4. LECTURE NOTES
STRUCTURAL ENGINEERING – I (RCC)

UNIT-I: DESIGN CONCEPTS

In the design and analysis of reinforced concrete members, you are presented with a problem unfamiliar to most of you: ―The mechanics of members consisting of two materials.‖ To compound this problem, one of the materials (concrete) behaves differently in tension than in compression, and may be considered to be either elastic or inelastic, if it is not neglected entirely.

Although we will encounter some peculiar aspects of behavior of concrete members, we will usually be close to a solution for most problems if we can apply the following three basic ideas:

- Geometry of deformation of sections will be consistent under given types of loading; i.e., moment will always cause strain to vary linearly with distance from neutral axis, etc.
- Mechanics of materials will allow us to relate stresses tostrains.
- Sections will be in equilibrium: external moments will be resisted by internal moment, external axial load will be equal to the sum of internal axial forces. (Many new engineers overly impressed speed and apparent accuracy of modern structural analysis computational procedures think less about equilibrium and details).

The overall goal is to be able to design reinforced concrete structures that are:

- Safe
- Economical
- Efficient

Reinforced concrete is one of the principal building materials used in engineered structures because:

- Low cost
- Weathering and fire resistance
- Good compressive strength
- Formability

Loads

Loads that act on structures can be divided into three general categories:

Dead Loads

Dead loads are those that are constant in magnitude and fixed in location throughout the

Life time of the structure such as: floor fill, finish floor, and plastered ceiling for buildings and wearing surface, sidewalks, and curbing for bridges

Live Loads

Live loads are those that are either fully or partially in place or not present at all, may also change in location; the minimum live loads for which the floors and roof of a building should be designed are usually specified in building code that governs at the site of construction (see Table1 - ―Minimum Design Loads for Buildings and Other Structure.‖)

Environmental Loads

Environmental Loads consist of wind, earthquake, and snow loads. such as wind, earthquake, and snow loads.

Serviceability

Serviceability requires that

- Deflections be adequately small;
- Cracks if any be kept to a tolerable limits;
- Vibrations be minimized

Safety

A structure must be safe against collapse; strength of the structure must be adequate for all Loads that might act on it. If we could build buildings as designed, and if the loads and their internal effects can be predicted accurately, we do not have to worry about safety. But there are uncertainties in:

- Actual loads;
- Forces/loads might be distributed in a manner different from what we assumed;
- The assumptions in analysis might not be exactly correct;
- Actual behavior might be different from that assumed;
- etc.

Finally, we would like to have the structure safe against

Concrete

Concrete is a product obtained artificially by hardening of the mixture of cement, sand, gravel and water in predetermined proportions.

Depending on the quality and proportions of the ingredients used in the mix the properties of concrete vary almost as widely as different kinds of stones.

Concrete has enough strength in compression, but has little strength in tension. Due to this, concrete is weak in bending, shear and torsion. Hence the use of plain concrete is limited applications where great compressive strength and weight are the principal requirements and where tensile stresses are either totally absent or are extremely low.

Properties of Concrete

The important properties of concrete, which govern the design of concrete mix are as follows

(i) Weight

The unit weights of plain concrete and reinforced concrete made with sand, gravel of crushed natural stone aggregate may be taken as 24 KN/m^3 and 25 KN/m^3 respectively.

(ii) Compressive Strength

With given properties of aggregate the compressive strength of concrete depends primarily on age, cement content and the water cement ratio are given Table 2 of IS 456:2000. Characteristic strength are based on the strength at 28 days. The strength at 7 days is about two-thirds of that at 28 days with ordinary portland cement and generally good indicator of strength likely to be obtained.

(iii) Increase in strength with age

There is normally gain of strength beyond 28 days. The quantum of increase depends upon the grade and type of cement curing and environmental conditions etc.

(iv) Tensile strength of concrete

The flexure and split tensile strengths of various concrete are given in IS 516:1959 and IS 5816:1970 respectively when the designer wishes to use an estimate of the tensile strength from compressive strength, the following formula can be used

Flexural strength, $f_{cr}=0.7\sqrt{f_{ck}}$ N/mm2

(v) Elastic Deformation

The modulus of elasticity is primarily influenced by the elastic properties of the aggregate and to lesser extent on the conditions of curing and age of the concrete, the mix proportions and the type of cement. The modulus of elasticity is normally related to the compressive characteristic strength of concrete

$E_c=5000\sqrt{f_{ck}}$ N/mm2

Where E_c= the short-term static modulus of elasticity in N/mm2

f_{ck}=characteristic cube strength of concrete in N/mm2

(vi) Shrinkage of concrete

Shrinkage is the time dependent deformation, generally compressive in nature. The constituents of concrete, size of the member and environmental conditions are the factors on which the total shrinkage of concrete depends. However, the total shrinkage of concrete is most influenced by the total amount of water present in the concrete at the time of mixing for a given humidity and temperature. The cement content, however, influences the total shrinkage of concrete to a lesser extent. The approximate value of the total shrinkage strain for design is taken as 0.0003 in the absence of test data (cl.6.2.4.1).

(vii) Creep of concrete

The effective modulus of Ece of concrete is used only in the calculation of creep deflection. It is seen that the value of creep coefficient θ is reducing with the age of concrete at loading. It may also be noted that the ultimate creep strain does not include short term strain.

- Properties of concrete
- Water/cement ratio
- Humidity and temperature of curing
- Humidity during the period of use
- Age of concrete at first loading
- Magnitude of stress and its duration
- Surface-volume ratio of the member

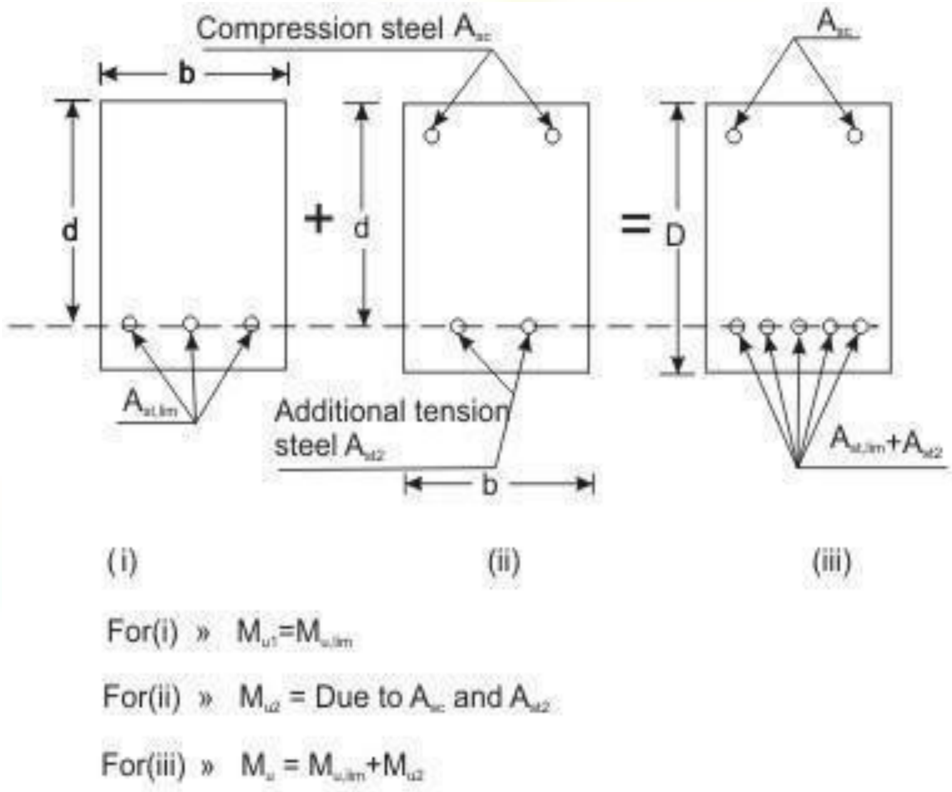

Fig. 4.8.1: Doubly reinforced beam

Concrete has very good compressive strength and almost negligible tensile strength. Hence, steel reinforcement is used on the tensile side of concrete. Thus, singly reinforced beams reinforced on the tensile face are good both in compression and tension. However, these beams have their respective limiting moments of resistance with specified width, depth and grades of concrete and steel. The amount of steel reinforcement needed is known as $A_{st,lim}$. Problem will arise, therefore, if such a section is subjected to bending moment greater than its limiting moment of resistance as a singly reinforced section.

There are two ways to solve the problem. First, we may increase the depth of the beam, which may not be feasible in many situations. In those cases, it is possible to increase both the compressive and tensile forces of the beam by providing steel reinforcement in compression face and additional reinforcement in tension face of the beam without increasing the depth (Fig. 4.8.1). The total compressive force of such beams comprises (i) force due to concrete in compression and (ii) force due to steel in compression. The tensile force also has two components: (i) the first provided by $A_{st,lim}$ which is equal to the compressive force of concrete in compression. The second part is due to the additional steel in tension - its force will be equal to the compressive force of steel in compression.

Such reinforced concrete beams having steel reinforcement both on tensile and compressive faces are known as doubly reinforced beams.

ly reinforced beams, therefore, have moment of resistance more than the singly reinforced beams of the same depth for particular grades of steel and concrete. In many practical situations, architectural or functional requirements may restrict the overall depth of the beams. However, other than in doubly reinforced beams compression steel reinforcement is provided when:

(i) some sections of a continuous beam with moving loads undergo change of sign of the bending moment which makes compression zone as tension zone or vice versa.

(ii) the ductility requirement has to be followed.

(iii) the reduction of long term deflection is needed.

It may be noted that even in so called singly reinforced beams there would be longitudinal hanger bars in compression zone for locating and fixing stirrups.

Assumptions

(i) The assumptions of sec. 3.4.2 of Lesson 4 are also applicable here.

(ii) Provision of compression steel ensures ductile failure and hence, the limitations of x/d ratios need not be strictly followed here.

(iii) The stress-strain relationship of steel in compression is the same as that in tension. So, the yield stress of steel in compression is 0.87fy.

Basic Principle

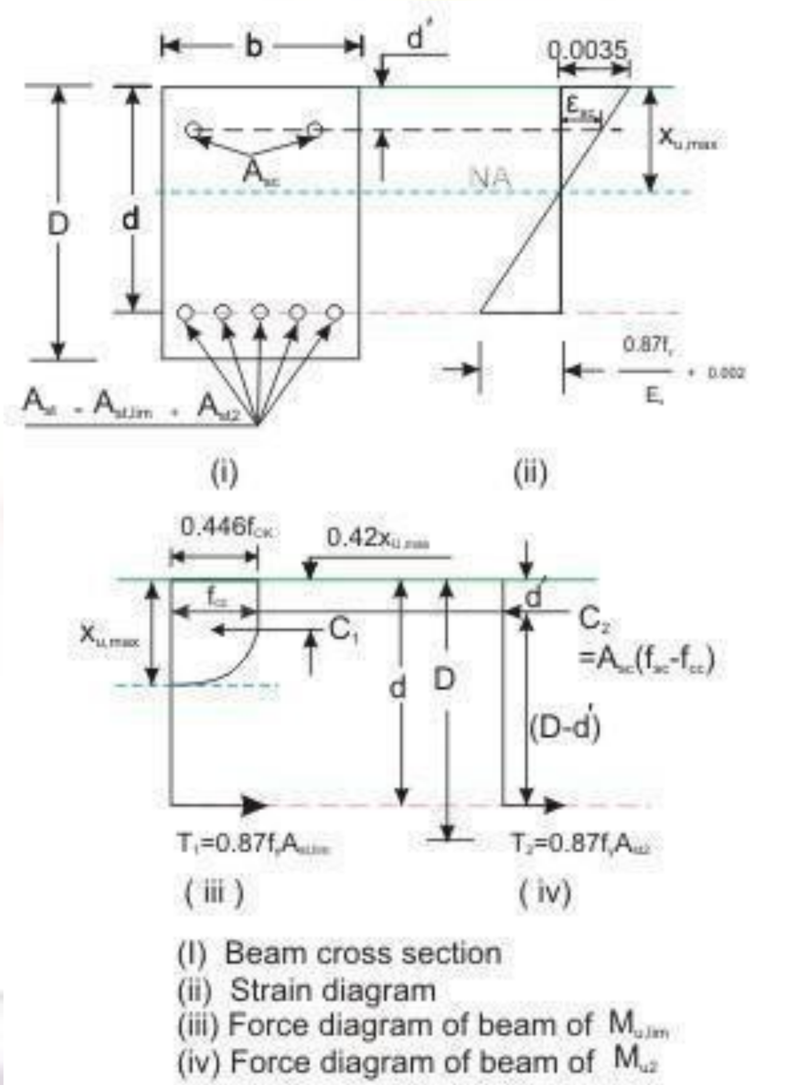

Fig. 4.8.2: Stress, strain and force diagrams of doubly reinforced beam

As mentioned in sec. 4.8.1, the moment of resistance M_u of the doubly reinforced beam consists of (i) $M_{u,lim}$ of singly reinforced beam and (ii) M_{u2} because of equal and opposite compression and tension forces ($C_2$and T_2) due to additional steel reinforcement on compression and tension faces of the beam

(Figs. 4.8.1 and 2). Thus, the moment of resistance $M_u$of a doubly reinforced beam is

$$M_u = M_{u,lim} + M_{u2} \quad (4.1)$$

The $M_{u,lim}$ is as given in Eq. 3.24 of Lesson 5, i.e.,

$$M_{u,lim} = 0.36 \left(\frac{x_{u,max}}{d}\right)\left(1 - 0.42\frac{x_{u,max}}{d}\right) b d^2 f_{ck} \quad (4.2)$$

Also, $M_{u,lim}$ can be written from Eq. 3.22 of Lesson 5, using $x_u = x_{u,max}$, i.e.,

$$M_{u,lim} = 0.87 A_{st,lim} f_y (d - 0.42 x_{u,max})$$

$$= 0.87 p_{t,lim}\left(1 - 0.42\frac{x_{u,max}}{d}\right) b d^2 f_y \quad (4.3)$$

The additional moment M_{u2} can be expressed in two ways (Fig. 4.8.2): considering (i) the compressive force C_2 due to compression steel and (ii) the tensile force T_2 due to additional steel on tension face. In both the equations, the lever arm is (d - d'). Thus, we have

$$M_{u2} = A_{sc}(f_{sc} - f_{cc})(d - d') \quad (4.4)$$

$$M_{u2} = A_{st2}(0.87 f_y)(d - d') \quad (4.5)$$

where A_{sc} = area of compression steel reinforcement

f_{sc} = stress in compression steel reinforcement

f_{cc} = compressive stress in concrete at the level of centroid of compression steel reinforcement

A_{st2} = area of additional steel reinforcement

Since the additional compressive force C_2 is equal to the additional tensile force T_2, we have

$$A_{sc}(f_{sc} - f_{cc}) = A_{st2}(0.87 f_y) \quad (4.6)$$

Any two of the three equations (Eqs. 4.4 - 4.6) can be employed to determine A_{sc} and A_{st2}.

The total tensile reinforcement A_{st} is then obtained from:

$$A_{st} = A_{st1} + A_{st2} \qquad (4.7)$$

$$\text{where } A_{st1} = \frac{p_{t,lim}\, b\, d}{100} = \frac{M_{u,lim}}{0.87 f_y (d - 0.42\, x_{u,max})} \qquad (4.8)$$

Determination of f_{sc} and f_{cc}

It is seen that the values of f_{sc} and f_{cc} should be known before calculating A_{sc}. The following procedure may be followed to determine the value of f_{sc} and f_{cc} for the design type of problems (and not for analysing a given section). For

the design problem the depth of the neutral axis may be taken as $x_{u,max}$ as shown in Fig. 4.8.2. From Fig. 4.8.2, the strain at the level of compression steel reinforcement ε_{sc} may be written as

$$\varepsilon_{sc} = 0.0035\left(1 - \frac{d'}{x_{u,max}}\right) \qquad (4.9)$$

The stress in compression steel f_{sc} is corresponding to the strain ε_{sc} of Eq. 4.9 and is determined for (a) mild steel and (b) cold worked bars Fe 415 and 500 as given below:

(a) Mild steel Fe250

The strain at the design yield stress of 217.39N/mm^2 (f_d = 0.87 f_y) is

0.0010869 (= 217.39/E_s). The f_{sc} is determined from the idealized stress-strain diagram of mild steel (Fig. 1.2.3 of Lesson 2 or Fig. 23B of IS 456) after computing the value of ε_{sc} from Eq. 4.9 as follows:

(i) If the computed value of $\varepsilon_{sc} \leq 0.0010869$, $f_{sc} = \varepsilon_{sc} E_s = 2\,(10^5)\varepsilon_{sc}$

(ii) If the computed value of $\varepsilon_{sc} > 0.0010869$, f_{sc} = 217.39N/mm^2.

(b) Cold worked bars Fe 415 and Fe500

The stress-strain diagram of these bars is given in Fig. 1.2.4 of Lesson 2 and in

Fig. 23A of IS 456. It shows that stress is proportional to strain up to a stress of 0.8 f_y. The stress-strain curve for the design purpose is obtained by substituting f_{yd} for f_y in the figure up to 0.8 f_{yd}. Thereafter, from 0.8 f_{yd} to f_{yd}, Table A of SP-16 gives the values of total strains and design stresses for Fe415

and Fe 500. Table 4.1 presents these values as a ready reference here.

Table4.1Valuesoff$_{sc}$ andε_{sc}

Stress level	Fe 415		Fe 500	
	Strain εsc	Stressf_{sc} (N/mm^2)	Strainε_{sc}	Stressf_{sc} (N/mm^2)
0.80 fyd	0.00144	288.7	0.00174	347.8
0.85 fyd	0.00163	306.7	0.00195	369.6
0.90 fyd	0.00192	324.8	0.00226	391.3
0.95 fyd	0.00241	342.8	0.00277	413.0
0.975 fyd	0.00276	351.8	0.00312	423.9
1.0 fyd	0.00380	360.9	0.00417	434.8

Linear interpolation may be done for intermediate values.

The above procedure has been much simplified for the cold worked bars by presenting the values of f_{sc} of compression steel in doubly reinforced beams for different values of d'/d only taking the practical aspects into consideration. In most of the doubly reinforced beams, d'/d has been found to be between 0.05 and 0.2. Accordingly, values of f_{sc} can be computed from Table 4.1 after determining the value of ε_{sc} from Eq. 4.9 for known values of d'/d as 0.05, 0.10, 0.15 and 0.2. Table F of SP-16 presents these values of f_{sc} for four values of d'/d (0.05, 0.10, 0.15 and 0.2) of Fe 415 and Fe 500. Table 4.2 below, however, includes Fe 250 also whose f_{sc} values are computed as laid down insec.

4.8.4(a) (i) and (ii) along with those of Fe 415 and Fe 500. This table is very useful and easy to determine the f_{sc} from the given value of d'/d. The table also includes strain values at yield which are explained below:

(i)The strain at yield of Fe 250=

$$= \frac{\text{Design Yield Stress}}{E_s} = \frac{250}{1.15\,(200000)} = 0.0010869$$

Here, there is only elastic component of the strain without any inelastic strain.

Table 4.2 Values of f_{sc} for different values of d'/d

f_y (N/mm^2)	d'/d				Strain at yield
	0.05	0.10	0.15	0.20	
250	217.4	217.4	217.4	217.4	0.0010869
415	355	353	342	329	0.0038043
500	412	412	395	370	0.0041739

Minimum and maximum steel

Minimum and maximum steel in compression

There is no stipulation in IS 456 regarding the minimum compression steel in doubly reinforced beams. However, hangers and other bars provided up to 0.2% of the whole area of cross section may be necessary for creep and shrinkage of concrete. Accordingly, these bars are not considered as compression reinforcement. From the practical aspects of consideration, therefore, the minimum steel as compression reinforcement should be at least 0.4% of the area of concrete in compression or 0.2% of the whole cross -sectional area of the beam so that the doubly reinforced beam can take care of the extra loads in addition to resisting the effects of creep and shrinkage ofconcrete.

The maximum compression steel shall not exceed 4 per cent of the whole area of cross- section of the beam as given in cl. 26.5.1.2 of IS 456.

Minimum and maximum steel in tension

As stipulated in cl. 26.5.1.1(a) and (b) of IS 456, the minimum amount of tensile reinforcement shall be at least ($0.85\ bd/f_y$) and the maximum area of tension reinforcement shall not exceed (0.04 bD).

It has been discussed in sec. 3.6.2.3 of Lesson 6 that the singly reinforced

beams shall have A_{st}normally not exceeding 75 to 80% of $A_{st,lim}$so that $x_u$remains less than $x_{u,max}$ with a view to ensuring ductile failure. However, in the

case of doubly reinforced beams, the ductile failure is ensured with the presence of compression steel. Thus, the depth of the neutral axis may be taken as $x_{u,max}$if the beam is over-reinforced.

Accordingly, the A_{st1} part of tension steel can go

up to $A_{st,lim}$ and the additional tension steel A_{st2} is provided for the additional moment M_u - $M_{u,lim}$. The quantities of A_{st1} and A_{st2} together form the total A_{st}, which shall not exceed 0.04 bD.

Types of problems and steps of solution

Similar to the singly reinforced beams, the doubly reinforced beams have two types of problems: (i) design type and (ii) analysis type. The different steps of solutions of these problems are taken up separately.

Design type of problems

In the design type of problems, the given data are b, d, D, grades of concrete and steel. The designer has to determine A_{sc} and A_{st} of the beam from the given factored moment. These problems can be solved by two ways: (i) use of the equations developed for the doubly reinforced beams, named here as direct computation method, (ii) use of charts and tables of SP-16.

(a) Direct computationmethod

Step 1: To determine $M_{u,lim}$ and $A_{st,lim}$ from Eqs. 4.2 and 4.8, respectively.

Step2: To determine M_{u2}, A_{sc}, A_{st2} and A_{st} from Eqs. 4.1, 4.4, 4.6 and 4.7, respectively.

Step 3: To check for minimum and maximum reinforcement in compression and tension as explained in sec. 4.8.5.

Step4: To select the number and diameter of bars from known valuesof A_{sc} and A_{st}.

(b) Use of SPtable

Tables 45 to 56 present the p_t and p_c of doubly reinforced sections for d'/d = 0.05, 0.10, 0.15 and 0.2 for different f_{ck} and f_y values against M_u/bd^2. The values of p_t and p_c are obtained directly selecting the proper table with known values of M_u/bd^2 and d'/d.

Analysis type of problems

In the analysis type of problems, the data given are b, d, d', D, f_{ck}, f_y, A_{sc} and A_{st}. It is required to determine the moment of resistance M_u of such beams.

These problems can be solved: (i) by direct computation method and (ii) by using tables of SP-16.

As mentioned earlier Tables 45 to 56 are needed for the doubly reinforced beams. First, the needed parameters d'/d, p_t and p_c are calculated. Thereafter, M_u/bd^2 is computed in two stages: first, using d'/d and p_t and then using d'/d and p_c. The lower value of M_u is the moment of resistance of the beam.