UNIT - V

INFLUENCE LINES FOR INDETERMINATE BEAMS

Influence lines are important in thedesign of structures that resist large live loads.shear and moment diagrams are important in determining the maximum internal force in a structure.If a structure is subjected to a ***live*** or ***moving load***, the variation in shear andmoment is best described using ***influencelines***.

An ***influence line*** represents the variation of the reaction, shear, moment, or deflection at a ***specific point*** in a member as a concentrated force moves over the member.

Once the ***influence line*** is drawn, the location of the live load which will cause the greatest influence on the structure can be found very quickly.Therefore, ***influence lines*** are important in the design of a structure where the loads move along the span (bridges, cranes, conveyors, etc.).

Although the procedure for constructing an ***influence line*** is rather simple, it is importantto remember the difference between constructing an influence line and constructing a shear or moment diagram.***Influence lines*** represent the effect of amoving load **only at a specified point** on a member, whereas shear and moment diagrams represent the effect of fixed loads at **allpoints**along the member.

Procedure for determining the***influence line*** at a point**P**foranyfunction (reaction, shear, or moment).

1. Place a unit load (a load whose magnitude is equal to one) at a point, *x*, along the member.
2. Use the equations of equilibrium to find the value of the function (reaction, shear, or moment) at a specific point **P** due the concentrated load at*x*.
3. Repeat steps 1 and 2 for various values of *x* over the wholebeam.
4. Plot the values of the reaction, shear, or moment for themember.

Construction of Influence Lines using Equilibrium Methods

The most basic method of obtaining influence line for a specific response parameter is to solve the static equilibrium equations for various locations of the unit load. The general procedure for constructing an influence line is described below.

1. Define the positive direction of the response parameter under consideration through a free body diagram of the wholesystem.

2..For a particular location of the unit load, solve for the equilibrium of the whole system and if required, as in the case of an internal force, also for a part of the member to obtain the response parameter for that location of the unit load.This gives the ordinate of the influence line at that particular location of theload.

3. Repeat this process for as many locations of the unit load as required to determine the shape of the influence line for the whole length of the member. It is often helpful if we can consider a generic location (or several locations) *x* of the unitload.

4. Joining ordinates for different locations of the unit load throughout the length of the member,we get the influence line for that particular responseparameter.

The following three examples show how to construct influence lines for a support reaction, a shear force and a bending moment for the simply supported beam *AB* .

Example .1 Draw the influence line for R_A (vertical reaction at *A*) of beam *AB* in Fig. E6.1.

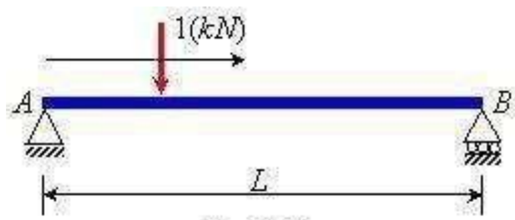
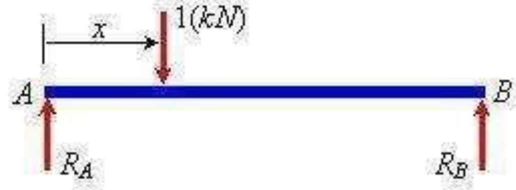


Fig. E6.1

Solution:

Free body diagram of *AB* :

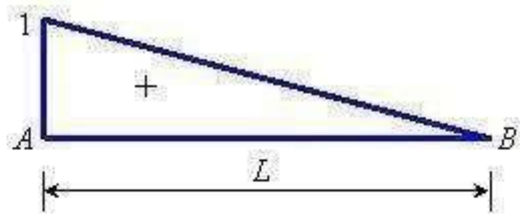


$$\sum F_y = 0 \Rightarrow R_A = 1 - R_B$$

$$\sum M(about B) = 0 \Rightarrow R_A(L) = 1(L - x)$$

$$\Rightarrow R_A = 1 - \frac{x}{L}$$

So the influencelineof R_A:



Example 6.2 Draw the influence line for V_C(shear force at mid point) ofbeam *AB* in Fig. E6.2.

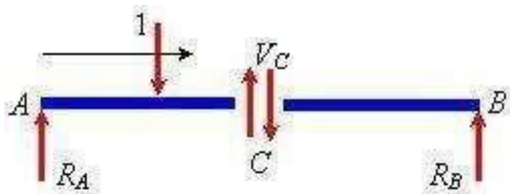
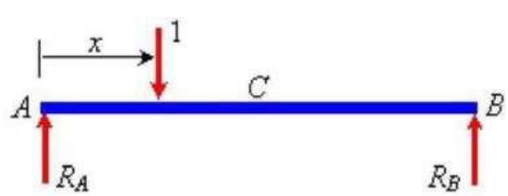


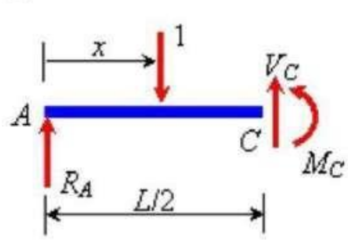
Fig. E6.2

Solution:



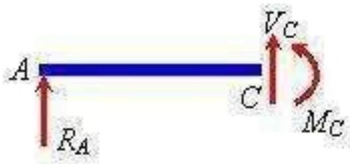
$$\sum M(about\, B) = 0 \Rightarrow R_A = 1 - \frac{x}{L}$$

$$x < \frac{L}{2}$$



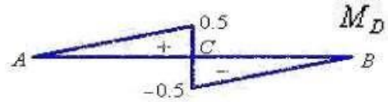
$$\sum F_y = 0 \Rightarrow V_C = 1 - R_A = \frac{x}{L}$$

For $x > \frac{L}{2}$



$$\sum F_y = 0 \Rightarrow V_C = -R_A = \frac{x}{L} - 1$$

So the influence line for V_C



Example 6.3 Draw the influencelinefor M_D (bendingmomentat $x = 2L/3$) for beam *AB* in Fig.E6.3.

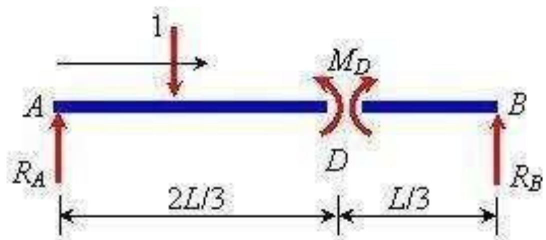
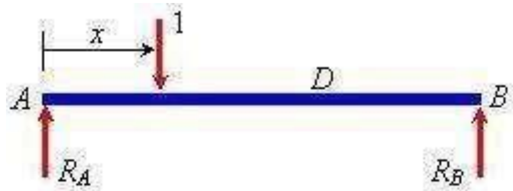
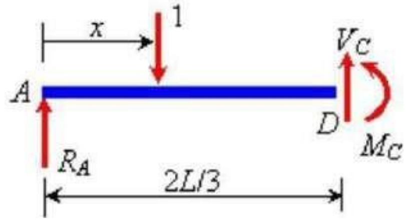


Fig. E6.3

Solution:



$$\sum M(about\ B) = 0 \Rightarrow R_A = 1 - \frac{x}{L}$$

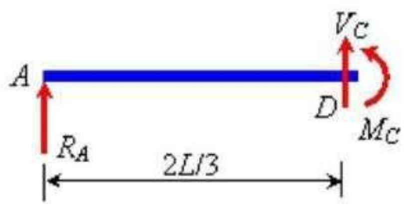


For $x < \frac{2L}{3}$

$$\sum M(about\ D) = 0$$

$$\Rightarrow M_D = R_A\left(\frac{2L}{3}\right) - 1\left(\frac{2L}{3} - x\right) = \left(1 - \frac{x}{L}\right)\left(\frac{2L}{3}\right) - \frac{2L}{3} + x$$

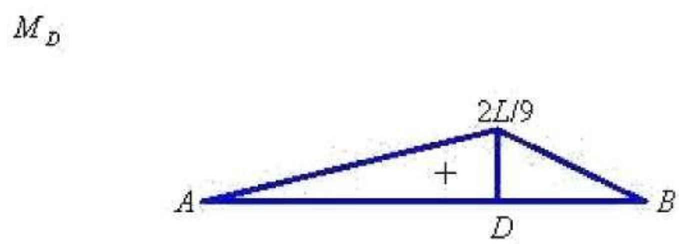
$$= -\frac{2x}{3} + x = \frac{x}{3}$$



For $x > \frac{2L}{3}$

$$\sum M(about\,D) = 0 \Rightarrow M_D = R_A\left(\frac{2L}{3}\right) = \frac{2L}{3} - \frac{2x}{3}$$

So, the influenceof M_D :



MÜLLER BRESLAU PRINCIPLE FOR QUALITATIVE INFLUENCE LINES

In 1886, Heinrich Müller Breslau proposed a technique to draw influence lines quickly. The Müller Breslau Principle states that the ordinate value of an influence line for any function on any structure is proportional to the ordinates of the deflected shape that is obtained by removing the restraint corresponding to the function from the structure and introducing a force that causes a unit displacement in the positive direction.

Let us say, our objective is to obtain the influence line for the support reaction at A for the beam shown in Figure 38.1.

Simply supported beam

First of all remove the support corresponding to the reaction and apply a force (Figure 38.2) in the positive direction that will cause a unit displacement in the direction of R_A. The resulting deflected shape will be proportional to the true influence line (Figure 38.3) for the support reaction at A.

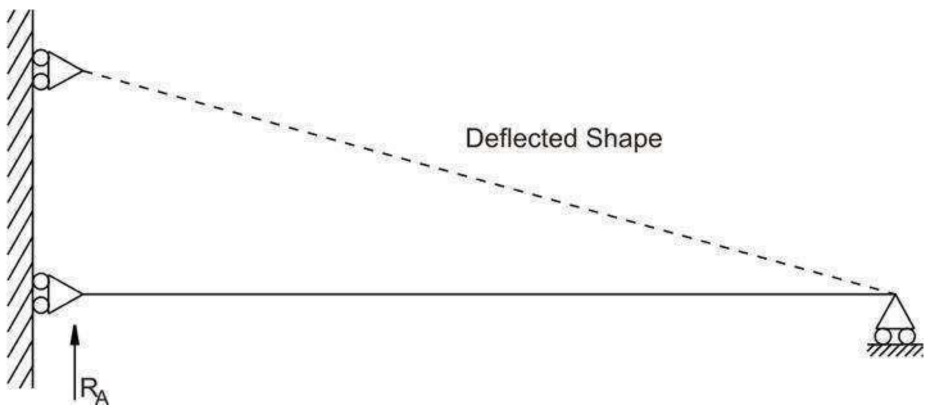


Figure 38.2: Deflected shape of beam

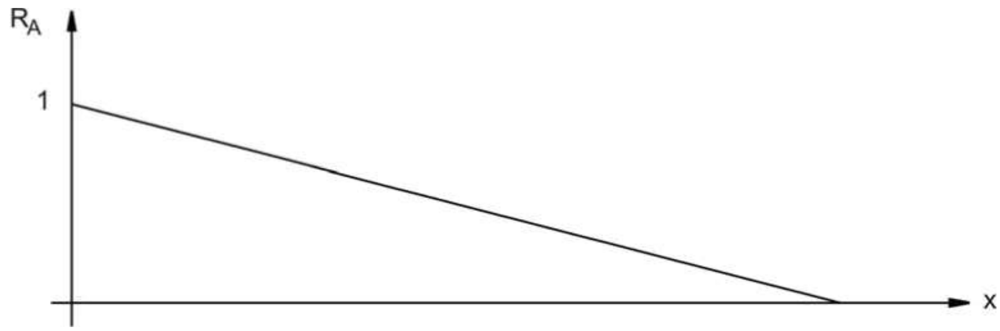


Figure 38.3: Influence line for support reaction A

The deflected shape due to a unit displacement at A is shown in Figure 38.2 and matches with the actual influence line shape as shown in Figure 38.3. Note that the deflected shape is linear, i.e., the beam rotates as a rigid body without any curvature. This is true only for statically determinate systems.

Similarly some other examples are given below.

Here we are interested to draw the qualitative influence line for shear at section

C of overhang beam as shown in Figure 38.4.

Figure 38.4: Overhang beam

As discussed earlier, introduce a roller at section C so that it gives freedom to the beam in vertical direction as shown in Figure 38.5.

Now apply a force in the positive direction that will cause a unit displacement in the direction of V_C. The resultant deflected shape is shown in Figure 38.5. Again, note that the deflected shape is linear. Figure 38.6 shows the actual influence, which matches with the qualitative influence.

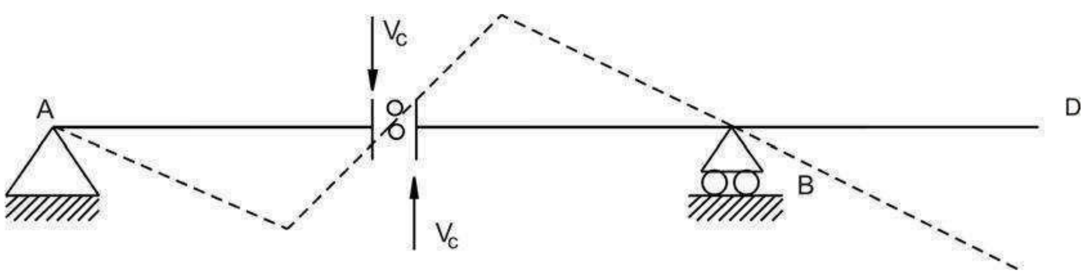


Figure 38.5: Deflected shape of beam

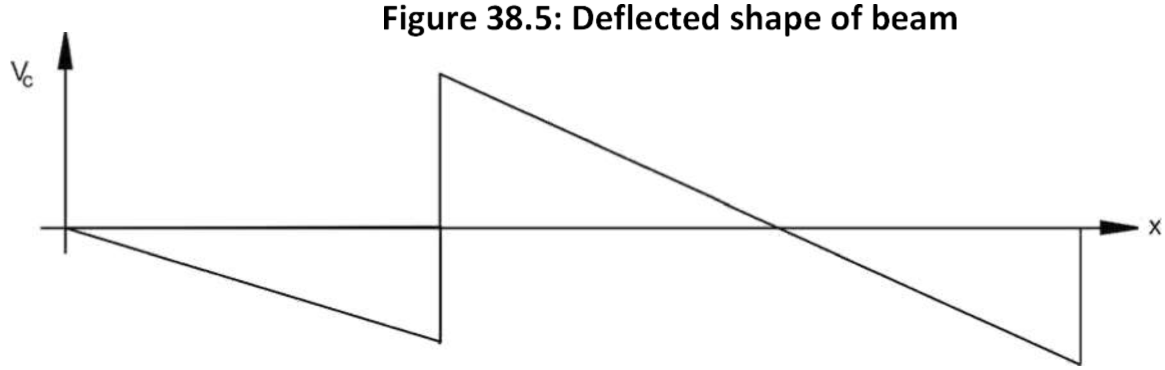


Figure 38.6: Influence line for shear at section C

In this second example, we are interested to draw a qualitative influence line for moment at C for the beam as shown in Figure 38.7.

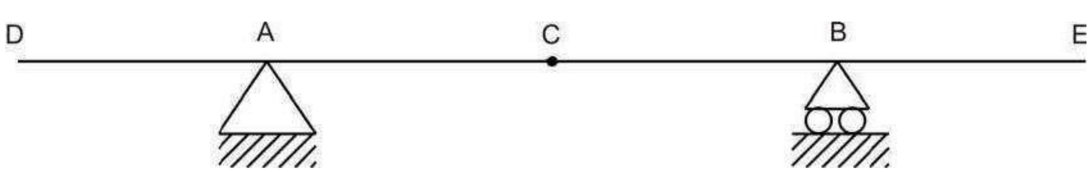


Figure 38.7: Beam structure

In this example, being our objective to construct influence line for moment, we will introduce hinge at C and that will only permit rotation at C. Now apply moment in the positive direction that will cause a unit rotation in the direction of

M_c. The deflected shape due to a unit rotation at C is shown in Figure 38.8 and matches with the actual shape of the influence line as shown in Figure 38.9.

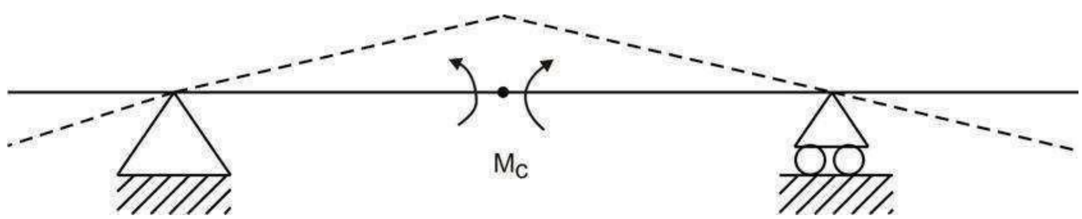


Figure 38.8: Deflected shape of beam

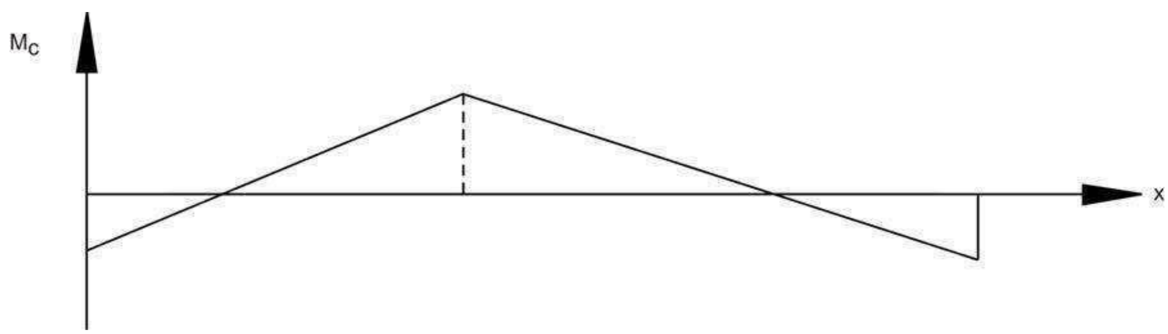


Figure 38.9: Influence line for moment at section C

Maximum shear in beam supporting UDLs

If UDL is rolling on the beam from one end to other end then there are two possibilities. Either Uniformly distributed load is longer than the span or uniformly distributed load is shorter than the span. Depending upon the length of the load and span, the maximum shear in beam supporting UDL will change. Following section will discuss about these two cases. It should be noted that for maximum values of shear, maximum areas should be loaded.

UDL LONGER THAN THE SPAN

Let us assume that the simply supported beam as shown in Figure 38.10 is loaded with UDL of w moving from left to right where the length of the load is longer than the span. The influence lines for reactions R_A, R_B and shear at section C located at x from support A will be as shown in Figure 38.11, 38.12 and 38.13 respectively. UDL of intensity w per unit for the shear at supports A and B will be given by

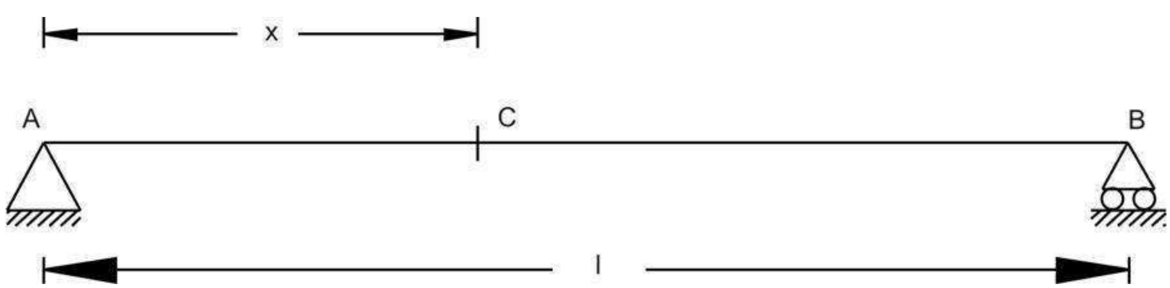


Figure 38.10: Beam Structure

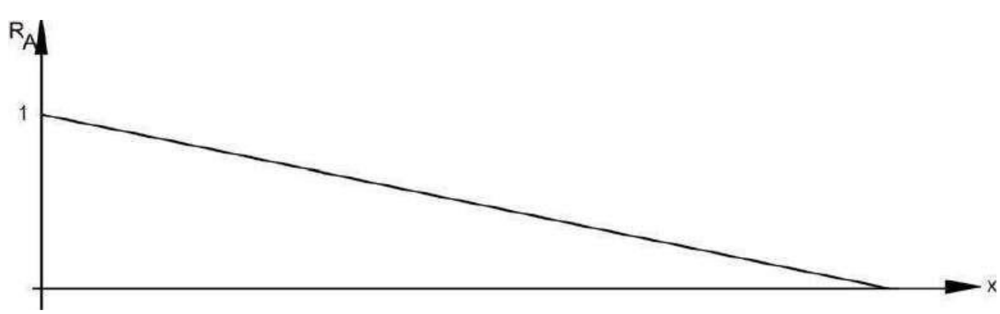


Figure 38.11: Influence line for support reaction at A

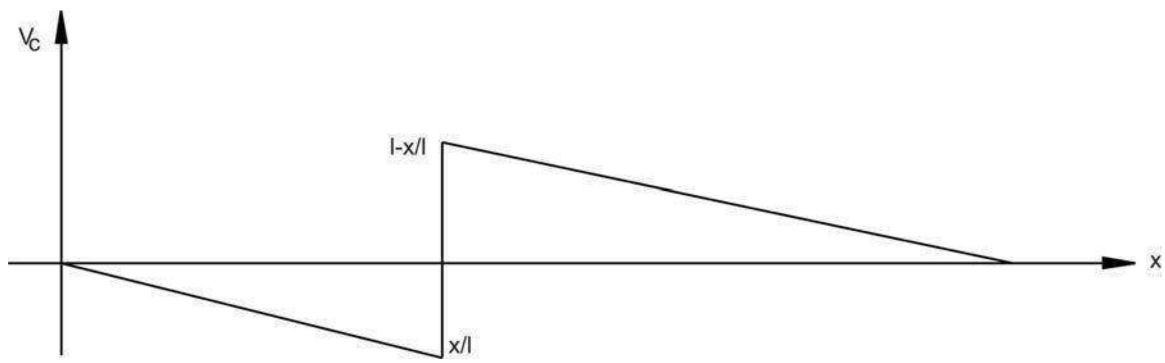


Figure 38.13: Influence line for shear at section C

$$R_A = w \times \frac{1}{2} \times l \times 1 = \frac{wl}{2}$$

$$R_B = -w \times \frac{1}{2} \times l \times 1 = \frac{-wl}{2}$$

Suppose we are interested to know shear at given section at C. As shown in Figure 38.13, maximum negative shear can be achieved when the head of the load is at the section C. And maximum positive shear can be obtained when the tail of the load is at the section C. As discussed earlier the shear force is computed by intensity of the load multiplied by the area of influence line diagram covered by load. Hence, maximum negative shear is given by $= -\frac{1}{2} \times x \times \frac{x}{l} \times w = -\frac{wx^2}{2l}$

and maximum positive shear is given by

$$= \frac{1}{2} \times \left(\frac{l-x}{l}\right) \times (l-x) \times w = -\frac{w(l-x)^2}{2l}$$

UDL SHORTER THAN THE SPAN

When the length of UDL is shorter than the span, then as discussed earlier, maximum negative shear can be achieved when the head of the load is at the section. And maximum positive shear can be obtained when the tail of the load is at the section. As discussed earlier the shear force is computed by the load intensity multiplied by the area of influence line diagram covered by load. The example is demonstrated in previous lesson.

Maximum bending moment at sections in beams supporting UDLs.

Like the previous section discussion, the maximum moment at sections in beam supporting UDLs can either be due to UDL longer than the span or due to ULD shorter than the span. Following paragraph will explain about computation of moment in these twocases.

UDL longer than the span

Let us assume the UDL longer than the span is traveling from left end to right hand for the beam as shown in Figure 38.14. We are interested to know maximum moment at C located at x from the support A. As discussed earlier, the maximum bending moment is given by the load intensity multiplied by the area of influence line (Figure 38.15) covered. In the present case the load will cover the completed span and hence the moment at section C can be givenby

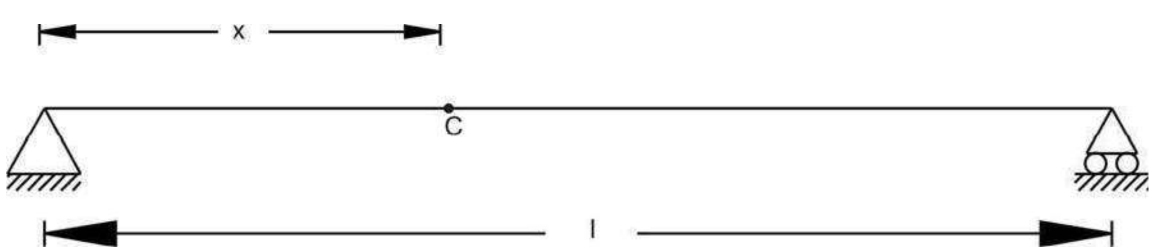


Figure 38.14: Beam structure

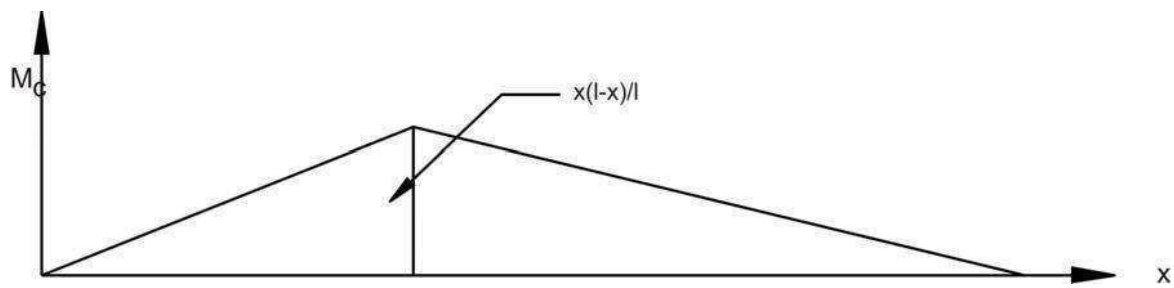


Figure 38.15: Influence line for moment at section C

$$w\times\frac{1}{2}\times l\times\frac{x(l-x)}{l}=-\frac{wx(l-x)}{2}$$

Suppose the section C is at mid span, then maximum moment is given by

$$\frac{w\times\frac{l}{2}\times\frac{l}{2}}{2}=\frac{wl^2}{8}$$

UDL shorter than the span

As shown in Figure 38.16, let us assume that the UDL length y is smaller than the span of the beam AB. We are interested to find maximum bending moment at section C located at x from support A. Let say that the mid point of UDL is located at D as shown in Figure 38.16 at distance of zfrom support A. Take moment with reference to A and it will bezero.

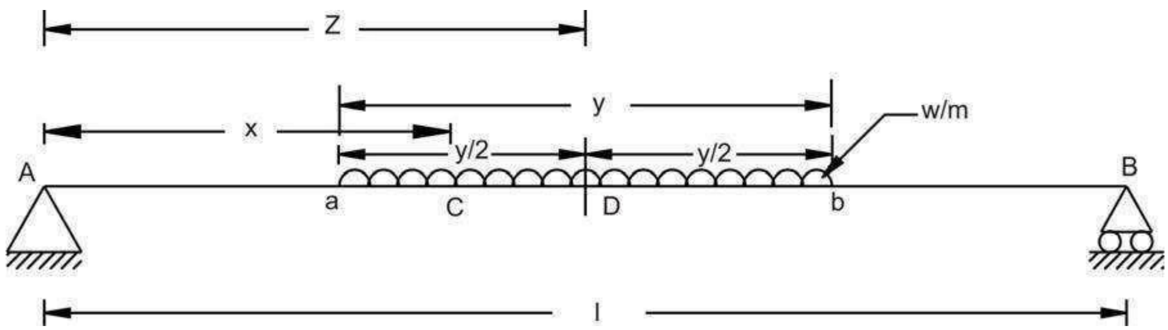


Figure 38.16: Beam loaded with UDL shorter in length than span

Hence, the reaction at B is given by

$$R_B = w \times y \times \frac{z}{l} = -\frac{wx(l-x)}{2}$$

And moment at C will be

$$M_c = R_B(l-x) - \frac{w}{2}(z + \frac{y}{2} - x)^2$$

Substituting value of reaction B in above equation, we can obtain

$$M_c = \frac{wyz}{l}(l-x) - \frac{w}{2}(z + \frac{y}{2} - x)^2$$

To compute maximum value of moment at C, we need to differentiate above given equation with reference to z and equal to zero.

$$\frac{dM_c}{dz} = \frac{wy}{l}(l-x) - w(z + \frac{y}{2} - x) = 0$$

Therefore,

$$\frac{y}{l}(l-x) = (z + \frac{y}{2} - x)$$

Using geometric expression, we can state that

$$\frac{ab}{AB} = \frac{Cb}{CB}$$

$$\therefore \frac{CB}{Cb} = \frac{AB}{ab} = \frac{AB - CB}{ab - Cb} = \frac{AC}{aC}$$

$$\therefore \frac{aC}{Cb} = \frac{AC}{CB}$$

The expression states that for the UDL shorter than span, the load should be placed in a way so that the section divides it in the same proportion as it divides the span. In that case, the moment calculated at the section will give maximum momentvalue.