STRCUTURAL ANALYSIS- II

UNIT - I

PART A : TWO HINGED ARCHES

INTRODUCTION

Mainly three types of arches are used in practice: three-hinged, two-hinged and hingeless arches. In the early part of the nineteenth century, three-hinged arches were commonly used for the long span structures as the analysis of such arches could be done with confidence. However, with the development in structural analysis, for long span structures starting from late nineteenth century engineers adopted two-hinged and hingeless arches. Two-hinged arch is the statically indeterminate structure to degree one. Usually, the horizontal reaction is treated as the redundant and is evaluated by the method of least work. In this lesson, the analysis of two-hinged arches is discussed and few problems are solved to illustrate the procedure for calculating the internal forces.

ANALYSIS OF TWO-HINGED ARCH

A typical two-hinged arch is shown in Fig. 33.1a. In the case of two-hinged arch, we have four unknown reactions, but there are only three equations of equilibrium available. Hence, the degree of statical indeterminacy is one for two- hingedarch.

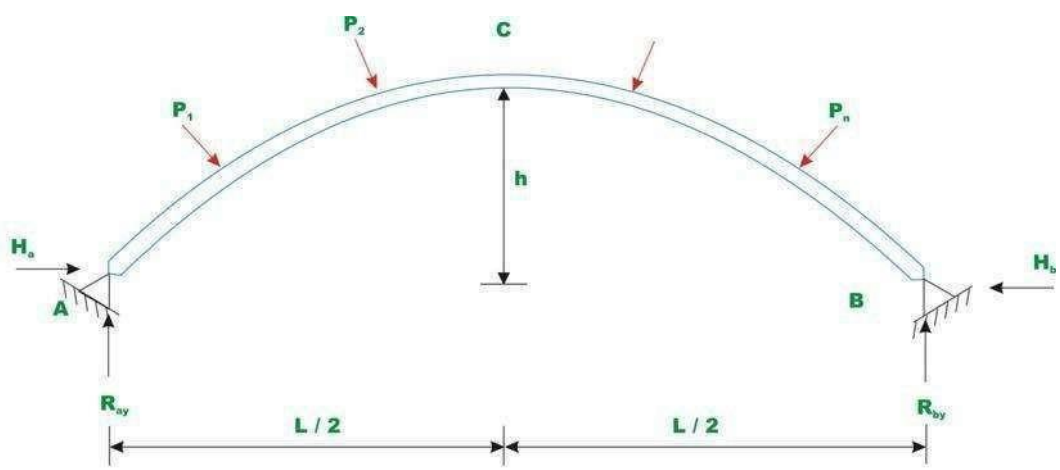
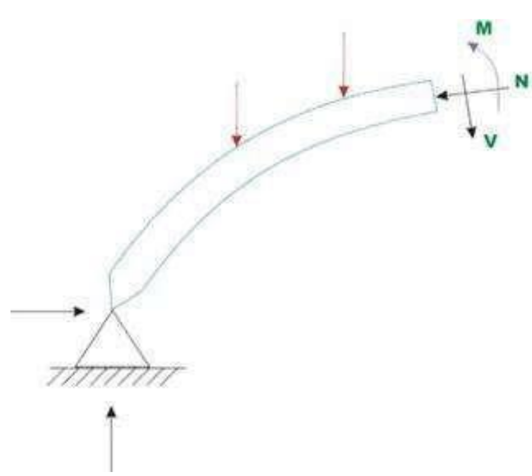


Fig. 33.1a Two - hinged arch.



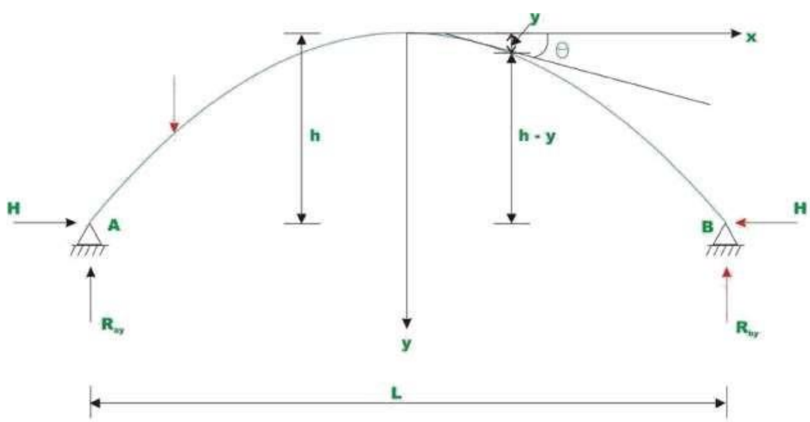


Fig. 33.2a

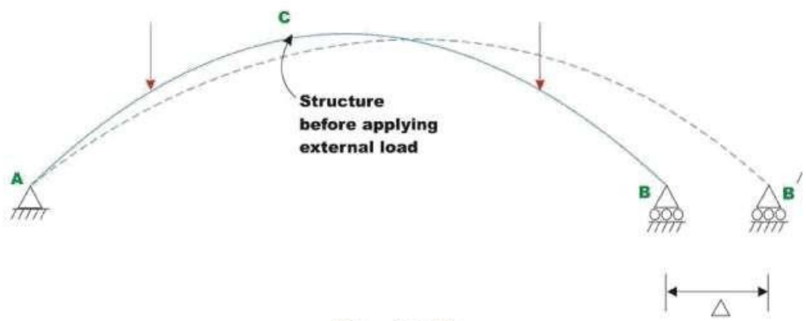


Fig. 33.2b.

The fourth equation is written considering deformation of the arch. The unknown redundant reaction H_b is calculated by noting that the horizontal displacement of hinge B is zero. In general the horizontal reaction in the two hinged arch is evaluated by straightforward application of the theorem of least work (see module 1, lesson 4), which states that the partial derivative of the strain energy of a statically indeterminate structure with respect to statically indeterminate action should vanish. Hence to obtain, horizontal reaction, one must develop an expression for strain energy. Typically, any section of the arch (vide Fig 33.1b) is subjected to shear force V, bending moment M and the axial compression N . The strain energy due to bending U_b is calculated from the following expression.

$$U_b = \int_0^s \frac{M^2}{2EI} ds \qquad (33.1)$$

The above expression is similar to the one used in the case of straight beams. However, in this case, the integration needs to be evaluated along the curved arch length. In the above equation, s is the length of the centerline of the arch, I is the moment of inertia of the arch cross section, E is the Young's modulus of the arch material. The strain energy due to shear is small as compared to the strain energy due to bending and is usually neglected in the analysis. In the case of flat arches, the strain energy due to axial compression can be appreciable and is givenby,

$$U_a = \int_0^s \frac{N^2}{2AE} ds \qquad (33.2)$$

The total strain energy of the arch is given by

$$U = \int_0^s \frac{M^2}{2EI} ds + \int_0^s \frac{N^2}{2AE} ds \qquad (33.3)$$

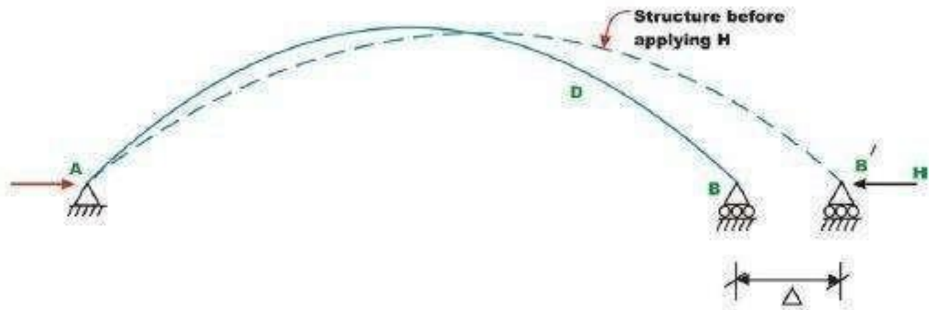


Fig. 33.2c.

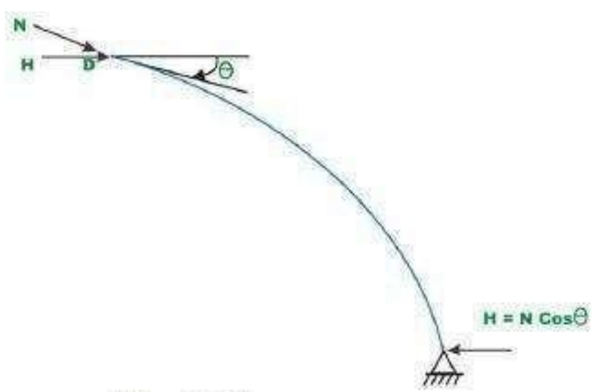


Fig. 33.2d.

From Fig. 33.2b and Fig 33.2c, the bending moment at any cross section of the arch (say D), may be written as

$$M = M_0 - H(h - y) \tag{33.5}$$

The axial compressive force at any cross section (say D) may be written as

$$N = N_0 + H\cos\theta \tag{33.6}$$

Where θ is the angle made by the tangent at D with horizontal (vide Fig 33.2d). Substituting the value of M and N in the equation (33.4),

$$\frac{\partial U}{\partial H} = 0 = -\int_0^s \frac{M_0 - H(h-y)}{EI}(h-y)\,ds + \int_0^s \frac{N_0 + H\cos\theta}{EA}\cos\theta\,ds \tag{33.7a}$$

Let, $\tilde{y} = h - y$

$$-\int_0^s \frac{M_0 - H\tilde{y}}{EI}\tilde{y}\,ds + \int_0^s \frac{N_0 + H\cos\theta}{EA}\cos\theta\,ds = 0 \tag{33.7b}$$

Solving for H, yields

$$-\int_0^s \frac{M_0}{EI}\tilde{y}\,ds + \int_0^s \frac{H\tilde{y}^2}{EI}ds + \int_0^s \frac{N_0}{EA}\cos\theta\,ds + \int_0^s \frac{H\cos^2\theta}{EA}ds = 0$$

$$H = \frac{\int_0^s \frac{M_0}{EI}\tilde{y}\,ds - \int_0^s \frac{N_0}{EA}\cos\theta\,ds}{\int_0^s \frac{\tilde{y}^2}{EI}ds + \int_0^s \frac{\cos^2\theta}{EA}ds} \tag{33.8}$$

Using the above equation, the horizontal reaction H for any two-hinged symmetrical arch may be calculated. The above equation is valid for any general type of loading. Usually the above equation is further simplified. The second term in the numerator is small compared with the first terms and is neglected in the analysis. Only in case of very accurate analysis second term s considered. Also for flat arched, $\cos\theta \cong 1$ as θ is small. The equation (33.8) is now written as,

$$H = \frac{\int_0^s \frac{M_0}{EI}\tilde{y}\,ds}{\int_0^s \frac{\tilde{y}^2}{EI}ds + \int_0^s \frac{ds}{EA}} \tag{33.9}$$

As axial rigidity is very high, the second term in the denominator may also be neglected. Finally the horizontal reaction is calculated by the equation

$$H = \frac{\int_0^s \frac{M_0}{EI} \tilde{y}\, ds}{\int_0^s \frac{\tilde{y}^2}{EI} ds} \qquad (33.10)$$

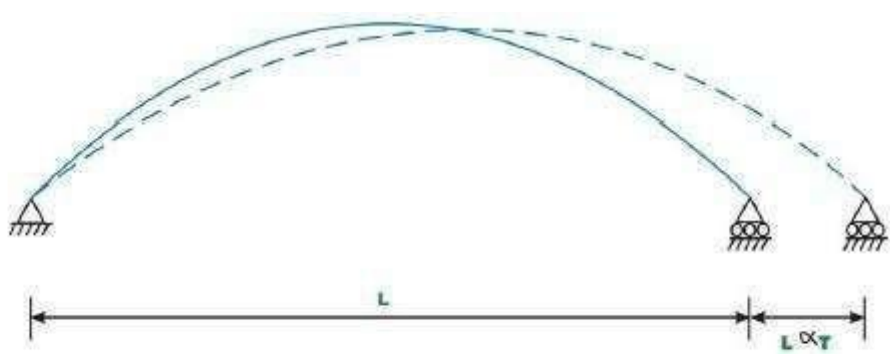
For an arch with uniform cross section EI is constant and hence,

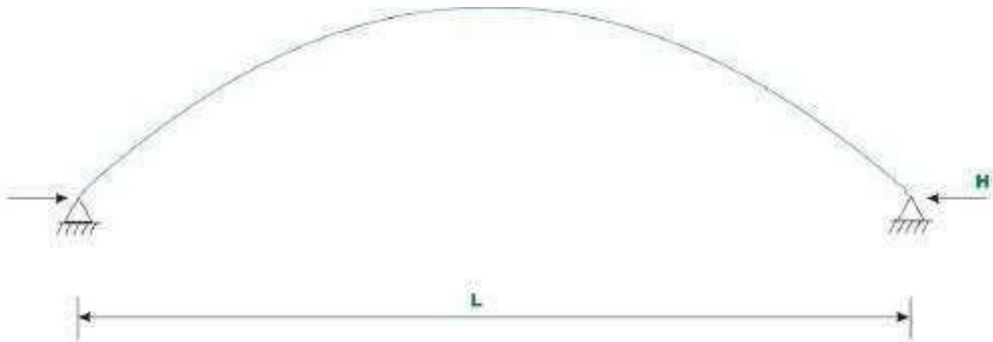
$$H = \frac{\int_0^s M_0 \tilde{y}\, ds}{\int_0^s \tilde{y}^2\, ds} \qquad (33.11)$$

In the above equation, M_0 is the bending moment at any cross section of the arch when one of the hinges is replaced by a roller support. $\tilde{y}$ is the height of the arch as shown in the figure. If the moment of inertia of the arch rib is not constant, then equation (33.10) must be used to calculate the horizontal reaction H.

TEMPERATURE EFFECT

Consider an unloaded two-hinged arch of span L. When the arch undergoes a uniform temperature change of T, then its span would increase by $C^\circ TL\alpha$ if it were allowed to expand freely (vide Fig 33.3a). α is the co-efficient of thermal expansion of the arch material. Since the arch is restrained from the horizontal movement, a horizontal force is induced at the support as the temperature is increase





Now applying the Castigliano's first theorem,

$$\frac{\partial U}{\partial H} = \alpha LT = \int_0^s \frac{H\tilde{y}^2}{EI} ds + \int_0^s \frac{H\cos^2\theta}{EA} ds$$

Solving for *H*,

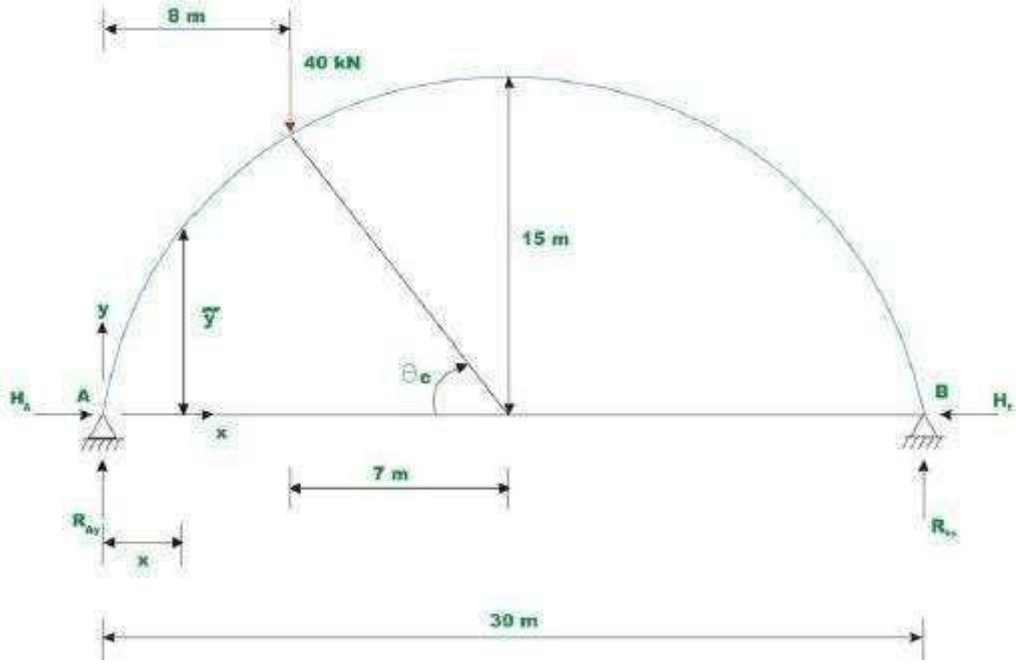
$$H = \frac{\alpha LT}{\int_0^s \frac{\tilde{y}^2}{EI} ds + \int_0^s \frac{\cos^2\theta}{EA} ds}$$

The second term in the denominator may be neglected, as the axial rigidity is quite high. Neglecting the axial rigidity, the above equation can be written as

$$H = \frac{\alpha LT}{\int^{s} \tilde{y}^2 \, ds}$$

Example

A semicircular two hinged arch of constant cross section is subjected to a concentrated load as shown in Fig. Calculate reactions of the arch and draw bending moment diagram.

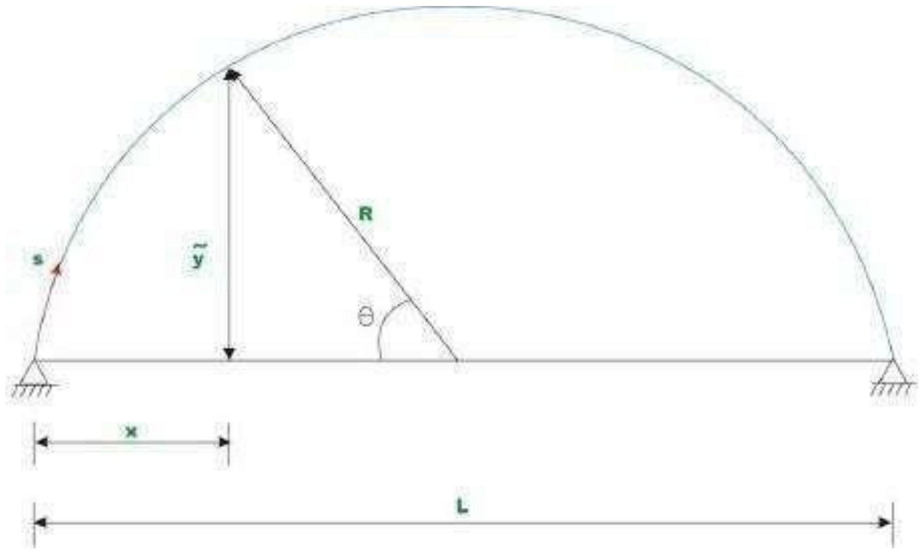


Solution:

Taking moment of all forces about hinge B leads to,

$$R_{ay} = \frac{40 \times 22}{30} = 29.3\ \ 3\ \ \text{kN}\ (\uparrow)$$

$$\sum Fy = 0 \qquad \Rightarrow R_{by} = 10.67\ \ \text{kN}\ (\uparrow)$$



From figure,

$$\tilde{y} = R\sin\theta$$

$$x = R(1-\cos\theta)$$

$$ds = R\,d\theta \qquad (2)$$

$$\tan\theta_c = \frac{13.267}{7} \qquad \Rightarrow \theta_c = 62.18^\circ = \pi/2.895\,rad$$

Now, the horizontal reaction H may be calculated by the following expression,

$$H = \frac{\int_0^s M_0 \tilde{y}\,ds}{\int_0^s \tilde{y}^2\,ds} \qquad (3)$$

Now M_0 the bending moment at any cross section of the arch when one of the hinges is replaced by a roller support is given by,

$$M_0 = R_{ay}x = R_{ay}R(1-\cos\theta) \qquad 0 \le \theta \le \theta_c$$

and,

$$\begin{aligned} M_0 &= R_{ay}R(1-\cos\theta) - 40(x-8) \\ &= R_{ay}R(1-\cos\theta) - 40\{R(1-\cos\theta)-8\} \qquad \theta_c \le \theta \le \pi \end{aligned} \tag{4}$$

Integrating the numerator in equation (3),

$$\int_0^s M_0 \tilde{y}\,ds = \int_0^{\theta_c} R_{ay}R^3(1-\cos\theta)\sin\theta\,d\theta + \int_{\theta_c}^{\pi}[R_{ay}R(1-\cos\theta) - 40\{R(1-\cos\theta)-8\}]R\sin\theta\,R\,d\theta$$

$$= R_{ay}R^3\int_0^{\pi/2.895}(1-\cos\theta)\sin\theta\,d\theta + R^2\int_{\pi/2.895}^{\pi}[R_{ay}R(1-\cos\theta)\sin\theta - 40\{R(1-\cos\theta)\sin\theta - 8\sin\theta\}]\,d\theta$$

$$= R_{ay}R^3[-\cos\theta]_0^{\pi/2.895} + R^2\left[[R_{ay}R(-\cos\theta)]_{\pi/2.895}^{\pi} - [40R(-\cos\theta)]_{\pi/2.895}^{\pi} + [40\times 8(-\cos\theta)]_{\pi/2.895}^{\pi}\right]$$

$$= 0.533R_{ay}R^3 + R^2[[1.4667R_{ay}R] - [40R(1.4667)] + [40\times 8(1.4667)]]$$

$$= 52761.00 + 225(645.275 - 410.676) = 105545.775 \tag{5}$$

The value of denominator in equation (3), after integration is,

$$\begin{aligned} \int_0^s \tilde{y}^2\,ds &= \int_0^{\pi}(R\sin\theta)^2 R\,d\theta \\ &= R^3\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta = R^3\left(\frac{\pi}{2}\right) = 5301.46 \end{aligned} \tag{6}$$

Hence, the horizontal thrust at the support is,

$$H = \frac{105545.775}{5301.46} = 19.90 \text{ kN} \tag{7}$$

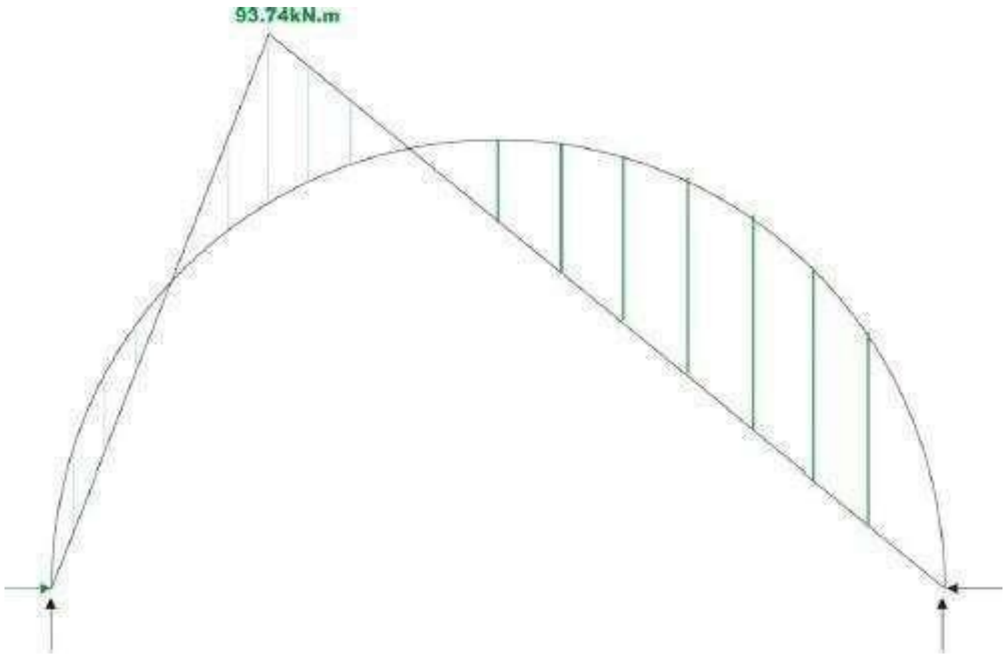
Bending moment diagram

Bending moment M at any cross section of the arch is given by,

$$\begin{aligned} M &= M_0 - H\tilde{y} \\ &= R_{ay}R(1-\cos\theta) - HR\sin\theta \qquad 0 \le \theta \le \theta_c \\ &= 439.95(1-\cos\theta) - 298.5\sin\theta \end{aligned} \tag{8}$$

$$M = 439.95(1-\cos\theta) - 298.5\sin\theta - 40(15(1-\cos\theta)-8) \qquad \theta_c \le \theta \le \pi \tag{9}$$

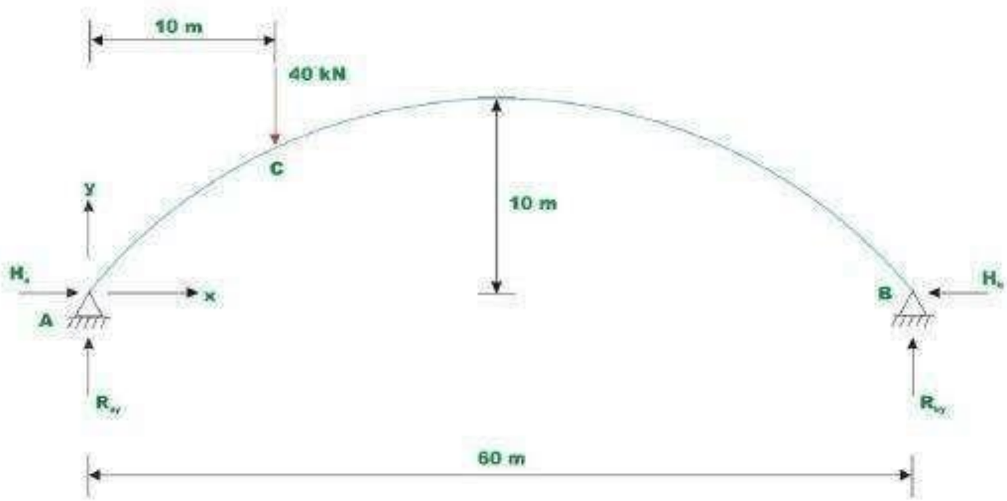
Using equations (8) and (9), bending moment at any angle θ can be computed. The bending moment diagram is shown in Fig.



Example

A two hinged parabolic arch of constant cross section has a span of 60m and a rise of 10m. It is subjected to loading as shown in Fig.. Calculate reactions of the arch if the temperature of the arch is raised by. Assume co-efficient of thermal expansion as

$\alpha = 12 \times 10^{-6} / °C.$



Taking A as the origin, the equation of two hinged parabolic arch may be written as,

$$y = \frac{2}{3}x - \frac{10}{30^2}x^2 \qquad (1)$$

The given problem is solved in two steps. In the first step calculate the horizontal reaction due to 40kN load applied at C. In the next step calculate the horizontal reaction due to rise in temperature. Adding both, one gets the horizontal reaction at the hinges due to 40kN combined external loading and temperature change. The horizontal reaction due to load may be calculated by the following equation,

$$H_1 = \frac{\int_0^s M_0 y \, ds}{\int_0^s \tilde{y}^2 \, ds} \qquad (2a)$$

For temperature loading, horizontal reaction is given by,

$$H_2 = \frac{\alpha L T}{\int_0^s \frac{y^2}{EI} ds} \qquad (2b)$$

Where L is the span of the arch.

For 40 kN load,

$$\int_0^s M_0 y \, ds = \int_0^{10} R_{ay} \, x y \, dx + \int_{10}^{60} \left[R_{ay} x - 40(x-10) \right] y \, dx \qquad (3)$$

Please note that in the above equation, the integrations are carried out along the x- axis instead of the curved arch axis. The error introduced by this change in the variables in the case of flat arches is negligible. Using equation (1), the above equation (3) can be easily evaluated.

The vertical reaction A is calculated by taking moment of all forces about B. Hence,

$$R_{ay} = \frac{1}{60}\left[40 \times 50\right] = 33.33 \ \ \text{kN}$$

$$R_{by} = 6.67 \ \ \text{kN}.$$

Now consider the equation (3),

$$\int_0^l M_0 y\,dx = \int_0^{10} (33.33)x(\frac{2}{3}x - \frac{10}{30^2}x^2)\,dx + \int_{10}^{60} [(33.33)x - 40(x-10)](\frac{2}{3}x - \frac{10}{30^2}x^2)\,dx$$

$$= 6480.76 + 69404.99 = 74885.75 \qquad (4)$$

$$\int_0^l y^2\,dx = \int_0^{60} \left[\frac{2}{3}x - \frac{10}{30^2}x^2\right]^2 dx$$

$$= 3200 \qquad (5)$$

Hence, the horizontal reaction due to applied mechanical loads alone is given by,

$$H_1 = \frac{\int_0^l M_0 y \ \ dx}{\int_0^l y^2 \ \ dx} = \frac{75885.75}{3200} = 23.71 \ \ \text{kN} \qquad (6)$$

The horizontal reaction due to rise in temperature is calculated by equation (2b),

$$H_2 = \frac{12 \times 10^{-6} \times 60 \times 40}{3200/EI} = \frac{EI \times 12 \times 10^{-6} \times 60 \times 40}{3200}$$

Taking $E = 200$ kN/mm^2 and $I = 0.0333 m^4$

$H_2 = 59.94$ kN. (7)

Hence the total horizontal thrust $H = H_1 + H_2 = 83.65$ kN.

When the arch shape is more complicated, the integrations $\int_0^s \frac{M_0 y}{EI} ds$ and $\int_0^s \frac{y^2}{EI} ds$ are accomplished numerically. For this purpose, divide the arch span in to n equals divisions. Length of each division is represented by $(\Delta s)_i$ (vide Fig.33.5b). At the midpoint of each division calculate the ordinate y_i by using the equation $y = \frac{2}{3}x - \frac{10}{30^2}x^2$. The above integrals are approximated as,

$$\int_0^s \frac{M_0 y}{EI} ds = \frac{1}{EI} \sum_{i=1}^{n} (M_0)_i y_i (\Delta s)_i \quad (8)$$

$$\int_0^s \frac{y^2}{EI} ds = \frac{1}{EI} \sum_{i=1}^{n} (y)_i^2 (\Delta s)_i \quad (9)$$

The complete computation for the above problem for the case of external loading is shown in the following table.

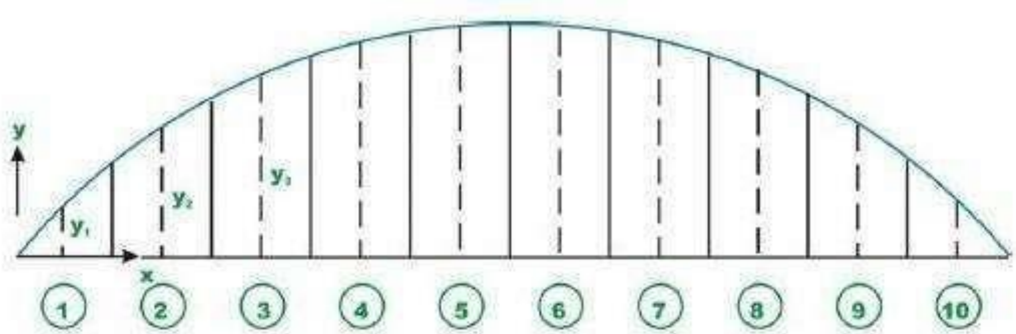


Table 1. Numerical integration of equations (8) and (9)

Segment No	Horizontal distance x Measured from A (m)	Corresponding y_i (m)	Moment at that Point $(M_0)_i$ (kNm)	$(M_0)_i y_i (\Delta s)_i$	$(y)_i^2 (\Delta s)_i$
1	3	1.9	99.99	1139.886	21.66
2	9	5.1	299.97	9179.082	156.06
3	15	7.5	299.95	13497.75	337.5
4	21	9.1	259.93	14192.18	496.86
5	27	9.9	219.91	13062.65	588.06
6	33	9.9	179.89	10685.47	588.06
7	39	9.1	139.87	7636.902	496.86
8	45	7.5	99.85	4493.25	337.5
9	51	5.1	59.83	1830.798	156.06
10	57	1.9	19.81	225.834	21.66
			$\sum$	75943.8	3300.3

$$H_1 = \frac{\sum (M_0)_i y_i (\Delta s)}{\sum (y)_i^2 (\Delta s)_i} = \frac{75943.8}{3200.3} = 23.73 \text{ kN} \qquad (10)$$

This compares well with the horizontal reaction computed from the exact integration.

UNIT - I

ANALYSIS OF PLANE FRAMES

PART-B MOMENT DISTRIBUTION METHOD

MOMENT DISTRIBUTION METHOD

This method of analyzing beams and frames was developed by Hardy Cross in 1930. Moment distribution method is basically a displacement method of analysis. But this method side steps the calculation of the displacement and instead makes it possible to apply a series of converging corrections that allow direct calculation of the end moments. This method of consists of solving slope deflection equations by successive approximation that may be carried out to any desired degree of accuracy. Essentially, the method begins by assuming each joint of a structure is fixed. Then by unlocking and locking each joint in succession, the internal moments at the joints are distributed and balanced until the joints have rotated to their final or nearly final positions. This method of analysis is both repetitive and easy to apply. Before explaining the moment distribution method certain definitions and concepts must beunderstood.

Sign convention: In the moment distribution table clockwise moments will be treated+ve and anticlockwise moments will be treated –ve. But for drawing BMD moments causing concavity upwards (sagging) will be treated +ve and moments causing convexity upwards (hogging) will be treated –ve.

Fixed end moments: The moments at the fixed joints of loaded member are called fixedend moment. FEM for few standards cases are given in previous chapter.

Distribution factors: If a moment 'M' is applied to a rigid joint 'o', as shown in figure, theconnecting members will each supply a portion of the resisting moment necessary to satisfy moment equilibrium at the joint. Distribution factor is that fraction which when multiplied with applied moment 'M' gives resisting moment supplied by the members. To obtain itsvalue imagine the joint is rigid joint connected to different members. If applied moment M cause the joint to rotate an amount 'θ' , then each member rotates by same amount.

From equilibrium requirement

$M = M_1 + M_2 + M_3 + \ldots\ldots$

$= K_1\theta + K_2\theta + K_3\theta = \theta\sum K$

$DF_1 = \frac{M1}{M} = \frac{K_1\theta}{\theta\sum K} = \frac{K_1}{\sum K}$

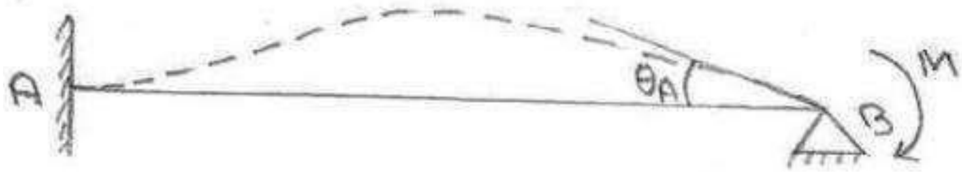
In general $DF = \frac{K}{\sum K}$ ....(6)

Member relative stiffness factor: In majority of the cases continuous beams and frames willbe made from the same material so that their modulus of electricity E will be same for all members. It will be easier to determine member stiffness factor by removing term 4E & 3E from equation (4) and (5) then will be called as relative stiffness factor.

$K_r = \frac{I}{L}$ for far end fixed

$K_r = \frac{3I}{4L}$ for far end hinged

Carry over factors: Consider the beam shown infigure



We have shown that

$$M = \frac{4EI}{L}\theta_A, R_B = \frac{3M}{2L}$$

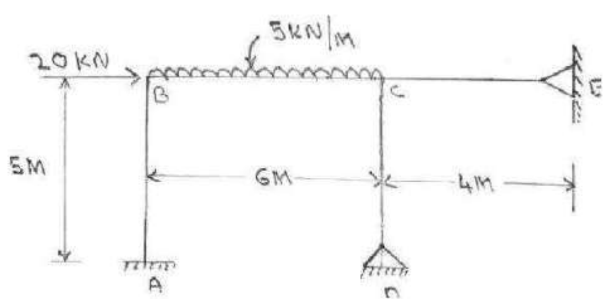
BM at A $\left(EI\frac{d^2y}{dx^2}\right)_{at\ x=L} = [(3M/2L)x - M]_{x=L} = \frac{M}{2}$

+ve BM of at A indicates clockwise moment of at A. In other words the moment 'M' at the pin induces a moment of at the fixed end. The carry over factor represents the fraction of M that is carried over from hinge to fixed end. Hence the carry over factor for the caseof far end fixed is +. The plus sign indicates both moments are in the samedirection.

MOMENT DISTRIBUTION FOR FRAMES WITH NO SIDE SWAY

The analysis of such a frame when the loading conditions and the geometry of the frame is such that there is no joint translation or sway, is similar to that given for beams.

Q. Analysis the frame shown in figure by moment distribution method and draw



BMD assume EI isconstant.

FEMS

$$M_{FAB=} M_{FBA=} M_{FCD=} M_{FDC=} M_{FCE=} M_{FEC=} 0$$

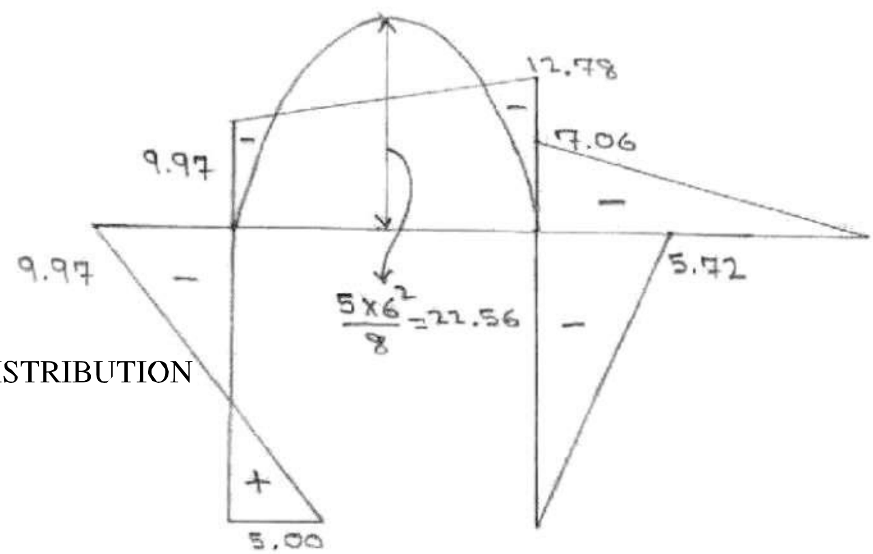
$$M_{FBC=-\frac{5\times6^2}{12}=-15KNm}$$

$$M_{FCB=\frac{5\times6^2}{12}=15KNm}$$

Jt.	Member	Relative stiffness (K)	ΣK	DF = $\frac{K}{\sum K}$
B	BA	I/5	$\frac{11}{30}I$	0.55
	BC	I/6		0.45
C	CB	I/6 = 0.17 I	0.51 I	0.33
	CD	$\frac{3}{4}$I/5 = 0.15 I		0.3
	CE	$\frac{3}{4}x\frac{I}{4}$= 0.19 I		0.37

MOMENT DISTRIBUTION

Jt	A	B		C			D	E
Member	AB	BA	BC	CB	CD	CE	DC	EC
D.F	0	0.55	0.45	0.33	0.3	0.37	1	1
FEM	0	0	-15	+15	0	0	0	0
Balance		8.25	6.75	-4.95	-4.5	-5.55		
C.O	4.13		-2.48	3.38				
Balance		1.36	1.12	-1.12	-1.01	-1.25		
C.O	0.68		0.56	0.56				
Balance		0.31	0.25	-0.18	-0.17	-0.21		
C.O	0.16		-0.09	0.13				
Balance		0.05	0.04	-0.04	-0.04	-0.05		
C.O	0.03							
Final moments	**5**	**9.97**	**-9.97**	**12.78**	**-5.72**	**-7.06**	**0**	**0**

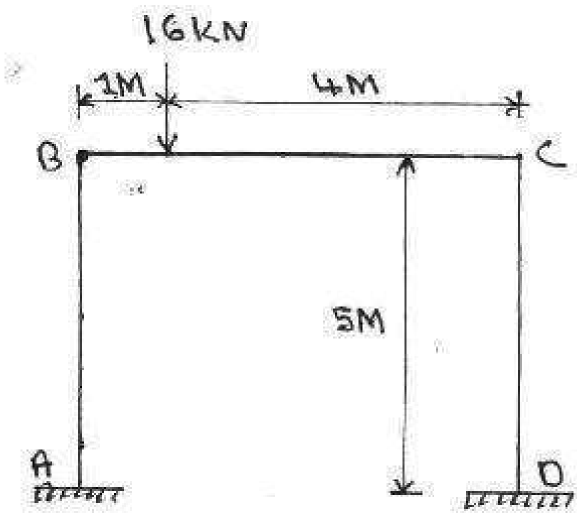


DISTRIBUTION

MOMENT DISTRIBUTION METHOD FOR FRAMES WITH SIDE SWAY

Frames that are non symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to non-symmetrical loading have a tendency to side sway.

Analyze the frame shown in figure by moment distribution method. Assume EI is constant.



A. Non SwayAnalysis:

First consider the frame without side sway

$M_{FAB} = M_{FBA} = M_{FCD} = 0$

$M_{FBC} = -\frac{16\times1\times4^2}{5^2} = -10.24KNm$

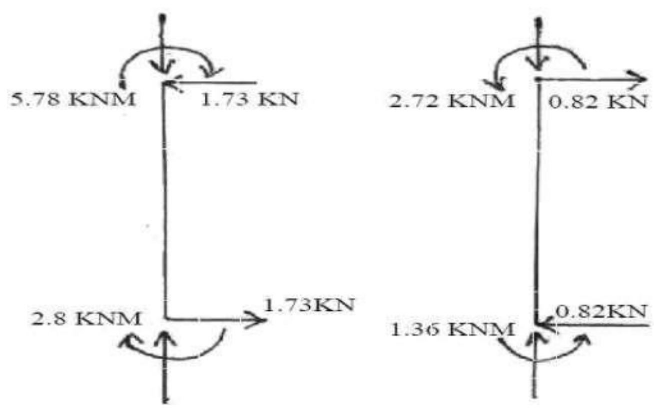
$M_{FCB} = \frac{16\times4\times1^2}{5^2} = 2.56KNm$

DISTRIBUTION FACTOR

Jt.	Member	Relative stiffness K	ΣK	$DF = \frac{K}{\sum K}$
B	BA BC	I/5 = 0.2 I I/5 = 0.2 I	0.4 I	0.5 0.5
C	CB CD	I/5 = 0.2I I/5 = 0.2I	0.4I	0.5 0.5

DISTRIBUTION OF MOMENTS FOR NON-SWAY ANALYSIS

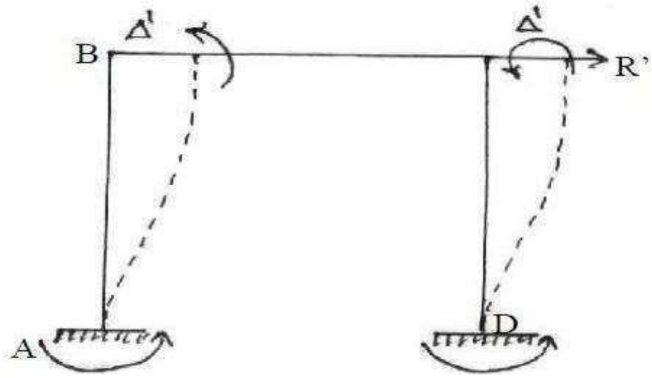
Joint	A		B		C	D	
Member	AB	BA	BC	CB	CD	DC	
D.F	0	0.5	0.5	0.5	0.5	0	
FEM	0		-10.24	2.56	0	0	
Balance		5.12	5.12	-1.28	-1.28		
CO	2.56		-0.64	2.56		-0.64	
Balance		0.32	0.32	-0.08	-0.08		
CO	0.16		-0.64	0.16		-0.64	
Balance		0.32	0.32	-0.08	-0.08		
C.O	0.16		-0.04	0.16		-0.04	
Balance		0.02	0.02	-0.08	-0.08		
C.O	0.01					-0.04	
Final moments	**2.89**	**5.78**	**-5.78**	**2.72**	**-2.72**	**-1.36**	



By seeing of the FBD of columns R = 1.73 – 0.82

(Using F_x =0 for entire frame) = 0.91 KN ←

Now apply R = 0.91 KN acting opposite as shown in the above figure for the sway analysis. Sway analysis: For this we will assume a force R` is applied at C causing the frame to deflect as shown in the following figure.



Since both ends are fixed, columns are of same length & I and assuming joints B & C are temporarily restrained from rotating and resulting fixed end moment are

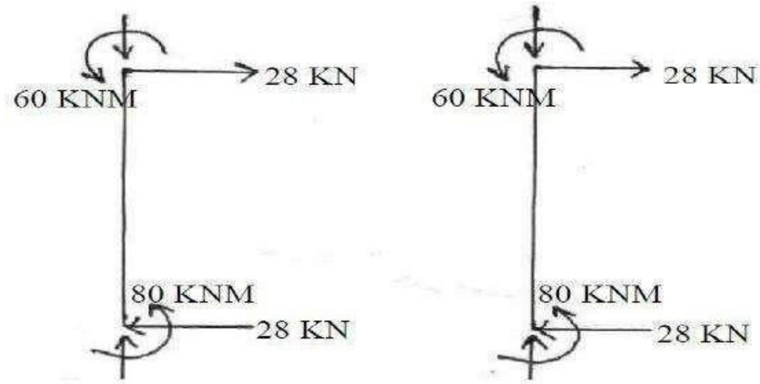
Assume

$$M'_{AB} = M'_{BA} = M'_{CD} = M'_{DC} = \frac{6EI}{L^2}\Delta$$
$$M'_{BA} = -100KNm$$
$$M'_{AB} = M'_{CD} = M'_{DC} = -100KNm$$

Moment distribution table for sway analysis:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	01	0.5	0.5	0.5	0.5	0
FEM	−100	−100	0	0	−100	−100
Balance		50	50	50	50	
CO	25		25	25		25
Balance		−12.5	−12.5	−12.5	−12.5	
CO	-6.25		-6.25	-6.25		-6.25
Balance		3.125	3.125	3.125	3.125	
C.O	1.56		1.56	1.56		1.56
Balance		−0.78	−0.78	−0.78	−0.78	
C.O	-0.39		−0.39	−0.39		0.39
Balance		0.195	0.195	0.195	0.195	
C.O	0.1					0.1
Final moments	**- 80**	**- 60**	**60**	**60**	**– 60**	**– 80**

Free body diagram of columns



Using F,x = 0 for the entire frame R = 28 + 28 = 56 KN

Hence R' = 56KN creates the sway moments shown in above moment distribution table. Corresponding moments caused by R = 0.91KN can be determined by proportion. Thus final moments are calculated by adding non sway moments and sway.

Moments calculated for R = 0.91KN, as shown below.

$$M_{AB} = 2.89 + \frac{0.91}{56}(-80) = 1.59KNm$$

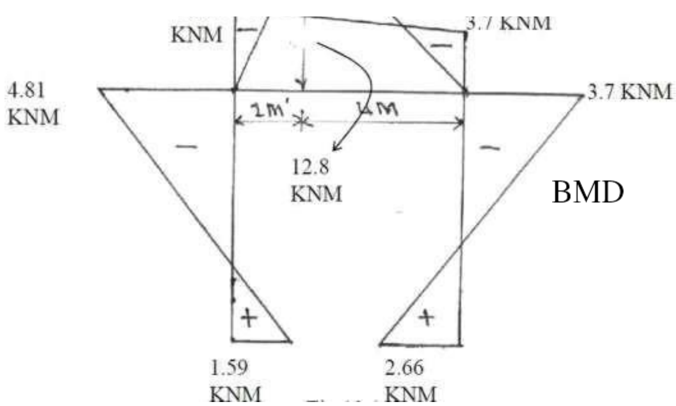
$$M_{BA} = 5.78 + \frac{0.91}{56}(-60) = 4.81KNm$$

$$M_{BC} = -5.78 + \frac{0.91}{56}(60) = -4.81KNm$$

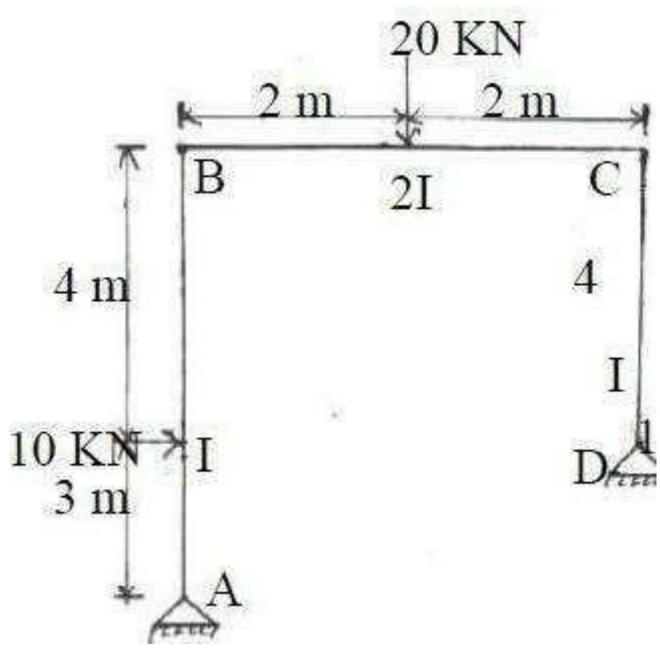
$$M_{CB} = 2.72 + \frac{0.91}{56}(60) = 3.7KNm$$

$$M_{CD} = -2.72 + \frac{0.91}{56}(-60) = -3.7KNm$$

$$M_{DC} = -1.36 + \frac{0.91}{56}(-80) = -2.66KNm$$



5.Q. Analysis the rigid frame shown in figure by moment distribution method and draw BMD



A. Non SwayAnalysis:

First consider the frame held from side sway

FEMS

$$M_{FAB=} - \frac{10 \times 3 \times 4^2}{7^2} = -9.8\text{KNm}$$

$$M_{FBA=} \frac{10 \times 4 \times 3^2}{7^2} = 7.3\text{KNm}$$

$$M_{FBC=} -\frac{20 \times 4}{8} = -10\text{KNm}$$

$$M_{FCB=} \frac{20 \times 4}{8} = 10\text{KNm}$$

$$M_{FCD=}\ M_{FDC=}\ 0$$

DISTRIBUTION FACTOR

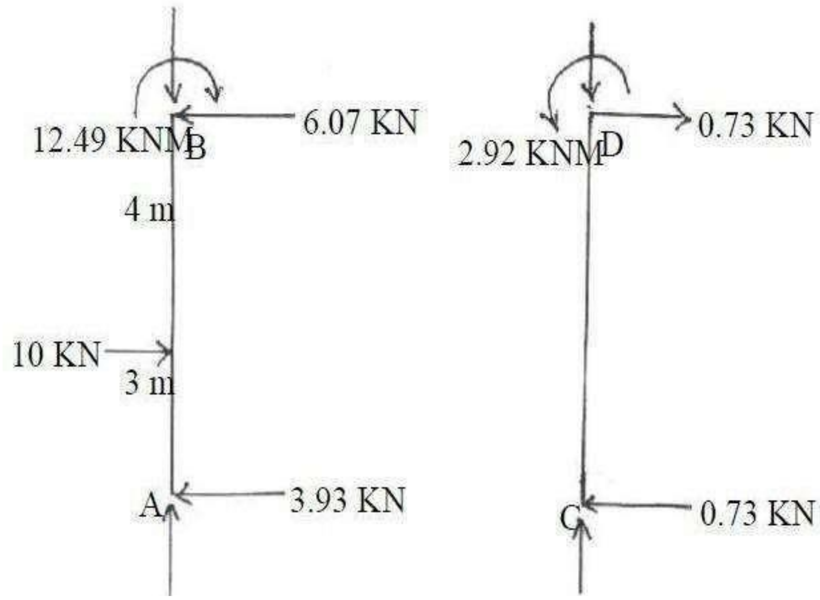
Joint	Member	Relative stiffness k	Σk	DF = $\frac{K}{\sum K}$
B	BA	$\frac{3}{4}\times\frac{I}{7}=0.11I$	0.61 I	0.18
	BC	2I/4 = 0.5I		0.82
C	CB	2I/4 = 0.5I		0.72
	CD	$\frac{3}{4}\times\frac{I}{4}=0.19\ I$	0.69 I	0.28

DISTRIBUTION OF MOMENTS FOR NON-SWAY ANALYSIS

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	1	0.18	0.82	0.72	0.28	1
FEM	-9.8	7.3	-10	10	0	0
Release jt. 'D'	+9.8					
CO		4.9				
Initial moments	0	12.2	-10	10	0	0
Balance		-0.4	-1.8	-7.2	-2.8	
CO			-3.6	-0.9		
Balance		0.65	2.95	0.65	0.25	
C.O			0.33	1.48		
Balance		-0.06	-0.27	-1.07	-0.41	
C.O			-0.54	-0.14		
Balance		0.1	0.44	0.1	0.04	
Final moments	**0**	**12.49**	**-12.49**	**2.92**	**-2.92**	**0**

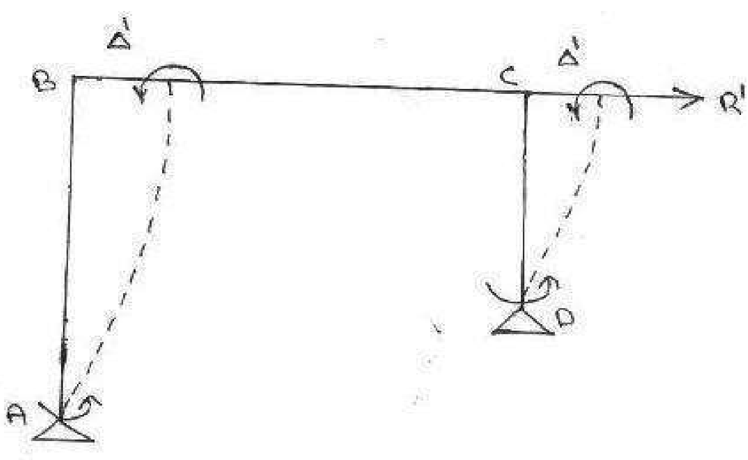
FREE BODY DIAGRAM OF COLUMNS

Applying Fx = 0 for frame as a Whole, R = 10 – 3.93 – 0.73 = 5.34 KN ←



Now apply R = 5.34KN acting opposite

Sway analysis: For this we will assume a force R' is applied at C causing the frame to deflect as shown in figure



Since ends A & D are hinged and columns AB & CD are of different lengths

$M'_{BA} = -\frac{3EI}{L_1^2}\Delta', \quad M'_{CD} = -\frac{3EI}{L_2^2}\Delta',$

$$\frac{M'_{BA}}{M'_{CD}} = \frac{\frac{3EI}{L_1^2}\Delta'}{\frac{3EI}{L_2^2}\Delta'} = \frac{L_2^2}{L_1^2} = \frac{4^2}{7^2} = \frac{16}{49}$$

Assume

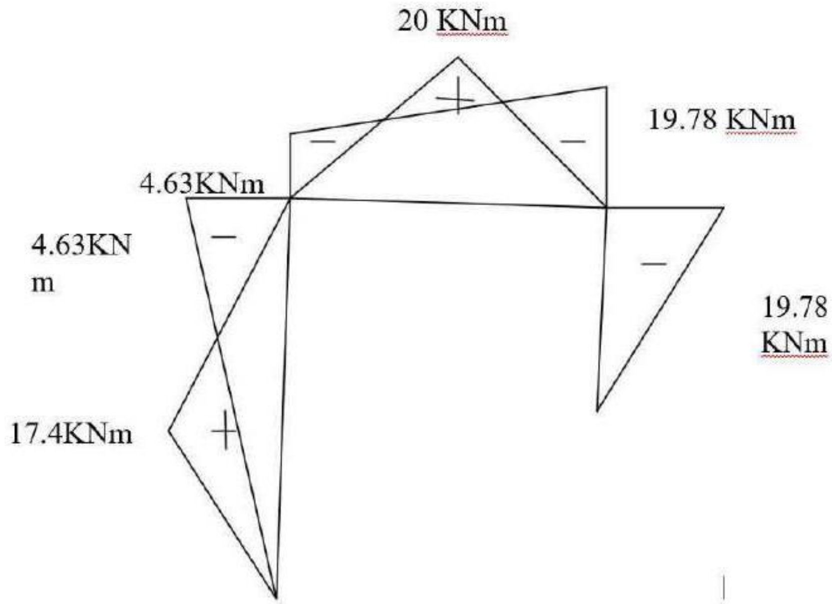
$$M'_{BA} = -16KNm, M'_{AB} = 0$$
$$M'_{CD} = -49KNm, \ M'_{DC} = 0$$

MOMENT DISTRIBUTION FOR SWAY ANALYSIS

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	1	0.18	0.82	0.72	0.28	1
FEM	0	-16	0	0	-49	0
Balance		2.88	13.12	35.28	13.72	
CO			17.64	6.56		
Balance		-3.18	-14.46	-4.72	-1.84	
CO			-2.36	-7.23		
Balance		0.42	1.94	5.21	2.02	
C.O			2.61	0.97		
Balance		-0.47	-2.14	-0.7	-0.27	
C.O			0.35	-1.07		
Balance		0.06	0.29	0.77	0.3	
C.O			0.39	0.15		
Balance		-0.07	-0.32	-0.11	-0.04	
Final moments	**0**	**-16.36**	**16.36**	**35.11**	**-35.11**	**0**

Using Fx = 0 for the entire frame R' = 11.12 kN ⟶

Hence R' = 11.12 KN creates the sway moments shown in the above moment distribution table. Corresponding moments caused by R = 5.34 kN can be determined by proportion. Thus final moments are calculated by adding non-sway moments and sway moments determined for R = 5.34 KN as shown below.



B.M.D

$M_{AB} = 0$

$$M_{BA} = 12.49 + \frac{5.34}{11.12}(-16.36) = 4.63\text{KNm}$$

$$M_{BC} = -12.49 + \frac{5.34}{11.12}(16.36) = -4.63\text{KNm}$$

$$M_{CB} = 2.92 + \frac{5.34}{11.12}(35.11) = 19.78\text{KNm}$$

$$M_{CD} = -2.92 + \frac{5.34}{11.12}(-35.11) = -19.78\text{KNm}$$

$M_{DC} = 0$

20 KNm