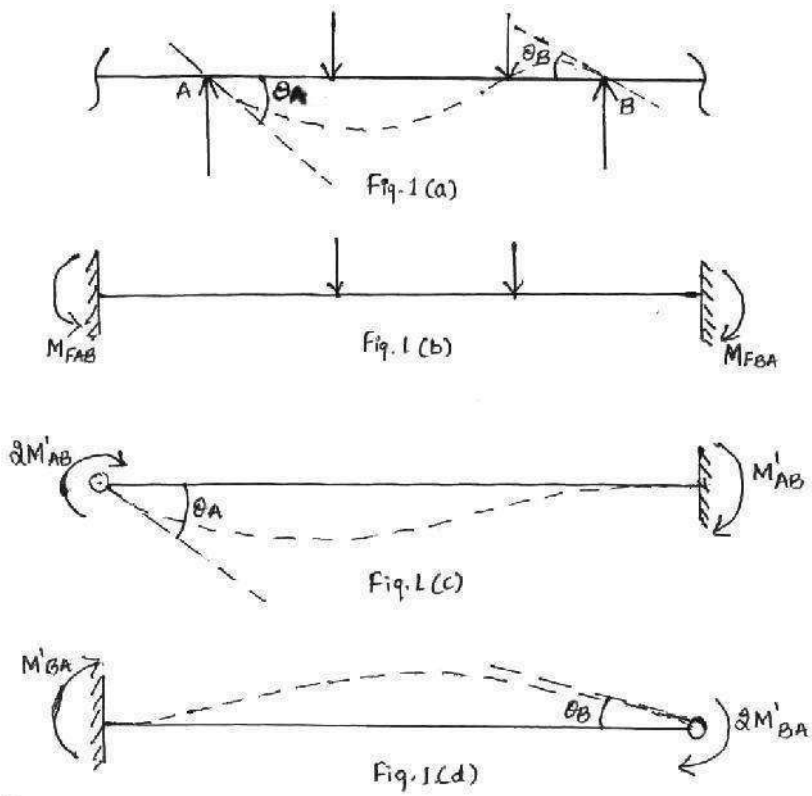
UNIT - II

PART A : KANI'S METHOD OF ANALYSIS

This method was developed by Dr. Gasper Kani of Germany in 1947. This method offers an iterative scheme for applying slope deflection method. We shall now see the application of Kani's method for different cases.

BEAMS WITH NO TRANSLATION OF JOINTS:



Let AB represent a beam in a frame, or a continuous structure under transverse loading, as show in fig. 1 (a) let the M_{AB} & M_{BA} be the end moment at ends A & B respectively.

Sign convention used will be: clockwise moment +ve and anticlockwise moment –ve.

The end moments in member AB may be thought of as moments developed due to a superposition of the following three components of deformation.

Thus the final moment M_{AB} & M_{BA} can be expressed as super position of three moments

$$\left.\begin{aligned} M_{AB} &= M_{FAB} + 2M'_{AB} + M'_{BA} \\ M_{BA} &= M_{FBA} + 2M'_{BA} + M'_{AB} \end{aligned}\right\} \qquad \ldots\ldots\ldots(1)$$

For member AB we refer end 'A' as near end and end 'B' as far end. Similarly when we refer to moment M_{BA}, B is referred as near end and end A as far end.

1. The member 'AB' is regarded as completely fixed. (Fig. 1 b). The fixed end moments for this condition are written as M_{FAB} & M_{FBA}, at ends A & B respectively.
2. The end A only is rotated through an angle θ_A by a moment $2M'_{AB}$ inducing a moment M'_{AB} at fixed end B.
3. Next rotating the end B only through an angle θ_B by moment $2M'_{BA}$ while keeping end 'A' as fixed. This induces a moment M'_{BA} at end A.

Hence above equations can be stated as follows. The moment at the near end of a member is the algebraic sum of (a) fixed end moment at near end. (b) Twice the rotation moment of the near end (c) rotation moment of the far end.

Rotation factors:

Fig. 2 shows a multistoried frame.

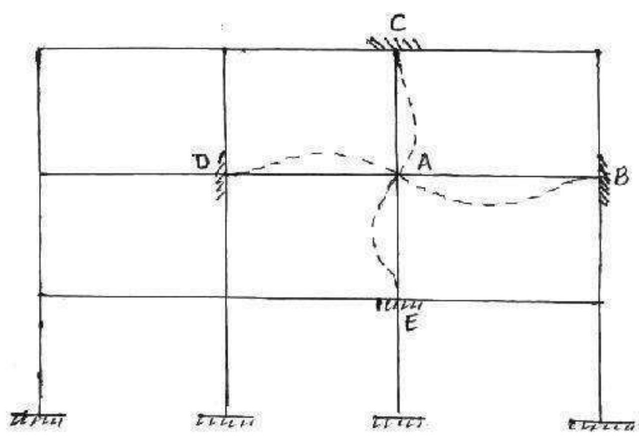


Fig. 2

Consider various members meeting at joint A. If no translations of joints occur, applying equation (1), for the end moments at A for the various members meeting at A are given by:

$$M_{AB} = M_{FAB} + 2M'_{AB} + M'_{BA}$$
$$M_{AC} = M_{FAC} + 2M'_{AC} + M'_{CA}$$
$$M_{AD} = M_{FAD} + 2M'_{AD} + M'_{DA}$$
$$M_{AE} = M_{FAE} + 2M'_{AE} + M'_{EA}$$

For equilibrium of joint A, $\Sigma M_A = 0$

$\therefore \Sigma M_{FAB} + 2\Sigma M'_{AB} + \Sigma M'_{BA} = 0$ ....(2)

where ,

ΣM_{FAB} =Algebraic sum of fixed end moments at A of all members meeting at A.

$\Sigma M'_{AB}$ = Algebraic sum of rotation moments at A of all member meeting at A.

$\Sigma M'_{BA}$ = Algebraic sum of rotation moments of far ends of the members meeting at A.

from equation (2)

$\Sigma M'_{AB} = -\frac{1}{2}\left[\sum M_{FAB} + \sum M'_{BA}\right]$ .... (3)

We know that $2M'_{AB} = \frac{4EI_{AB}}{L_{AB}}\theta_A = 4EK_{AB}\,\theta_A$

Where $K_{AB} = \frac{I_{AB}}{L_{AB}}$, relative stiffness of member AB

$M'_{AB} = 2E\,K_{AB}\,\theta_A$ ....(4)

$\therefore\ \Sigma M'_{AB} = 2E\theta_A\,\Sigma K_{AB}$ ....(5) (At rigid joint A all the members undergo same rotation θ_A)

Dividing Equation (4)/(5) gives

$$\frac{M'_{AB}}{\Sigma M'_{AB}} = \frac{K_{AB}}{\Sigma K_{AB}}$$

$\therefore\ M'_{AB} = \frac{K_{AB}}{\Sigma K_{AB}}\Sigma M'_{AB}$ ....(5)

Substituting value of $\Sigma M'_{AB}$ from (3) in (5)

$$M'_{AB} = \left(-\frac{1}{2}\right)\frac{K_{AB}}{\Sigma K_{AB}}\left[\Sigma M_{FAB} + \Sigma M'_{BA}\right]$$

$= U_{AB}\left[\Sigma M_{FAB} + \Sigma M'_{BA}\right]$..................(6)

where $U_{AB} = -\frac{1}{2}\ \frac{K_{AB}}{\Sigma K_{AB}}$ is called as rotation factor for member AB at joint A.

Analysis Method:

In equation (6) ΣM_{FAB} is a known quantity. To start with the far end rotation moments M'_{BA} are not known and hence they may be taken as zero. By a similar approximation the rotation moments at other joints are also determined. With the approximate values of rotation moments computed, it is possible to again determine a more correct value of the rotation moment at A from member AB using equation (6).

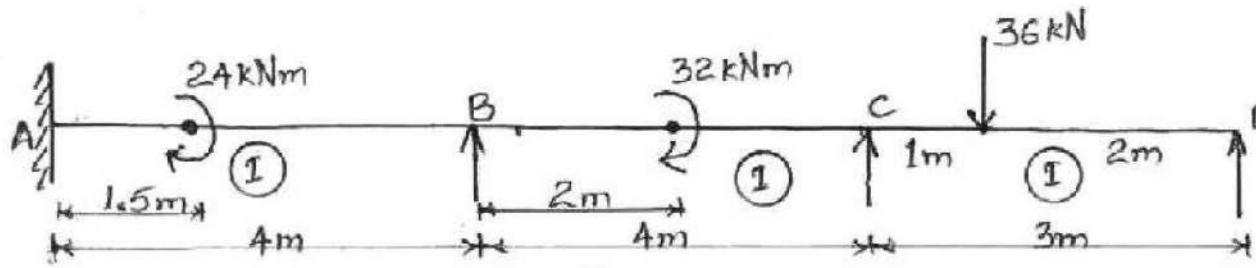
The process is carried out for sufficient number of cycles until the desired degree of accuracy is achieved.

The final end moments are calculated using equation (1).

EXAMPLES:

Ex.1:

Analyze the continuous beam shown in fig.



Solution:

a) <u>Fixed end moments:</u>

$$M_{FAB} = \frac{b(3a-1)}{l^2}M_o = \frac{2.5(3\times1.5-4)}{4^2}\times 24 = 1.88 \text{ kNm}$$

$$M_{FBA} = \frac{a(3b-1)}{l^2}M_o = \frac{1.5(3\times2.5-4)}{4^2}\times 24 = 7.88 \text{ kNm}$$

$$M_{FBC} = \frac{M_o}{4} = \frac{32}{4} = 8 \text{ kNm}$$

$$M_{FCB} = \frac{M_o}{4} = 8 \text{ kNm}$$

$$M_{FCD} = -\frac{36\times1\times2^2}{3^2} = -16 \text{ kNm}$$

$$M_{FDC} = \frac{36\times1^2\times2}{3^2} = 8 \text{ kNm}$$

b) Modification in fixed end moments:

Actually end 'D' is a simply supported. Hence moment at D should be zero. To make moment at D as zero apply –8 kNm at D and perform the corresponding carry over to CD.

Modified $M_{FDC} = 8 - 8 = 0$

Modified $M_{FCD} = -16 + \frac{1}{2}(-8) = -20$ kNm

Now joint D will not enter the iteration process.

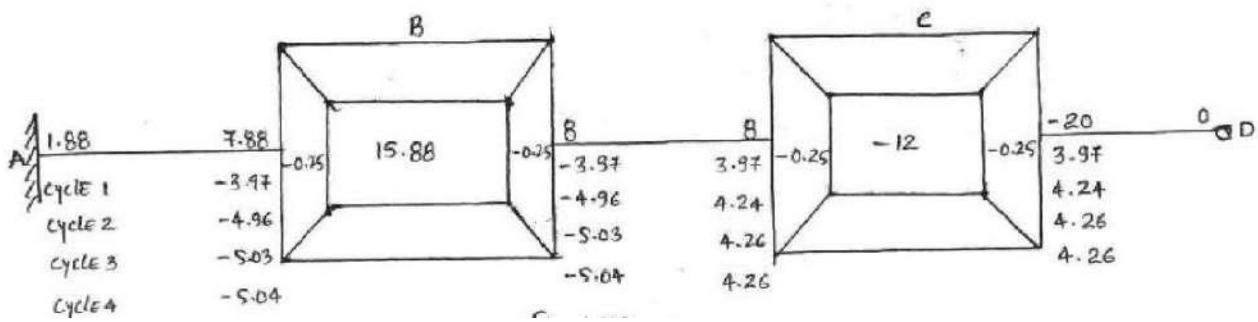
c) Rotation Factors:

Joint	Member	Relative stiffness (K)	ΣK	Rotation Factor $U = -\frac{1}{2} x \frac{K}{\sum K}$
B	BA	I/4 = 0.25I	0.5 I	– 0.25
	BC	I/4 = 0.25I		–0.25
C	CB	I/4 = 0.25I		–0.25
	CD	$\frac{3}{4} \times \frac{I}{3} = 0.25I$	0.5I	–0.25

d) Sum of fixed end moments at joints:

$\Sigma M_{FB} = 7.88 + 8 = 15.88$ kNm

$\Sigma M_{FC} = 8 - 20 = -12$kNm



e) Iteration process

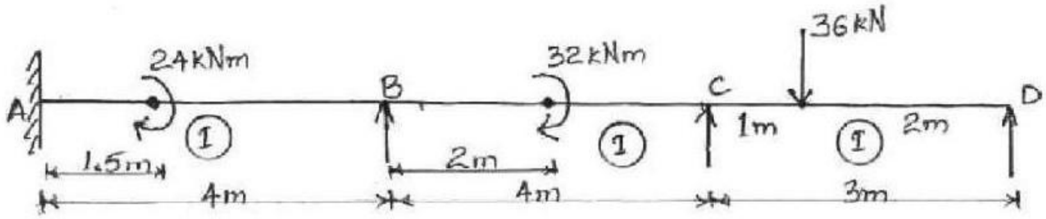
Joint	B		C	
Rotation Contribution	M'_{BA}	M'_{BC}	M'_{CB}	M'_{CD}
Rotation factor	-0.25	-0.25	-0.25	-0.25
Iteration 1 started at B assuming $M'_{CB} = 0$ & taking $M'_{AB} = 0'$ $M'_{DC} = 0.$	$-0.25 \times (15.88 + 0 + 0) = -3.97$	$-0.25 \times (15.88 + 0 + 0) = -3.97$	$-0.25 \times (-12 -3.97 + 0) = 3.97$	$-0.25 \times (-12 -3.97 + 0) = 3.97$
Iteration 2	$-0.25 \times (15.88 + 0 + 3.97) = -4.96$	$-0.25 \times (15.88 + 0 + 3.97) = -4.96$	$-0.25 \times (-12 -4.96 + 0) = 4.24$	$-0.25 (-12 -4.96 + 0) = 4.24$
Iteration 3	$-0.25 (15.88 + 0 + 4.24) = -5.03$	$-0.25 (15.88 + 0 + 4.24) = -5.03$	$-0.25 \times (-12 -5.03 + 0) = 4.26$	$-0.25 (-12 -5.03 + 0) = 4.26$
Iteration 4	$-0.25 (15.88 + 0 + 4.26) = -5.04$	$-0.25 (15.88 + 0 + 4.26) = -5.04$	$-0.25 \times (-12 -5.03 + 0) = 4.26$	$-0.25 (-12 -5.03 + 0) = 4.26$

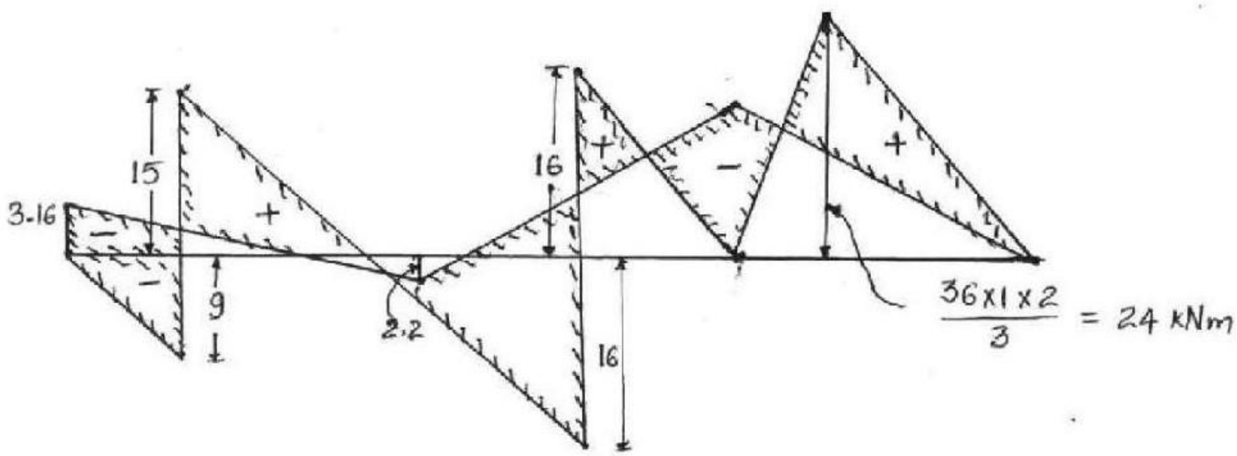
Iteration process has been stopped after 4th cycle since rotation contribution values are becoming almost constant. Values of fixed end moments, sum of fixed end moments, rotation factors along with rotation contribution values after end of each cycle in appropriate places has been shown in fig. 4 (b).

f) Final moments

Member (ij)	**FEM M_{Fij} (kNm)**	$2M'_{ij}$ **(kNm)**	M^1_{ji} **(kNm)**	**Final moments** $\left(= M_{Fij} + 2M'_{ij} + M'_{ji}\right)$ **(kNm)**
AB	1.88	0	-5.04	-3.16
BA	7.88	2 (-5.04) = 10.08	0	-2.2
BC	8	2 (-504) = −10.08	4.26	+2.2
CB	8	2 × 4.26 = 8.52	-5.04	11.48
CD	-20	2 x 4.26 = 8.52	0	-11.48

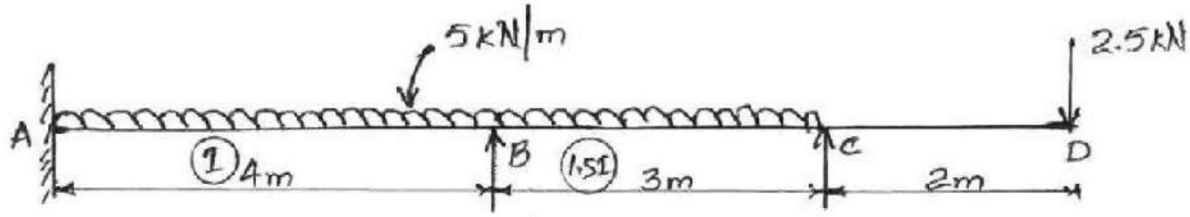
BMD is shown below:





Ex.2:

Analyze the continuous beam shown in fig.



Solution:

a) Fixed end moments:

$$M_{FAB} = -\frac{5\times4^2}{12} = -6.67 \text{ kNm}$$

$$M_{FBA} = +6.67 \text{ kNm}$$

$$M_{FBC} = \frac{-5\times3^2}{12} = -3.75 \text{ kNm}$$

$$M_{FCB} = +3.75 \text{ kNm}$$

$$M_{CD} = -2.5 \times 2 = -5 \text{ kNm}$$

b) Modification in fixed end moments:

Since $M_{CD} = -5$ kNm; $M_{CB} = +5$kNm, for this add 1.25 kNm to M_{FCB} and do the corresponding carry over to M_{FBC}

$\therefore$ Now $M_{CB} = 5$ kNm

Modified $M_{FBC} = -3.75 + \frac{1}{2}(1.25) = -3.13$ kNm

Now joint C will not enter in the iteration process.

c) Rotation factors:

Jt.	Member	Relative stiffness (K)	ΣK	Rotation Factor $U = -\frac{1}{2} x \frac{K}{\sum K}$
B	BA	I/4 = 0.25I		− 0.2
	BC	$\frac{3}{4} \times \frac{1.5I}{3} = 0.375I$	0.625I	−0.3
C	CB	1.5I/3 = 0.5I		−0.5
	CD	0	0.5I	0

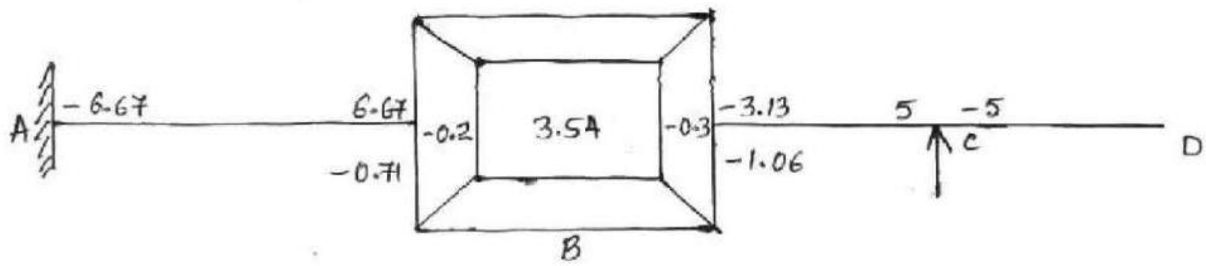
d) Sum of fixed end moments at joints:

$\Sigma M_{FB} = 6.67 - 3.13 = 3.54$ kNm

e) Iteration Process

Joint	B	
Rotation Contribution	M'_{BA} (kNm)	M'_{BC} (kNm)
Rotation factor	− 0.2	−0.3
Iteration 1 started at B taking $M'_{AB} = 0$ & $M'_{CB} = 0$	$-0.2 \times (3.54 + 0 + 0) = -0.71$	$-0.3 \times (3.54 + 0 + 0) = -1.06$

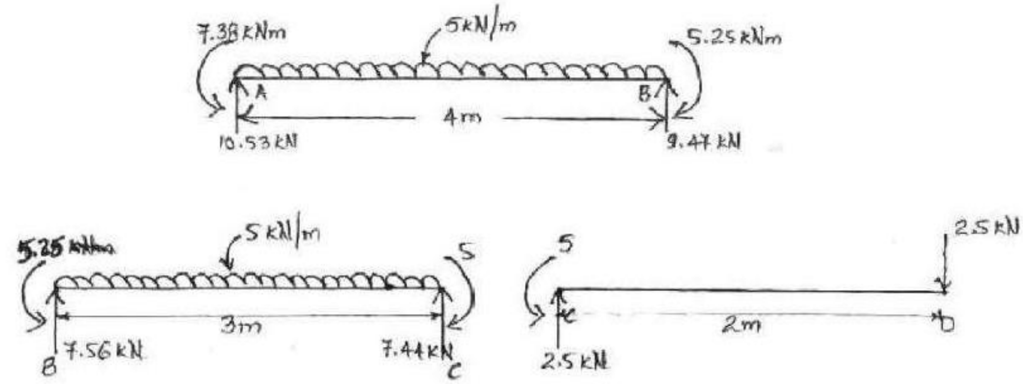
Since 'B' is the only joint needing rotation correction, the iteration process will stop after first iteration. Value of FEMs, sum of FEM at joint, rotation factors along with rotation contribution values in appropriate places is shown in fig. 5 (b)



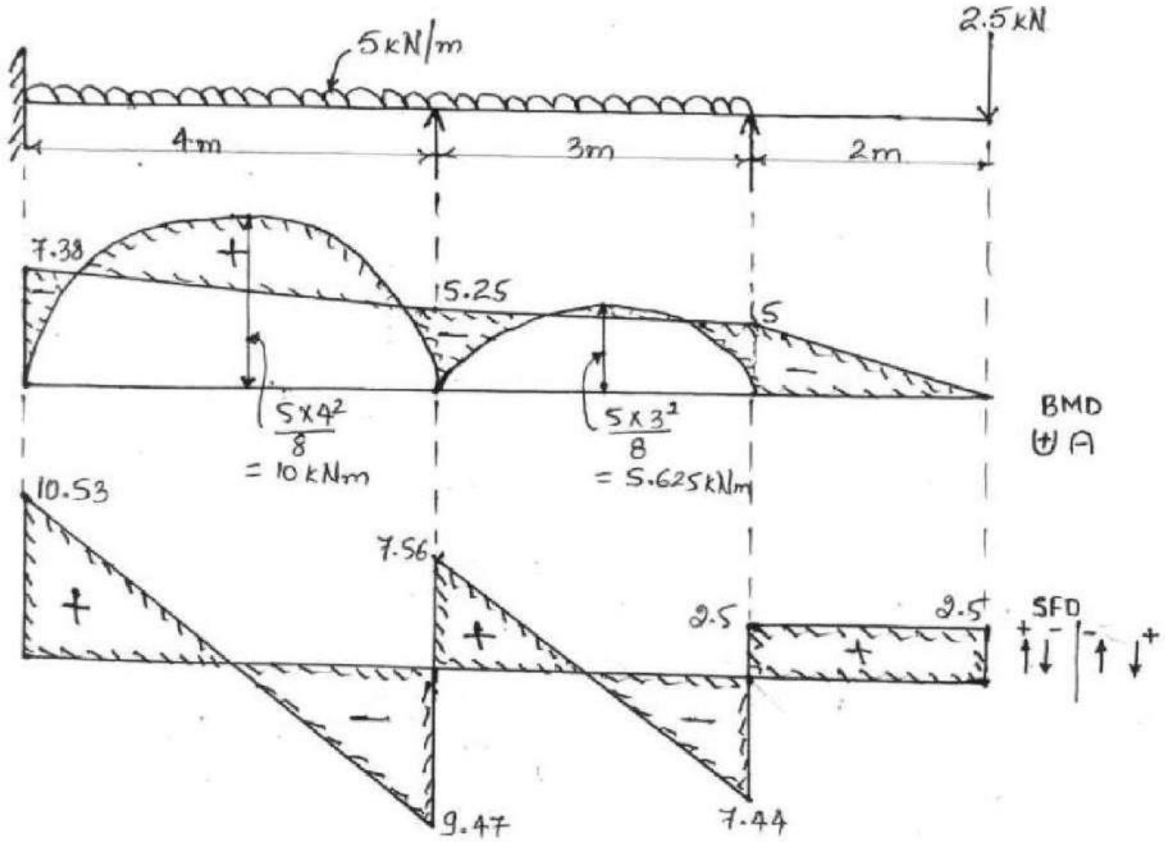
(f) Final moments:

Member (ij)	FEM M_{Fij} (kNm)	$2M'_{ij}$ (kNm)	M^1_{ji} (kNm)	Final moments $\left(= M_{Fij} + 2M'_{ij} + M'_{ji}\right)$ (kNm)
AB	− 6.67	0	−0.71	−7.38
BA	6.67	2 × (-0.71) =	0	5.25
BC	−3.13	2 × (−1.06)	0	−5.25
CB				+ 5
CD	-	-	-	− 5
DC				0

FBD of each span along with reaction values which have been calculated from statics are shown below:

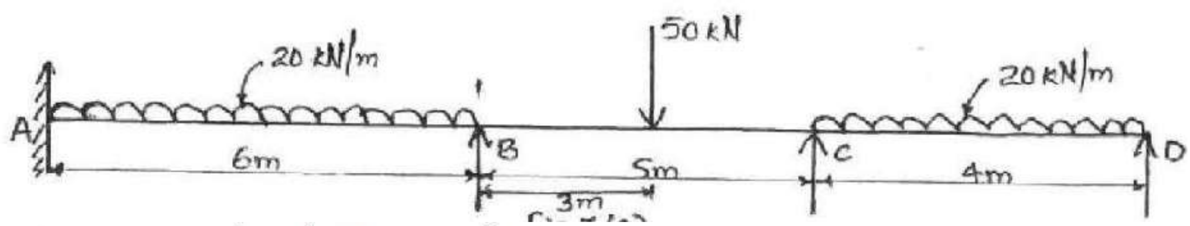


BMD and SFD are shown below



Ex.2:

In a continuous beam shown in fig. The support 'B' sinks by 10mm. Determine the moments by Kani's method & draw BMD.



Take I = 1.2 x 10^{-4} mm^4 & E = 2 x 10^5 N/mm^2

Solution:

(a) Calculation of FEM:

$$M_{FAB} = -\frac{20\times6^2}{12} - \frac{6\times2\times10^5\times1.2\times10^{-4}\times10^{12}\times10}{(6000)^2\times10^6}$$

$= -60 - 40$

$= -100$ kNm

M_{FBA} = +60 −40= 20 kNm

$$M_{FBC} = -\frac{50\times3\times2^2}{5^2} + \frac{6\times 2\times10^5 \times1.2\times10^{-4}\times10^{12}\times10}{(5000)^2\times10^6}$$

$= -24 + 57.6$

$= 33.6$ kNm

$$M_{FCB} = +\frac{50\times3^2\times2}{5^2} + \frac{6\times2\times10^5 \times1.2\times10^{-4}\times10^{12}\times10}{(5000)^2\times10^6}$$

$= 36 + 57.6$

$= 93.6$ kNm

C & D are at same level

$$\therefore M_{FCD} = -\frac{20\times4^2}{12} = -26.67 \text{ kNm}$$

$M_{FDC} = +26.67$ kNm

b) Modification in fixed end moments:

Since end 'D' is a simply supported, moment at 'D' is zero. To make moment at D as zero apply a moment of – 26.67 kNm at end D and perform the corresponding carry over to CD.

$\therefore$ Modified $M_{FDC} = +26.67 - 26.67 = 0$

Modified $M_{FCD} = -26.67 + \frac{1}{2}(-26.67)$

$= -40$ kNm

Other FEMs will be same as calculated earlier. Now joint 'D' will not enter the iteration process.

c) Rotation factors:

Joint	Member	Relative stiffness (K)	ΣK	Rotation Factor $U = -\frac{1}{2}\times\frac{K}{\sum K}$
B	BA	I/6 = 0.17 I	0.37 I	−0.23
	BC	I/5 = 0.2 I		−0.27
C	CB	I/5 = 0.2I	0.39I	−0.26
	CD	$\frac{3}{4}$ x I/4 = 0.19 I		−0.24

d) Sum of fixed end moments:

$$\Sigma M_{FB} = 20 + 33.6 = 53.6 \text{ kNm}$$
$$\Sigma M_{FC} = 93.6 - 40 = 53.6 \text{ kNm}$$

e) Iteration process:

Joint	B		C	
Rotation Contribution	M'_{BA} (kNm)	M'_{BC} (kNm)	M'_{CB} (kNm)	M'_{CD} (kNm)
Rotation factor	−0.23	−0.27	−0.26	−0.24
Iteration 1 (Started at B by taking $M'_{AB} = 0$ and assuming $M'_{CB} = 0$	−0.23 × (53.6 + 0 + 0) = −12.33	−0.27 x (53.6 + 0+0)= −14.47	−0.26×(53.6 – 14.47+ 0) = −10.17	−0.24 (53.6 − 14.47) 10.96= −9.39
Iteration 2	−0.23 (53.6 – 10.17) = −10.00	−0.27 (53.6 −10.17) = −11.73	−0.26 (53.6 −11.73) = −10.89	−0.24 (53.6 −11.73) =−10.05
Iteration 3	−0.23 (53.6−10.89) = −9.82	−0.27 (53.6 −10.89) =−11.53	−0.26 (53.6 −11.53) = −10.94	−0.24 (53.6 −11.53) =−10.10
Iteration 4	−0.23 (53.6 −10.94) = −9.81	−0.27 (53.6 −10.94) = −11.52	−0.26 (53.6 −11.52) = −10.94	−0.24 (53.6 −11.52) = −10.1

Iteration process has been stopped after fourth cycle since rotation contribution values are becoming almost constant. Values of FEMs, sum of fixed end moments, rotation factors along with rotation contribution values after end of each cycle in appropriate places has been shown in Fig. 7 (b).

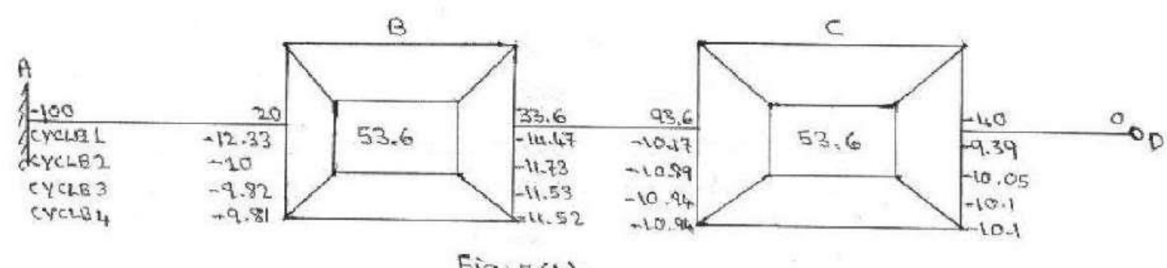
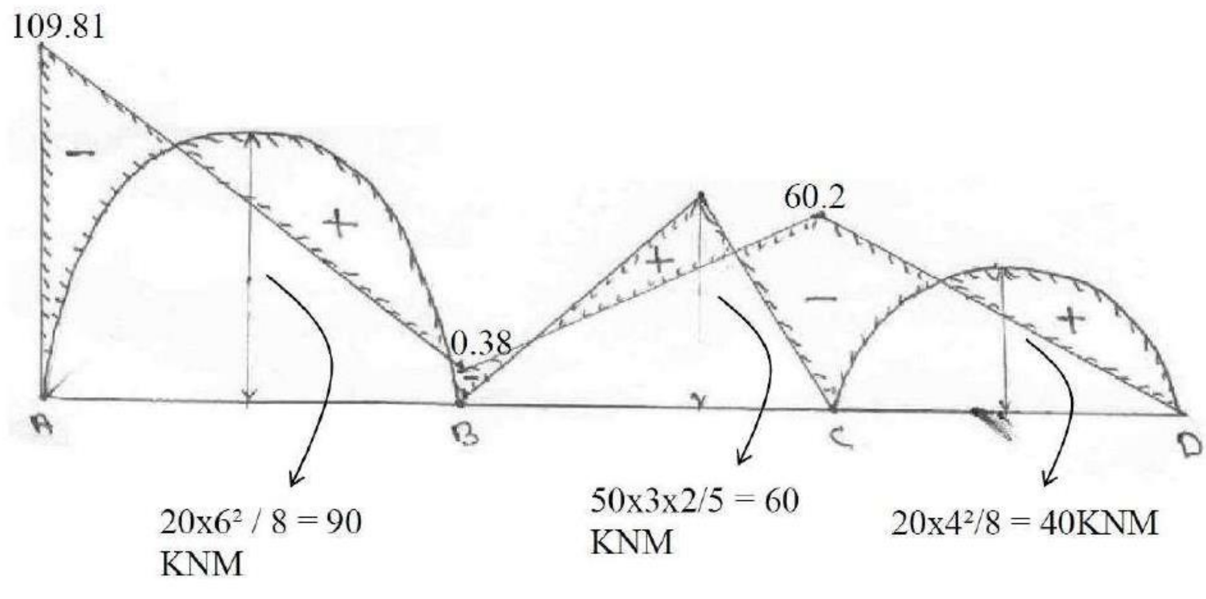


Fig: 7(b)

f) Final moments:

Member (ij)	**FEM M_{Fij} (kNm)**	$2M'_{ij}$ **(kNm)**	M^1_{ji} **(kNm)**	**Final moments** $\left(= M_{Fij} + 2M'_{ij} + M'_{ji}\right)$ **(kNm)**
AB	− 100	0	−9.81	−109.81
BA	20	2 × (-9.81)= -19.62	0	+0.38
BC	33.6	2 × (−11.52) = −23.04	−10.94	−0.38
CB	93.6	2 × (−10.94) = −21.88	−11.52	60.2
CD	−40	2 × (−10.1) = −20.2	0	−60.2
DC	0	0	0	0

g) BMD is shown below:



ANALYSIS OF FRAMES WITH NO TRANSLATION OF JOINTS

The frames, in which lateral translations are prevented, are analyzed in the same way as continuous beams. The lateral sway is prevented either due to symmetry of frame and loading or due to support conditions. The procedure is illustrated in following example.

Example-5. Analyze the frame shown in Figure 8 (a) by Kani's method. Draw BMD.

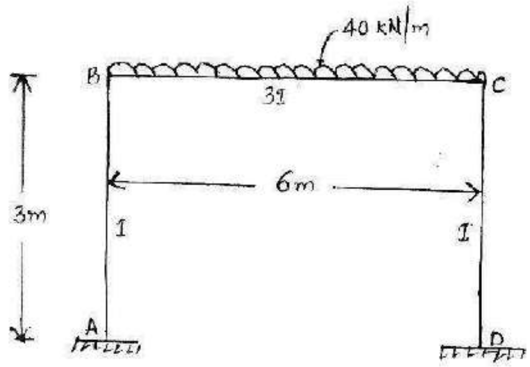


Fig-8(a)

Solution:

(a) Fixed endmoments:

$$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$$

$$M_{FBC} = \frac{-40x6^2}{12} = -120kNm.$$

$$M_{FCB} = +120kNm.$$

(b) Rotationfactors:

Joint	Member	Relative Stiffness (k)	?k	Rotation factor =-½k/?
B	BC	3I/6 = 0.5I	0.83I	-0.3
	BA	I/3 = 0.33I		-0.2
C	CB	3I/6 = 0.5I	0.83I	-0.3
	CD	I/3 = 0.33I		-0.2

(c) Sum of FEM:

$$\Sigma M_{FB} = -120+0 = -120$$

$$\Sigma M_{FC} = 120+0 = +120$$

(d) **Iteration process:**

Joint	B		C	
Rotation Contribution	**M'BA**	**M'BC**	**M'CB**	**M'CD**
Rotation Factor	**-0.2**	**-0.3**	**-0.3**	**-0.2**
Iteration 1 Stated with end B taking M'AB=0 and Assuming M'CB=0	-0.2(-120+0) =24	-0.3(-120+0) =36	-0.2(120+36+0) = -46.8	-0.2(120+36+0) = -31.2
Iteration 2	-0.2(-120-46.8) =33.6	-0.3(-120-46.8) =50.04	-0.3(120+50.04) = -51.01	-0.2(120+50.04) = -34.01
Iteration 3	-0.2(-120-51.01) =34.2	-0.3(-120-51.01) =51.3	-0.3(120+51.3) = -51.39	-0.2 (120+51.3) = -34.26
Iteration 4	-0.2(-120-51.39) =34.28	-0.3(-120-51.39) =51.42	-0.3(120+51.42) = -51.43	-0.2 (120+51.42) = -34.28

The fixed end moments, sum of fixed and moments, rotation factors along with rotation contribution values at the end of each cycle in appropriate places is shown in figure 8(b).

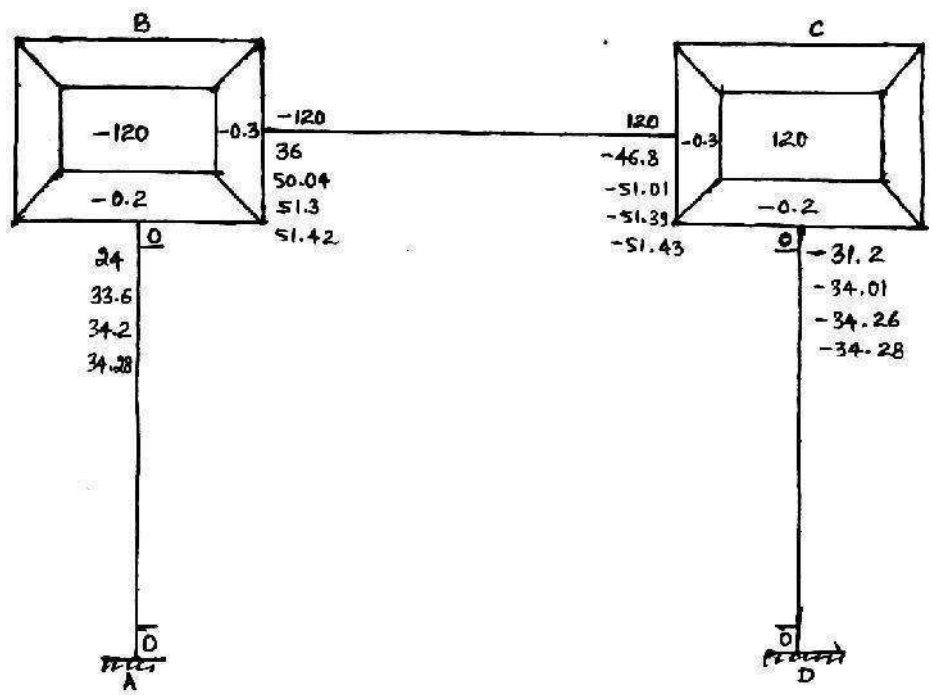
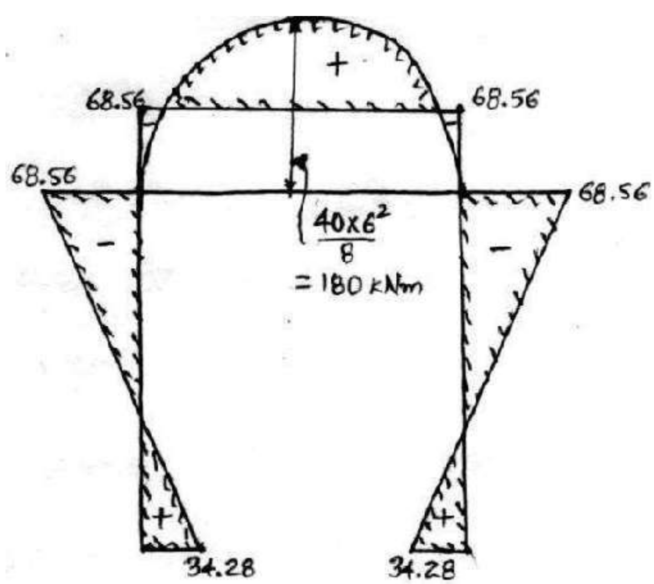


Fig-8(b)

(e) **Final moments:**

Member (ij)	M_{Fij}	$2M'_{ij}$(kNm)	M'_{ji}(kNm)	(kNm) Final moment = $M_{Fij}+ 2M'_{ij}+ M'_{ji}$
AB	0	0	34.28	34.28
BA	0	2 x 34.28	0	68.56
BC	-120	2 x 51.42	-51.43	-68.59
CB	120	2 x (-51.43)	51.42	68.56
CD	0	2 x (-34.28)	0	-68.56
DC	0	0	-34.28	-34.28

BMD is shown below in figure-8 (c)



PART - B CABLES AND BRIDGES

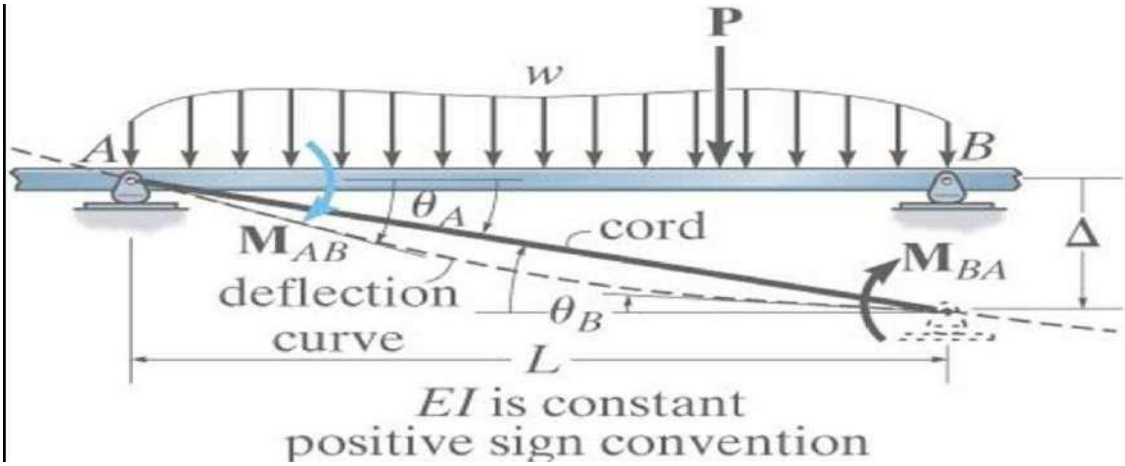
BY USING THE SLOPE DEFLECTION METHOD

In the slope-deflection method, the relationship is established between moments at the ends of the members and the corresponding rotations and displacements.

The slope-deflection method can be used to analyze statically determinate and indeterminate beams and frames. In this method it is assumed that all deformations are due to bending only. In other words deformations due to axial forces are neglected. In the force method of analysis compatibility equations are written in terms of unknown reactions. It must be noted that all the unknown reactions appear in each of the compatibility equations making it difficult to solve resulting equations. The slope-deflection equations are not that lengthy in comparison. The basic idea of the slope deflection method is to write the equilibrium equations for each node in terms of the deflections and rotations. Solve for the generalized displacements. Using moment-displacement relations, moments are then known. The structure is thus reduced to a determinate structure. The slope-deflection method was originally developed by Heinrich Manderla and Otto Mohr for computing secondary stresses in trusses. The method as used today was presented by G.A.Maney in 1915 for analyzing rigid jointed structures.

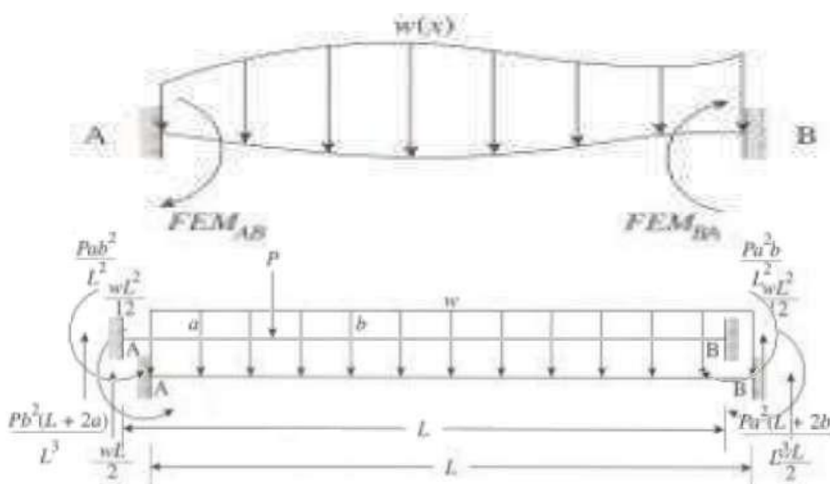
FUNDAMENTAL SLOPE-DEFLECTION EQUATIONS:

The slope deflection method is so named as it relates the unknown slopes and deflections to the applied load on a structure. In order to develop general form of slope deflection equations, we will consider the typical span AB of a continuous beam which is subjected to arbitrary loading and has a constant EI. We wish to relate the beams internal end moments in terms of its three degrees of freedom, namely its angular displacements and linear displacement which could be caused by relative settlements between the supports. Since we will be developing a formula, moments and angular displacements will be considered positive, when they act clockwise on the span. The linear displacement will be considered positive since this displacement causes the chord of the span and the span's chord angle to rotate clockwise. The slope deflection equations can be obtained by using principle of superposition by considering separatelythemomentsdevelopedateachsupportsduetoeachofthedisplacements θ_A& θ_B & Δ and then the load.



Case A: fixed-end moments

$M_{AB} = FEM_{AB}$, $M_{BA} = FEM_{BA}$

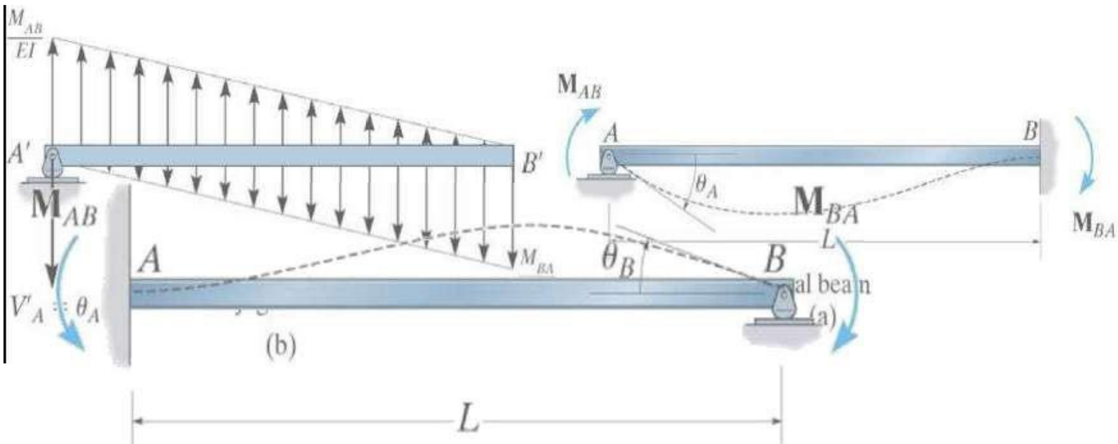


$FEM_{AB} = -\frac{wL^2}{12}$, $FEM_{BA} = \frac{wL^2}{12}$

$FEM_{AB} = -\frac{Pab^2}{L^2}$, $FEM_{BA} = \frac{Pa^2b}{L^2}$

Case B: rotation at A, (angular displacement at A)

Consider node A of the member as shown in figure to rotate while its far end B is fixed. To determine the moment needed to cause the displacement, we will use conjugate beam method. The end shear at A acts downwards on the beam since is clockwise.



$$\Sigma M'_A = 0, \quad \left[\frac{1}{2}\frac{M_{AB}}{EI}L\right]\frac{L}{3} - \left[\frac{1}{2}\frac{M_{BA}}{EI}L\right]\frac{2L}{3} = 0$$

$$\Sigma M'_B = 0, \quad \left[\frac{1}{2}\frac{M_{BA}}{EI}L\right]\frac{L}{3} - \left[\frac{1}{2}\frac{M_{BA}}{EI}L\right]\frac{2L}{3} + \theta_A L = 0$$

$$M_{AB} = \frac{4EI}{L}\theta_A,\ M_{BA} = \frac{2EI}{L}\theta_A$$

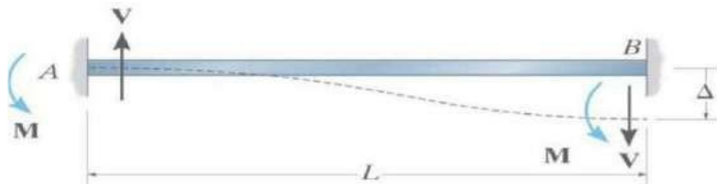
Case C: rotationatB, (angular displacement atB)

In a similar manner if the end B of the beam rotates to its final position, while end A is held fixed. We can relate the applied moment to the angular displacement and the reaction moment

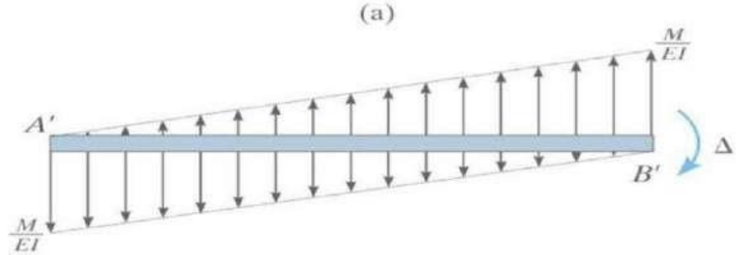
$$M_{AB} = \frac{2EI}{L}\theta_B,\ M_{BA} = \frac{4EI}{L}\theta_B$$

Case D: displacement of end B related to end A

If the far node B of the member is displaced relative to A so that so that the chord of the member rotates clockwise (positive displacement) .The moment M can be related to displacement by using conjugate beam method. The conjugate beam is free at both the ends as the real beam is fixed supported. Due to displacement of the real beam at B, the momentat the end B of the conjugate beam must have a magnitude of .Summing moments about B we have,



real beam
(a)



conjugate beam
(b)

$$\Sigma M_B = 0, \quad \left[\frac{1}{2}\frac{M}{EI}(L)\right]\frac{2L}{3} - \left[\frac{1}{2}\frac{M}{EI}(L)\right]\frac{L}{3} - \Delta = 0$$

$$M_{AB} = M_{BA} = M = -\frac{6EI}{L^2}\Delta$$

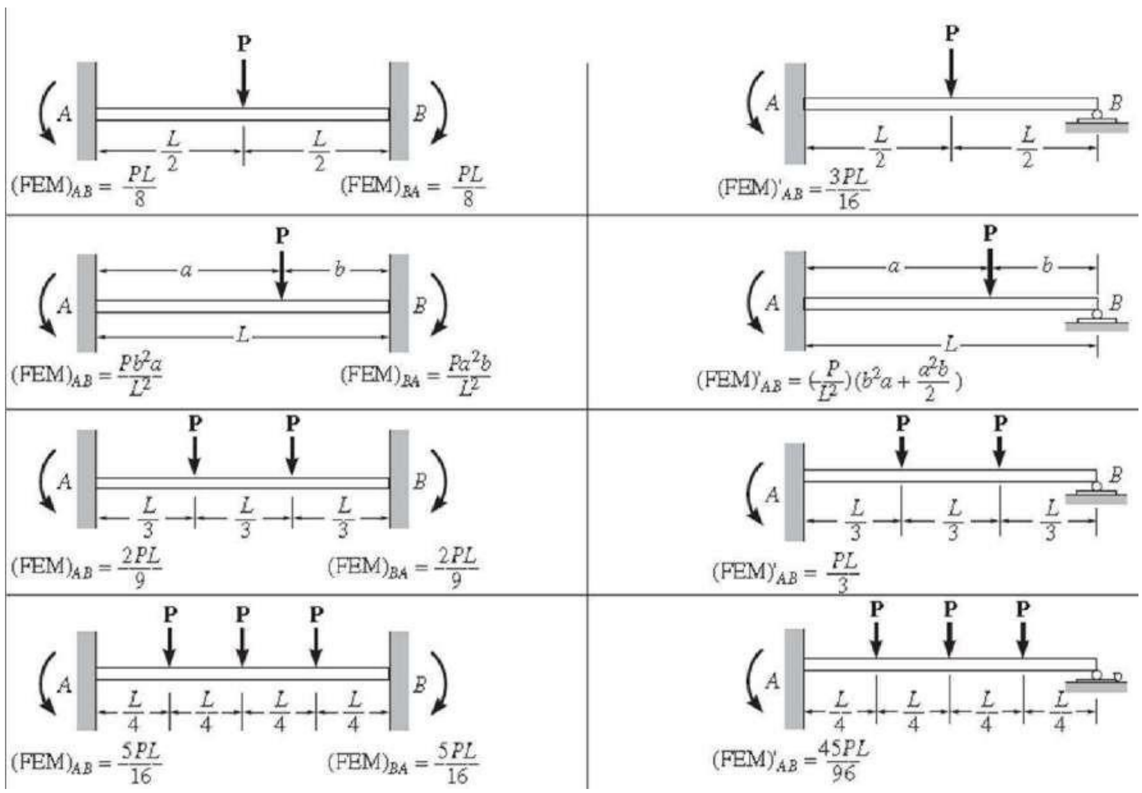
By our sign convention the induced moment is negative, since for equilibrium it acts counter clockwise on the member.

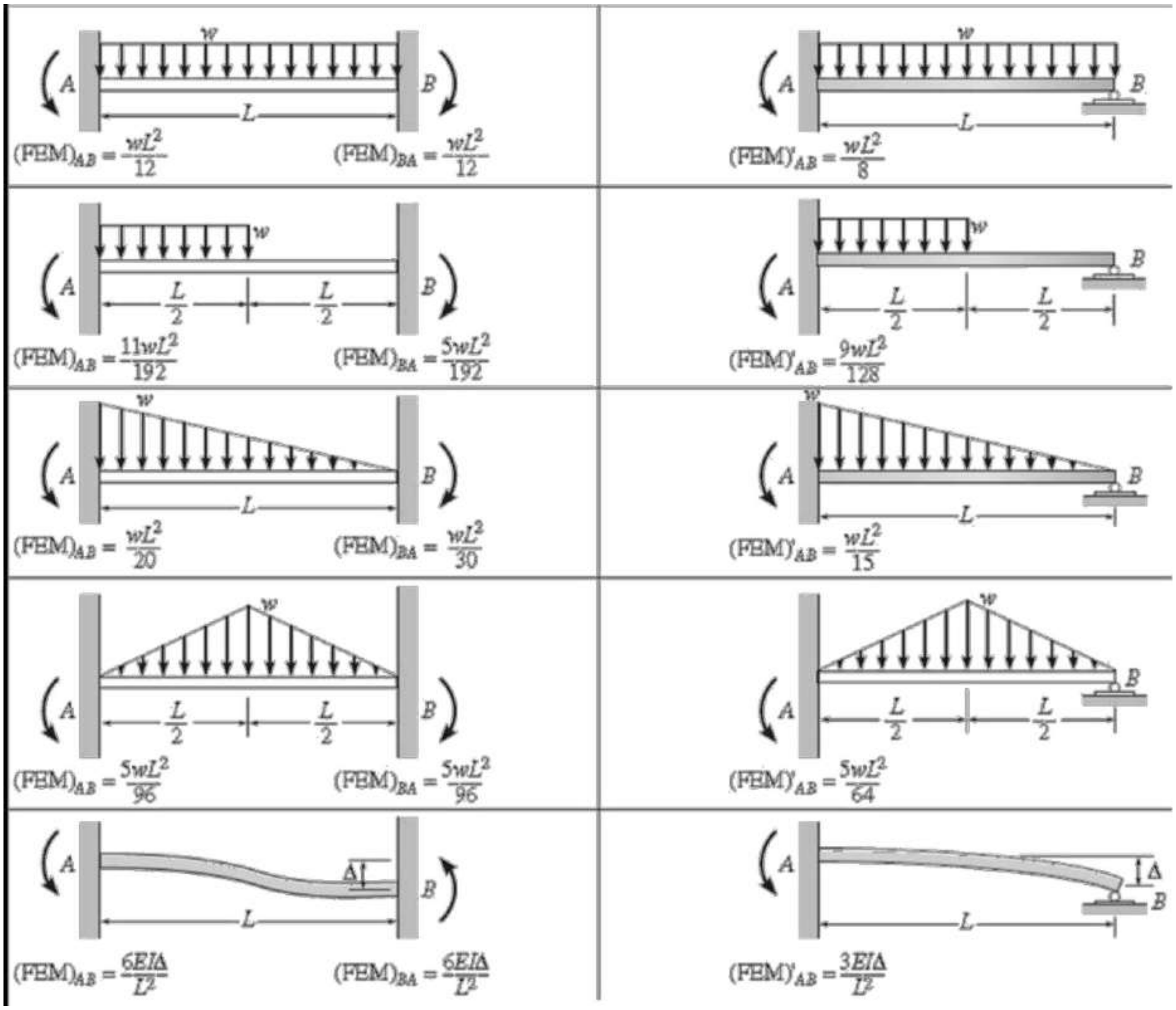
If the end moments due to the loadings and each displacements are added together, then the resultant moments at the ends can be writtenas,

$$M_{AB} = \frac{2EI}{L}[2\theta_A + \theta_B - \frac{3\Delta}{L}] + FEM_{AB}$$

$$M_{BA} = \frac{2EI}{L}[2\theta_B + \theta_A - \frac{3\Delta}{L}] + FEM_{BA}$$

FIXED END MOMENT TABLE





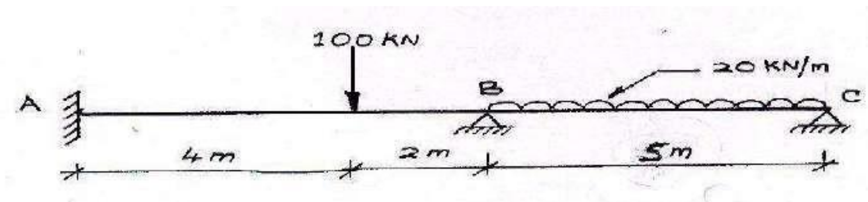
w
A
B
L
$(FEM)_{AB} = \frac{wL^2}{12}$
$(FEM)_{BA} = \frac{wL^2}{12}$
w
A
B
L
$(FEM)'_{AB} = \frac{wL^2}{8}$
w
A
B
$\frac{L}{2}$
$\frac{L}{2}$
$(FEM)_{AB} = \frac{11wL^2}{192}$
$(FEM)_{BA} = \frac{5wL^2}{192}$
w
A
B
$\frac{L}{2}$
$\frac{L}{2}$
$(FEM)'_{AB} = \frac{9wL^2}{128}$
w
A
B
L
$(FEM)_{AB} = \frac{wL^2}{20}$
$(FEM)_{BA} = \frac{wL^2}{30}$
w
A
B
L
$(FEM)'_{AB} = \frac{wL^2}{15}$
w
A
B
$\frac{L}{2}$
$\frac{L}{2}$
$(FEM)_{AB} = \frac{5wL^2}{96}$
$(FEM)_{BA} = \frac{5wL^2}{96}$
w
A
B
$\frac{L}{2}$
$\frac{L}{2}$
$(FEM)'_{AB} = \frac{5wL^2}{64}$
A
Δ
B
L
$(FEM)_{AB} = \frac{6EI\Delta}{L^2}$
$(FEM)_{BA} = \frac{6EI\Delta}{L^2}$
A
Δ
B
L
$(FEM)'_{AB} = \frac{3EI\Delta}{L^2}$

GENERAL PROCEDURE OF SLOPE-DEFLECTION METHOD

- Find the fixed end moments of each span (both ends left &right).
- Apply the slope deflection equation on each span & identify the unknowns.
- Write down the joint equilibrium equations.
- Solve the equilibrium equations to get the unknown rotation & deflections.
- Determine the end moments and then treat each span as simply supported beam subjected to given load & end moments so we can work out the reactions & draw the bending moment & shear force diagram.

Numerical Examples

1. Q. Analyze two span continuous beam ABC by slope deflection method.Then draw Bending moment & Shear force diagram. Take EIconstant.



Fixed end moments are

$$M_{AB=} -\frac{Wab^2}{L^2} = -\frac{100 \times 4 \times 2^2}{6^2} = -44.44 \text{ KNm}$$

$$M_{BA=} \frac{Wa^2b}{L^2} = \frac{100 \times 4^2 \times 2}{6^2} = 88.89\text{KNm}$$

$$M_{BC=} -\frac{wL^2}{12} = -\frac{20 \times 5^2}{12} = -41.67\text{KNm}$$

$$M_{CB=} \frac{wL^2}{12} = \frac{20 \times 5^2}{12} = 41.67 \text{ KNm}$$

Since A is fixed $\Theta_A = 0 \ \& \ \Theta_B \& \ \Theta_c \neq 0$

Slope deflection equations are

$$M_{AB} = M_{FAB} + \frac{2EI}{L}[2\theta_A + \theta_B] = -44.44 + \frac{2EI}{6}\theta_B = -44.44 + \frac{EI}{3}\theta_B \quad \cdots\cdots(1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}[2\theta_B + \theta_A] = 88.89 + \frac{4EI}{6}\theta_B = 88.89 + \frac{2EI}{3}\theta_B \quad \cdots\cdots(2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}[2\theta_B + \theta_C] = -41.67 + \frac{4EI}{5}\theta_B + \frac{2EI}{5}\theta_C \quad \cdots\cdots(3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}[2\theta_C + \theta_B] = 41.67 + \frac{4EI}{5}\theta_C + \frac{2EI}{5}\theta_B \quad \cdots\cdots(4)$$

In all the above 4 equations there are only2unknowns θ_B& θ_C and accordinglythe boundary

conditions are

$$M_{BA} + M_{BC} = 0$$

$M_{CB} = 0$ as end C is simply supported.

Solving the equations (5) & (6), we get

$$\theta_B = -\frac{20.83}{EI}$$

$$\theta_C = -\frac{41.67}{EI}$$

Substituting the values in the slope deflections we have,

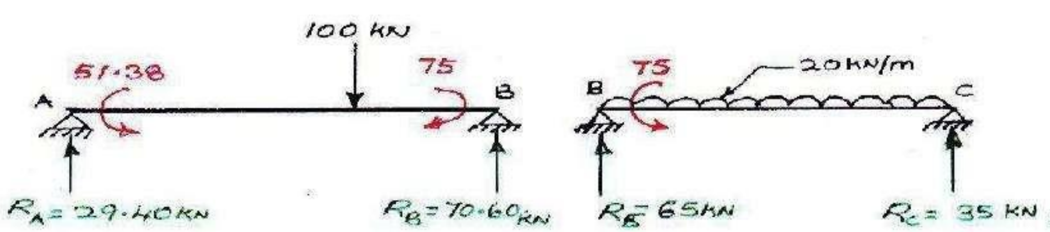
$$M_{AB} = -44.44 + \frac{EI}{3}\left(-\frac{20.83}{EI}\right) = -51.38 \text{ KNm}$$

$$M_{BA} = 88.89 + \frac{2EI}{3}\left(-\frac{20.83}{EI}\right) = 75 \text{ KNm}$$

$$M_{BC} = -41.67 + \frac{4EI}{5}\left(-\frac{20.83}{EI}\right) + \frac{2EI}{5}\left(-\frac{41.67}{EI}\right) = -75 \text{ KNm}$$

$$M_{CB} = 41.67 + \frac{4EI}{5}\left(-\frac{41.67}{EI}\right) + \frac{2EI}{5}\left(-\frac{20.83}{EI}\right) = 0$$

Reactions: Consider the free body diagram of the beam



Find reactions using equations of equilibrium.

Span AB: $M_A = 0$, $R_B\times 6 = 100\times 4+75-51.38$

$R_B = 70.60$ KN

$V = 0$, $R_A+R_B = 100$KN

$R_A = 100-70.60=29.40$ KN

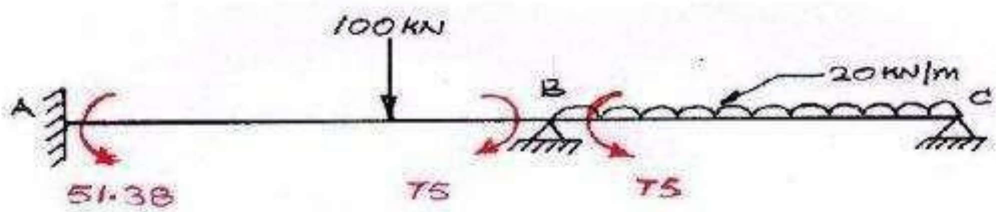
Span BC: $M_C = 0$, $R_B\times 5 = 20\times 5\times \pm 75$

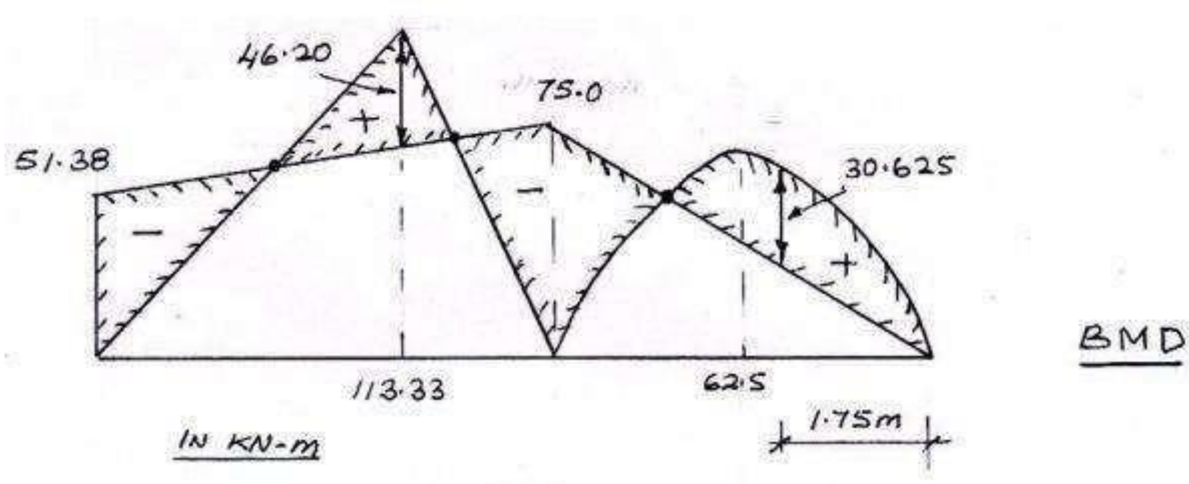
$R_B = 65$ KN

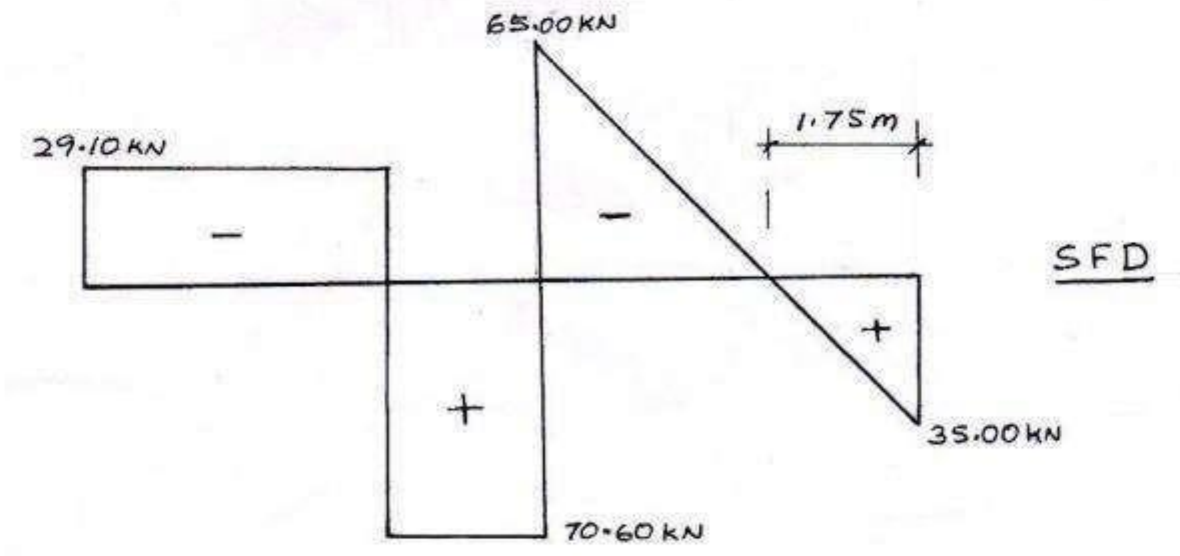
$V=0$ $R_B+R_C = 20\times 5 = 100$KN

$R_C = 100-65 = 35$ KN

Using these data BM and SF diagram can be drawn







Max BM:

Span AB: Max BM in span AB occurs under point load and can be found geometrically,

$$M_{max}=113.33-51.38-\left(\frac{75-51.38}{6}\right)\times 4=46.20KNm$$

Span BC: Max BM in span BC occurs where shear force is zero or changes its sign. Henceconsider SF equation w.r.t C

$S_x = 35\text{-}20x = 0$

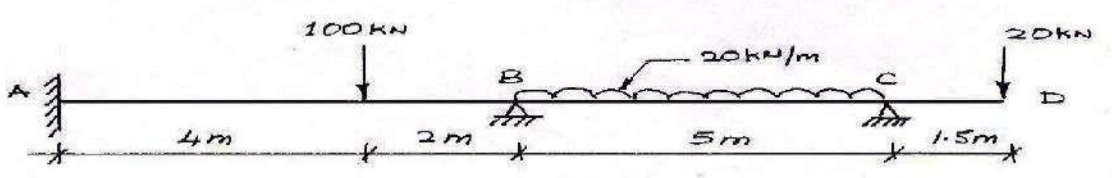
$x = \frac{35}{20}$=1.75m

—

Max BM occurs at 1.75m
from

$$\therefore M_{max} = 35 \times 1.75 - 20\times \frac{1.75^2}{2} = 30.625\ KNm$$

2. Q. Analyze continuous beam ABCD by slope deflection method and thendraw bending moment diagram. Take EIconstant.



$$\theta_A = 0 \ \& \ \theta_B \ \& \ \theta_c \neq 0$$

$$M_{AB=} -\frac{Wab^2}{L^2} = -\frac{100 \times 4 \times 2^2}{6^2} = -44.44 \text{ KNm}$$

$$M_{BA=} \frac{Wa^2b}{L^2} = \frac{100 \times 4^2 \times 2}{6^2} = 88.89\text{KNm}$$

$$M_{BC=} -\frac{wL^2}{12} = -\frac{20 \times 5^2}{12} = -41.67\text{KNm}$$

$$M_{CB=} \frac{wL^2}{12} = \frac{20 \times 5^2}{12} = 41.67 \text{ KNm}$$

$$M_{CD} = -20 \times 1.5 = -30 \text{ KNm}$$

Slope deflection equations are

$$M_{AB} = M_{FAB} + \frac{2EI}{L}[2\theta_A + \theta_B] = -44.44 + \frac{2EI}{6}\theta_B = -44.44 + \frac{EI}{3}\theta_B \quad \cdots\cdots(1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}[2\theta_B + \theta_A] = 88.89 + \frac{4EI}{6}\theta_B = 88.89 + \frac{2EI}{3}\theta_B \quad \cdots\cdots(2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}[2\theta_B + \theta_C] = -41.67 + \frac{4EI}{5}\theta_B + \frac{2EI}{5}\theta_C \quad \cdots\cdots(3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}[2\theta_C + \theta_B] = 41.67 + \frac{4EI}{5}\theta_C + \frac{2EI}{5}\theta_B \quad \cdots\cdots(4)$$

$$M_{CD} = -20 \times 1.5 = -30 \text{ KNm}$$

In all the above equations there are only 2 unknowns and accordingly the boundary conditions are

$$M_{BA} + M_{BC} = 0$$
$$M_{CB} + M_{CD} = 0$$

$$M_{BA} + M_{BC} = 88.89 + \frac{2EI}{3}\theta_B - 41.67 + \frac{4EI}{5}\theta_B + \frac{2EI}{5}\theta_C = 47.22 + \frac{22}{15}EI\theta_B + \frac{2}{5}EI\theta_C = 0 \quad \ldots\ldots \quad (5)$$

$$M_{CB} + M_{CD} = 41.67 + \frac{4EI}{5}\theta_C + \frac{2EI}{5}\theta_B - 30 = 11.67 + \frac{2EI}{5}\theta_B + \frac{4EI}{5}\theta_C = 0 \quad \ldots\ldots \quad (6)$$

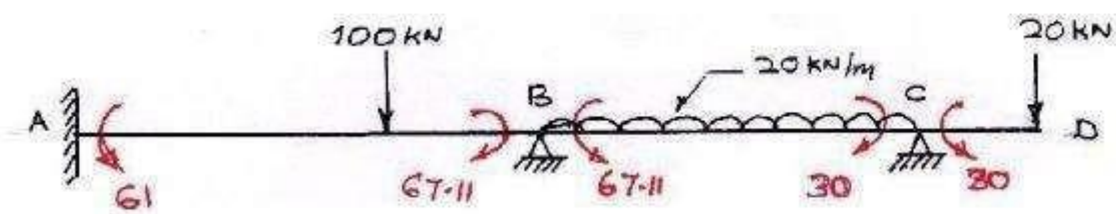
Solving equations (5) & (6),

$$\theta_B = -\frac{32.67}{EI}$$
$$\theta_C = \frac{1.75}{EI}$$

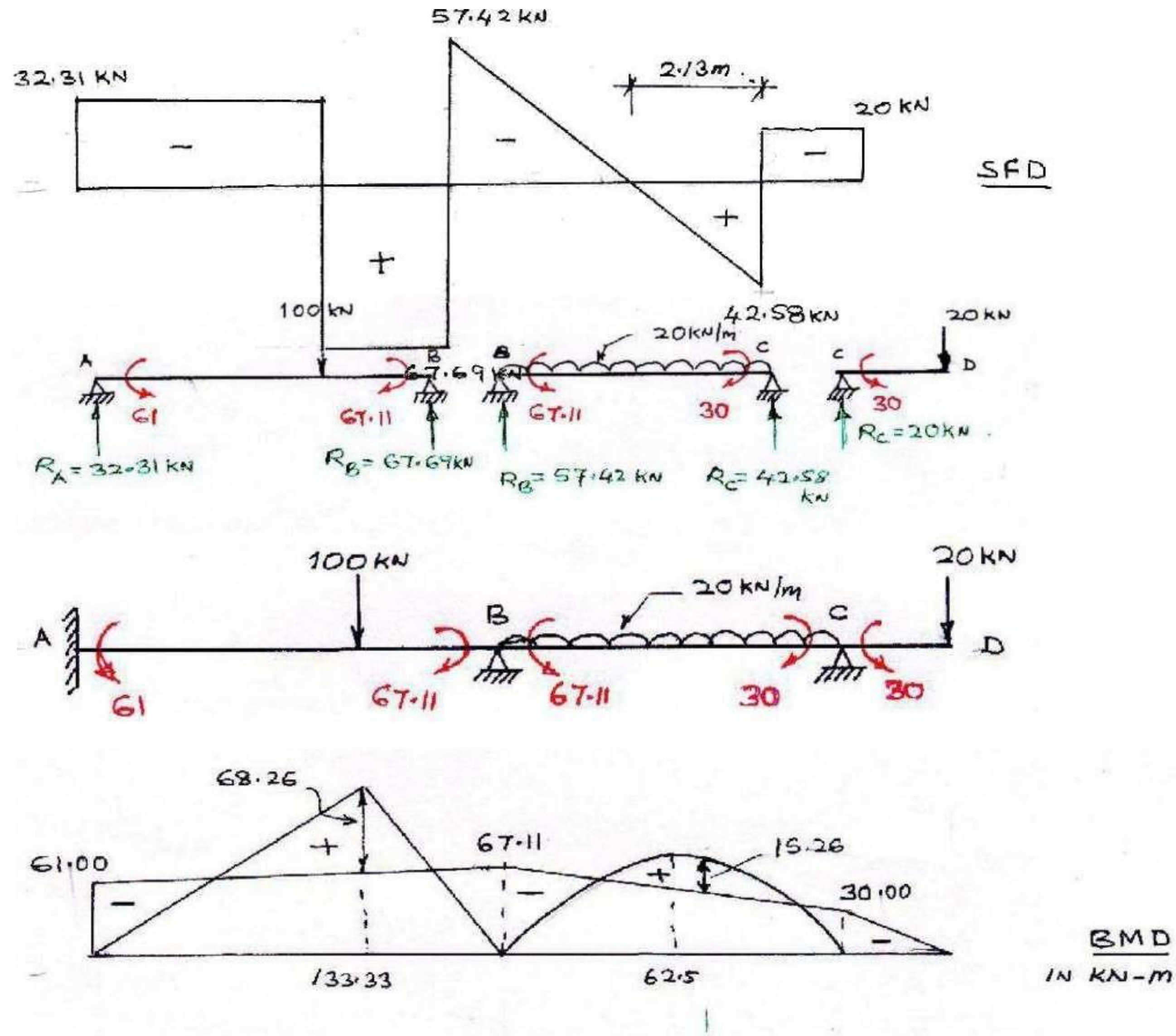
Substituting the values in the slope deflections we have,

$$M_{AB} = -44.44 + \frac{EI}{3} \times \left(-\frac{32.67}{EI}\right) = -61 \text{ KNm}$$

$$M_{BA} = 88.89 + \frac{2EI}{3} \times \left(-\frac{32.67}{EI}\right) = 67.11\text{KNm}$$
$$M_{BC} = -41.67 + \frac{4EI}{5}\left(-\frac{32.67}{EI}\right) + \frac{2EI}{5}\left(\frac{1.75}{EI}\right) = -67.11\text{KNm}$$
$$M_{CB} = 41.67 + \frac{4EI}{5}\left(\frac{1.75}{EI}\right) + \frac{2EI}{5}\left(-\frac{32.67}{EI}\right) = 30\text{KNm}$$
$$M_{CD} = -30\text{KNm}$$



Reactions: Consider free body diagram of beam AB, BC and CD as shown



Span AB:

$R_B \times 6 = 100 \times 4 + 67.11 - 61$

$R_B = 67.69KN$

$R_A = 100 - R_B = 32.31$ KN

Span BC:

$R_C \times 5 = 20 \times \frac{5}{2} \times 5 + 30 - 67.11$

$R_C = 42.58$ KN

$R_B = 20 \times 5 - R_C = 57.42KN$

Maximum Bending Moments:

Span AB: Occurs under point load

$M_{max}= 133.33 - 61 - \frac{67.11-61}{6} \times 4 = 68.26KNm$

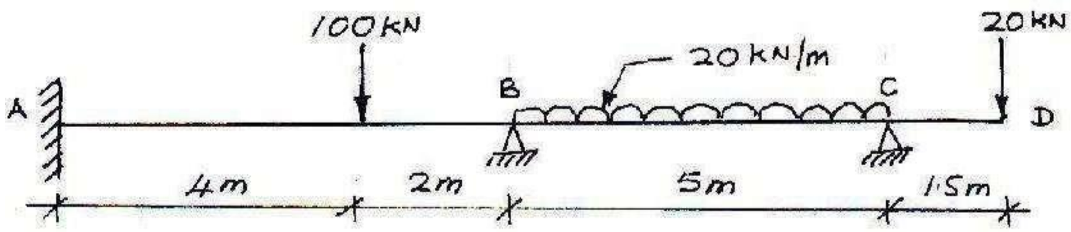
Span BC: Where SF=0, consider SF equation with C as reference

$S_x = 42.58\text{-}20x = 0$

$x = \frac{42.58}{20} = 2.13m$

$M_{max}= 42.58 \times 2.13 - 20 \times \frac{2.13^2}{2} - 30 = 15.26KNm$

3. Q. Analyse the continuous beam ABCD shown in figure by slope deflectionmethod. The support B sinks by 15mm.Take E =200 10^5 KN/m^2 and I =12010 $^{-6}$$m^4$



A. $\Theta_A = 0 \,\&\, \Theta_B \& \Theta_c \neq 0 \;\Delta = 15mm$

$$M_{AB=} -\frac{Wab^2}{L^2} = -\frac{100 \times 4 \times 2^2}{6^2} = -44.44\,KNm$$

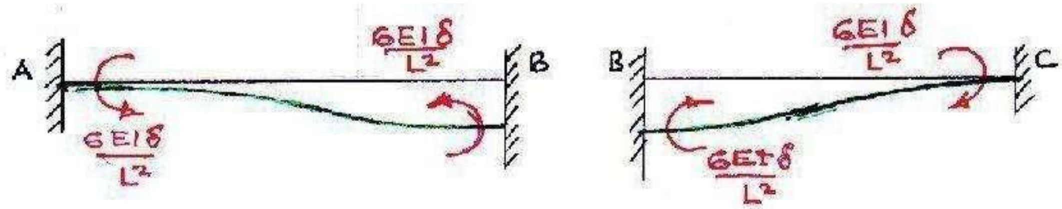
$$M_{BA=} \frac{Wa^2b}{L^2} = \frac{100 \times 4^2 \times 2}{6^2} = 88.89KNm$$

$$M_{BC=} -\frac{wL^2}{12} = -\frac{20 \times 5^2}{12} = -41.67KNm$$

$$M_{CB=} \frac{wL^2}{12} = \frac{20 \times 5^2}{12} = 41.67\,KNm$$

$$M_{CD} = -20 \times 1.5 = -30\,KNm$$

FEM due to yield of support B



For span AB:

$$M_{AB} = M_{BA} = -\frac{6EI}{L^2}\Delta = -\frac{6\times200\times10^5\times120\times10^{-6}}{6^2}\frac{15}{1000} = -6KNm$$

For span BC:

$$M_{BC} = M_{CB} = \frac{6EI}{L^2}\Delta = \frac{6\times200\times10^5\times120\times10^{-6}}{5^2}\frac{15}{1000} = 8.64KNm$$

Slope deflection equations are

$$M_{AB} = M_{FAB} + \frac{2EI}{6}\left[2\theta_A + \theta_B - 3\frac{\Delta}{6}\right] = -44.44 + \frac{EI}{3}\theta_B - 6 = -50.44 + \frac{EI}{3}\theta_B \cdots\cdots (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{6}\left[2\theta_B + \theta_A - 3\frac{\Delta}{6}\right] = 88.89 + \frac{2EI}{3}\theta_B - 6 = 82.89 + \frac{2EI}{3}\theta_B \cdots\cdots (2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{5}\left[2\theta_B + \theta_C + 3\frac{\Delta}{5}\right] = -41.67 + \frac{4EI}{5}\theta_B + \frac{2EI}{5}\theta_C + 8.64$$

$$= -33.03 + \frac{4EI}{5}\theta_B + \frac{2EI}{5}\theta_C \cdots\cdots (3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{5}\left[2\theta_C + \theta_B + 3\frac{\Delta}{5}\right] = 41.67 + \frac{4EI}{5}\theta_C + \frac{2EI}{5}\theta_B + 8.64$$

$$= 50.31 + \frac{4EI}{5}\theta_C + \frac{2EI}{5}\theta_B \cdots\cdots (4)$$

$$M_{CD} = -20 \times 1.5 = -30\ KNm$$

In all the above equations there are only2unknowns and accordinglythe boundary

conditions are

$$M_{BA} + M_{BC} = 0$$
$$M_{CB} + M_{CD} = 0$$

$$M_{BA} + M_{BC} = 82.89 + \frac{2EI}{3}\theta_B - 33.03 + \frac{4EI}{5}\theta_B + \frac{2EI}{5}\theta_C = 49.86 + \frac{22}{15}EI\theta_B + \frac{2}{5}EI\theta_C = 0 \cdots\cdots (5)$$

$$M_{CB} + M_{CD} = 50.31 + \frac{4EI}{5}\theta_C + \frac{2EI}{5}\theta_B - 30 = 20.31 + \frac{2EI}{5}\theta_B + \frac{4EI}{5}\theta_C = 0 \cdots\cdots (6)$$

Solving equations (5) & (6),

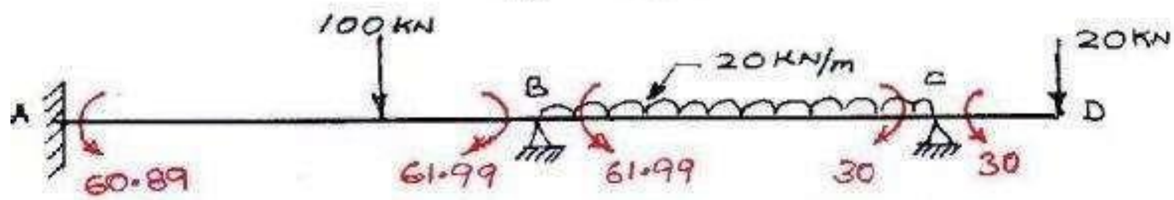
$$\theta_B = -\frac{31.35}{EI}$$
$$\theta_C = -\frac{9.71}{EI}$$

Substituting the values in the slope deflections we have,

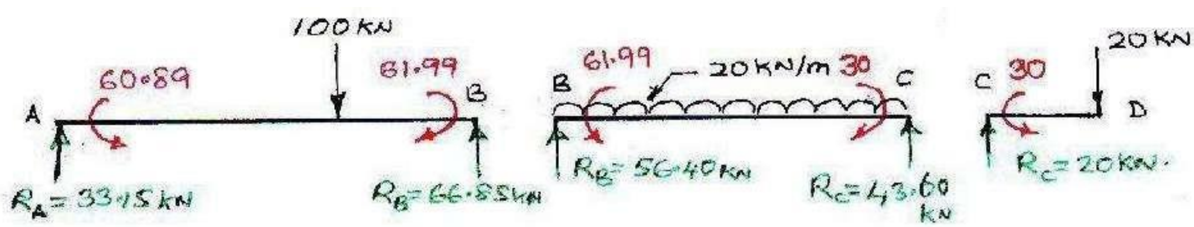
$$M_{AB} = -50.44 + \frac{EI}{3} \times \left(-\frac{31.35}{EI}\right) = -60.89 \text{ KNm}$$

$$M_{BA} = 82.89 + \frac{2EI}{3} \times \left(-\frac{31.35}{EI}\right) = 61.99KNm$$

$$M_{BC} = -33.03 + \frac{4EI}{5}\left(-\frac{31.35}{EI}\right) + \frac{2EI}{5}\left(\frac{-9.71}{EI}\right) = -61.99KNm$$
$$M_{CB} = 50.31 + \frac{4EI}{5}\left(\frac{-9.71}{EI}\right) + \frac{2EI}{5}\left(-\frac{31.35}{EI}\right) = 30KNm$$
$$M_{CD} = -30KNm$$



Consider the free body diagram of continuous beam for finding reactions



REACTIONS

Span AB:

$R_B \times 6 = 100 \times 4 + 61.99 - 60.89$

$R_B = 66.85KN$

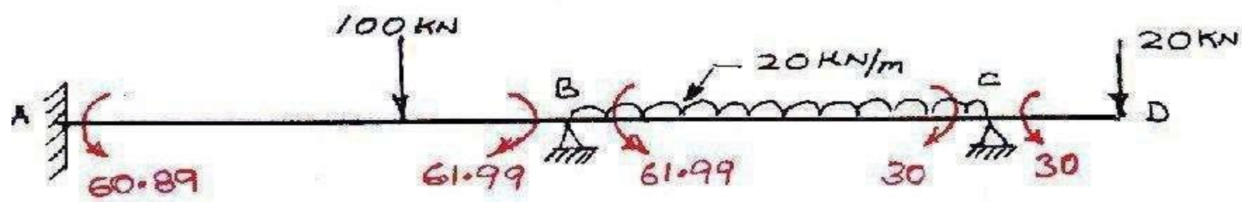
$R_A = 100 - R_B = 33.15\ KN$

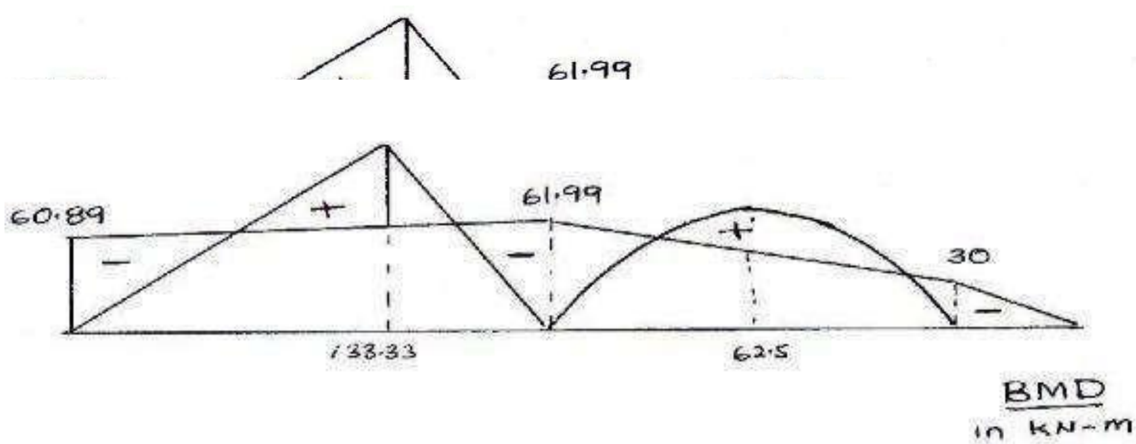
Span BC:

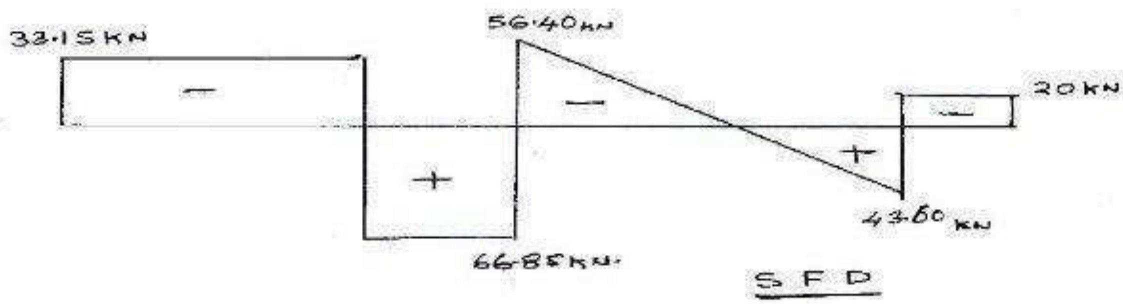
$R_B \times 5 = 20 \times \frac{5}{2} \times 5 + 61.99 - 30$

$R_B = 56.40\ KN$

$R_C = 20 \times 5 - R_B = 43.60KN$

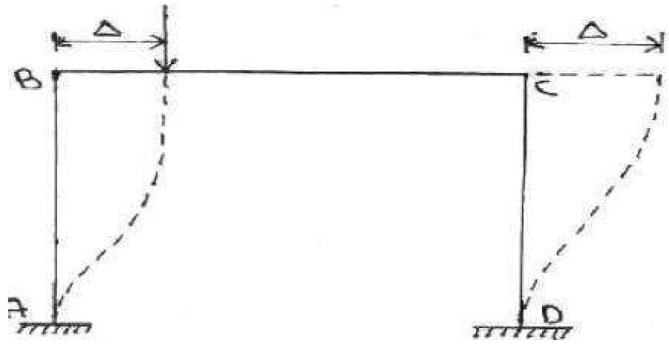




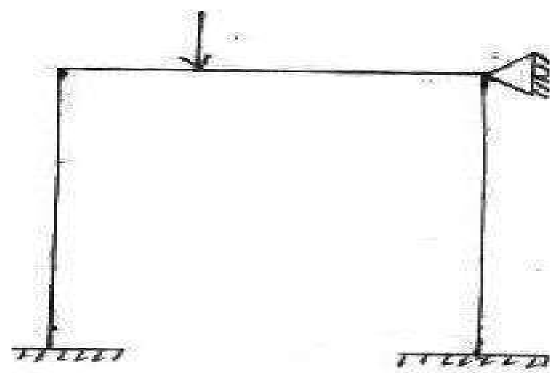


ANALYSIS OF FRAMES (WITHOUT & WITH SWAY)

The side movement of the end of a column in a frame is called sway. Sway can be prevented by unyielding supports provided at the beam level as well as geometric or load symmetry about vertical axis.

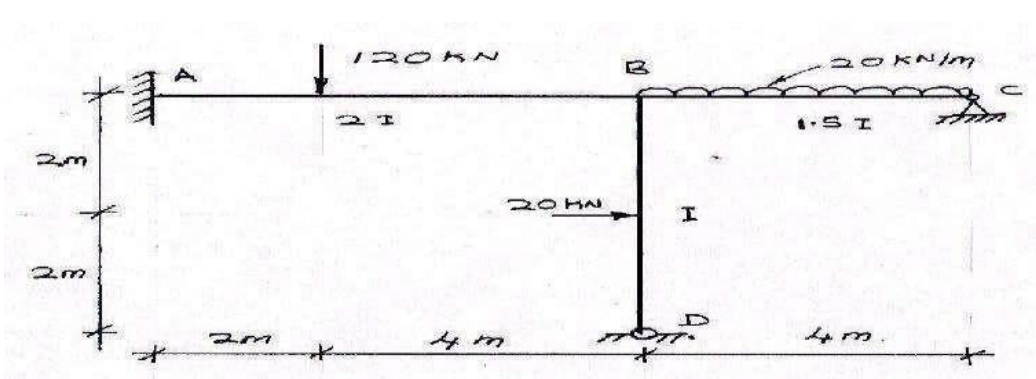


Frame with sway



Sway prevented by unyielding support

4. Q. Analyse the simple frame shown in figure. End A is fixed and ends B & Care hinged. Draw the bending momentdiagram.



$$M_{BA} + M_{BC} + M_{BD} = 0$$
$$M_{CB} = 0$$
$$M_{DB} = 0$$

$M_{BA} + M_{BC} + M_{BD} = 53.33 + \frac{4EI}{3}\theta_B - 26.67 + \frac{3EI}{2}\theta_B + \frac{3EI}{4}\theta_C + 10 + EI\theta_B + \frac{EI}{2}\theta_D =$

$36.66 + \frac{23}{6}EI\theta_B + \frac{3}{4}EI\theta_C + \frac{EI}{2}\theta_D = 0$ …… (7)

$M_{CB} = 26.67 + \frac{3EI}{2}\theta_C + \frac{3EI}{4}\theta_B = 0$ …… (8)

$M_{DB} == -10 + EI\theta_D + \frac{EI}{2}\theta_B = 0$ …… (9)

$$M_{BC=} - \frac{}{12} = \frac{}{12} = -26.67\,KNm$$

$$M_{CB=} \frac{wL^2}{12} = \frac{20 \times 4^2}{12} = 26.67\,KNm$$

$$M_{DB} = -\frac{wL}{8} = -\frac{20 \times 4}{8} = -10\,KNm$$

$$M_{BD} = \frac{wL}{8} = \frac{20 \times 4}{8} = 10\,KNm$$

Slope deflection equations are

$$M_{AB} = M_{FAB} + \frac{2EI}{L}[2\theta_A + \theta_B] = -106.67 + \frac{2E(2I)}{6}\theta_B$$
$$= -106.67 + \frac{2EI}{3}\theta_B \quad \cdots\cdots (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}[2\theta_B + \theta_A] = 53.33 + \frac{2E(2I)}{6}2\theta_B = 53.33 + \frac{4EI}{3}\theta_B \quad \cdots\cdots (2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}[2\theta_B + \theta_C] = -26.67 + \frac{3EI}{2}\theta_B + \frac{3EI}{4}\theta_C \quad \cdots\cdots (3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}[2\theta_C + \theta_B] = 26.67 + \frac{3EI}{2}\theta_C + \frac{3EI}{4}\theta_B \quad \cdots\cdots (4)$$

$$M_{BD} = M_{FBD} + \frac{2EI}{L}[2\theta_B + \theta_D] = 10 + \frac{2EI}{4}2\theta_B + \frac{2EI}{4}\theta_D$$
$$= 10 + EI\theta_B + \frac{EI}{2}\theta_D \quad \cdots\cdots (5)$$

$$M_{DB} = M_{FDB} + \frac{2EI}{L}[2\theta_D + \theta_B] = -10 + \frac{2EI}{4}2\theta_D + \frac{2EI}{4}\theta_B$$
$$= -10 + EI\theta_D + \frac{EI}{2}\theta_B \quad \cdots\cdots (6)$$

In all the above equations there are only 3 unknowns and accordingly the boundary conditions are

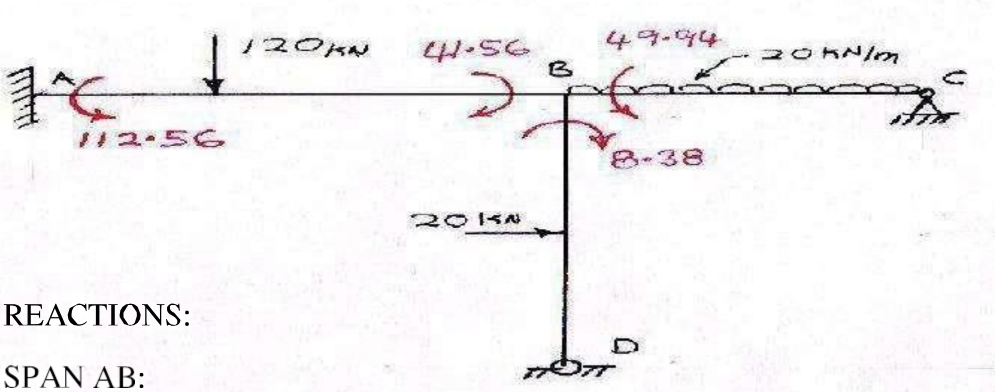
Solving equations (7) & (8) & (9),

$$\theta_B = -\frac{8.83}{EI}$$
$$\theta_C = -\frac{13.36}{EI}$$
$$\theta_D = \frac{14.414}{EI}$$

Substituting the values in the slope deflections we have,

$$M_{AB} = -106.67 + \frac{2}{3}(-8.83) = -112.56KNm$$
$$M_{BA} = 53.33 + \frac{4}{3}(-8.83) = 41.56KNm$$
$$M_{BC} = -26.67 + \frac{3}{2}(-8.83) + \frac{3}{4}(-13.36) = -49.94KNm$$
$$M_{CB} = 26.67 + \frac{3}{2}(-13.36) + \frac{3}{4}(-8.83) = 0$$

$$M_{BD} = 10 - 8.83 + \frac{1}{2}(14.414) = 8.38KNm$$
$$M_{DB} = -10 + \frac{1}{2}(-8.83) + (14.414) = 0$$



REACTIONS:

SPAN AB:

$$R_B = \frac{41.56 - 112.56 + 120 \times 2}{6} = 28.17KN$$
$$R_A = 120 - R_B = 91.83KN$$

SPAN BC:

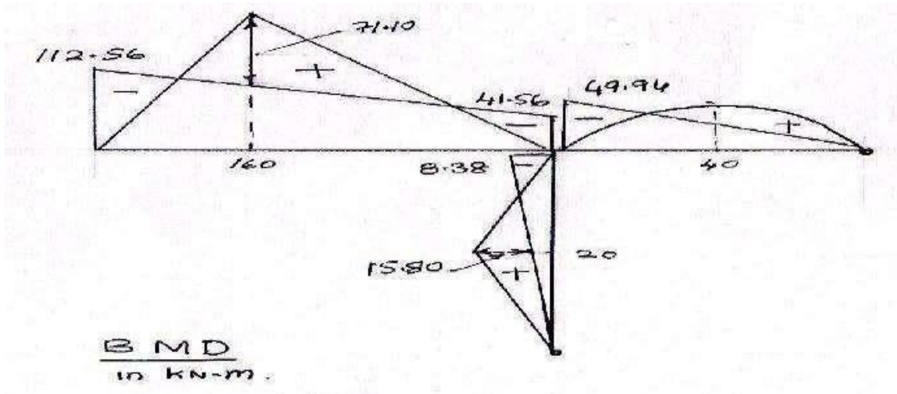
$$R_B = \frac{49.94 + 20 \times 4 \times 2}{4} = 52.485KN$$

$$R_C = 20 \times 4 - R_B = 27.515KN$$

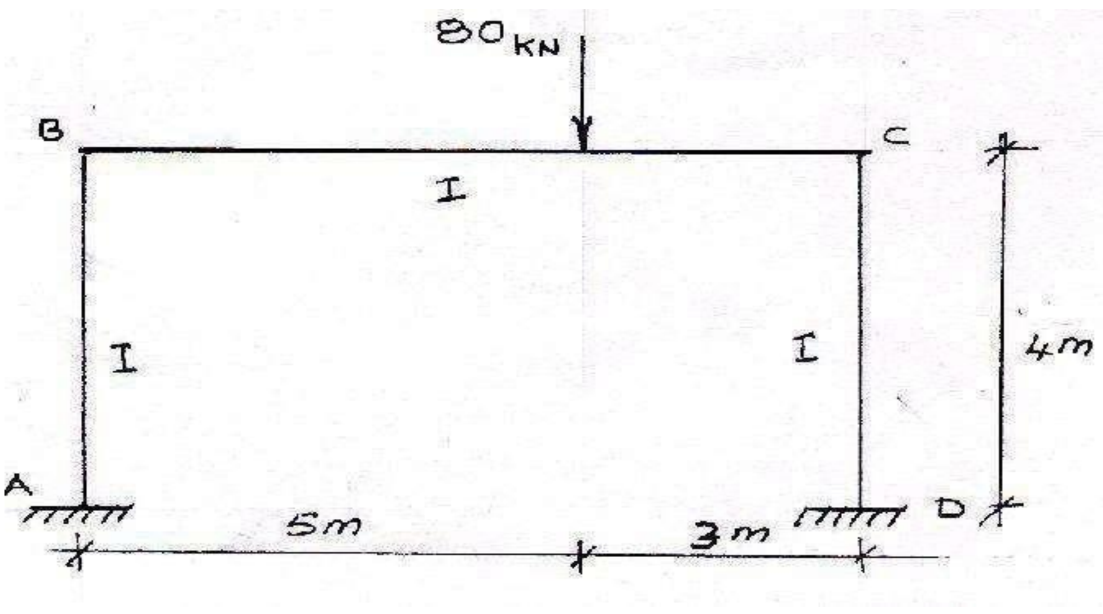
Column BD:

$$H_D = \frac{20 \times 2 - 8.38}{4} = 7.92KN$$

$$H_B = 12.78KN$$



Analyse the portal frame and then draw the bending momentdiagram



A. This is a symmetrical frame and unsymmetrically loaded, thus it is an unsymmetrical problem and there is a sway ,assume sway toright

$$\theta_A = 0, \theta_D = 0, \theta_B \neq 0, \theta_C \neq 0$$

FEMS:

$$M_{BC=} -\frac{wab^2}{L^2} = -\frac{80 \times 5 \times 3^2}{8^2} = -56.25KNm$$

$$M_{CB=} \frac{wa^2b}{L^2} = \frac{80 \times 3 \times 5^2}{8^2} = 93.75\ KNm$$

Slope deflection equations are

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left[2\theta_A + \theta_B - 3\frac{\Delta}{L}\right] = 0 + \frac{2EI}{4}\left(0 + \theta_B - 3\frac{\Delta}{L}\right)$$
$$= \frac{EI}{2}\theta_B - \frac{3EI\Delta}{8} \qquad \cdots\cdots(1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left[2\theta_B + \theta_A - 3\frac{\Delta}{L}\right]$$
$$= 0 + \frac{2EI}{4}\left(2\theta_B + 0 - 3\frac{\Delta}{4}\right) = EI\theta_B - \frac{3EI}{8}\Delta \qquad \cdots\cdots(2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}[2\theta_B + \theta_C] = -56.25 + \frac{2EI}{8}(2\theta_B + \theta_C)$$
$$= -56.25 + \frac{EI}{2}\theta_B + \frac{EI}{4}\theta_C \qquad \cdots\cdots(3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}[2\theta_C + \theta_B] = 93.75 + \frac{2EI}{8}[2\theta_C + \theta_B]$$
$$= 93.75 + \frac{EI}{2}\theta_C + \frac{EI}{4}\theta_B \qquad \cdots\cdots(4)$$

$$M_{CD} = M_{FCD} + \frac{2EI}{L}\left[2\theta_C + \theta_D - 3\frac{\Delta}{L}\right] = 0 + \frac{2EI}{4}\left(2\theta_C + 0 - 3\frac{\Delta}{L}\right)$$
$$= EI\theta_C - \frac{3EI\Delta}{8} \qquad \cdots\cdots(5)$$

$$M_{DC} = M_{FDC} + \frac{2EI}{L}\left[2\theta_D + \theta_C - 3\frac{\Delta}{L}\right]$$
$$= 0 + \frac{2EI}{4}\left(0 + \theta_C - 3\frac{\Delta}{4}\right) = \frac{EI}{2}\theta_C - \frac{3EI}{8}\Delta \qquad \cdots\cdots(6)$$

In the above equation there are three unknowns, θ_B, θ_C& Δ,accordingly the boundary conditions are,joint conditions , $M_{BA} + M_{BC} = 0, M_{CB} + M_{CD} = 0$
shear condition, $H_A + H_D + \sum P_H = 0,\ \frac{M_{AB}+M_{BA}}{4} + \frac{M_{CD}+M_{DC}}{4} = 0$

$$\therefore M_{AB} + M_{BA} + M_{CD} + M_{DC} = 0$$

Now, $M_{BA} + M_{BC} = EI\theta_B - \frac{3EI}{8}\Delta - 56.25 + \frac{EI}{2}\theta_B + \frac{EI}{4}\theta_C = -56.25 + \frac{3EI}{2}\theta_B + \frac{EI}{4}\theta_C - \frac{3EI}{8}\Delta = 0$ ……(7)

$M_{CB} + M_{CD} = 93.75 + \frac{EI}{2}\theta_C + \frac{EI}{4}\theta_B + EI\theta_C - \frac{3EI\Delta}{8} = 93.75 + \frac{EI}{4}\theta_B + \frac{3EI}{2}\theta_C - \frac{3EI\Delta}{8} = 0$ ……(8)

$M_{AB} + M_{BA} + M_{CD} + M_{DC} = \frac{EI}{2}\theta_B - \frac{3EI\Delta}{8} + EI\theta_B - \frac{3EI}{8}\Delta + EI\theta_C - \frac{3EI\Delta}{8} + \frac{EI}{2}\theta_C - \frac{3EI}{8}\Delta = \frac{3EI}{2}\theta_B + \frac{3EI}{2}\theta_C - \frac{3EI}{2}\Delta = 0$

$\therefore EI\Delta = EI\theta_B + EI\theta_C$ ……(9)

Substitute in (7) & (8), equation (9),

$-56.25 + \frac{3EI}{2}\theta_B + \frac{EI}{4}\theta_C - \frac{3EI}{8}(\theta_B + \theta_C) = 0$

$-56.25 + \frac{9EI}{8}\theta_B - \frac{EI}{8}\theta_C = 0$ ……(10)

$93.75 + \frac{EI}{4}\theta_B + \frac{3EI}{2}\theta_C - \frac{3EI}{8}(\theta_B + \theta_C) = 0$

$93.75 - \frac{EI}{8}\theta_B + \frac{9EI}{8}\theta_C = 0$ ……(11)

Solving equations (10) & (11), we get

$$\theta_B = \frac{41.25}{EI}$$

By equation (10),

$$EI\theta_C = 8\left[-56.25 + \frac{9EI}{8}\theta_B\right] = 8\left[-56.25 + \frac{9}{8}(41.25)\right] = -78.75$$

$$\theta_C = \frac{-78.75}{EI}$$

$EI\Delta = EI\theta_B + EI\theta_C = 41.25 - 78.75 = -37.5$

$$\Delta = \frac{-37.5}{EI}$$

Substituting these values in slope deflection equation, we have,

$$M_{AB} = \frac{1}{2}(41.25) - \frac{3}{8}(-37.5) = 34.69KNm$$

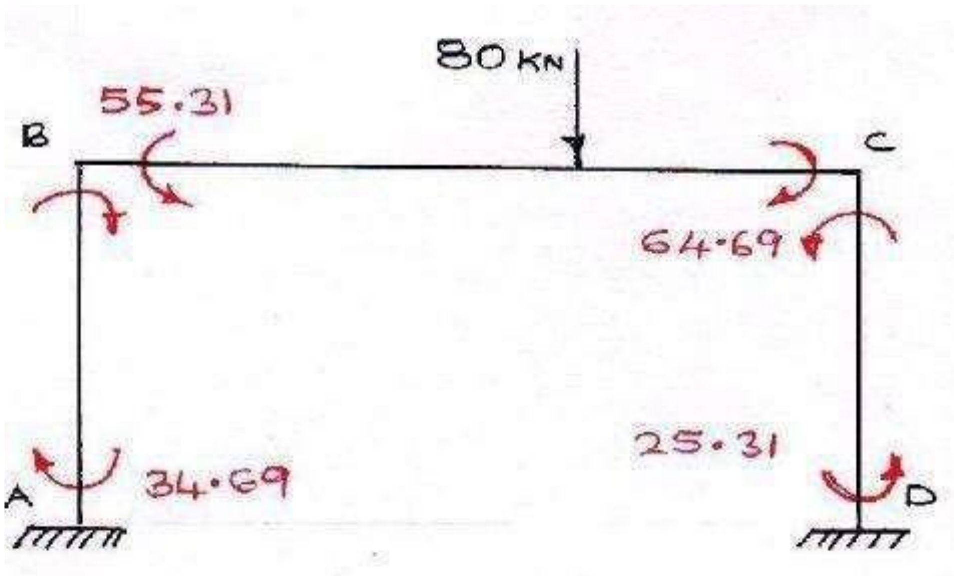
$$M_{BA} = 41.25 - \frac{3}{8}(-37.5) = 55.31KNm$$

$$M_{BC} = -56.25 + \frac{1}{2}(41.25) + \frac{1}{4}(-78.75) = -55.31KNm$$

$$M_{CB} = 93.75 + \frac{1}{2}(-78.75) + \frac{1}{4}(41.25) = 64.69KNm$$

$$M_{CD} = -78.75 - \frac{3}{8}(-37.5) = -64.69KNm$$

$$M_{DC} = \frac{1}{2}(-78.75) - \frac{3}{8}(-37.5) = -25.31KNm$$



Reactions: consider the free body diagram of beam and columns

Column AB:

$$H_A = \frac{34.69 + 55.31}{4} = 22.5KN$$

Span BC:

$$R_B = \frac{55.31 - 64.69 + 80 \times 3}{8} = 28.83KN$$

$$R_C = 80 - R_B = 51.17KN$$

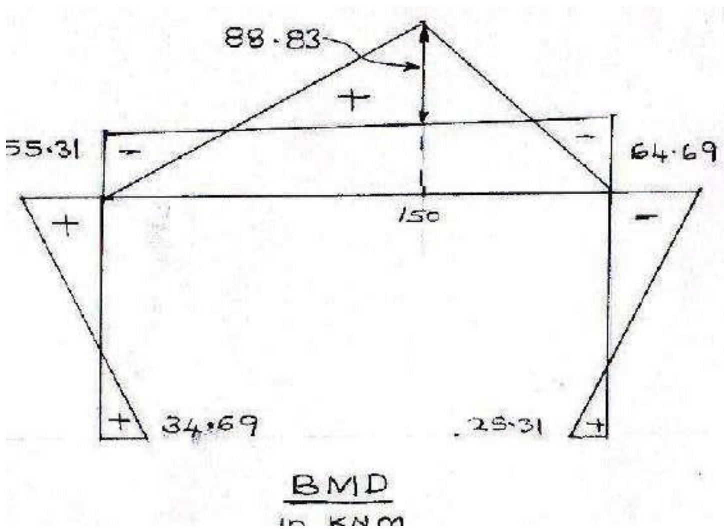
Column CD:

$$H_D = \frac{64.69 + 25.31}{4} = 22.5KN$$

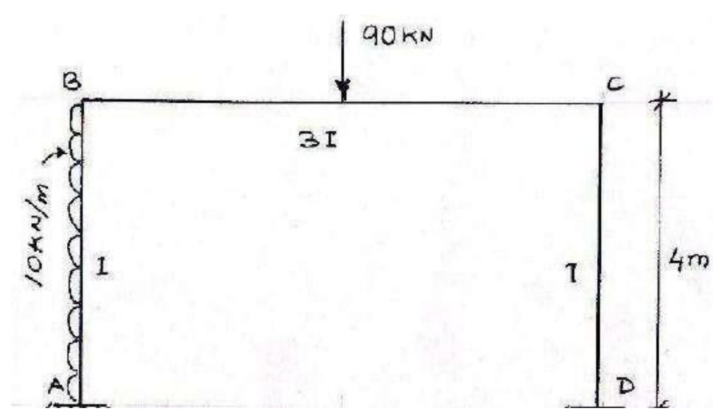
Check:

$\Sigma H = 0$, $H_A + H_D = 0$, $22.5 - 22.5 = 0$

Hence okay



6. Q. Frame ABCD is subjected to a horizontal force of 20 KN at joint C as shown in figure. Analyse and draw bending moment diagram.



The frame is symmetrical but loading is unsymmetrical. Hence there is a sway, assume sway towards right. In this problem

FEMS

$$M_{FAB=} -\frac{wL^2}{12} = -\frac{10 \times 4^2}{12} = -13.33KNm$$

$$M_{FBA=} \frac{wL^2}{12} = \frac{10 \times 4^2}{12} = 13.33\ KNm$$

$$M_{FBC=} -\frac{wL}{8} = -\frac{90 \times 10}{8} = -112.5KNm$$

$$M_{FCB=} \frac{wL}{8} = \frac{90\times10}{8} = 112.5KNm$$

Slope deflection equations:

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left[2\theta_A + \theta_B - 3\frac{\Delta}{L}\right] = -13.33 + \frac{2EI}{4}\left(0 + \theta_B - 3\frac{\Delta}{4}\right)$$
$$= -13.33 + \frac{EI}{2}\theta_B - \frac{3EI\Delta}{8} \qquad \cdots\cdots(1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left[2\theta_B + \theta_A - 3\frac{\Delta}{L}\right]$$
$$= 13.33 + \frac{2EI}{4}\left(2\theta_B + 0 - 3\frac{\Delta}{4}\right) = 13.33 + EI\theta_B - \frac{3EI}{8}\Delta \qquad \cdots\cdots(2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}[2\theta_B + \theta_C] = -112.5 + \frac{2E3I}{10}(2\theta_B + \theta_C)$$
$$= -112.5 + \frac{6EI}{5}\theta_B + \frac{3EI}{5}\theta_C \qquad \cdots\cdots(3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}[2\theta_C + \theta_B] = 112.5 + \frac{2E3I}{10}[2\theta_C + \theta_B]$$
$$= 112.5 + \frac{6EI}{5}\theta_C + \frac{3EI}{5}\theta_B \qquad \cdots\cdots(4)$$

$$M_{CD} = M_{FCD} + \frac{2EI}{L}\left[2\theta_C + \theta_D - 3\frac{\Delta}{L}\right] = 0 + \frac{2EI}{4}\left(2\theta_C + 0 - 3\frac{\Delta}{4}\right)$$
$$= EI\theta_C - \frac{3EI\Delta}{8} \qquad \cdots\cdots(5)$$

$$M_{DC} = M_{FDC} + \frac{2EI}{L}\left[2\theta_D + \theta_C - 3\frac{\Delta}{L}\right]$$
$$= 0 + \frac{2EI}{4}\left(0 + \theta_C - 3\frac{\Delta}{4}\right) = \frac{EI}{2}\theta_C - \frac{3EI}{8}\Delta \qquad \cdots\cdots(6)$$

In the above equation there are three unknowns, θ_B, θ_C & Δ, accordingly the boundary conditions are,

Joint conditions , $M_{BA} + M_{BC} = 0, M_{CB} + M_{CD} = 0$

Shear condition, $H_A + H_D + \sum P_H = 0, \quad H_A + H_D + 40 = 0$

$$H_A \times 4 = M_{AB} + M_{BA} - 10 \times 4 \times \frac{4}{2}$$

$$H_A = \frac{M_{AB} + M_{BA} - 80}{4}$$

$$H_D \times 4 = M_{CD} + M_{DC}$$

$$H_D = \frac{M_{CD} + M_{DC}}{4}$$

$$\therefore \frac{M_{AB} + M_{BA} - 80}{4} + \frac{M_{CD} + M_{DC}}{4} + 40 = 0$$

$$M_{AB} + M_{BA} + M_{CD} + M_{DC} + 80 = 0$$

Now, $\quad M_{BA} + M_{BC} = 0$

$$13.33 + EI\theta_B - \frac{3EI}{8}\Delta - 112.5 + \frac{6EI}{5}\theta_B + \frac{3EI}{5}\theta_C = 0$$

$$2.2EI\theta_B + 0.6EI\theta_C - 0.375\,EI\Delta - 99.17 = 0 \qquad \cdots\cdots(7)$$

$$M_{CB} + M_{CD} = 0$$

$$112.5 + \frac{6EI}{5}\theta_C + \frac{3EI}{5}\theta_B + EI\theta_C - \frac{3EI\Delta}{8} = 0$$

$$112.5 + 2.2\,EI\theta_C + 0.6EI\theta_B - 0.375\,EI\Delta = 0 \qquad \cdots\cdots(8)$$

$$M_{AB} + M_{BA} + M_{CD} + M_{DC} + 80 = 0$$

$$-13.33 + \frac{EI}{2}\theta_B - \frac{3EI\Delta}{8} + 13.33 + EI\theta_B - \frac{3EI}{8}\Delta + EI\theta_C - \frac{3EI\Delta}{8} + \frac{EI}{2}\theta_C - \frac{3EI}{8}\Delta + 80 = 0$$

$$1.5\,EI\theta_B + 1.5\,EI\theta_C - 1.5\,EI\Delta + 80 = 0 \qquad \cdots\cdots(9)$$

Solving equations (7), (8)& (9)

$$\theta_B = \frac{72.65}{EI}$$

$$\theta_C = -\frac{59.64}{EI}$$

$$\Delta = \frac{66.34}{EI}$$

Substituting these values in slope deflection equation, we have,

$$M_{AB} = -13.33 + \frac{1}{2}(72.65) - \frac{3}{8}(66.34) = -1.88KNm$$

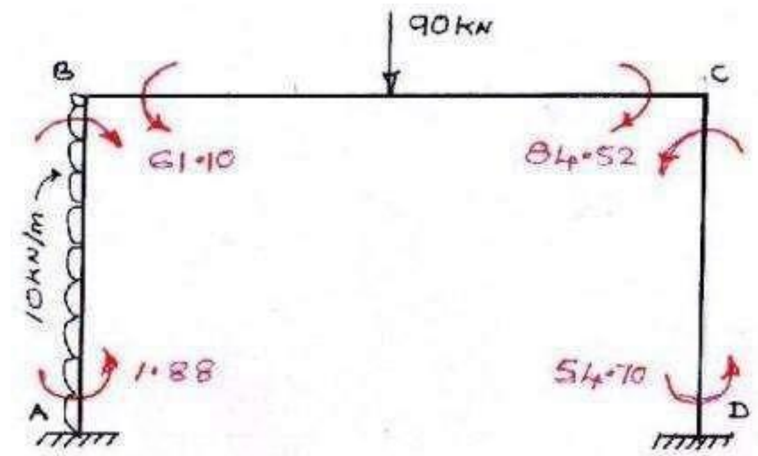
$$M_{BA} = 72.65 - \frac{3}{8}(66.34) = 61.10KNm$$

$$M_{BC} = -112.5 + \frac{6}{5}(72.65) + \frac{3}{5}(-59.64) = -61.10KNm$$

$$M_{CB} = 112.5 + \frac{6}{5}(-59.64) + \frac{3}{5}(72.65) = 84.52KNm$$

$$M_{CD} = -59.64 - \frac{3}{8}(66.34) = -84.52KNm$$

$$M_{DC} = \frac{1}{2}(-59.64) - \frac{3}{8}(66.34) = -54.70KNm$$



Reactions: Consider the free body diagram of various members

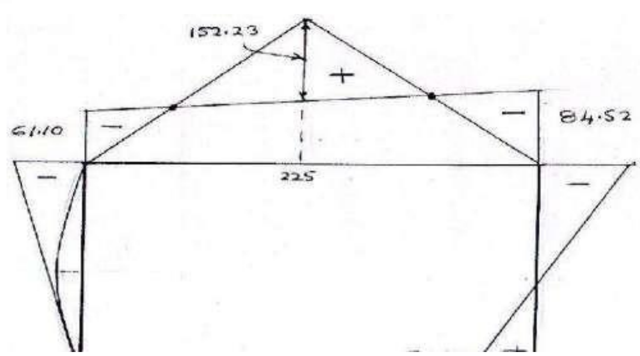
Member AB:

$$H_A = \frac{61.10 - 1.88 - 10 \times 4 \times 2}{4} = -5.195\ \text{KN}$$

Span BC:

$$R_C = \frac{84.52 - 61.10 + 90 \times 5}{10} = 47.34\ \text{KN}$$

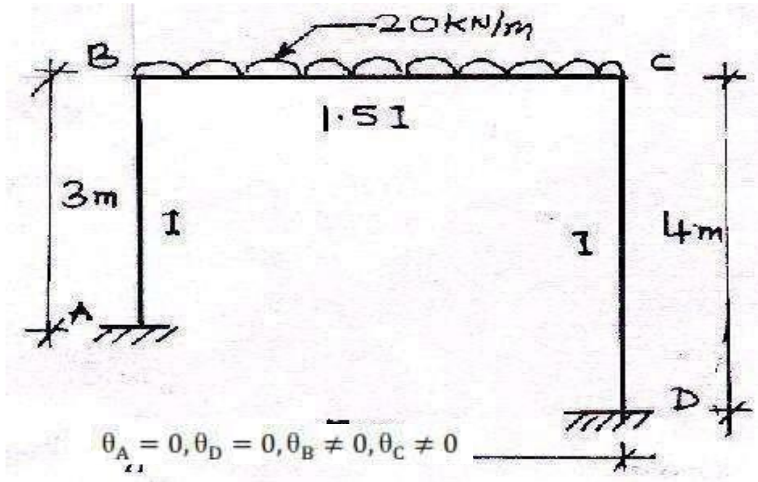
$$R_B = 90 - R_C = 38.34\text{KN}$$



Column CD:

$$H_D = \frac{84.52 + 54.7}{4} = 34.81\text{KN}$$

Analyse the portal frame and draw theB.M.D.



A. It is an unsymmetrical problem, hence there is a sway be towards right.

FEMS:

$$M_{FAB} = M_{FBA} = 0 = M_{FCD} = M_{FDC}$$

$$M_{FBC} = -\frac{wL^2}{12} = -\frac{20 \times 5^2}{12} = -41.67KNm$$

$$M_{FCB} = \frac{wL^2}{12} = \frac{20 \times 5^2}{12} = 41.67KNm$$

Slope deflection equations:

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left[2\theta_A + \theta_B - 3\frac{\Delta}{L}\right] = \frac{2EI}{3}\left(0 + \theta_B - 3\frac{\Delta}{3}\right)$$

$$= \frac{2EI}{3}\theta_B - \frac{2EI\Delta}{3} \qquad \cdots\cdots(1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left[2\theta_B + \theta_A - 3\frac{\Delta}{L}\right]$$

$$= \frac{2EI}{3}\left(2\theta_B + 0 - 3\frac{\Delta}{3}\right) = \frac{4EI}{3}\theta_B - \frac{2EI}{3}\Delta \qquad \cdots\cdots(2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}[2\theta_B + \theta_C] = -41.67 + \frac{2E(1.5)I}{5}(2\theta_B + \theta_C)$$

$$= -41.67 + \frac{6EI}{5}\theta_B + \frac{3EI}{5}\theta_C \qquad \cdots\cdots(3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}[2\theta_C + \theta_B] = 41.67 + \frac{2E(1.5)I}{5}[2\theta_C + \theta_B]$$

$$= 41.67 + \frac{6EI}{5}\theta_C + \frac{3EI}{5}\theta_B \qquad \cdots\cdots(4)$$

$$M_{CD} = M_{FCD} + \frac{2EI}{L}\left[2\theta_C + \theta_D - 3\frac{\Delta}{L}\right] = 0 + \frac{2EI}{4}\left(2\theta_C + 0 - 3\frac{\Delta}{4}\right)$$

$$= EI\theta_C - \frac{3EI\Delta}{8} \qquad \cdots\cdots(5)$$

$$M_{DC} = M_{FDC} + \frac{2EI}{L}\left[2\theta_D + \theta_C - 3\frac{\Delta}{L}\right]$$

$$= 0 + \frac{2EI}{4}\left(0 + \theta_C - 3\frac{\Delta}{4}\right) = \frac{EI}{2}\theta_C - \frac{3EI}{8}\Delta \qquad \cdots\cdots(6)$$

In the above equation there are three unknowns, $\theta_B, \theta_C \& \Delta$, accordingly the boundary conditions are,

Joint conditions , $M_{BA} + M_{BC} = 0, M_{CB} + M_{CD} = 0$

Shear condition, $H_A + H_D = 0, \ \therefore \frac{M_{AB} + M_{BA}}{3} + \frac{M_{CD} + M_{DC}}{4} + 40 = 0$

$$4(M_{AB} + M_{BA}) + 3(M_{CD} + M_{DC}) = 0$$

Now, $M_{BA} + M_{BC} = 0$

$$\frac{4EI}{3}\theta_B - \frac{2EI}{3}\Delta - 41.67 + \frac{6EI}{5}\theta_B + \frac{3EI}{5}\theta_C = 0$$

$$2.53EI\theta_B + 0.6EI\theta_C - \frac{2EI}{3}\Delta - 41.67 = 0 \qquad \cdots\cdots(7)$$

$$M_{CB} + M_{CD} = 0$$

$$41.67 + \frac{6EI}{5}\theta_C + \frac{3EI}{5}\theta_B + EI\theta_C - \frac{3EI\Delta}{8} = 0$$

$41.67+2.2\ EI\theta_C+0.6EI\theta_B - 0.375\ EI\Delta= 0$ ……(8)

$$4(M_{AB} + M_{BA}) + 3(M_{CD} + M_{DC}) = 0$$

$$4\left(\frac{2EI}{3}\theta_B - \frac{2EI\Delta}{3} + \frac{4EI}{3}\theta_B - \frac{2EI}{3}\Delta\right) + 3(EI\theta_C - \frac{3EI\Delta}{8} + \frac{EI}{2}\theta_C - \frac{3EI}{8}\Delta) = 0$$

$8\ EI\theta_B + 4.5\ EI\theta_C - 7.53\ EI\Delta= 0$ ……(9)

Solving equations (7), (8)& (9)

$$\theta_B = \frac{25.46}{EI}$$

$$\theta_C = \frac{-23.17}{EI}$$

$\Delta=\frac{12.8}{EI}$

Substituting these values in slope deflection equation, we have,

$$M_{AB} = \frac{2}{3}(25.46) - \frac{2}{3}(12.8) = 8.44KNm$$

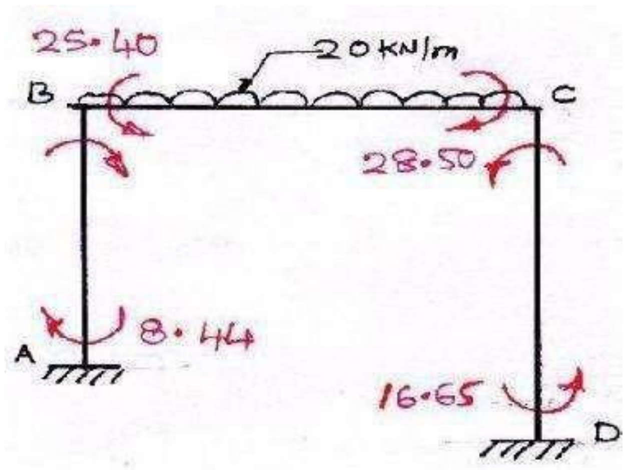
$$M_{BA} = \frac{4}{3}(25.46) - \frac{2}{3}(12.8) = 25.4KNm$$

$$M_{BC} = -41.67 + \frac{6}{5}(25.46) + \frac{3}{5}(-23.17) = -25.4KNm$$

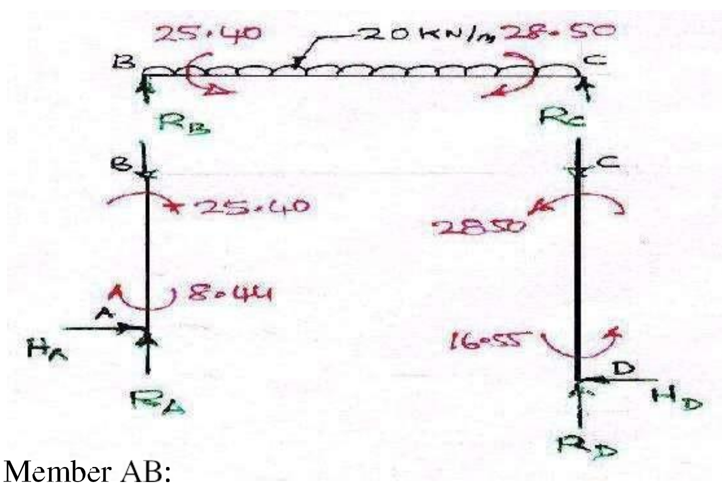
$$M_{CB} = 41.67 + \frac{6}{5}(-23.17) + \frac{3}{5}(20.46) = 28.5KNm$$

$$M_{CD} = -23.17 - \frac{3}{8}(12.8) = -28.5KNm$$

$$M_{DC} = \frac{1}{2}(-23.17) - \frac{3}{8}(12.8) = -16.65KNm$$



Reactions: Consider the free body diagram



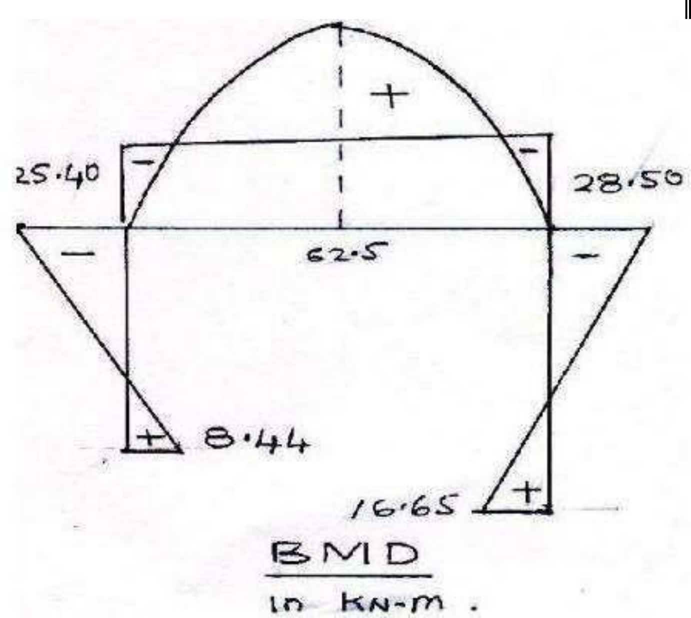
Member AB:

$$H_A = \frac{25.4 + 8.44}{3} = 11.28\ \text{KN}$$

Span BC:

$$R_C = \frac{28.5 - 25.4 + 20 \times 5 \times \frac{5}{2}}{5} = 51.64\ \text{KN}$$

$$R_B = 20 \times 5 - 51.64 = 48.36\text{KN}$$



Column CD:

$$H_D = \frac{28.5 + 16.65}{4} = 11.28\text{KN}$$

Check:

$\Sigma H = 0,\ H_A + H_D = 0$ Satisfied, hence okay