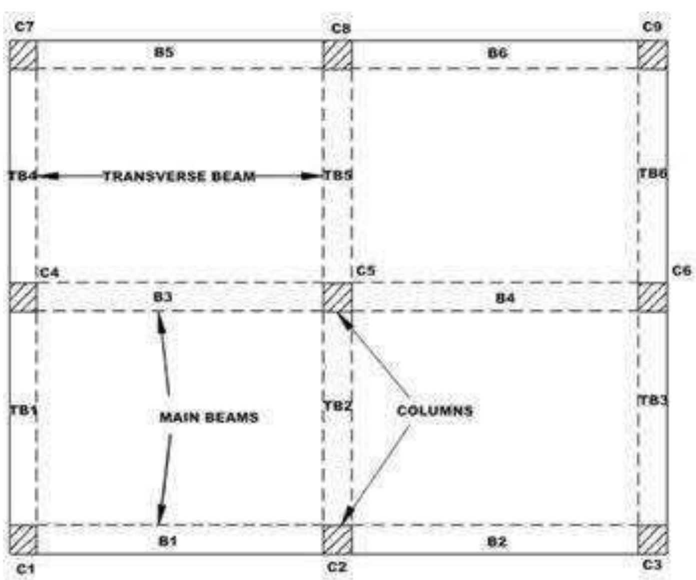
UNIT - III

APPROXIMATE METHODS OFANALYSIS OF BUILDING FRAMES

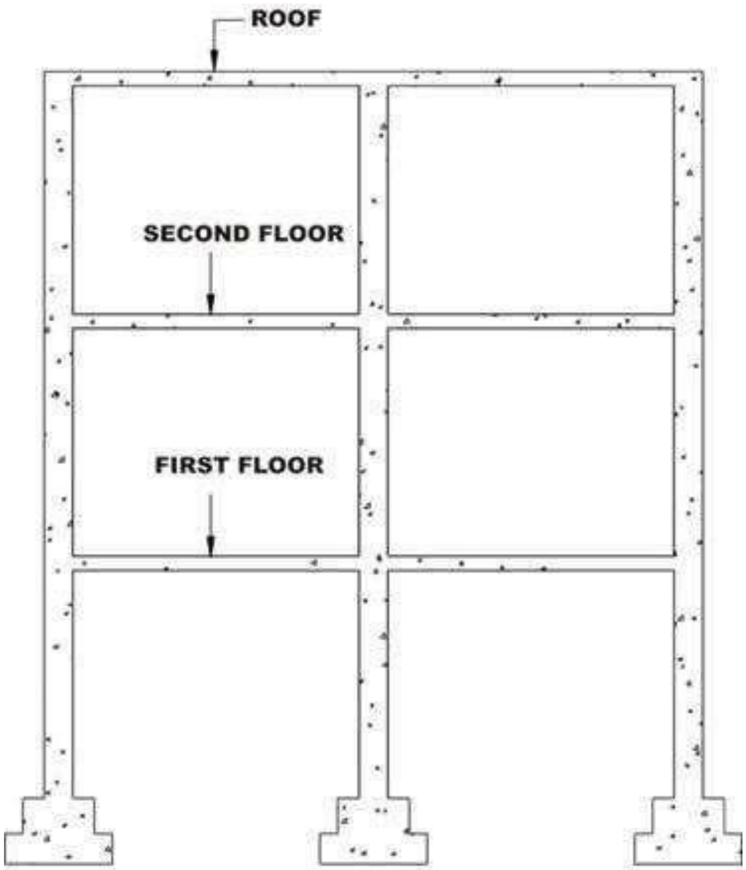
INTRODUCTION

The building frames are the most common structural form, an analyst/engineer encounters in practice. Usually the building frames are designed such that the beam column joints are rigid. A typical example of building frame is the reinforced concrete multistory frames. A two-bay, three-storey building plan and sectional elevation are shown in Fig. In principle this is a three dimensional frame.

However, analysis may be carried out by considering planar frame in two perpendicular directions separately for both vertical and horizontal loads as shown in Fig. 36.2 and finally superimposing moments appropriately. In the case of building frames, the beam column joints are monolithic and can resist bending moment, shear force and axial force. Any exact method, such as slope-deflection method, moment distribution method or direct stiffness method may be used to analyse this rigid frame. However, in order to estimate the preliminary size of different members, approximate methods are used to obtain approximate design values of moments, shear and axial forces in various members. Before applying approximate methods, it is necessary to reduce the given indeterminate structure to a determinate structure by suitable assumptions. These will be discussed in this lesson. In next section, analysis of building frames to vertical loads is discussed and in section after that, analysis of building frame to horizontal loads will be discussed.



Plan



Sectional Elevation Along C_1 - C_3

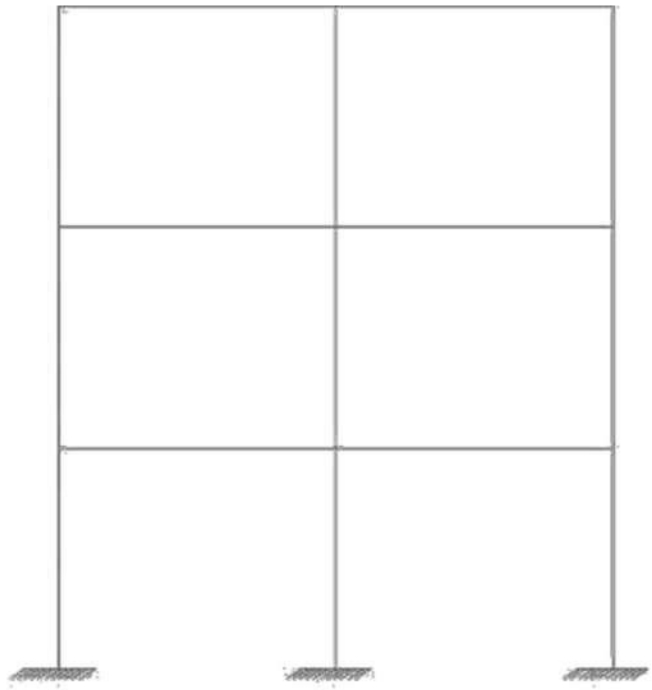


Fig.36.2 Idealized frame for analysis

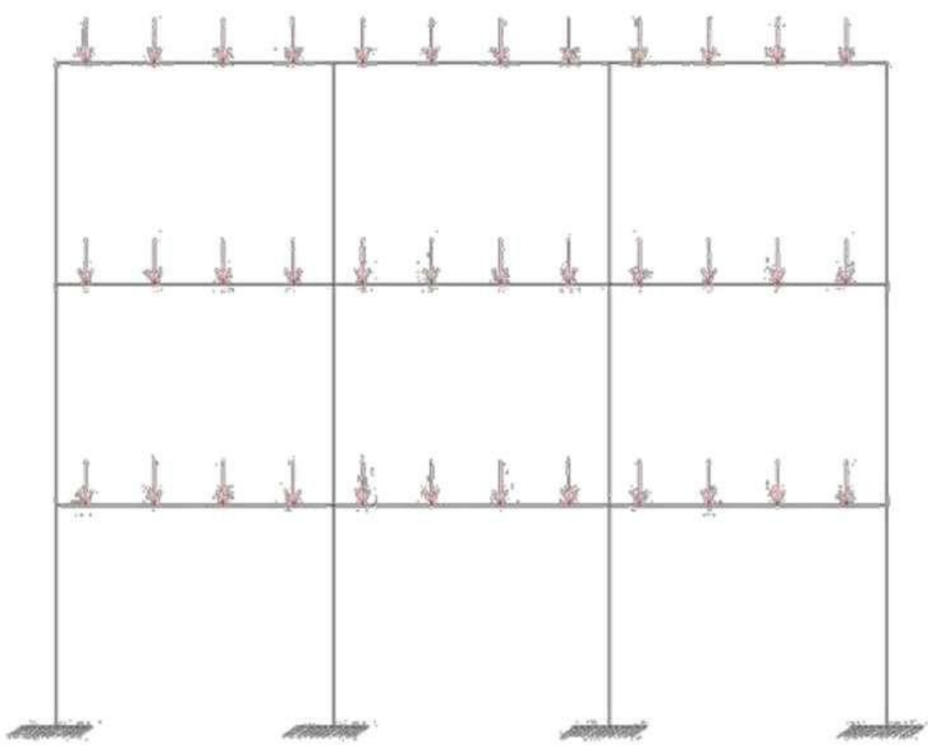


Fig.36.3 Building frame subjected to vertical loads

SUBSTITUTE FRAME METHOD

Consider a building frame subjected to vertical loads as shown in Fig.36.3. Any typical beam, in this building frame is subjected to axial force, bending moment and shear force. Hence each beam is statically indeterminate to third degree and hence 3 assumptions are required to reduce this beam to determinate beam.

Before we discuss the required three assumptions consider a simply supported beam. In this case zero moment (or point of inflexion) occurs at the supports as shown in Fig.36.4a. Next consider a fixed-fixed beam, subjected to vertical loads as shown in Fig. 36.4b. In this case, the point of inflexion or point of zero moment occurs at $0.21L$ from both ends of the support.

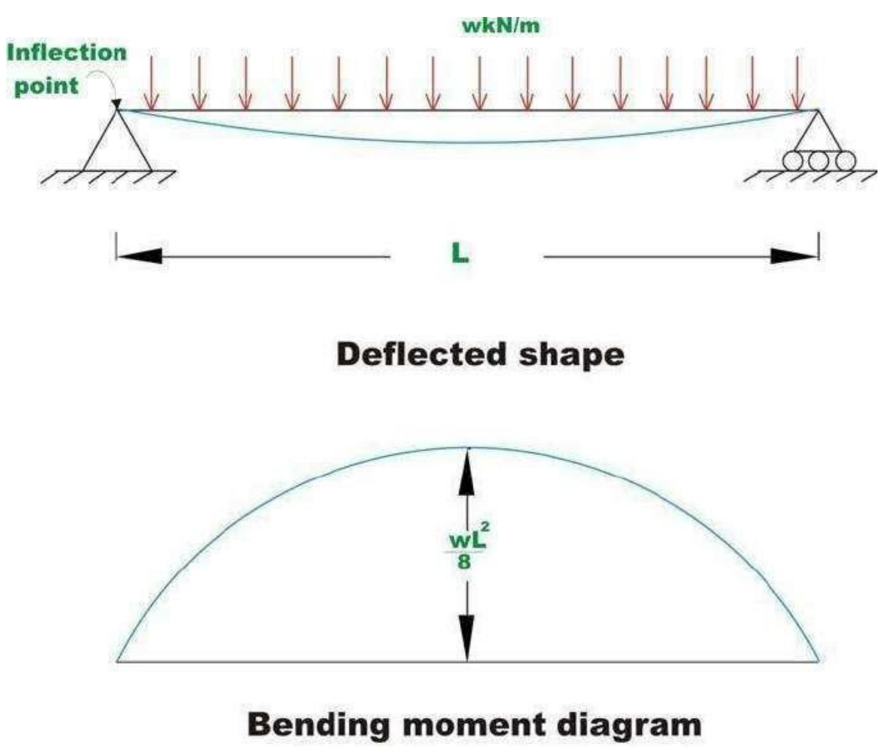
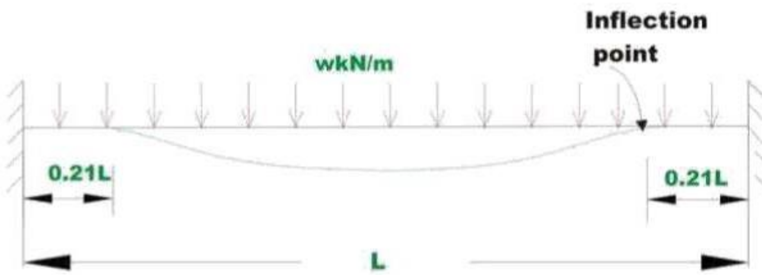
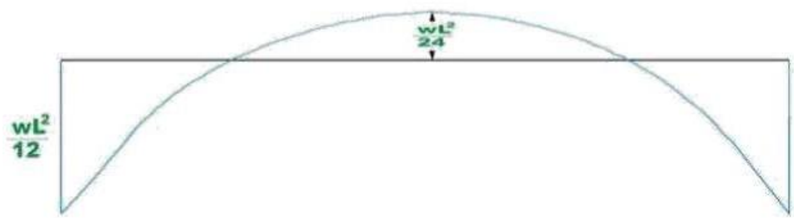


Fig.36. 4a Simply Supported beam



Deflected shape



Bending moment diagram

Fig.36. 4b Fixed - Fixed beam

Now consider a typical beam of a building frame as shown in Fig.36.4c. In this case, the support provided by the columns is neither fixed nor simply supported.

For the purpose of approximate analysis the inflexion point or point of zero moment is assumed to occur at $\left(\frac{0+0.21L}{2}\right) \approx 0.1L$ from the supports. In reality the point of zero moment varies depending on the actual rigidity provided by the columns. Thus the beam is approximated for the analysis as shown in Fig.

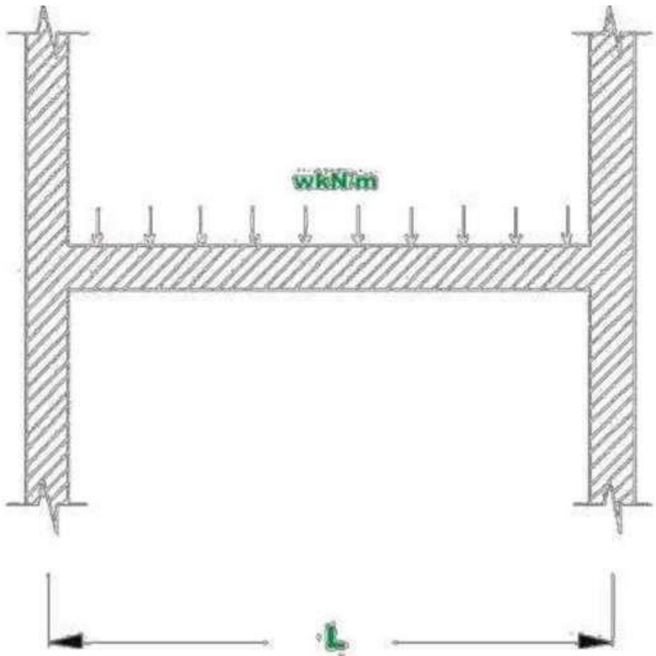
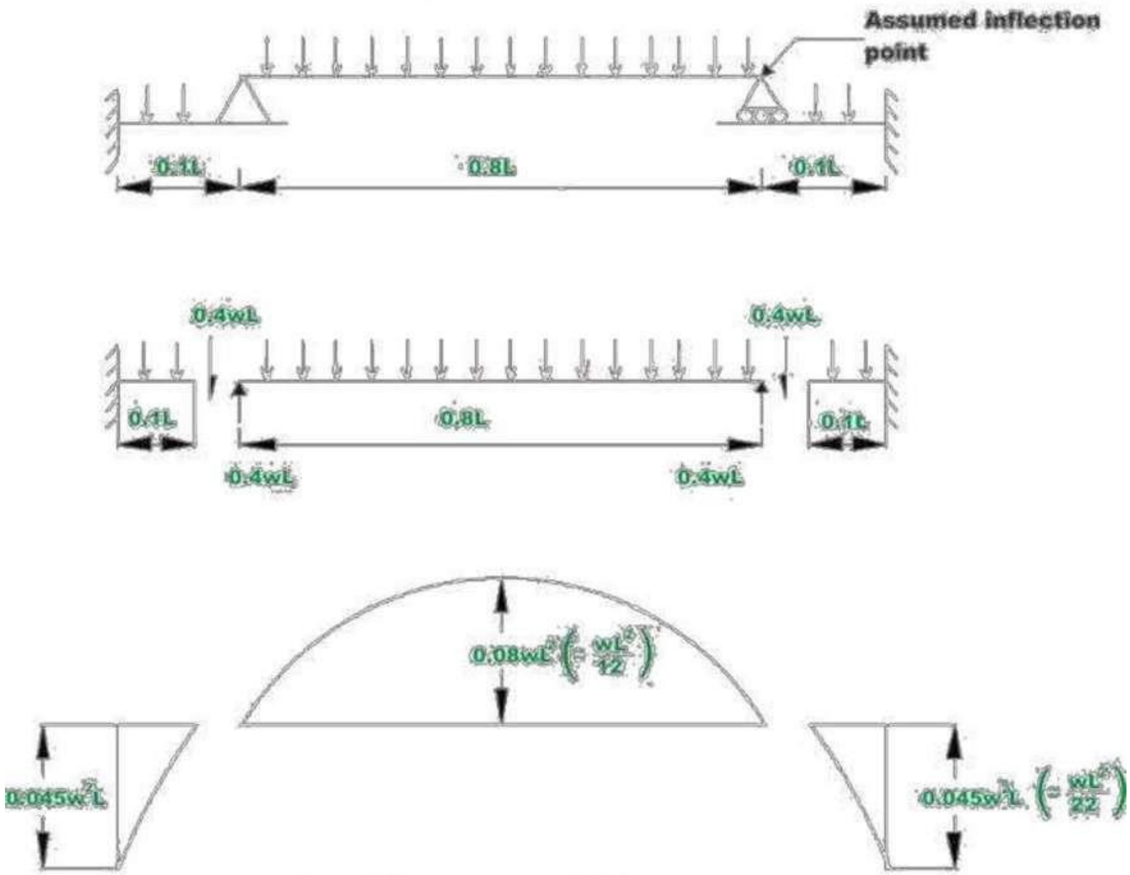


Fig.36.4c



Bending moment diagram

Fig.36.4d

For interior beams, the point of inflexion will be slightly more than $0.1L$. An experienced engineer will use his past experience to place the points of inflexion appropriately. Now redundancy has reduced by two for each beam. The third assumption is that axial force in the beams is zero. With these three assumptions one could analyse this frame for vertical loads.

Example 36.1

Analyse the building frame shown in Fig. 36.5a for vertical loads using approximate methods.

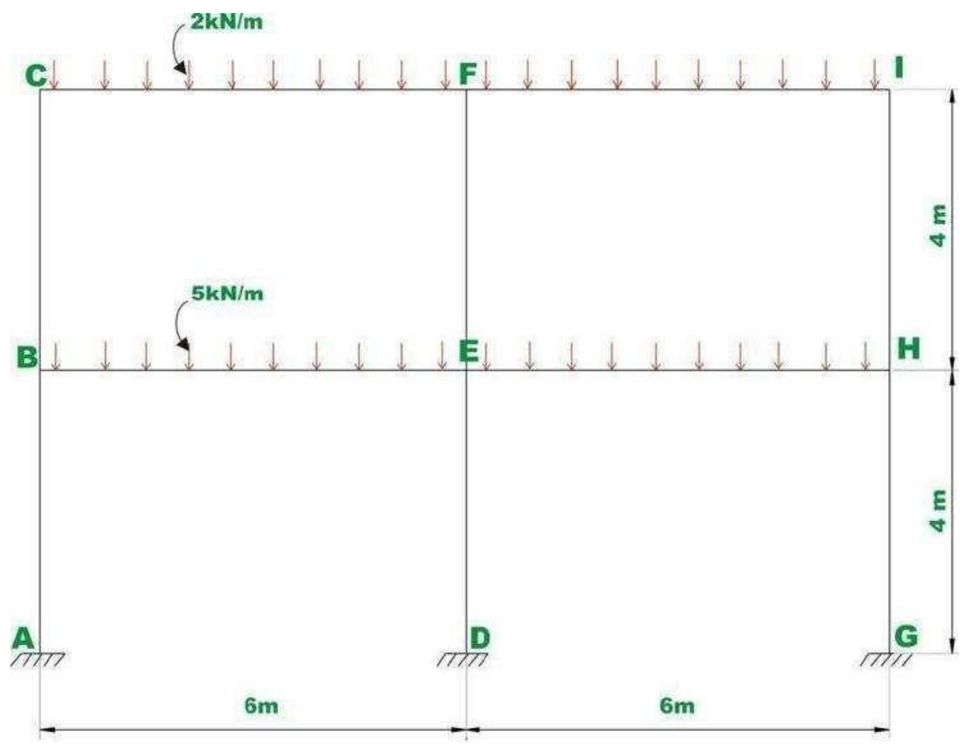


Fig.36.5a

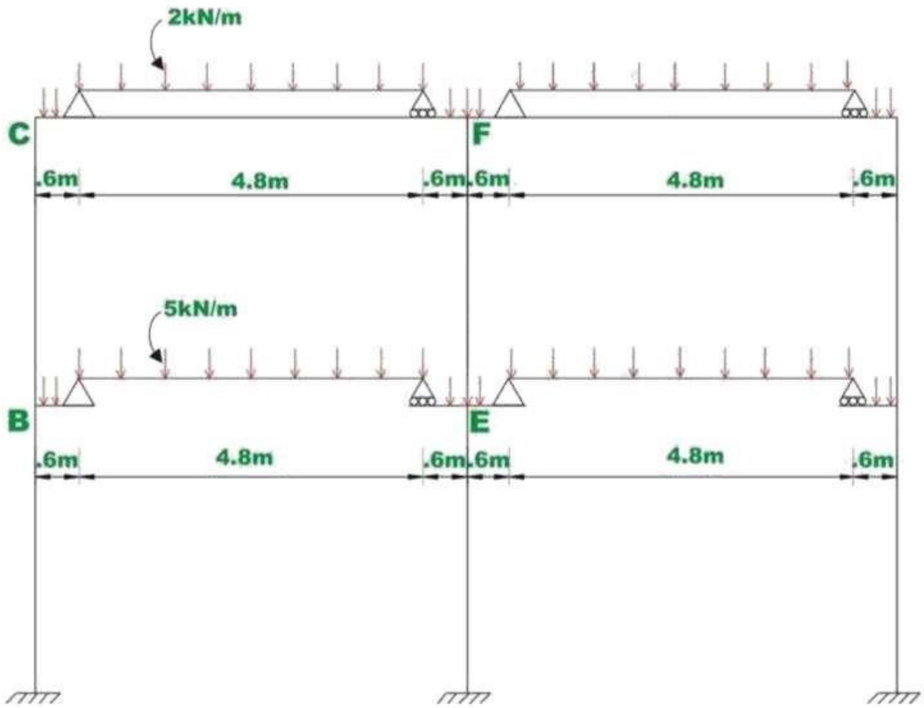
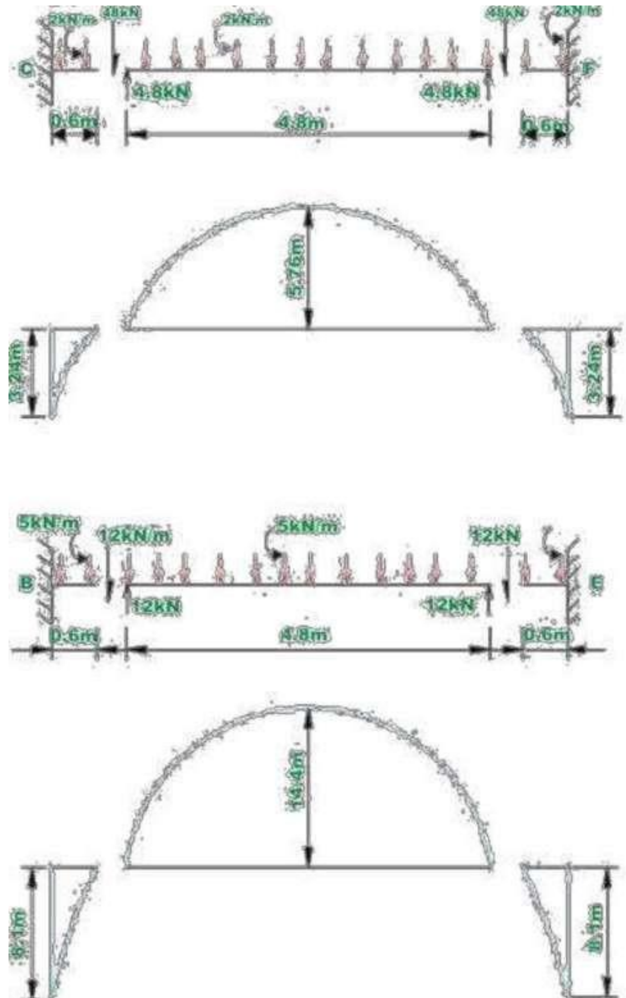


Fig.36.5 b

Solution:

In this case the inflexion points are assumed to occur in the beam at $0.1L(=0.6m)$ from columns as shown in Fig. 36.5b. The calculation of beam moments is shown in Fig. 36.5c.



Bending moment diagrams

Fig.36.5c

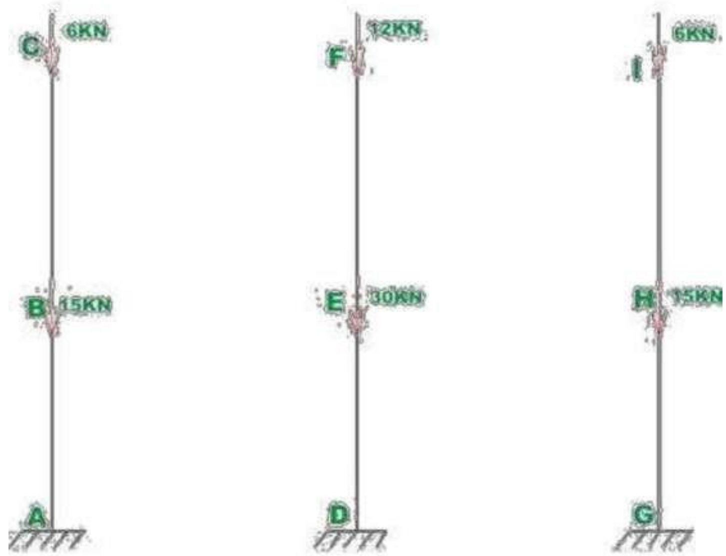


Fig.36.5d Axial force in columns

ANALYSIS OF BUILDING FRAMES TO LATERAL (HORIZONTAL) LOADS

A building frame may be subjected to wind and earthquake loads during its life time. Thus, the building frames must be designed to withstand lateral loads. A two-storey two-bay multistory frame subjected to lateral loads is shown in Fig.36.6. The actual deflected shape (as obtained by exact methods) of the frame is also shown in the figure by dotted lines. The given frame is statically indeterminate to degree 12.

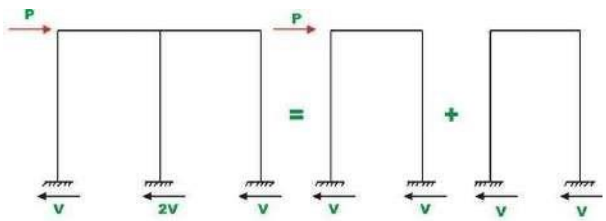


Fig.36.6 Shear in columns

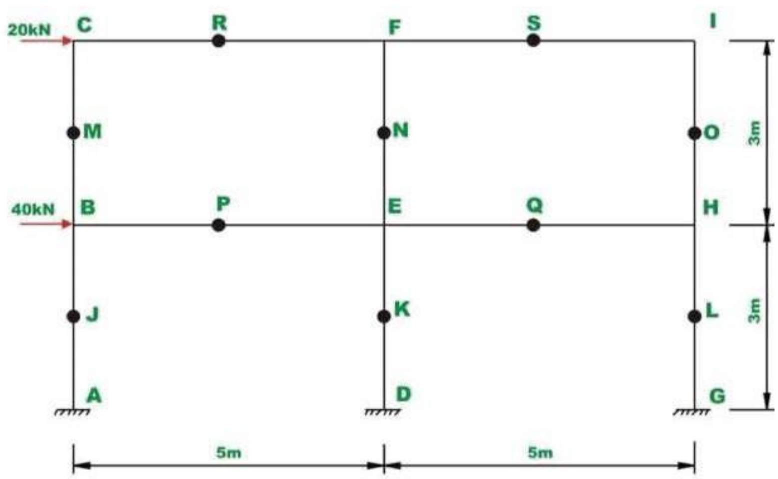


Fig.36.7a Two storey building frame subjected to lateral load of Example 36.2

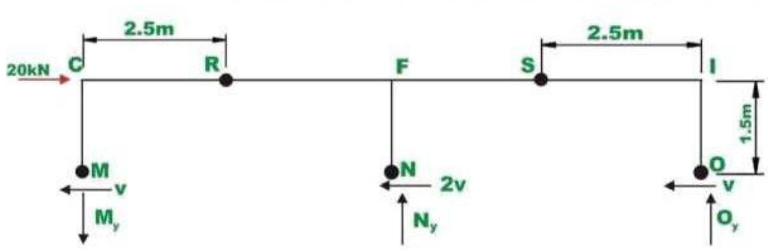


Fig.36.7b

Hence it is required to make 12 assumptions to reduce the frame in to a statically determinate structure. From the deformed shape of the frame, it is observed that inflexion point (point of zero moment) occur at mid height of each column and mid point of each beam. This leads to 10 assumptions. Depending upon how the remaining two assumptions are made, we have two different methods of analysis: *i*) Portal method and *ii*) cantilever method. They will be discussed in the subsequent sections.

PORTAL METHOD

In this method following assumptions are made.

1) An inflexion point occurs at the mid height of eachcolumn.
2) An inflexion point occurs at the mid point of eachgirder.
3) The total horizontal shear at each storey is divided between the columns of that storey such that the interior column carries twice the shear of exterior column.

The last assumption is clear, if we assume that each bay is made up of a portal thus the interior column is composed of two columns (Fig. 36.6). Thus the interior column carries twice the shear of exterior column. This method is illustrated in example 36.2.

Example

Analyse the frame shown in Fig. 36.7a and evaluate approximately the column end moments, beam end moments and reactions.

Solution:

The problem is solved by equations of statics with the help of assumptions made in the portal method. In this method we have hinges/inflexion points at mid height of columns and beams. Taking the section through column hinges $M.N,O$weget, (ref. Fig. 36.7b).

$$\sum F_X = 0 \qquad \Rightarrow \qquad V + 2V + V = 20$$

or $V = 5\text{kN}$

Taking moment of all forces left of hinge R about R gives,

$$V \times 1.5 - M_y \times 2.5 = 0$$

$$M_y = 3 \text{ kN}(\downarrow)$$

Column and beam moments are calculates as,

$$M_{CB} = 5 \times 1.5 = 7.5\text{kN.m} \ ; \ M_{IH} = +7.5\text{kN.m}$$

$$M_{CF} = -7.5\text{kN.m}$$

Taking moment of all forces left of hinge S about S gives,

$$5 \times 1.5 - O_y \times 2.5 = 0$$

$$O_y = 3\text{kN}(\uparrow)$$

$$N_y = 0$$

Taking a section through column hinges $J.K.L$ we get, (ref. Fig. 36.7c).

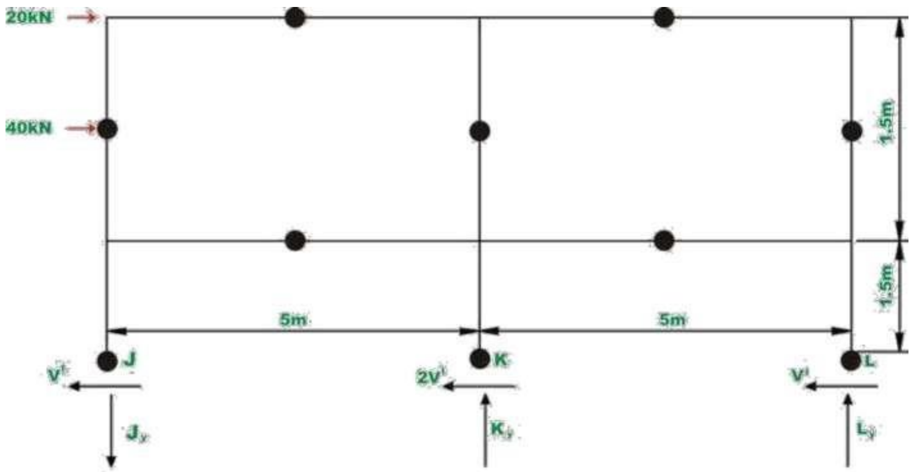


Fig.36.7c

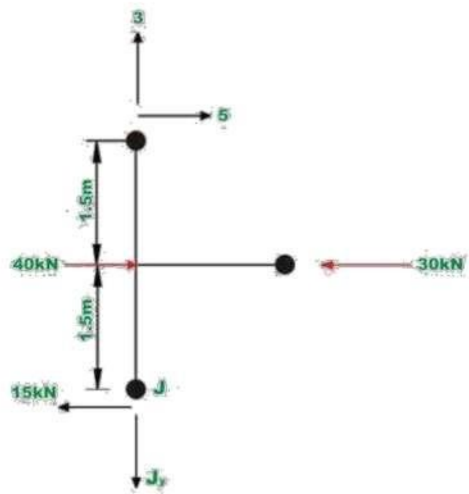


Fig.36.7d

$$\sum F_X = 0 \qquad \Rightarrow \qquad V'+2V'+V'=60$$

or $V' = 15$ kN

Taking moment of all forces about P gives (vide Fig. 36.7d)

$$\sum M_p = 015\times1.5+5\times1.5+3\times2.5-J_y\times2.5=0$$

$$J_y = 15 \;\; \text{kN}(\downarrow)$$

$$L_y = 15 \;\; \text{kN}(\uparrow)$$

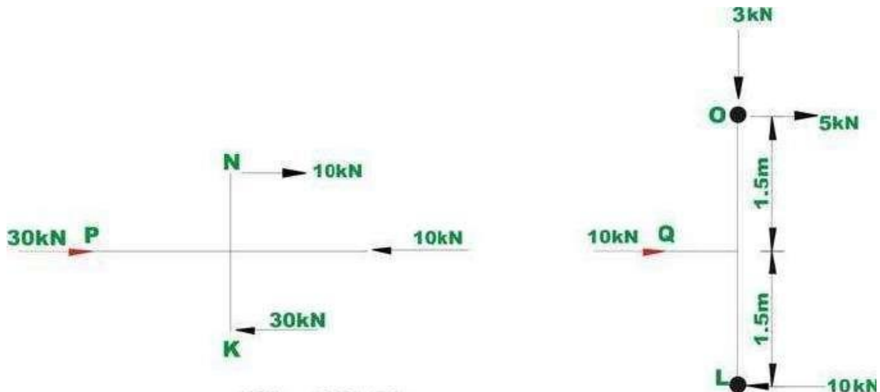


Fig.36.7e

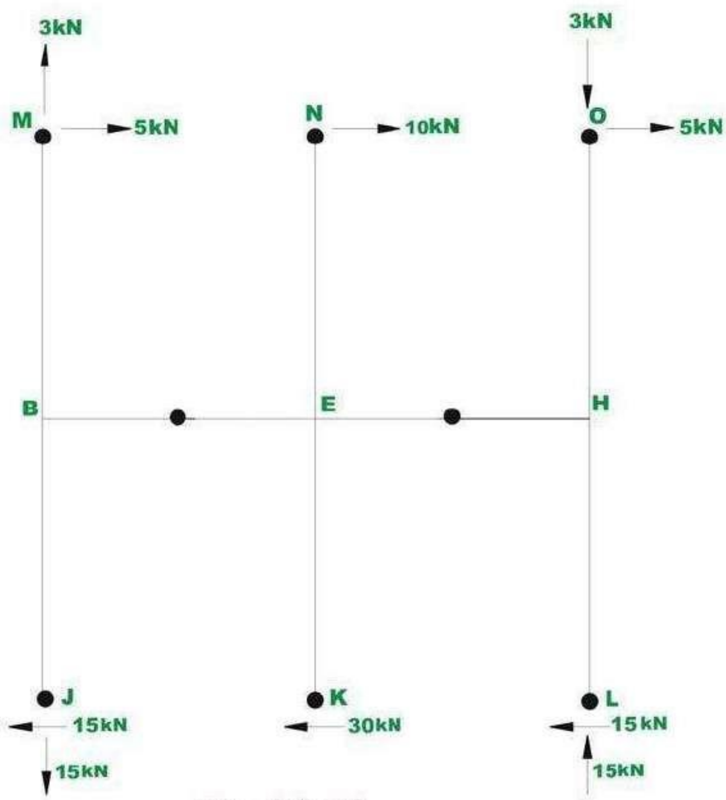


Fig.36.7f

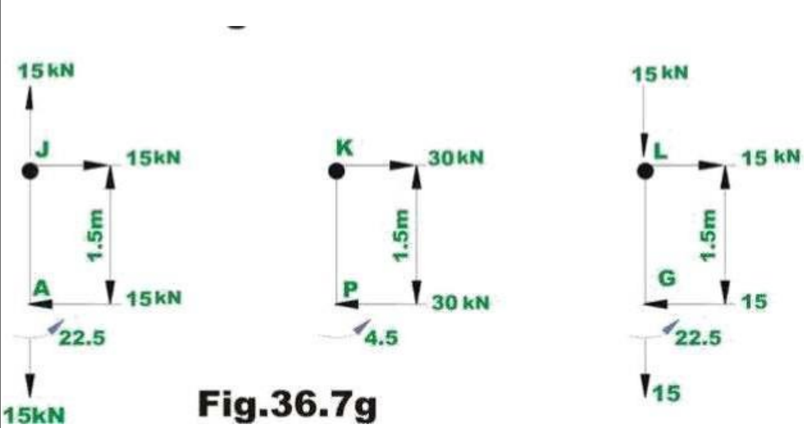


Fig.36.7g

Column and beam moments are calculated as, (ref. Fig. 36.7f)

M_{BC}=5x1.5=7.5 $kN.m$;M_{BA}=15x1.5=22.5 kN.m

M_{BE}=–30 kN.m

M_{EF}=10x1.5=15 kN.m ;M_{ED}=30x1.5=45 kN.m

M_{EB}=–30kN.m M_{EH}=–30kN.m

M_{HI}=5x1.5=7.5 kN.m ;M_{HG}=15x1.5=22.5 kN.m

M_{HE}=–30 kN.m

Reactions at the base of the column are shown in Fig. 36.7g**.**

CANTILEVER METHOD

The cantilever method is suitable if the frame is tall and slender. In the cantilever method following assumptions are made.

An inflexion point occurs at the mid point of eachgirder.An inflexion point occurs at mid height of eachcolumn.In a storey, the intensity of axial stress in a column is proportional to its horizontal distance from the center of gravity of all the columns in that storey. Consider a cantilever beam acted by a horizontal load P as shown in Fig. 36.8. In such a column the bending stress in the column cross section varies linearlyfrom its neutral axis. The last assumption in the cantilever method is based on this fact. The method is illustrated in example36.3.

Example 36.3

Estimate approximate column reactions, beam and column moments using cantilever method of the frame shown in Fig. 36.8a. The columns are assumed to have equal cross sectional areas.

Solution:

This problem is already solved by portal method. The center of gravity of all column passes through centre column.

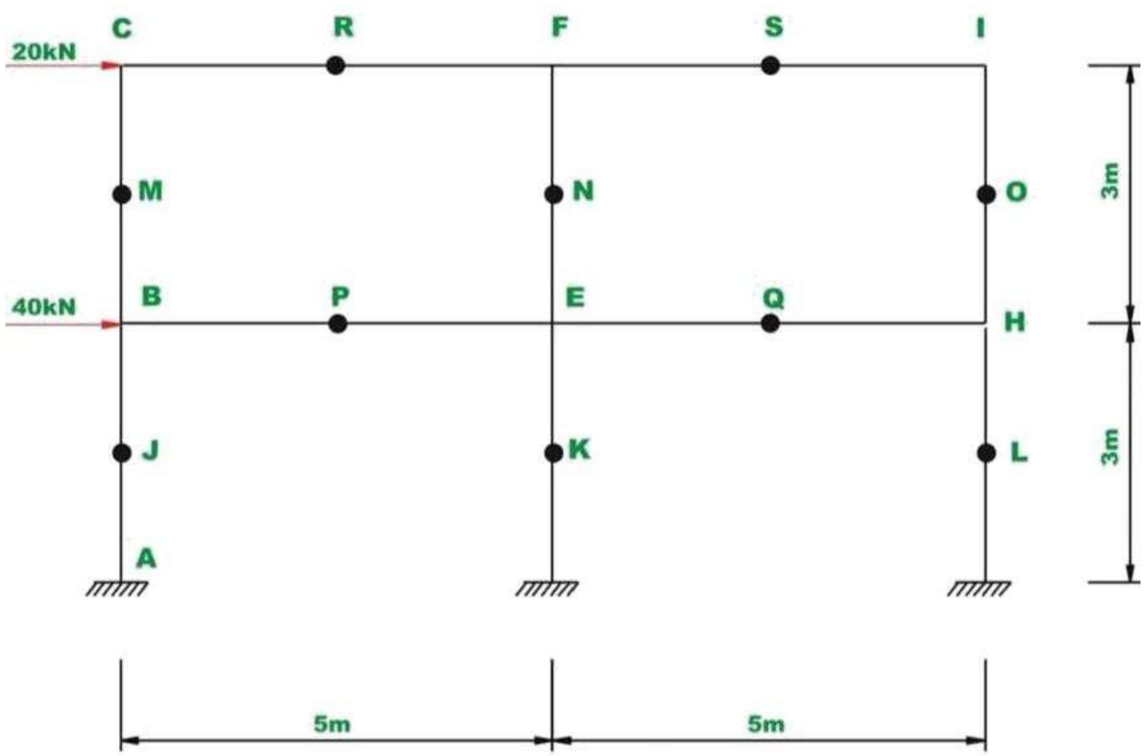


Fig.36.8b

Taking a section through first storey hinges gives us the free body diagram as shown in Fig. 36.8b. Now the column left of C.G. *i.e.*CBmust be subjected to tension and one on the right is subjected to compression.

From the third assumption,

$$\frac{M_y}{5\times A} = -\frac{O_y}{5\times A} \qquad \Rightarrow \qquad M_y = -O_y$$

Taking moment about O of all forces gives,

$$20\times 1.5 - M_y \times 10 = 0$$

$$M_y = 3\,\text{kN}(\downarrow) \quad ; \qquad O_y = 3\,\text{kN}(\uparrow)$$

Taking moment about R of all forces left of R ,

$$V_M \times 1.5 - 3\times 2.5 = 0$$

$$V_M = 5 \ \ \text{kN}(\leftarrow)$$

Taking moment of all forces right of S about S ,

$$V_O \times 1.5 - 3\times 2.5 = 0 \quad \Rightarrow \qquad V_O = 5\,\text{kN}.$$

$$\sum F_X = 0 \qquad V_M + V_N + V_O - 20 = 0$$

$$V_N = 10\,\text{kN}.$$

Moments

M_{CB}=5·1.5=7.5 kN.m
M_{CF}=−7.5 kN.m
M_{FE}=15 kN.m
M_{FC}=−7.5 kN.m
M_{FI}=−7.5 kN.m
M_{IH}=7.5 kN.m
M_{IF}=−7.5 kN.m

Tae a section through hinges J,K,L(ref. Fig. 36.8c). Since the center of gravity passes through centre column the axial force in that column is zero.

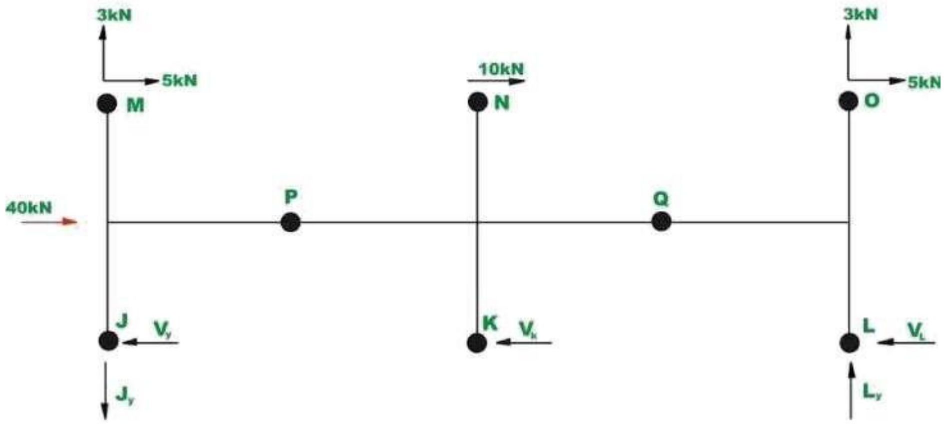


Fig.36.8c

Taking moment about hinge L, J_y can be evaluated. Thus,

$$20 \times 3 + 40 \times 1.5 + 3 \times 10 - J_y \times 10 = 0$$

$$J_y = 15\,\mathrm{kN}(\downarrow) \quad ; \quad L_y = 15\,\mathrm{kN}(\uparrow)$$

Taking moment of all forces left of P about P gives,

$$5 \times 1.5 + 3 \times 2.5 - 15 \times 2.5 + V_j \times 1.5 = 0$$

$$V_J = 15\,\mathrm{kN}(\leftarrow)$$

Similarly taking moment of all forces right of Q about Q gives,

$$5 \times 1.5 + 3 \times 2.5 - 15 \times 2.5 + V_L \times 1.5 = 0$$

$$V_L = 15\,\mathrm{kN}(\leftarrow)$$

$$\sum F_X = 0 \qquad V_J + V_K + V_L - 60 = 0$$

$$V_K = 30 \;\; \mathrm{kN.}$$

Moments

$M_{BC}=5\cdot 1.5=7.5$ kN.m ; $M_{BA}=15\cdot 1.5=22.5$ kN.m

$M_{BE}=-30$ kN.m

$M_{EF}=10\cdot 1.5=15$ kN.m ; $M_{ED}=30\cdot 1.5=45$ kN.m

$M_{EB}=-30$ kN.m $M_{EH}=-30$ kN.m

$M_{HI}=5\cdot 1.5=7.5$ kN.m ; $M_{HG}=15\cdot 1.5=22.5$ kN.m

$M_{HE}=-30$ kN.m

PART B

Approximate Lateral Load Analysis by Portal Method

Portal Frame

Portal frames, used in several Civil Engineering structures like buildings, factories, bridges have the primary purpose of transferring horizontal loads applied at their tops to their foundations. Structural requirements usually necessitate the use of statically indeterminate layout for portal frames, and approximate solutions are often used in their analyses.

Assumptions for the Approximate Solution

In order to analyze a structure using the equations of statics only, the number of independent force components must be equal to the number of independent equations of statics.

If there are *n*more independent force components in the structure than there are independent equations of statics, the structure is statically indeterminate to the *nth* degree. Therefore to obtain an approximate solution of the structure based on statics only, it will be necessary to make *n*additional independent assumptions. A solution based on statics will not be possible by making fewer than *n* assumptions, while more than *n* assumptions will not in general be consistent.

Thus, the first step in the approximate analysis of structures is to find its degree of statical indeterminacy (dosi) and then to make appropriate number of assumptions.

For example, the dosi of portal frames shown in (i), (ii), (iii) and (iv) are 1, 3, 2 and 1 respectively. Based on the type of frame, the following assumptions can be made for portal structures with a *vertical axis of symmetry* that are *loaded horizontally at thetop*

1. The horizontal support reactions areequal

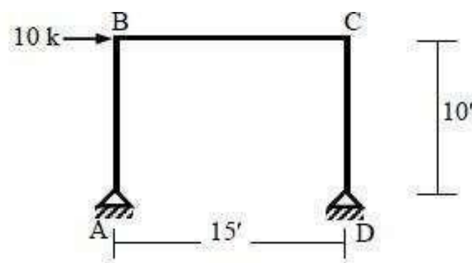
2. There is a point of inflection at the center of the unsupported height of each fixed basedcolumn

Some additional assumptions can be made in order to solve the structure approximately for different loading and support conditions.

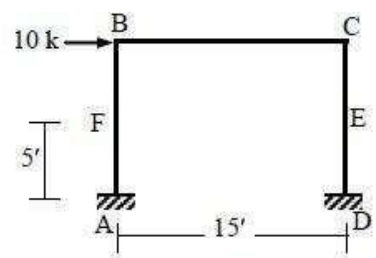
3. Horizontal body forces not applied at the top of a column can be divided into two forces (i.e., applied at the top and bottom of the column) based on simple supports

4. For hinged and fixed supports, the horizontal reactions for fixed supports can be assumed to be four times the horizontal reactions for hinged supports Example

Draw the axial force, shear force and bending moment diagrams of the frames loaded as shown below.



(i)



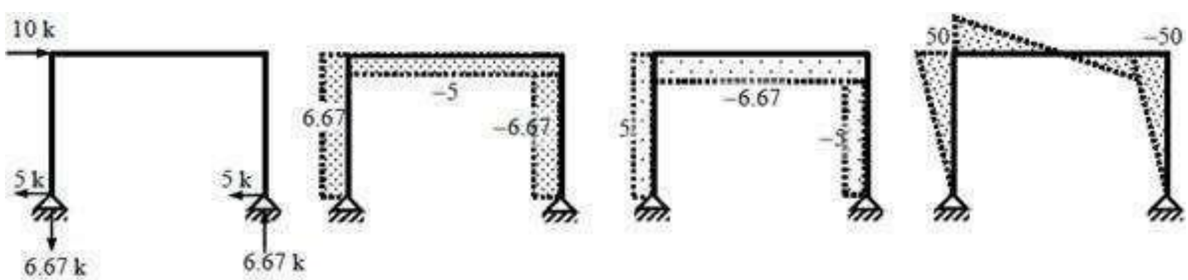
(ii)

Solution

(i) For this frame, dosi = 3 × 3 + 4 – 3 × 4 = 1; i.e., Assumption 1 ⇒ $H_A = H_D = 10/2 = 5$ k

$\therefore \sum M_A = 0 \Rightarrow 10 \times 10 - V_D \times 15 = 0 \Rightarrow V_D = 6.67$ k

$\therefore \sum F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -6.67$ k



Reactions AFD (k) SFD (k) BMD

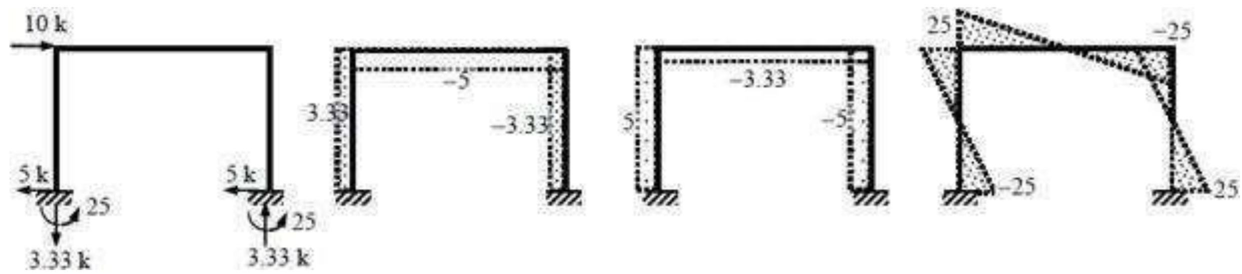
(ii) dosi = 3 × 3 + 6 – 3 × 4 = 3

Assumption 1 ⇒ $H_A = H_D = 10/2 = 5$ k, Assumption 2 ⇒ $BM_E = BM_F = 0$

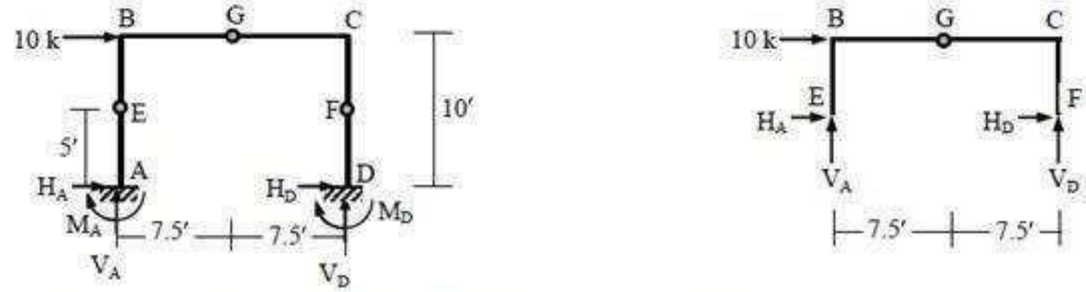
$\therefore BM_F = 0 \Rightarrow H_A \times 5 + M_A = 0 \Rightarrow M_A = -25$ k-ft; Similarly $BM_E = 0 \Rightarrow M_D = -25$

$\therefore \sum M_A = 0 \Rightarrow -25 - 25 + 10 \times 10 - V_D \times 15 = 0 \Rightarrow V_D = 3.33$ k

$\therefore \sum F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -3.33$ k



(iii)



dosi = $3 \times 4 + 6 - 3 \times 5 - 1 = 2$; $\therefore$ Assumption 1 and 2 $\Rightarrow BM_E = BM_F = 0$

$\therefore BM_E = 0$ (bottom) $\Rightarrow -H_A \times 5 + M_A = 0 \Rightarrow M_A = 5H_A$; Similarly $BM_F = 0 \Rightarrow M_D = 5H_D$

Also $BM_E = 0$ (free body of EBCF) $\Rightarrow 10 \times 5 - V_D \times 15 = 0 \Rightarrow V_D = 3.33$ k

$\therefore \Sigma F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -V_D = -3.33$ k

$BM_G = 0$ (between E and G) $\Rightarrow V_A \times 7.5 - H_A \times 5 = 0 \Rightarrow H_A = -5$ k $\Rightarrow M_A = 5H_A = -25$

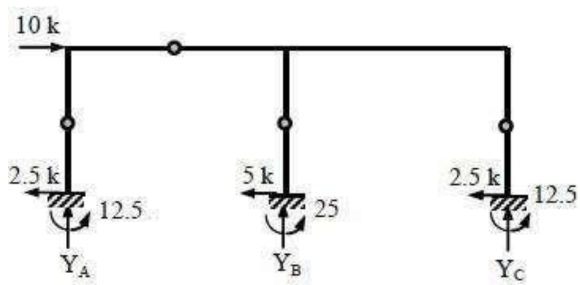
$\Sigma F_x = 0$ (entire structure) $\Rightarrow H_A + H_D + 10 = 0 \Rightarrow -5 + H_D + 10 = 0 \Rightarrow H_D = -5$ k $\Rightarrow M_D = 5H_D = -25$

(iv) dosi = $3 \times 5 + 9 - 3 \times 6 = 6 \Rightarrow 6$ Assumptions needed to solve the structure

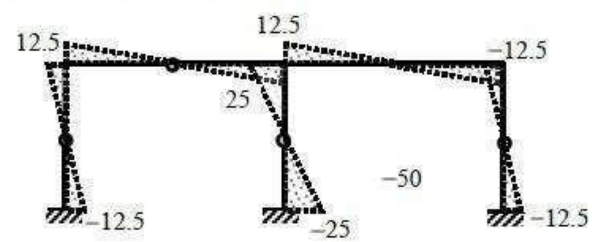
Assumption 1 and 2 $\Rightarrow H_A: H_B: H_C = 1: 2: 1 \Rightarrow H_A = 10/4 = 2.5$ k, $H_B = 5$ k, $H_C = 2.5$ k

$\therefore M_A = M_C = 2.5 \times 5 = 12.5$ k-ft, $M_B = 5 \times 5 = 25$

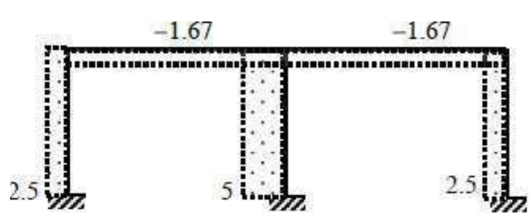
The other 4 assumptions are the assumed internal hinge locations at midpoints of columns and one beam



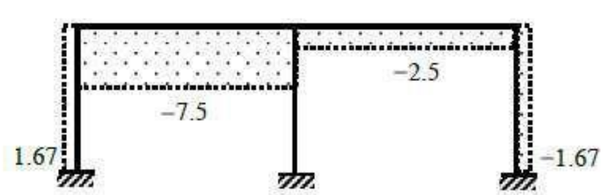
Reactions



BMD



SFD (k)



AFD (k)

Analysis of Multi-storied Structures by Portal Method

Approximate methods of analyzing multi-storied structures are important because such structures are statically highly indeterminate. The number of assumptions that must be made to permit an analysis by statics alone is equal to the degree of statical indeterminacy of the structure.

Assumptions

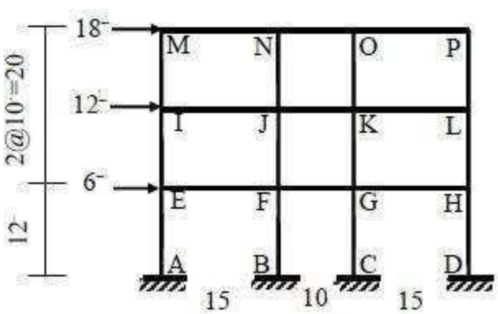
The assumptions used in the approximate analysis of portal frames can be extended for the lateral load analysis of multi-storied structures. The *Portal Method* thus formulated is based on three assumptions

1. The shear force in an interior column is twice the shear force in an exterior column.
2. There is a point of inflection at the center of eachcolumn.
3. There is a point of inflection at the center of eachbeam.

Assumption 1 is based on assuming the interior columns to be formed by columns of two adjacent bays or portals. Assumption 2 and 3 are based on observing the deflected shape of thestructure.

Example

Use the Portal Method to draw the axial force, shear force and bending moment diagrams of the three-storied frame structure loaded as shown below.



Column shear forces are at the ratio of 1:2:2:1.

∴ Shear force in (V) columns IM, JN, KO, LP are $[18 \times 1/(1+2+2+1) =]\ 3$, $[18 \times 2/(1+2+2+1) =]\ 6$, 6, 3 respectively. Similarly,

$V_{EI} = 30\times1/(6) = 5$, $V_{FJ} = 10$, $V_{GK} = 10$, $V_{HL} = 5$; and

$V_{AE} = 36\times1/(6) = 6$, $V_{BF} = 12$, $V_{CG} = 12$, $V_{DH} = 6$

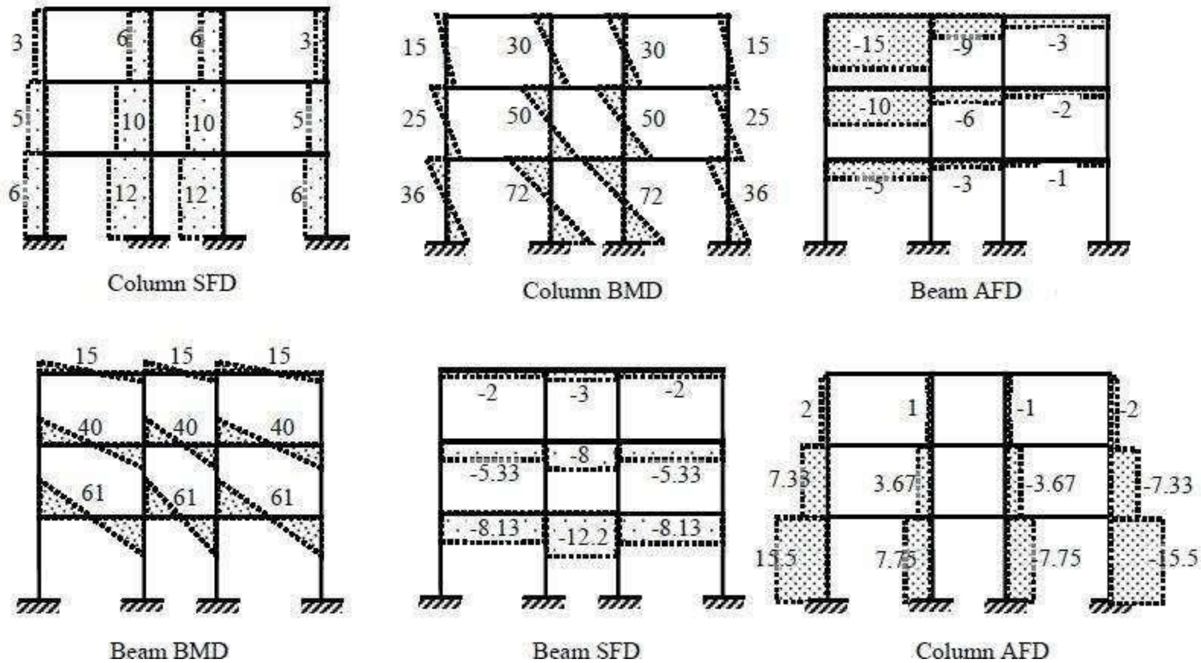
Bending moments are

$M_{IM} = 3\times10/2 = 15$, $M_{JN} = 30$, $M_{KO} = 30$, $M_{LP} = 15$

$M_{EI} = 5\times10/2 = 25$, $M_{FJ} = 50$, $M_{GK} = 50$, $M_{HL} = 25$

$M_{AE} = 6\times10/2 = 30$, $M_{FJ} = 60$, $M_{GK} = 60$, $M_{HL} = 30$

The rest of the calculations follow from the free-body diagrams



Analysis of Multi-storied Structures by Cantilever Method

Although the results using the *Portal Method* are reasonable in most cases, the method suffers due to the lack of consideration given to the variation of structural response due to the difference between sectional properties of various members. The *Cantilever Method* attempts to rectify this limitation by considering the cross- sectional areas of columns in distributing the axial forces in various columns of a story.

Assumptions

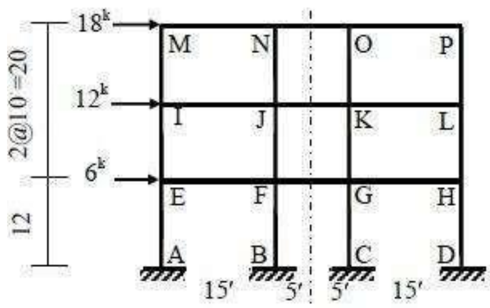
The *Cantilever Method* is based on three assumptions

1. The axial force in each column of a storey is proportional to its horizontal distance from the centroidal axis of all the columns of thestorey.
2. There is a point of inflection at the center of eachcolumn.
3. There is a point of inflection at the center of eachbeam.

Assumption 1 is based on assuming that the axial stresses can be obtained by a method analogous to that used for determining the distribution of normal stresses on a transverse section of a cantilever beam. Assumption 2 and 3 are based on observing the deflected shape of the structure.

Example

Use the Cantilever Method to draw the axial force, shear force and bending moment diagrams of the three -storied frame structure loaded as shown below.



The dotted line is the column centerline (at all floors)

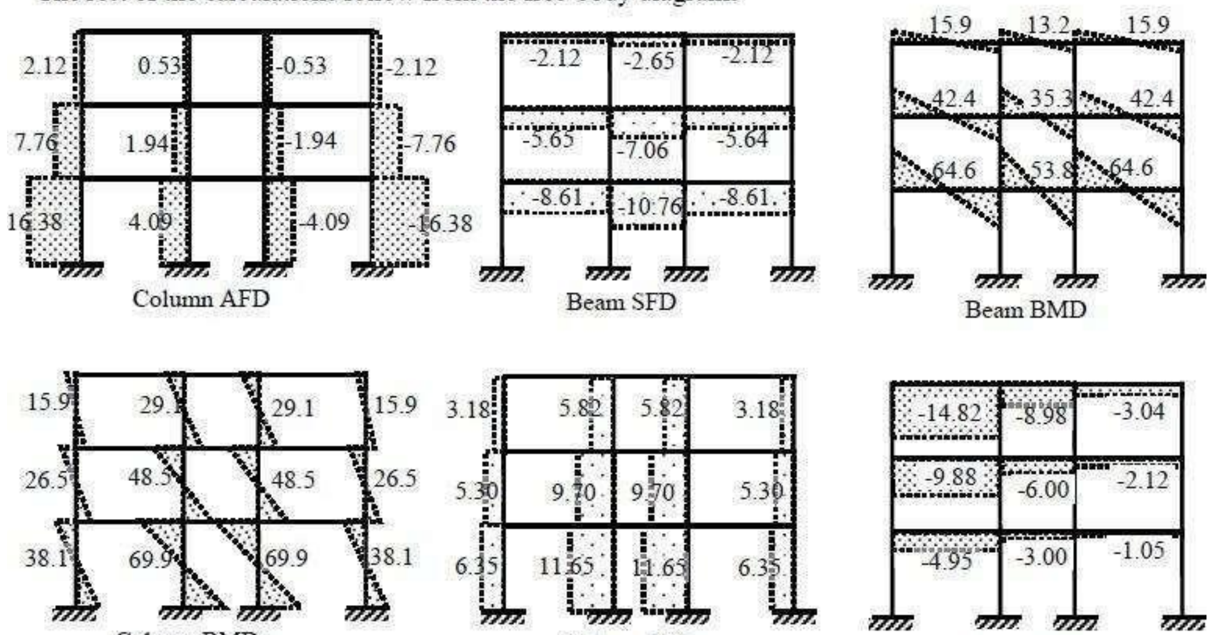
∴ Column axial forces are at the ratio of 20: 5: −5: −20.

∴ Axial force in (P) columns IM, JN, KO, LP are $[18 \times 5 \times 20/\{20^2 + 5^2 + (-5)^2 + (-20)^2\} =]$ 2.12, $[18 \times 5 \times 5/(20^2 + 5^2 + (-5)^2 + (-20)^2\} =]$ 0.53, −0.53, −2.12 respectively.

Similarly, $P_{EI} = 330 \times 20/(850) = 7.76$, $P_{FJ} = 1.94$, $P_{GK} = -1.94$, $P_{HL} = -7.76$; and

$P_{AE} = 696 \times 20/(850) = 16.38$, $P_{BF} = 4.09$, $P_{CG} = -4.09$, $P_{DH} = 16.38$

The rest of the calculations follow from the free-body diagrams



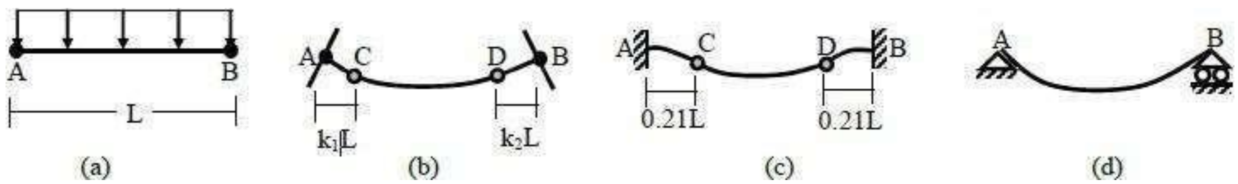
Column AFD — Beam SFD — Beam BMD

Column BMD — Column SFD — Beam AFD

Approximate Vertical Load Analysis

Approximation based on the Location of Hinges

If a beam AB is subjected to a uniformly distributed vertical load of w per unit length [Fig. (a)], both the joints A and B will rotate as shown in Fig. (b), because although the joints A and B are partly restrained against rotation, the restraint is not complete. Had the joints A and B been completely fixed against rotation [Fig. (c)] the points of inflection would be located at a distance 0.21L from each end. If, on the other hand, the joints A and B are hinged [Fig. (d)], the points of zero moment would be at the end of the beam. For the actual case of partial fixity, the points of inflection can be assumed to be somewhere between 0.21 L and 0 from the end of the beam. *For approximate analysis, they are often assumed to be located at one-tenth (0.1 L) of the span length from each end joint.*



Depending on the support conditions (i.e., hinge ended, fixed ended or continuous), a beam in general can be statically indeterminate up to a degree of three. Therefore, to make it statically determinate, the following three assumptions are often made in the vertical load analysis of a beam

1. The axial force in the beam iszero
2. Points of inflection occur at the distance 0.1 L measured along the span from the left and rightsupport.

Bending Moment and Shear Force from Approximate Analysis

Based on the approximations mentioned (i.e., points of inflection at a distance 0.1 L from the ends), the maximum positive bending moment in the beam is calculated to be

$M(+) = w(0.8L)^2/8 = 0.08\ wL^2$, at the midspan of the beam The maximum negative bending moment is

$M() = wL^2/8\ 0.08\ wL^2 = 0.045\ wL^2$, at the joints A and B of the beam

The shear forces are maximum (positive or negative) at the joints A and B and are calculated to be

$VA = wL/^2$, and $VB = wL/2$

Moment and Shear Values using ACI Coefficients

Maximum allowable LL/DL = 3, maximum allowable adjacent span difference = 20%

1. PositiveMoments

(i) For EndSpans

(a) If discontinuous end is unrestrained, $M(+) = wL^2/11$

(b) If discontinuous end is restrained, $M(+) = wL^2/14$

(ii) For Interior Spans, $M(+) = wL^2/16$

2. NegativeMoments

(i) At the exterior face of first interiorsupports

(a) Two spans, $M(-) = wL^2/9$

(b) More than two spans, $M(-) = wL^2/10$

(ii) At the other faces of interior supports, $M(-) = wL^2/11$

(iii) Forspansnotexceeding10,ofwherecolumnsaremuchstifferthanbeams,

M(-) = wL2/12

(iv) At the interior faces of exteriorsupports

(a) If the support is a beam, $M(-) = wL^2/24$

(b) If the support is a column, $M(-) = wL^2/16$

3. ShearForces

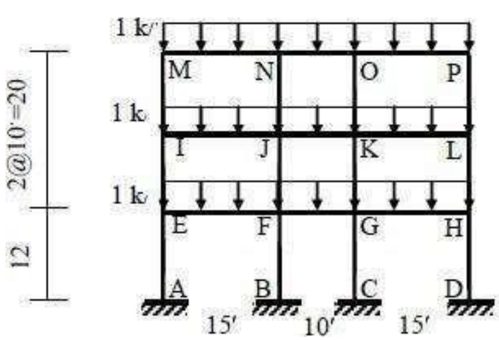
(i) In end members at first interior support, $V = 1.15wL/2$

(ii) At all other supports, $V = wL/2$

[where L = clear span for M(+) and V, and average of two adjacent clear spans for M(-)]

Example

Analyze the three-storied frame structure loaded as shown below using the approximate location of hinges to draw the axial force, shear force and bending moment diagrams of the beams and columns.



The maximum positive and negative beam moments and shear forces are as follows.

For the 15' beam, $M_{(+)} = 0.08 \times 1 \times 15^2 = 18$

$M_{(-)} = 0.045 \times 1 \times 15^2 = 10.13$

$V_{(\pm)} = 1 \times 15/2 = 7.5$ k

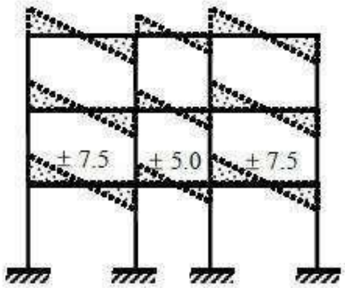
For the 10' beam, $M_{(+)} = 0.08 \times 1 \times 10^2 = 8$

$M_{(-)} = 0.045 \times 1 \times 10^2 = 4.5$

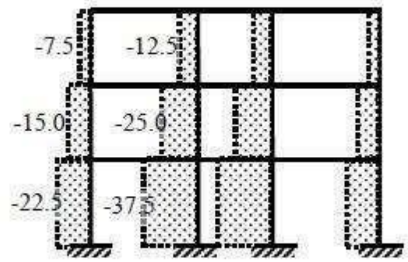
$V_{(\pm)} = 1 \times 10/2 = 5$ k

Axial Force P in all the beams = 0

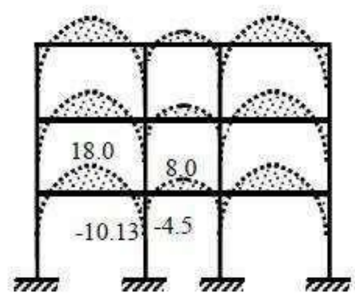
The rest of the calculations follow from the free-body diagrams



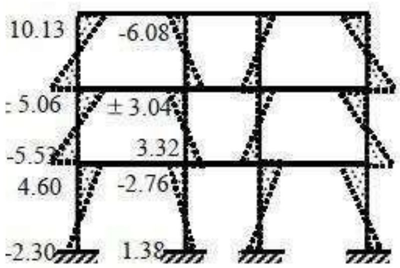
Beam SFD (k)



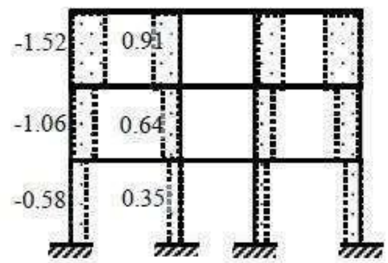
Column AFD (k)



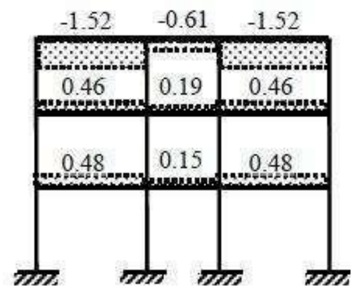
Beam BMD



Column BMD



Column SFD (k)



Beam AFD (k)