced beam. It rather prevents the beam from tension failure. That is why over-reinforced beams are to be avoided. However, if the M_u has to be increased beyond 413.87 kNm, the flanged beam may be doubly reinforced.

Use of SP-16 for the Analysis Type of Problems

Using the two governing parameters (b_f /b_w) and (D_f /d), the $M_{u,lim}$ of balanced flanged beams can be determined from Tables 57-59 of SP-16 for the three grades of steel (250, 415 and 500). The value of the moment coefficient $M_{u,lim}/b_w d^2 f_{ck}$ of Ex.2, as obtained from SP-16, is presented in Table 5.2 making linear interpolation for both the parameters, wherever needed. $M_{u,lim}$ is then calculated from the moment coefficient.

Table 5.2 $M_{u,lim}$ of Example 2 using Table 58 of SP-16

Parameters:(i) b_f/b_w = 1000/300 =3.33

(ii) D_f/d = 100/450 = 0.222

$(M_{u,lim}/b_w d^2 f_{ck})$ in N/mm^2			
D_f/d	b_f/b_w		
	3	4	3.33
0.22	0.309	0.395	
0.23	0.314	0.402	
0.222	0.31*	0.3964*	0.339*

*by linear interpolation

$M_{u,lim}$ as obtained from SP-16 is close to the earlier computed value of $M_{u,lim}$ = 413.87 kNm(see Table5.1).

5.11.6 Practice Questions and Problems with Answers

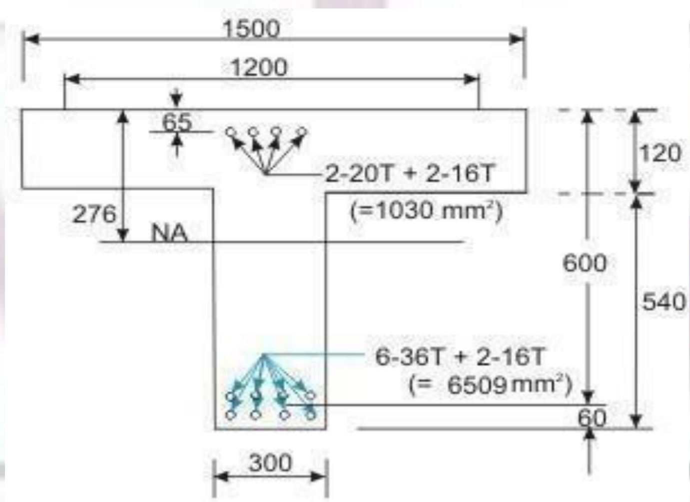


Fig. 5.11.6: Q. 1