<u>**UNIT - IV**</u>

Groundwater Hydrology: Occurrence, movement and application of groundwater, aquifers – types, Specific Yield, Permeability, Storage coefficient, Transmissibility, Darcy's Law. Well Hydraulics - Steady radial flow into well for confined and unconfined aquifers, Recuperation tests. Well constants. Crop water requirements – Water requirements of crops – crops and crop seasons in India, cropping pattern, duty and delta; Quality of irrigation water; Soil-water relationships, root zones oil water, infiltration, consumptive use, irrigation requirement, frequency of irrigation; Methods of applying water to the fields: surface, sub-surface, Micro irrigation.

GROUNDWATER HYDROLOGY

In the previous chapters various aspects of surface water hydrology that deal with surface runoff have been discussed. Study of subsurface flow is equally important since about 30% of the world's freshwater resources exist in the form of groundwater.

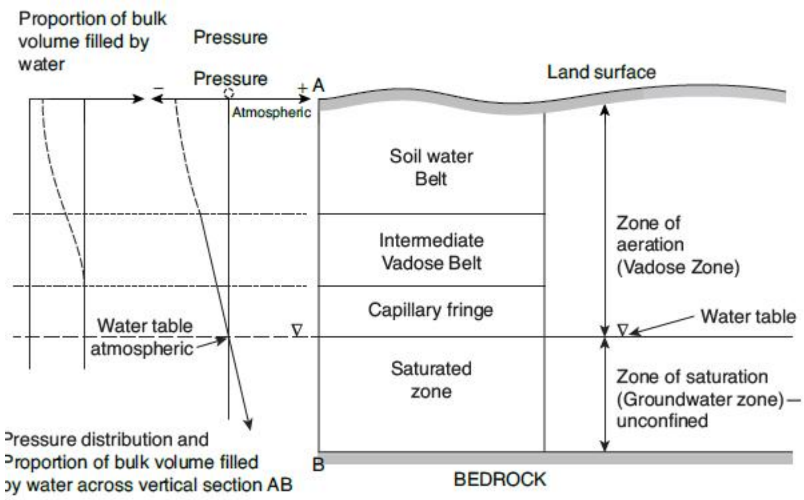
Sources and Forms of Groundwater

Sources

Groundwater originates from precipitation and recharge from surface and man-made water bodies. Infiltrated rainwater and water from streams, lakes, ponds, canals, and artificial recharge systems percolate downward under gravity, filling the soil's pore spaces and forming groundwater.

Forms

The subsurface water is considered in two major zones as (i) Zone of aeration and (ii) Zone of saturation. The subsurface water that exists in saturated zone is popularly called groundwater and hence the Zone of saturation is also known as Groundwater zone.



1. Zone of Aeration (Vadose Zone)

The **zone of aeration (Vadose zone)** lies just below the ground surface, where soil pores contain both air and water. Water moves downward by **percolation** under gravity. This zone includes:

- **Soil Water Belt** – the top layer (about 0.9–1.2 m) from which plants absorb water through roots.
- **Intermediate Vadose Belt** – lies between the soil water belt and capillary fringe; thickness varies with soil type and moisture.
- **Capillary Fringe** – the layer just above the water table where water rises by capillary action; thickness depends on soil pore size.

The zone of aeration contains **hygroscopic, capillary, and gravitational water**. Key soil properties include **field capacity** (maximum water soil can retain after drainage) and **permanent wilting point** (when plants can no longer extract water).

2. Zone of Saturation

The **zone of saturation**, or **groundwater zone**, extends from the **water table** (where soil is fully saturated) to the bottom of the porous layer. Here, all pores are filled with water, and pressure increases with depth below the water table. The **water table** level rises or falls based on groundwater storage. Groundwater flows in the direction of the lowest hydraulic gradient. In fractured rock areas, groundwater occupies the rock's **joints and fractures**, but the same zonal structure applies.

Types of Geological Formation

Saturated formations are classified into four categories:

- Aquifer
- Aquitard
- Aquiclude
- Aquifuge

Aquifer

An aquifer is a saturated formation of earth material which not only stores water but yields it in sufficient quantity. Thus, an aquifer transmits water relatively easily due to its high permeability. Unconsolidated deposits of sand and gravel form good aquifers. (Formation of Sand and Gravels).

Aquitard

It is a formation through which only seepage is possible and thus the yield is insignificant compared to an aquifer. It is partly permeable. A sandy clay unit is an example of aquitard. Through an aquitard appreciable quantities of water may leak to an aquifer below it. (Formation of Sandy Clay)

Aquiclude

It is a geological formation which is essentially impermeable to the flow of water. It may be considered as closed to water movement even though it may contain large amounts of water due to its high porosity. Clay is an example of an aquiclude. (Formation of Clay)

Aquifuge

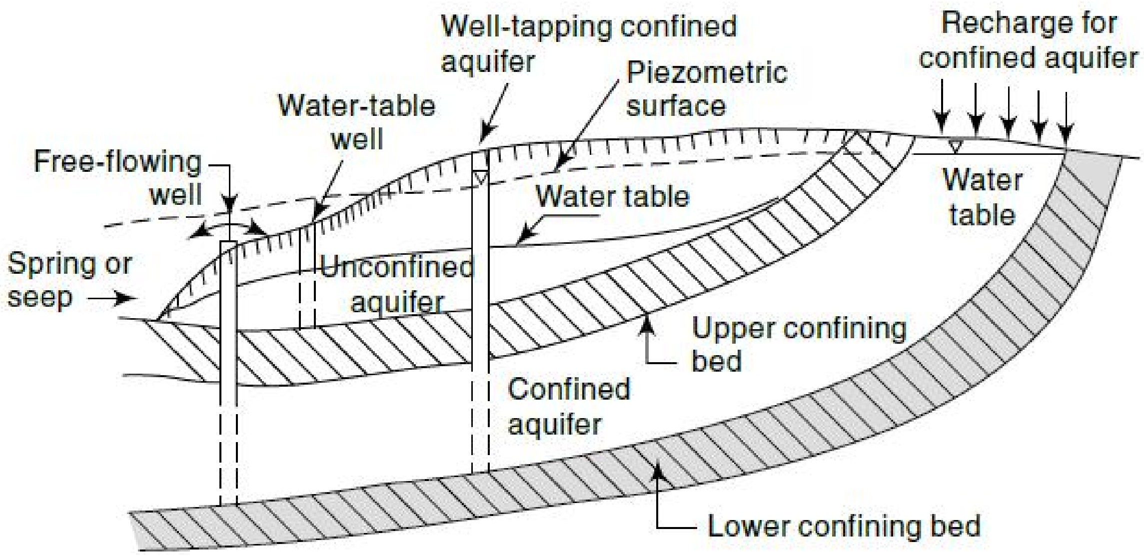
It is a geological formation which is neither porous nor permeable. There are no interconnected openings and hence it cannot transmit water. Massive compact rock without any fractures is an aquifuge.

The definitions of aquifer, aquitard and aquiclude as above are relative. A formation which may be considered as an aquifer at a place where water is at a premium (e.g. arid zones) may be classified as an aquitard or even aquiclude in an area where plenty of water is available. The availability of groundwater from an aquifer at a place depends upon the rates of withdrawal and replenishment (recharge).

Types of Aquifer

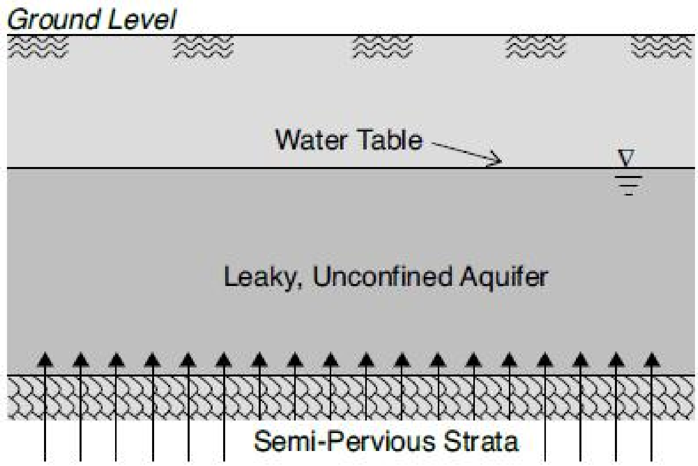
Aquifers play the roles of both a transmission conduit and a storage. Aquifers are classified as unconfined aquifers and confined aquifers on the basis of their occurrence and field situation.

Unconfined Aquifer: An unconfined aquifer (also known as water table aquifer) is one in which a free water surface, i.e. a water table. Only the saturated zone of this aquifer is of importance in groundwater studies. Recharge of this aquifer takes place due to infiltration of the precipitation at the ground surface. A well driven into an unconfined aquifer, when not pumping, will indicate the static water level corresponding to the water table at that location.



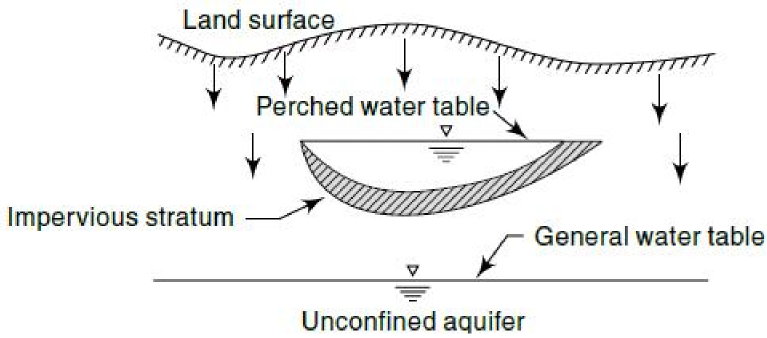
Confined Aquifer: A confined (artesian) aquifer is a water-bearing layer trapped between two impervious beds. It gets recharged only where it's exposed at the surface. The water is under pressure, so the piezometric level is higher than the aquifer top — sometimes even above the ground surface, causing free-flowing wells without pumps. The term artesian comes from Artois, France, where such wells were first found.

Leaky Aquifer: A leaky aquifer (also known as semi-confined aquifer) is an aquifer whose upper and lower boundaries or one boundary is an aquitard, the other one is an aquiclude. In a general sense, existence of leaky aquifer is more expected than confined or unconfined aquifers. This is due to the fact that it is hard to find an aquifer that is completely confined or unconfined. Leaky aquifers are common in alluvial valleys, plains and former lake basins.

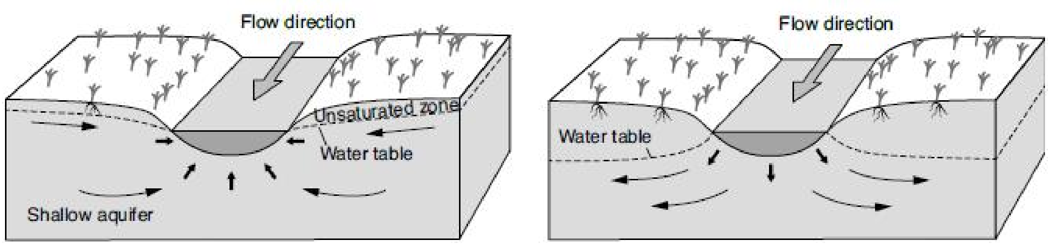


Water Table

A water table is the free surface of groundwater in an unconfined aquifer, showing the level where soil is saturated. It rises or falls with recharge and discharge, pumping, or seasonal changes, and generally follows surface topography. Where it meets the land, springs or seepage occur. A perched water table forms when an impervious layer traps water above the main water table; it is local and yields little water, often mistaken for the main water table.



The position of the water table relative to the water level in a stream determines whether the stream contributes water to the groundwater storage or the other way around. If the bed of the stream is below the groundwater table, during periods of low flows in the stream, the water surface may go down below the general water table elevation and the groundwater contributes to the flow in the stream. Such streams which receive groundwater flow are called effluent streams (gaining streams). Perennial rivers and streams are of this kind. If, however, the water table is below the bed of the stream, the stream water percolates to the groundwater storage and a hump is formed in the groundwater table. Such streams which contribute to the groundwater are known as influent streams (losing streams). Intermittent rivers and streams which go dry during long periods of dry spell (i.e. no rain periods) are of this kind.



Aquifer Properties

The important properties of an aquifer are its capacity to release the water held in its pores and its ability to transmit the flow easily. These properties essentially depend upon the composition of the aquifer.

Porosity

The amount of pore space per unit volume of the aquifer material is called porosity. It is expressed as

$$n = \frac{V_v}{V_0}$$

where n = porosity, V_v = volume of voids and V_0 = volume of the porous medium. In an unconsolidated material the size distribution, packing and shape of particles determine the porosity. In hard rocks the porosity is dependent on the extent, spacing and the pattern of fracturing or on the nature of solution channels. In qualitative terms porosity greater than 20% is considered as large, between 5 and 20% as medium and less than 5% as small.

Specific Yield (S_y)

The actual volume of water that can be extracted by the force of gravity from a unit volume of aquifer material is known as specific yield.

Specific Retention (S_r)

The fraction of water held back in the aquifer is known as specific retention.

$$n = S_y + S_r$$

Darcy's Law

In 1856, Henry Darcy, a French hydraulic engineer, on the basis of his experimental findings proposed a law relating the velocity of flow in a porous medium. This law, known as Darcy's law, can be expressed as

$$V = Ki$$

where V = apparent velocity of seepage = Q/A in which Q = discharge and A = cross-sectional area of the porous medium. V is also known as Darcy velocity and also as Discharge Velocity $i = -\frac{dh}{ds}$ = hydraulic gradient, in which h = piezometric head and

s = distance measured in the direction of flow: the negative sign emphasises that the piezometric head drops in the direction of flow

K = a coefficient, called the Coefficient of permeability.

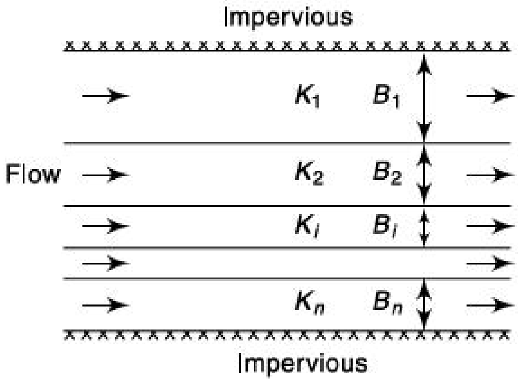
The discharge Q can be expressed as

$$Q = K\,i\,A$$
$$= KA\left(-\frac{\Delta H}{\Delta s}\right)$$

Stratification

Sometimes the aquifers may be stratified, with different permeabilities in each strata. Two kinds of flow situations are possible in such a case.

a. When the flow is parallel to the stratification



$$K_e = \frac{\sum_1^n K_i B_i}{\sum_1^n B_i}$$

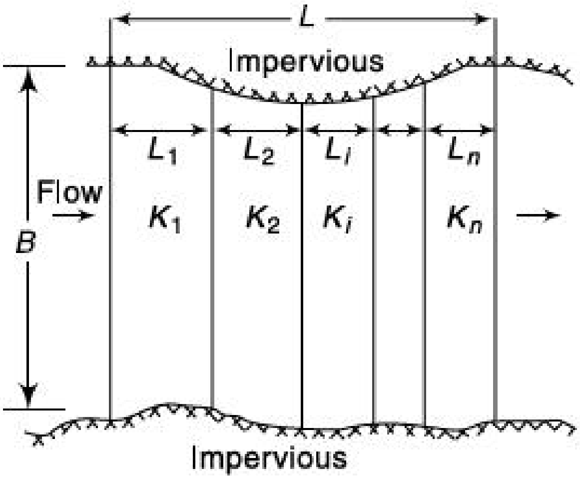
The transmissibility of the formation is

$$T = K_e\,\Sigma B_i = \sum_1^n K_i B_i$$

b. When the flow is normal to the stratification

$$K_e = \frac{\sum_1^n L_i}{\sum_1^n (L_i/K_i)}$$

[**Note:** In this case L is the length of seepage and the thickness B of the aquifer does not come into the picture in calculating the equivalent permeability.]



The transmissibility of the aquifer is T = K_e × B.

i). In a certain aquifer of $3 km^2$ the water table was at an elevation of 102m. due to natural recharge in a wet season, it level rows to 103.20 a volume of 1.5 mm^3 was water was then pumped out of the aquifer ~~Cath~~ crossing the aquifer to reach a level of 101.20m assuming the water-table in the entire aquifer to respond to similar way estimate (a) the specific yield of the aquifer (b) the volume of recharge during the wet season.

Sol:- $D = S_y + S_R$.

volume of pumped out = Area × sy × height.

$1.5 \times 10^6 = 3 \times 10^6 \times S_y \times (103.20 - 101.20)$

a). $\boxed{S_y = 0.25}$

b). volume = $3 \times 10^6 \times (103.20 - 102) \times 0.25$

$\boxed{V = 9 \times 10^5 m^3}$

A field test for permeability consists in observing the time required for a tracer to travel between two observation wells. A tracer was found to take 10 h to travel between two wells 50 m apart when the difference in the water surface elevation in them was 0.5 m. The mean particle size of the aquifer was 2 mm and the porosity of the medium 0.3. If v = 0.01 cm2/s estimate (a) the coefficient of permeability and intrinsic permeability of the aquifer and (b) the Reynolds number of the flow.

a). $K = C d^2 m \frac{\gamma}{\mu}$

$K = K_0 \cdot \frac{g}{\nu}$

Bulk pore velocity

$V_a = \frac{V}{\eta} = \frac{50 \times 100}{10 \times 60 \times 60} = 0.139 \text{ cm/s}$

$i = \frac{h}{L} = \frac{0.5}{50} = 0.01$

$V = ki$

$V_a = \frac{V}{\eta} \quad V = \eta \times V_a = 0.3 \times 0.139$

$\boxed{V = 0.0417 \text{ cm/s}}$

$V = ki$

$0.0417 = K \times 0.01$

$\frac{0.0417}{0.01} = K \quad \boxed{K = 4.17 \text{ cm/s}}$

$K_0 = K \times \frac{\nu}{g}$

$= 4.17 \times \frac{0.01}{9810} = 4.25 \times 10^{-5} \text{ cm}^2$

$\boxed{K_0 = 4.25 \times 10^{-5} \text{ cm}^2}$

In Darcy:

$= \frac{4.25 \times 10^{-5}}{9.87 \times 10^{-9}} = 4305.97 \text{ days}$

$\therefore 1 \text{ cm} = 9.87 \times 10^{-13} \text{ m}^2$

$cm^2 = (100)^2$

$(10^{-13}) \times 10^{4}$

Renold's number (Re) $= \frac{Vd}{\nu}$

$= \frac{0.0417 \times 0.2}{0.01} = \cancel{8.34}$

$= 0.834$

10 mm = 10 cm

$\boxed{Re = 0.834}$

In a field test a time of 6 hours was Required for a tracer to travel Between two Observation wells 42 m apart. If difference in water table Elevations in these wells were 0.85m and the Porosity of the aquifer is 20%. Calculate the co-efficient of Permeability of the aquifer.

$$V_a = \frac{42 \times 100}{6 \times 60 \times 60} = 0.1944 \text{ cm/s}$$

$$V = V_a \times n = 0.1944 \times 0.20 = 0.0388 \text{ cm/s}$$

$$V = ki \Rightarrow K = \frac{V}{i} = \frac{0.0388}{\frac{0.85}{42}} = 1.92 \text{ cm/s}$$

Q

A Confined aquifer is 25m thick and 2km wide two Observation wells located 2km apart in the Direction of flow indicate heads of 45 and 39.5 m. If the co-efficient of permeability of the aquifer is 30 m/day.

Calculate

1. total daily flow through aquifer
2. the piezometric head at an Observation well located 300m from the upstream well.

①

$K = 30 \text{m/day}$

$\text{Area} = 2000 \times 25 \text{ m}^2$

$Q = KiA$

$$i = \frac{h}{L} = \frac{45 - 39.5}{2000}$$

$$= 0.00275$$

$Q = KiA$

$$= 30 \times 2000 \times 25 \times 0.0275$$

$$\boxed{Q = 4125 \text{ m}^3/\text{day}}$$

②

$$= 45 - \left[\frac{45 - 39.5}{2000} \times 300\right]$$

$$= 44.175 \text{m}$$

What is the intrinsic permeability, in darcys, of a water-saturated medium that has a hydraulic conductivity of 15.24 m/day? Assume the groundwater is at atmospheric pressure at 20°C and has a density of 998 kg/m^3 and viscosity of 1.002×10^{-3} kg/ms.

Solution

By Eq. (9.8), intrinsic permeability $K_0 = \dfrac{K\nu}{g} = \dfrac{K\mu}{g\rho}$

Here, given $K = 15.24 \text{ m/day} = \dfrac{15.24}{86{,}400} \text{ m/s}$

$\mu = 1.002 \times 10^{-3}$ kg/ms

$\rho = 998$ kg/m^3

Hence, intrinsic permeability $K_0 = \dfrac{15.24}{86{,}400} \times \dfrac{1}{9.81} \times \dfrac{1.002 \times 10^{-3}}{998}$

$$= 1.80 \times 10^{-11} \text{ m}^2$$

Since 1 darcy $= 9.87 \times 10^{-13}$ m^2

$$K_0 = \frac{1.80 \times 10^{-11}}{9.87 \times 10^{-13}} = \mathbf{18.29 \text{ darcy}}$$

At a certain point in an unconfined aquifer of 3 km^2 area, the water table was at an elevation of 102.00 m. Due to natural recharge in a wet season, its level rose to 103.20 m. A volume of 1.5 Mm^3 of water was then pumped out of the aquifer causing the water table to reach a level of 101.20m. Assuming the water table in the entire aquifer to respond in a similar way, estimate (a) the specific yield of the aquifer and (b) the volume of recharge during the wet season.

Solution

(a) Volume pumped out = area × drop in water table × specified yield S_y

$$1.5 \times 10^6 = 3 \times 10^6 \times (103.20 - 101.20) \times S_y$$

$$S_y = 0.25$$

(b) Recharge volume $= 0.25 \times (103.20 - 102.00) \times 3 \times 10^6 = 0.9$ Mm^3

An inert pollutant material spills into the subsurface and infiltrates the water table aquifer. The aquifer consists mainly of sand and gravel with hydraulic conductivity of 80 m/day and effective porosity of 0.30. The water level in a neighbouring well *A* is at 155 m above sea level. The water level in another well *B* directly 1200 m downgrade is 152 m above sea level. Determine the (a) Darcy velocity of the groundwater, (b) the bulk pore velocity of the groundwater and (c) the velocity of pollutant migration. (d) How long will it take for the pollutant from the well *A* to reach the well *B*?

Solution

Hydraulic gradient for the groundwater flow $= i = \dfrac{dh}{dl} = \dfrac{155-153}{1200} = 0.001667$

(a) Darcy velocity $V = Ki = 80 \times 0.001667 = 0.1336$ m/day

(b) Bulk pore velocity $= v_a = \dfrac{V}{\eta} = \dfrac{0.1336}{0.30} = 0.445$ m/day

(c) Since the pollutant is inert (i.e. it does not react with water), the bulk pore velocity = velocity of the pollutant = 0.445 m/day

(d) Time taken for the pollutant from the well *A* to reach the well *B*

$= t = \dfrac{1200}{0.445} = 2696$ days = **7.38 years**

A confined stratification aquifer has a total thickness of 12 m and a made up of 3 layers. The bottom layer has a K = 30m/day and thickness of 5m. the middle and top layer have permeability 20 m/day and 45 m/day respectively and are of equal thickness.

Calculate (a) Transmissibility, (b) Equivalent Permeability, if flow is along the stratification

$$K_e = \frac{k_1 b_1 + K_2 b_2 + k_3 b_3}{b_1 + b_2 + b_3}.$$

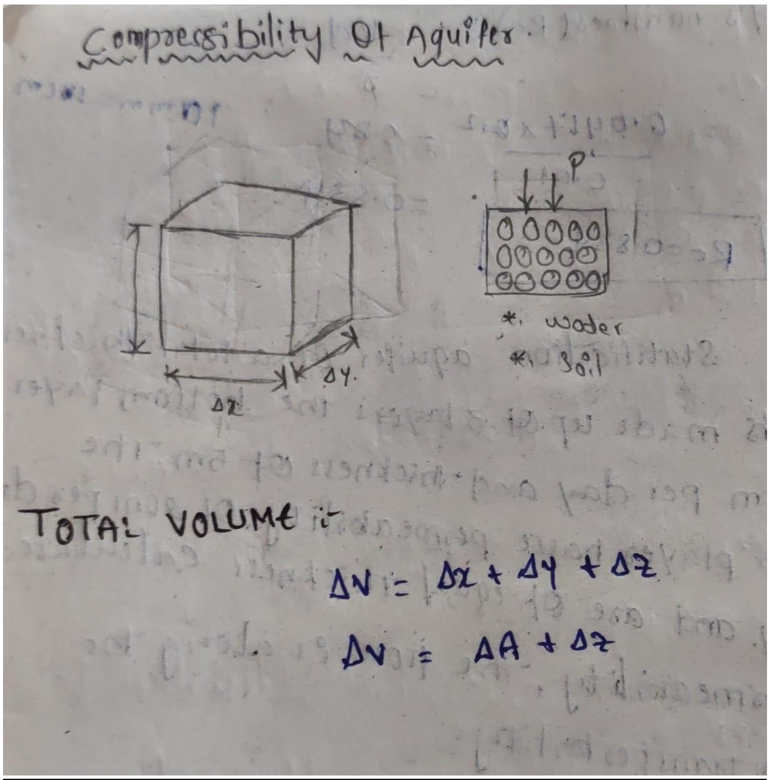
$K_1 = 45 m/d \quad K_2 = 20 m/d \quad k_3 = 30 m/d.$

$b_1 = 3.5 m \quad K_2 = 3.5 m \quad k_3 = 5 m.$

$$K_e = \frac{45 \times 3.5 + 20 \times 3.5 + 30 \times 5}{12} = 31.45.$$

$T = K_e B. \quad = 31.45 \times 12 = 377.4\ m^2/sec.$

Compressibility of Aquifers



Assumptions

The elemental volume is constraint in lateral dimensions and undergoes change of length in z-direction only. That is ΔA is constant

⇒ The pore water is compressible

⇒ The soil grains of the aquifer is incompressible for the pore structure is compressible.

Compressibity of water (β)

$$\boxed{\beta = \frac{-d\Delta V_w / \Delta V_w}{dp}} \rightarrow (1)$$

∴ V_w = volume of water (in soil)

p = pressure.

Density of water:- (ρ)

$\rho \Delta V_w$ = constant

thus;

$$\rho d(\Delta V_w) + \Delta V_w d\rho = 0.$$

$$\rho d(\Delta V_w) = -\Delta V_w d\rho$$

$$\boxed{\frac{d\rho}{\rho} = \frac{-d\Delta V_w}{\Delta V_w}}$$

Sub(2) in (1)

$$\beta = \frac{d\rho/\rho}{dp}$$

$$dp\beta = d\rho/\rho$$

$$\rho dp\beta = d\rho$$

$$\boxed{d\rho = \rho\beta dp} \rightarrow (3).$$

$$\boxed{\frac{d\rho}{\rho} = \beta dp}$$

compressibility of pore structure

$$\alpha = \frac{-d\Delta V/\Delta V}{d\sigma_z} \qquad \therefore \sigma_z = \text{Intergranular pressure}$$

∴ In this whole volume will not change only Δz will change

so, $$\alpha = \frac{-d\Delta z/\Delta z}{d\sigma_z}$$

Over burden weight $= p+\sigma_z$ = constant.

$$dp = -d\sigma_z$$

$$\alpha = \frac{+d\Delta z/\Delta z}{+dp} \Rightarrow \alpha\, dp\, \Delta z = d\Delta z$$

$$\alpha\, dp\, \Delta z = d\Delta z \rightarrow (4)$$

volume of soil (ΔV_s)

$$\Delta V_s = (1-n)\,\Delta A\, \Delta z.$$

∴ n = porosity of the Aquifer.

$$d\Delta V_s = (1-n)\, d\Delta z - \Delta z\, dn = 0$$

$$(1-n)\alpha\, dp\, \cancel{\Delta z} = dn\, \cancel{\Delta z}$$

$$\boxed{dn = (1-n)\alpha\, dp}$$

Mass of water (Δm)

$$\Delta m = \rho n \Delta A \Delta z.$$

$$d\Delta m = \Delta V\left[n\, d\rho + \rho\, dn + \rho n \frac{d\Delta z}{\Delta z}\right]$$

$$\frac{d\Delta m}{\rho \Delta V} = n\frac{d\rho}{\rho} + dn + \frac{d\Delta z}{\Delta z}$$

$$\boxed{n\frac{d\rho}{\rho} = a} \quad \boxed{dn = b} \quad \boxed{\frac{d\Delta z}{\Delta z} = c}$$

Substitute all the values a, b, c

$$\frac{d\Delta m}{\rho \Delta V} = n\beta\, dp + \alpha(1-n)\, dp + n\alpha\, dp$$

$$= n\beta\, dp + \alpha\, dp - \cancel{\alpha n\, dp} + \cancel{n\alpha\, dp}$$

$$= (n\beta+\alpha)\, dp$$

$$= \gamma(n\beta+\alpha)\, dh. \qquad \therefore dh = \text{small height}, \ \gamma = \text{unit weight}$$

$$\frac{d\Delta m}{\rho \Delta V} = S_s\, dh.$$

$$\therefore S_s = \gamma(n\beta+\alpha)$$

$$S = \gamma(n\beta+\alpha)B$$

where

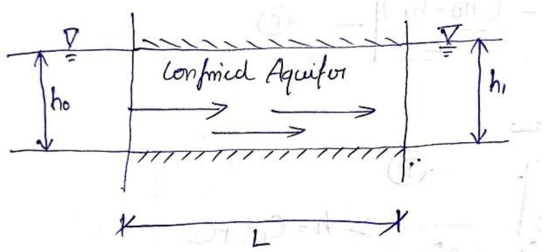
S_s = specific storage.

S = storage co-efficient

B = thickness of Aquifer.

Equation of Motion

Confined Ground water flow between two water bodies



$$\frac{\partial^2 h}{\partial x^2} = 0$$

If we Integrate twice,

$$h = C_1 x + C_2 \quad \text{—— (A)}$$

<u>Boundary condition - 1</u>

When $x = 0$ —— (1) $\because x$ = Length of Aquifer

$$h = h_0 \quad \text{—— (2)}$$

Put Equation (1) and (2) in (A)

$$h_0 = C_1 \times (0) + C_2$$

$$h_0 = 0 + C_2$$

$$\boxed{C_2 = h_0} \quad \text{—— (B)}$$

<u>Boundary Condition - 2</u>

When $x = L$ —— (3)

$$h = h_1 \quad \text{—— (4)}$$

Put Equation (3) and (4) in (A)

$$h = C_1 x + C_2$$

$$h_1 = C_1 \times (L) + h_0$$

$$\boxed{C_1 = -\frac{(h_0 - h_1)}{L}} \quad \text{—— (C)}$$

Discharge per unit width

According to Darcy law

$$\boxed{q = -KB\frac{\partial h}{\partial x}} \quad \text{—— (D)}$$

$\rightarrow h = C_1 x + C_2$

$$h = h_0 + \left(-\frac{(h_0 - h_1)}{L}\right)x$$

→ Sub (B) and (C) in (A)

$$h = \cancel{h_0}^{0} + \left(-\frac{(h_0 - h_1)}{L}\right)x$$

$$\boxed{\frac{\partial h}{\partial x} = -\frac{(h_0 - h_1)}{L}} \quad \text{—— (5)}$$

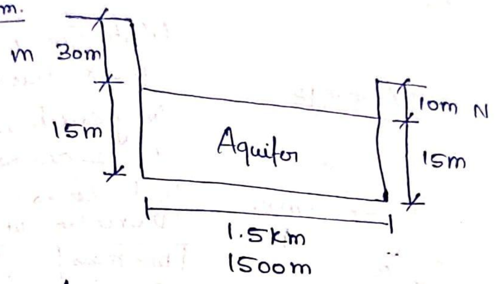
Sub (5) in (D)

$$q = -KB\left(-\frac{(h_0 - h_1)}{L}\right)$$

$$\boxed{q = \frac{h_0 - h_1}{L} KB}$$

A confined horizontal aquifer of thickness 15 m and permeability K = 20 m/day connects two reservoirs M and N situated 1.5 km apart. The elevations of the water surface in reservoirs M and N measured from the top of the aquifer are 30 m and 10 m respectively. If the reservoir M is polluted by a contaminant suddenly, how long will it take the contaminant to reach the reservoir N? Assume n = 0.30.

Problem.



$$h_0 = 15+30 = 45m$$
$$h_1 = 15+10 = 25m$$

Gross Velocity (V),

$$V = \frac{K(h_0 - h_1)}{L}$$
$$= \frac{20(45-25)}{1500}$$
$$\boxed{V = 0.267 m/day}$$

Actual Velocity (V_0)

$$V_0 = \frac{\text{Gross Velocity}}{\text{Porosity}} = \frac{V}{n} = \frac{0.267}{0.3} \Rightarrow \boxed{0.89 m/day = V_0}$$

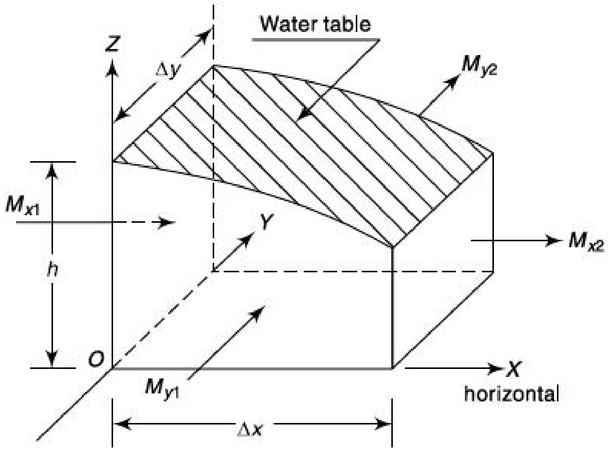
Length/actual Velocity

$$\text{Time taken} = \frac{1500}{0.89} = 1685.39 \text{ days} \Rightarrow \frac{1685.39}{365}$$
$$= 4.617 \text{ years.}$$

Unconfined Flow by Dupit's Assumption

A simplified approach based on the assumptions suggested by Dupit (1863) which gives reasonably good results is described below. The basic assumptions of Dupit are the following:

- The curvature of the free surface is very small so that the streamlines can be assumed to be horizontal at all sections.
- The hydraulic grade line is equal to the free surface slope and does not vary with depth.



Mass flux Entering the Element $M_{x_1} = \rho V_x (h \Delta y)$ — Density, Area, thickness, Velocity

" " Leaving the Element $M_{x_2} = \rho V_x h \Delta y + \frac{\partial}{\partial x}(\rho V_x h \Delta y)\Delta x$

Mass flux Entering the Element $M_{y_1} = \rho V_y h \Delta x$

" Leaving " " $M_{y_2} = \rho V_y h \Delta x + \frac{\partial}{\partial y}\left[\rho V_y h \Delta x\right]\Delta y$

The Net Mass efflux from the element in x - Direction

$$\Delta M_x = M_{x_1} - M_{x_2}$$

$$= \rho V_x h \Delta y - \left[\rho V_x h \Delta y + \frac{\partial}{\partial x}(\rho V_x h \Delta y)\Delta x\right]$$

$$= \cancel{\rho V_x h \Delta y} - \cancel{\rho V_x h \Delta y} - \frac{\partial}{\partial x}(\rho V_x h \Delta y)\Delta x$$

$$\boxed{\Delta M_x = -\frac{\partial}{\partial x}(\rho V_x h \Delta y)\Delta x}$$

Similary, the Net Mass efflux in y- Direction

$$\Delta M_y = M_{y_1} - M_{y_2}$$

$$= \rho V_y h \Delta x - \left[\rho V_y h \Delta x + \frac{\partial}{\partial y}(\rho V_y h \Delta x)\Delta y\right]$$

$$\boxed{\Delta M_y = -\frac{\partial}{\partial y}(\rho V_y h \Delta x)\Delta y}$$

Further, there is neither inflow nor outflow in the z-Direction. Thus for Steady, incompressible flow, By continuity

$$\Delta M_x + \Delta M_y = 0 \quad \text{——— (1)}$$

Sub ΔM_x and ΔM_y in (1) Simplifying

$$\frac{\partial}{\partial x}(V_x) + \frac{\partial}{\partial y}(V_y) = 0$$

According to Darcy's Law

$V_x = Ki$ Where i = Change in head along Length / Slope

$$V_x = -K\frac{\partial h}{\partial x}, \quad V_y = -K\frac{\partial h}{\partial y}$$

$$\frac{\partial}{\partial x}(V_x h) + \frac{\partial}{\partial y}(V_y h) = 0$$

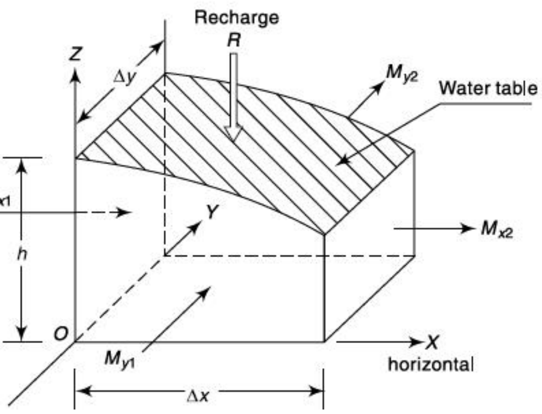
$$\frac{\partial}{\partial x}\left[-Kh\frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[-Kh\frac{\partial h}{\partial y}\right] = 0$$

Thus,

$$\boxed{\frac{\partial^2 h^2}{\partial x^2} + \frac{\partial^2 h^2}{\partial y^2} = 0}$$

Unconfined Flow with Recharge

If there is a recharge, i.e. infiltration of water from the top ground surface into the aquifer, at a rate of R (m^3/s per m^2 of horizontal area)



$$\Delta M_z = \rho R \Delta x \Delta y$$

$$\Delta M_x = -\frac{\partial}{\partial x}(\rho V_x h \Delta y)\Delta x$$

$$\Delta M_y = -\frac{\partial}{\partial y}(\rho V_y h \Delta x)\Delta y$$

for Steady, Incompressible flow, the continuity Relationship for the Element is

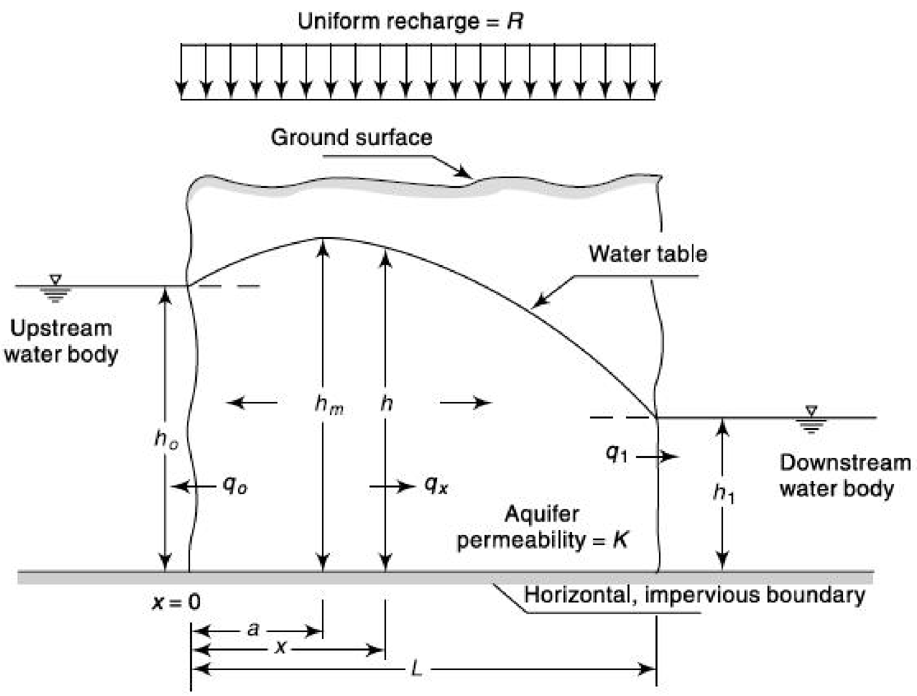
$$\Delta M_x + \Delta M_y + \Delta M_z = 0$$

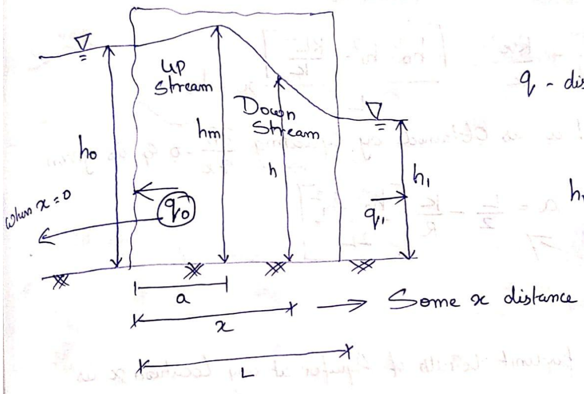
$$-\frac{\partial}{\partial x}\left(\rho V_x h \Delta x \Delta y\right) - \frac{\partial}{\partial y}\left(\rho V_y h \Delta x \Delta y\right) + \rho R \Delta x \Delta y = 0$$

Subsitituting $V_x = -k\frac{\partial h}{\partial x}$ and $V_y = -k\frac{\partial h}{\partial y}$ and Simplifying

$$\frac{\partial^2 h^2}{\partial x^2} + \frac{\partial^2 h^2}{\partial y^2} = -\frac{2R}{K}$$

One Dimensional Dupit's Flow with Recharge





q - discharge Per unit width

h_m - max height

$$\frac{\partial^2 h^2}{\partial x^2} = -\frac{2R}{K}$$

On Integrating with Respect to x

$$h^2 = -\frac{R}{k}x^2 + C_1 x + C_2 \quad \text{(1)}$$

Where C_1 and C_2 are constant of Integration

the bondary conditions are

(i) $x = 0$, $h = h_o$

Hence, $C_2 = h_o$

(ii) $x = L$, $h = h_1$

Hence

$$h_1^2 = -\frac{R}{k}L^2 + C_1 L + h_o^2$$

$$C_1 = -\frac{\left[h_o^2 - h_1^2 - \frac{RL^2}{k}\right]}{L}$$

Substitute C_1 & C_2 in Equation (1)

$$h^2 = -\frac{Rx^2}{k} - \frac{\left[h_o^2 - h_1^2 - \frac{RL^2}{k}\right]}{L}x + h_o^2$$

The Value of a is Obtained by equating $\frac{\partial h}{\partial x} = 0$ & is given

find the length of the Divide line / divide line a

$$a = \frac{L}{2} - \frac{K}{R}\left[\frac{h_o^2 - h_1^2}{2L}\right]$$

The Discharge per unit Width of Aquifer at any Location x is

$$q_x = -Kh\frac{\partial h}{\partial x}$$

$$= -K\left[-\frac{Rx}{k} - \frac{\left(h_o^2 - h_1^2 - \left(\frac{RL^2}{k}\right)\right)}{2L}\right]$$

$$q_x = R\left[x - \frac{L}{2}\right] + \frac{k}{2L}(h_0^2 - h_1^2)$$

It is obvious the discharge q_x varies with x. At the upstream water body, $x=0$ and

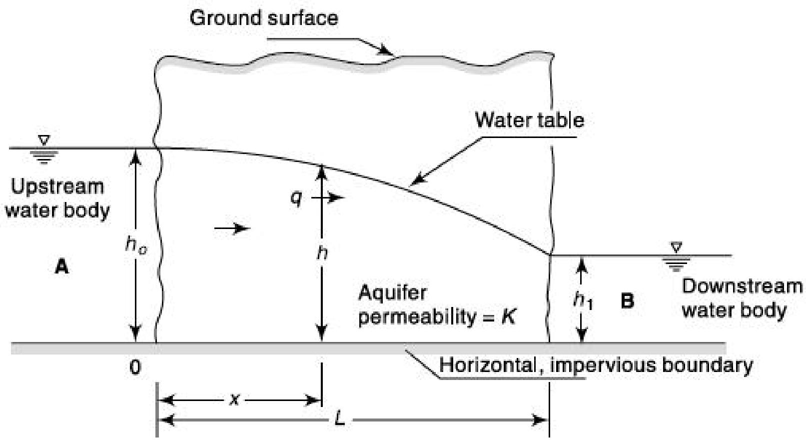
$$\text{Discharge} \quad q_0 = q_{x=0} = -\frac{RL}{2} + \frac{k}{2L}(h_0^2 - h_1^2)$$

At the downstream water body, $x = L$ and

$$q_1 = q_{x=L} = \frac{RL}{2} + \frac{k}{2L}(h_0^2 - h_1^2)$$

$$\boxed{q_1 = RL + q_0}$$

One Dimensional Flow with No Recharge



$\because R = 0$

$$h^2 = \left(\frac{h_0^2 - h_1^2}{L}\right)x + h_0^2$$

$$h^2 - h_0^2 = \left(\frac{h_0^2 - h_1^2}{L}\right)x$$

Differentiating with respect to x

$$2h\frac{\partial h}{\partial x} = \frac{h_0^2 - h_1^2}{L}$$

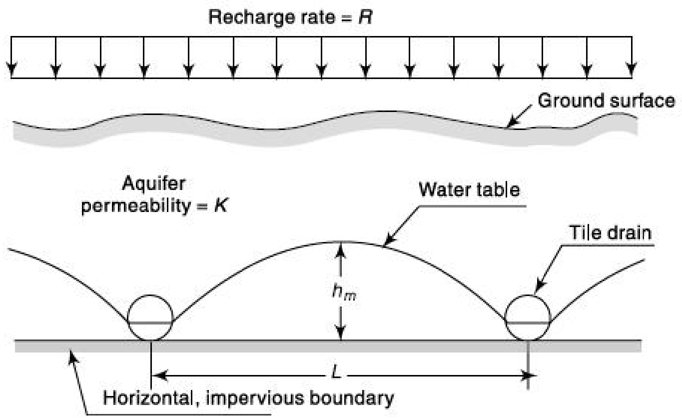
the Discharge q per unit width of the Aquifer is

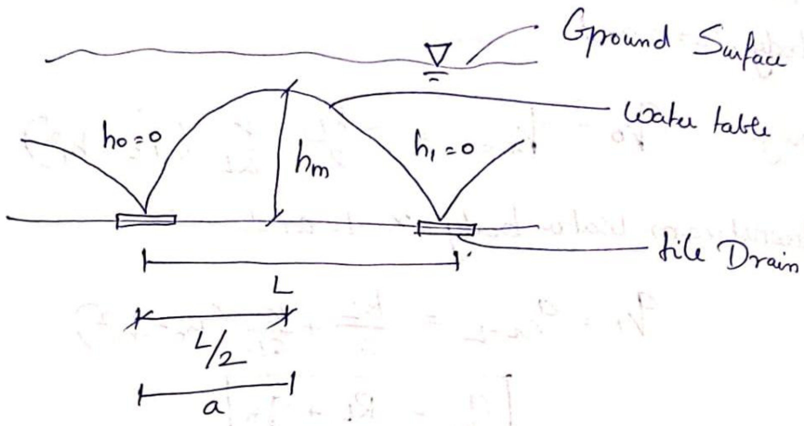
$$q = -kh\frac{\partial h}{\partial x}$$

$$\boxed{q = \frac{(h_0^2 - h_1^2)}{2L}k}$$

Tile Drain Problem

The provision of a drain system is one of the most widely used method of draining waterlogged areas, the object being to reduce the level of the water table. Figure shows a set of porous tile drains maintaining a constant recharge rate of R at the top ground surface.





$$h^2 = \frac{R}{k}(L-x)x$$

The Maximum height of the water table Occurs at $a = L/2$ and is of Magnitude

$$h_m^2 = \frac{R}{k}\left[L - \frac{L}{2}\right]\frac{L}{2}$$

Squaring on Both the sides

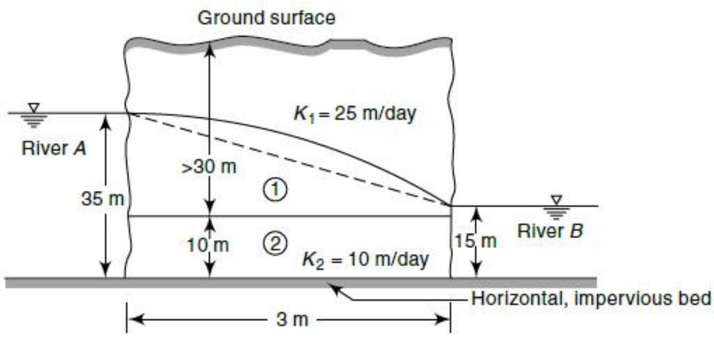
$$h_m = \frac{L}{2}\sqrt{R/K}$$

Considering a set of drains, Since the flow is Steady, the Discharge entering a drain per unit length of the drain is

$$q = \cancel{2}\left(R \cdot \frac{L}{\cancel{2}}\right)$$

$$\boxed{q = RL}$$

Two parallel rivers A and B are separated by a land mass as shown in Figure. Estimate the seepage discharge from River A to River B per unit length of the rivers.



$$q_2 = \left(\frac{h_0 - h_1}{L}\right) \times K \times B \Rightarrow \text{Confined Aquifer}$$

where,

$h_0 = 35m$

$h_1 = 15m$

$L = 3km / 3000m$

$b = 10m$

$K_2 = 10m/day$

$$q_2 = \left(\frac{35-15}{3000}\right) \times 10 \times 10$$

$q_2 = 0.667 \, m^3/day$ Per metre width

for the Confined Aquifer-2

$$q_1 = \left(\frac{h_0^2 - h_1^2}{2L}\right) k \Rightarrow \text{unconfined Aquifer without Recharge}$$

where,

$h_0 = 35m - 10m = 25m$

$h_1 = 15m - 10m = 5m$

$L = 3Km | 3000m$

$K_1 = 25m/day$

$$q_1 = \frac{25^2 - 5^2}{2 \times 3000} \times 25$$

$q_1 = 2.5 \, m^3/day$ per metre width

for the unconfined Aquifer-1

total Discharge from river A to river B

$q = q_1 + q_2$

$= 0.667 + 2.5$

$= 3.167 \, m^3/day$ per unit Length of the River.

A Canal and a Stream Run parallel to Each other at a Separation Distance of 400m. Both of them completely penetrate an unconfined Aquifer ($K = 3m/day$) located above a horizontal impervious bed. the Aquifer forms the Separation land mass Between the two water bodies. The water Surface Elevation in the Canal & the Stream are 5m and 3m, the datum Being the top of the Impervious layer

Calculate

1. The uniform infiltration rate that will create a water table divide at a distance of 100m from the Canal
2. The Elevation of water table divide
3. The Seepage discharge into the two water bodies

Given

$h_0 = 5m$ ∵ Unconfined Aquifer

$h_1 = 3m$

$L = 400m$

$K = 3m/day$

1. It is uniform Infiltration rate Means It is getting Recharged

So, $a = 100m$

$$a = \frac{L}{2} - \frac{K}{R}\frac{(h_0^2 - h_1^2)}{2L} \qquad \frac{400}{2}$$

$$100 = 200 - \frac{3}{R}\left[\frac{5^2-3^2}{800}\right] \rightarrow 2\times400$$

$$R = \frac{(5^2-3^2)\times3}{800\times100}$$

$$\boxed{R = 6\times10^{-4} m^3/day/m^2}$$

2. $h^2 = \frac{-Rx^2}{k} - \frac{\left[h_0^2 - h_1^2 - \frac{RL^2}{k}\right]}{L} x + h_0^2$

When $h = h_m$, $x = a$

$h_m^2 = -\frac{6\times10^{-4}\times100^2}{3} - \left(\frac{5^2 - 3^2 - \frac{6\times10^{-4}\times400^2}{3}}{400}\right)100 + 5^2$

$h_m^2 = -2 + 4 + 25$

$= 27$

$h_m = \sqrt{27}$

$\boxed{h_m = 5.196\,m}$

3. $q_x = R\left(x - \frac{L}{2}\right) + \frac{K}{2L}\left(h_0^2 - h_1^2\right)$

When $x = 0$, $q_x = q_0$

$q_0 = -\frac{RL}{2} + \frac{K}{2L}\left(h_0^2 - h_1^2\right)$

$= -\frac{6\times10^{-4}\times400}{2} + \frac{3}{2\times400}\left(5^2 - 3^2\right)$

$\boxed{q_0 = -0.06\,m^3/day/m\ width}$

$q_1 = RL + q_0$

$= 6\times10^{-4}\times400 - 0.06$

$\boxed{q_1 = 0.18\,m^3/day/m\ width}$

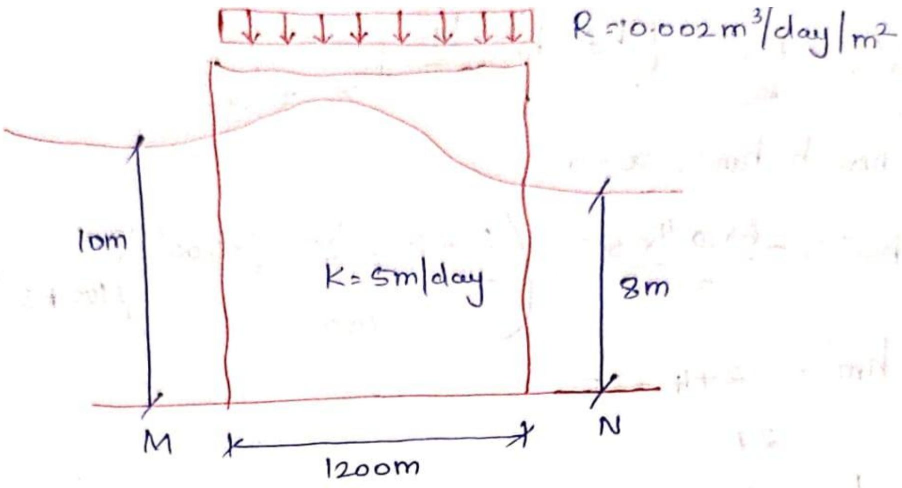
An unconfined aquifer (K = 5 m/day) situated on the top of a horizontal impervious layer connects two parallel water bodies M and N which are 1200 m apart. The water surface elevations of M and N, measured above the horizontal impervious bed, are 10.00 m and 8.00 m. If a uniform recharge at the rate of 0.002 m3/day per m2 of horizontal area occurs on the ground surface, estimate

(i) the water table profile,

(ii) the location and elevation of the water table divide,

(iii) the seepage discharges into the lakes and

(iv) the recharge rate at which the water table divide coincides with the upstream edge of the aquifer and the total seepage flow per unit width of the aquifer at this recharge rate.



(i)

$$h^2 = -\frac{R}{k}x^2 - \frac{\left(h_0^2 - h_1^2 - \frac{RL^2}{k}\right)}{L}x + h_0^2$$

$$h^2 = -\left(\frac{0.002}{5}\right)x^2 - \left(\frac{10^2 - 8^2 - \frac{0.002\times1200^2}{5}}{1200}\right)x + 10^2$$

$$h^2 = -0.0004x^2 + 0.45x + 100$$

(ii)

$$a = \frac{L}{2} - \frac{K}{R}\left(\frac{h_0^2 - h_1^2}{2L}\right)$$

$$= 600 - \left(\frac{5.0}{0.002}\right)\left(\frac{10^2 - 8^2}{2\times1200}\right) \qquad \frac{1200}{2}$$

$$\boxed{a = 562.50\text{m}}$$

At $x = a = 562.5$m, $h = h_m =$ height of water table divide

$$h_m^2 = -0.0004(562.5)^2 + 0.45\times562.5 + 100$$

$$= 226.56$$

$$h_m = \sqrt{226.56}$$

$$\boxed{h_m = 15.05\text{m}}$$

(iii) $q_x = R\left(x - \frac{L}{2}\right) + \frac{k}{2L}\left(h_0^2 - h_1^2\right)$

At $x = 0$, $q_0 = -R\frac{L}{2} + \frac{k}{2L}\left(h_0^2 - h_1^2\right)$

$$= -\frac{(0.002 \times 1200)}{2} + \frac{5}{2 \times 1200}\left[(10)^2 - (8)^2\right]$$

$$= -1.20 + 0.075$$

$$\boxed{q_0 = -1.125\, m^3/day/m \text{ width}}$$

At $x = L$, $q_1 = q_L$

q_1 = Discharge into water bodies N

$$q_L = RL + q_0$$

$$= 0.002 \times 1200 + (-1.125)$$

$$\boxed{q_L = 1.275\, m^3/day/m \text{ width}}$$

(iv) When the distance of the water table divide $a = 0$

$$a = \frac{L}{2} - \frac{k}{R}\left[\frac{h_0^2 - h_1^2}{2L}\right]$$

$$\frac{L}{2} = \frac{k}{R}\left[\frac{h_0^2 - h_1^2}{2L}\right]$$

$$R = \frac{k}{L^2}\left(h_0^2 - h_1^2\right)$$

$$= \frac{5.0}{1200^2}\left[(10)^2 - (8)^2\right]$$

$$\boxed{R = 1.25 \times 10^{-4}\, m^3/day/m^2}$$

Since $a = 0$, $q_0 = 0$

$$q_1 = q_L = RL$$

$$= 1.25 \times 10^{-4} \times 1200$$

$$\boxed{q_1 = 0.15\, m^3/day/m \text{ width}}$$

Well Hydraulics

Consider the water in an unconfined aquifer being pumped at a constant rate from a well. Prior to the pumping, the water level in the well indicates the static water table. A lowering of this water level takes place on pumping.

If the aquifer is homogeneous and isotropic and the water table initially horizontal, due to the radial flow into the well through the aquifer the water table assumes a conical shape called ***cone of depression***.

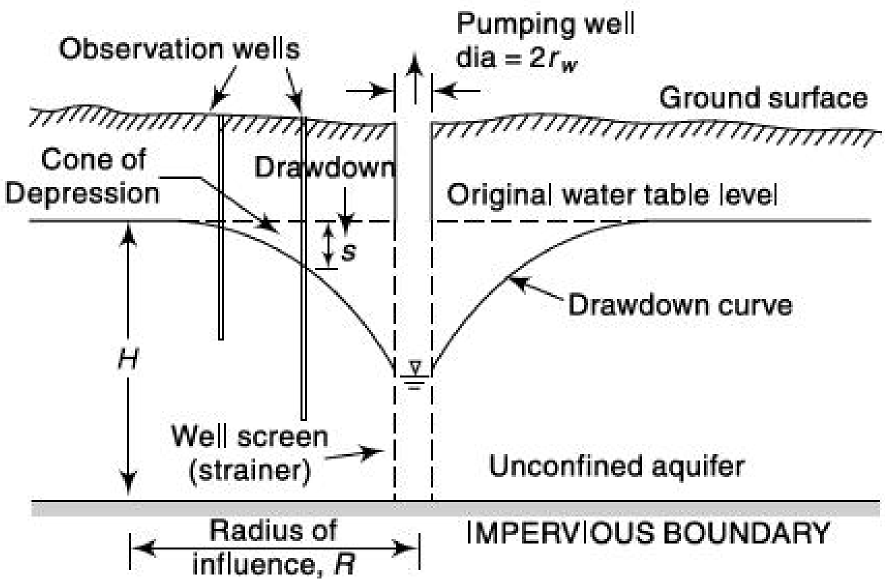
The drop in the water table elevation at any point from its previous static level is called drawdown.

The areal extent of the cone of depression is called area of influence and its radial extent ***radius of influence.***

At constant rate of pumping, the drawdown curve develops gradually with time due to the withdrawal of water from storage. This phase is called an ***unsteady flow*** as the water table elevation at a given location near the well changes with time.

On prolonged pumping, an equilibrium state is reached between the rate of pumping and the rate of inflow of groundwater from the outer edges of the zone of influence. The drawdown surface attains a constant position with respect to time when the well is known to operate under ***steady-flow conditions***.

As soon as the pumping is stopped, the depleted storage in the cone of depression is made good by groundwater inflow into the zone of influence. There is a gradual accumulation of storage till the original (static) level is reached. This stage is called ***recuperation or recovery*** and is an unsteady phenomenon. Recuperation time depends upon the aquifer characteristics.

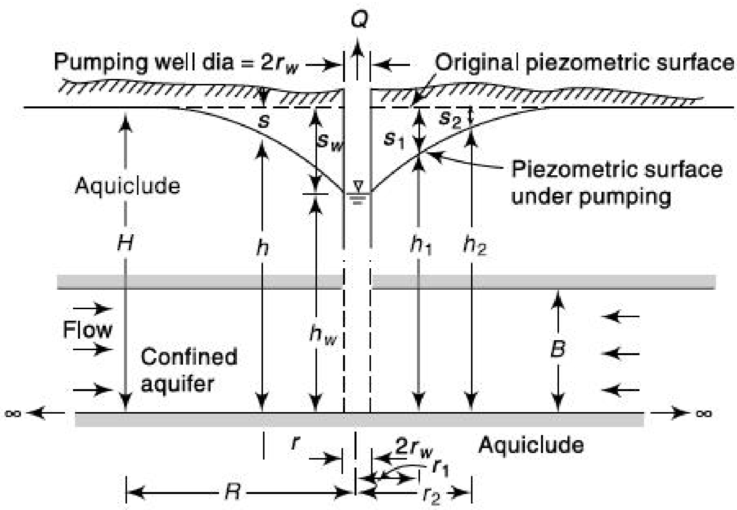


Steady Flow into a Well

Steady-state groundwater problems are relatively simpler. Expressions for the steady-state radial flow into a well under both confined and unconfined aquifer conditions are presented below.

Confined Flow

Figure shows a well completely penetrating a horizontal confined aquifer of thickness B. Consider the well to be discharging a steady flow, Q. The original piezometric head (static head) was H and the drawdown due to pumping is indicated in Figure.The piezometric head at the pumping well is h_w and the drawdown s_w.



At a radial distance r from the well, if h is the piezometric head, the velocity of flow by Darcy's law is

$$V_r = K\frac{dh}{dr}$$

The cylindrical surface through which this velocity occurs is $2p\,r\,B$. Hence by equating the discharge entering this surface to the well discharge,

$$Q = (2\pi rB)\left(K\frac{dh}{dr}\right)$$

$$\frac{Q}{2\pi KB}\frac{dr}{r} = dh$$

Integrating between limits r_1 and r_2 with the corresponding piezometric heads h_1 and h_2, respectively,

$$\frac{Q}{2\pi KB}\ln\frac{r_2}{r_1} = (h_2 - h_1)$$

or

$$Q = \frac{2\pi KB(h_2 - h_1)}{\ln\frac{r_2}{r_1}}$$

This is the equilibrium equation for the steady flow in a confined aquifer. This equation is popularly known as *Thiem's equation.*

If the drawdown s_1 and s_2 at the observation wells is known, then by noting that

$$s_1 = H - h_1,\ s_2 = H - h_2 \text{ and } KB = T$$

$$Q = \frac{2\pi T(s_1 - s_2)}{\ln \frac{r_2}{r_1}}$$

Further, at the edge of the zone of influence, $s = 0$, $r_2 = R$ and $h_2 = H$; at the well wall $r_1 = r_w$, $h_1 = h_w$ and $s_1 = s_w$. Equation (9.47) would then be

$$Q = \frac{2\pi T s_w}{\ln R/r_w}$$

A 20 cm diameter tube well taps an artesian aquifer. The length at the strainer is 30 m and the sufficient of permeability of the aquifer is 35 m/day. Assume the radius of influence = 300 m. Find the yield for the drawdown of 3 m.

If all other conditions remain same, find the percentage change in the yield under following cases:

1. **The diameter of well is 40 cm.**
2. **The drawdown is 6 m.**
3. **The permeability is 17.5 m/day.**

Given

$B = 30m$

$K = 35 m/day$, $K = 35/(24 \times 60 \times 60) = 4.051 \times 10^{-4} m/sec$

$R = 300m$

$r_w = 10cm \rightarrow 0.1m$, Diameter of the well = 20cm, $S_w = 3m$

$$Q = \frac{2\pi K B S_w}{\ln \frac{R}{r_w}}$$

$$= \frac{2 \times \pi \times (4.05 \times 10^{-4}) \times 30 \times 3}{\ln\left(\frac{300}{0.1}\right)}$$

$= 0.0286 \; m^3/sec$

$= 28.612$ litre/sec

$= 1716.723$ litre/minute

1. $\frac{Q_n}{Q} = \frac{\ln R/r_w}{\ln R/r_{w_n}} = \frac{\ln \frac{300}{0.1}}{\ln \frac{300}{0.2}} = 1.09478 = 9.478\%$ increase

2. $\frac{Q_n}{Q} = \frac{S_{w_n}}{S_w} = \frac{6}{3} = 2 = 100\%$ increase

3. $\frac{Q_n}{Q} = \frac{K_n}{K} = \frac{17.5}{35} = 0.5 = 50\%$ decrease

EXAMPLE 9.8 A 30 cm diameter well completely penetrates a confined aquifer of permeability 45 m/day. The length of the strainer is 20 m. Under steady state of pumping, the drawdown at the well was found to be 3.0 m and the radius of influence was 300 m. Calculate the discharge.

Solution

In this problem, referring to Fig. 9.16,

$$r_w = 0.15 \text{ m} \qquad R = 300 \text{ m} \qquad s_w = 3.0 \text{ m} \qquad B = 20 \text{ m}$$

$$K = 45/(60 \times 60 \times 24) = 5.208 \times 10^{-4} \text{ m/s}$$

$$T = KB = 10.416 \times 10^{-3} \text{ m}^2\text{/s}$$

By Eq. (9.48)

$$Q = \frac{2\pi T s_w}{\ln R/r_w} = \frac{2\pi \times 10.416 \times 10^{-3} \times 3}{\ln \dfrac{300}{0.15}} = 0.02583 \text{ m}^3\text{/s} = 25.83 \text{ lps}$$

$$= 1550 \text{ lpm}$$

EXAMPLE 9.9 For the well in the previous example, calculate the discharge (a) if the well diameter is 45 cm and all other data remain the same as in Example 9.8, (b) if the drawdown is increased to 4.5 m and all other data remain unchanged as in Example 9.8.

Solution

(a) $Q = \dfrac{2\pi T s_w}{\ln R/r_w}$

As T and s_w are constants, $\dfrac{Q_1}{Q_2} = \dfrac{\ln R/r_{w_2}}{\ln R/r_{w_1}}$

Putting $R = 300$ m, $Q_1 = 1550$ lpm, $r_{w1} = 0.15$ m and $r_{w2} = 0.225$ m.

$$Q_2 = 1550 \frac{\ln 300/0.15}{\ln 300/0.225} = 1637 \text{ lpm}$$

[*Note:* The discharge has increased by about 6% for 50% increase in the well diameter.]

(b) $Q = \dfrac{2\pi T s_w}{\ln R/r_w}$

$Q \propto s_w$ for constant T, R and r_w.

Thus, $\dfrac{Q_1}{Q_2} = \dfrac{s_{w_1}}{s_{w_2}}$

$$Q_2 = 1550 \times \frac{4.5}{3.0} = 2325 \text{ lpm}$$

[*Note:* The discharge increases linearly with the drawdown when other factors remain constant.]

EXAMPLE 9.10 The discharge from a fully penetrating well operating under steady state in a confined aquifer of 30 m thickness is 2100 litres/minute. The drawdowns observed at two observation wells located at 15 m and 150 m from the well are 3.2 m and 0.28, respectively. Determine the transmissibility and the permeability of the aquifer.

Solution

Here, $s_1 = 3.2$ m and $r_1 = 15$ m

$s_2 = 0.28$ m and $r_2 = 150$ m

$$Q = \frac{2100}{1000 \times 60} = 0.035 \text{ m}^3\text{/s}$$

$$Q = \frac{2\pi T(s_1 - s_2)}{\ln \frac{r_2}{r_1}}$$

$$0.035 = \frac{2 \times \pi \times T}{\ln \frac{150}{15}}(3.2 - 0.28) = 7.96795T$$

$$T = \frac{0.035}{7.96795} = 0.00439 \text{ m}^2\text{/s}$$

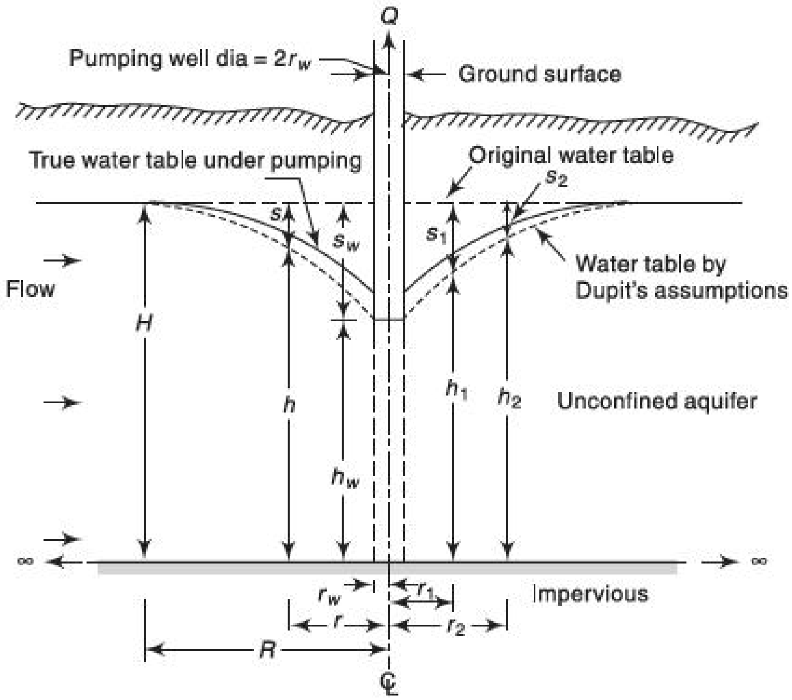
$$\text{Transmissibility } T = 0.00439 \times 3600 \times 24 = 379.5 \text{ m}^2\text{/day}$$

$$\text{Permeability } K = \frac{T}{B} = \frac{379.5}{30} = 12.65 \text{ m/day}$$

Radial Flow to a Well in an Unconfined Aquifer

Consider a steady flow from a well completely penetrating an unconfined aquifer. In this case, because of the presence of a curved free surface, the streamlines are not strictly radial straight lines. While a streamline at the free surface will be curved, the one at the bottom of the aquifer will be a horizontal line, both converging to the well. To obtain a simple solution, Dupit's assumptions as indicated are mentioned above are made. In the present case these are the following:

- For small inclinations of the free surface, the streamlines can be assumed to be horizontal and the equipotentials are thus vertical.
- The hydraulic gradient is equal to the slope of the free surface and does not vary with depth. This assumption is satisfactory in most of the flow regions except inthe immediate neighbourhood of the well.



Consider the well of radius r_w penetrating completely an extensive unconfined horizontal aquifer as shown in Fig. 9.19. The well is pumping a discharge Q. At any radial distance r, the velocity of radial flow into the well is

$$V_r = K\frac{dh}{dr}$$

where h is the height of the water table above the aquifer bed at that location. For steady flow, by continuity

$$Q = (2\pi rh)V_r = 2\pi rKh\,\frac{dh}{dr}$$

or

$$\frac{Q}{2\pi K}\frac{dr}{r} = h\,dh$$

Integrating between limits r_1 and r_2 where the water table depths are h_1 and h_2, respectively, and on rearranging

$$Q = \frac{\pi K\,(h_2^2 - h_1^2)}{\ln\dfrac{r_2}{r_1}}$$

This is the equilibrium equation for a well in an unconfined aquifer. As at the edge of the zone of influence of radius R, H = saturated thickness of the aquifer, can be written as

$$Q = \frac{\pi K\,(H^2 - h_w^2)}{\ln\dfrac{R}{r_w}}$$

EXAMPLE 9.11 A 30 cm well completely penetrates an unconfined aquifer of saturated depth 40 m. After a long period of pumping at a steady rate of 1500 lpm, the drawdown in two observation wells 25 and 75 m from the pumping well were found to be 3.5 m and 2.0 m, respectively. Determine the transmissibility of the aquifer. What is the drawdown at the pumping well?

Solution

(a) $Q = \dfrac{1500 \times 10^{-3}}{60} = 0.025 \text{ m}^3/\text{s}$

$h_2 = 40.0 - 2.0 = 38.0$ $\qquad r_2 = 75$ m

$h_1 = 40.0 - 3.5 = 36.5$ m $\qquad r_1 = 25$ m

From Eq. (9.49),

$$Q = \frac{\pi K (h_2^2 - h_1^2)}{\ln \dfrac{r_2}{r_1}}$$

$$0.025 = \frac{\pi K[(38)^2 - (36.5)^2]}{\ln \dfrac{75}{25}}$$

$$K = 7.823 \times 10^{-5} \text{ m/s}$$

$$T = KH = 7.823 \times 10^{-5} \times 40 = 3.13 \times 10^{-3} \text{ m}^2/\text{s}$$

(b) At the pumping well, $r_w = 0.15$ m

$$Q = \frac{\pi K (H_1^2 - h_w^2)}{\ln \dfrac{r_1}{r_w}}$$

$$0.025 = \frac{\pi \times 7.823 \times 10^{-5}[(36.5)^2 - h_w^2]}{\ln \dfrac{25}{0.15}}$$

$h_w^2 = 811.84$ and $h_w = 28.49$ m

Drawdown at the well, $s_w = 11.51$ m

A 45 cm well in an unconfined aquifer of saturated thickness of 45 m yields 600 lpm under a drawdown of 3m at pumping well?

Calculate:

(a) What will be the discharge under a drawdown of 6m.

(b) What will be the discharge in a 30 cm well under a drawdown of 3m.

Assume radius of influence R = 500m.

Given

$R = 500m$

$r_w = \frac{45}{2} = 22.5cm = 0.225m$

$H = 45m$

$S_w = 3m$

$h_w = H - S_w$

$= 45 - 3$

$= 42m$

$Q = 600 lpm$

$= \frac{600 \times 10^{-3}}{60}$

$= 0.01 m^3/s$

$$Q = \frac{\pi K (H^2 - h_w^2)}{\ln \frac{R}{r_w}}$$

$$0.01 = \frac{\pi K (45^2 - 42^2)}{\ln \frac{500}{0.225}}$$

$$\boxed{K = 9.4 \times 10^{-5} m/s}$$

a. $S_w = 6m$

$h_w = H - S_w$

$= 45 - 6$

$\boxed{h_w = 39m}$

$$Q = \frac{\pi K (H^2 - h_w^2)}{\ln \frac{R}{r_w}}$$

$$= \frac{\pi \times 9.4 \times 10^{-5} (45^2 - 39^2)}{\ln \frac{500}{0.225}}$$

$Q = 0.0193136 m^3/s$

$\simeq 1159 lpm$

b. $Q_1 \propto \frac{1}{\ln \frac{R}{r_{w1}}}$

$$\frac{Q_2}{Q_1} = \frac{\ln \frac{R}{r_{w1}}}{\ln \frac{R}{r_{w2}}}$$

$$\frac{Q_2}{600} = \frac{\ln \frac{500}{0.225}}{\ln \frac{500}{0.15}}$$

$$\boxed{Q_2 = 570 lpm}$$

For conducting tests on 30 cm diameter well in an unconfined aquifer two observation well A and B are bored at a distance of 25 m and 40 m respectively from the centre of pumping well. When water is pumped at a rate of 10 litres/sec the water depth in the pumping well is 10 m above the horizontal impervious layer upto which the well is driven. The median grain size of the aquifer is 2.5 mm and the permeability is known to be 0.1 cm/s.

Calculate:

(a) The depth of water above the impervious layer in the observation well A and B.

(b) The Reynold's number of flow at the pumping well and observation well A and B.

Assume kinematic viscosity of water = 0.01cm^2/s.

Given

$r_w = 30cm$

$r_A = 25m$

$r_B = 40m$

$h_w = 10m$

$d = 2.5mm$

$= 0.0025m$

$K = 0.1cm/s$

$K = 0.001m/s$

$Q = 10l/s$

$= 0.01m^3/s$

a. $$Q = \frac{\pi k(h_A^2 - h_w^2)}{\ln \frac{r_A}{r_w}}$$

$$0.01 = \frac{\pi \times 0.001 \times (h_A^2 - 10^2)}{\ln\left(\frac{25}{0.15}\right)}$$

$$\boxed{h_A = 10.7835m}$$

$$\frac{h_A^2 - h_w^2}{h_B^2 - h_w^2} = \frac{\ln \frac{r_A}{r_w}}{\ln \frac{r_B}{r_w}}$$

$$\frac{10.7825^2 - 10^2}{h_B^2 - 10^2} = \frac{\ln \frac{25}{0.15}}{\ln \frac{40}{0.15}} = 0.91536$$

$$\boxed{h_B = 10.85156m}$$

b. $$Re = \frac{\rho V d}{\mu} = \frac{Vd}{\nu}$$

$$Re = \frac{0.0025V}{10^{-6}} = 2500V$$

for well A

$$V = \frac{Q}{Area} = \frac{0.01}{2\pi \times 25 \times 10.7835} = 5.9036 \times 10^{-6} m/s$$

$Re = 0.01476$

for well B

$$V = \frac{0.01}{2\pi \times 40 \times 10.85156} = 3.67 \times 10^{-6} m/s$$

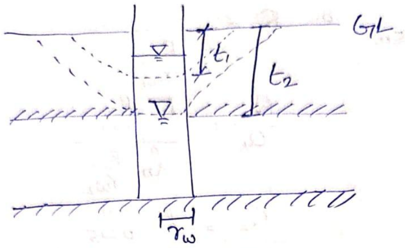
$Re = 0.00917$

for observation well

$$V = \frac{0.01}{2\pi \times 0.15 \times 10} = 1.061 \times 10^{-3}$$

$Re = 2.653$

Unsteady Flow in a Well Situated in a Confined Aquifer



$$s = \frac{Q}{4\pi T} \times \ln \frac{2.25Tt}{r^2 S}$$

s = drawdown
Q = discharge
T = transmissibility
S = co-efficient of Storage
t = time
r = Radial distance

Difference Between t_1 & t_2

$$S_2 - S_1 = \frac{Q}{4\pi T} \ln \frac{t_2}{t_1}$$

When $S = 0$, $t = t_0$ — Very little time

$$S = \frac{2.25 T t_0}{r^2}$$

to find the co-efficient of Storage

A 30 cm well penetrating a confined aquifer is pumped at a rate of 1200 lpm. The drawdown at an observation well at a radial distance of 30 m is as follows:

Time from Start (min)	1.0	2.5	5	10	20	50	100	200	500	1000
Drawdown (m)	0.2	0.5	0.8	1.2	1.8	2.5	3.0	3.7	4.4	5.0

Calculate the aquifer parameters S and T

$$S_2 - S_1 = \frac{Q}{4\pi T} \ln\left(\frac{t_2}{t_1}\right)$$

$\because$ $Q = 1200\, l/min$

$$= \frac{1200}{1000 \times 60} = 0.02 m^3/s$$

Lets take 100 to 1000 time
t_1 t_2

$\because$ $t_1 = 100m \rightarrow 100 \times 60 = Sec$

$t_2 = 1000m \rightarrow 1000 \times 60 = Sec$

$$5 - 3 = \frac{0.02}{4\pi T} \ln \frac{1000 \times \cancel{60}}{100 \times \cancel{60}}$$

$$5 - 3 = \frac{0.02}{4\pi T} 2.30$$

$$\boxed{T = 1.832 \times 10^{-3} m^3/s/m}$$

Assume $t_0 = 25 min \Rightarrow 25 \times 60 = 150$ Sec

$$S = \frac{2.25 T t_0}{r^2}$$

$$= \frac{2.25 \times (1.832 \times 10^{-3}) \times 150}{30^2}$$

$$\boxed{S = 6.87 \times 10^{-4}}$$

Water Requirements of Crops

The functions of soil moisture in plant growth are very important.

→ Water and nutrients are the two most important requirements of the crop. The following are the **main functions of irrigation water**:

(i) It acts as a solvent for the nutrients. Water forms the solution of the nutrients, and this solution is absorbed by the roots. Thus, water acts as the nutrient carrier.

(ii) The irrigation water supplies moisture which is essential for the life of bacteria, which in turn is beneficial to plant growth.

(iii) The irrigation water supplies moisture which is essential for the chemical action within the plant leading to its growth.

(iv) Some salts present in soil react to produce nourishing food products only in the presence of water.

(v) Water cools the soil and the atmosphere and makes a more favourable environment for healthy plant growth.

(vi) Irrigation water, with controlled supply, washes out (or) dilutes acids in the soil.

(vii) It reduces the hazard of soil piping.

(viii) It softens the tillage (The preparation of land for growing crops) pan.

Crops and Crop seasons in India

Crops can be classified in the following ways:

- Agricultural classification
- Classification based on crop seasons
- Classification based on irrigation requirement

Agricultural Classification

It consists of the following types of crop:

(i) **Field crops:** such as wheat, rice, maize, barley, oats, great millet, spiked millet, gram, pulses, etc.

(ii) **Commercial crops:** such as sugarcane, cotton, tobacco, hemp, sugar beet, etc.

(iii) **Oil seed crops:** such as mustard, groundnut, sesame, linseed, castor etc.

(iv) **Horticulture crops:** consisting of various fruit crops, various vegetable crops and flower crops.

(v) **Plantation crops:** such as tea, coffee, cocoa, coconut, rubber etc.

(vi) **Forage crops:** such as fodder, grass etc.

(vii) **Miscellaneous crops:** such as medicinal crops, aromatic crops, sericulture crops, condiments and spices.

Classification based on Crop seasons

Kharif/Rainy/Monsoon crops	**Rabi/winter/cold seasons crops**	**Summer/Zaid crops**
The crops grown in monsoon months	The crops grown in winter season	Crops grown in summer
They are sown before monsoon and harvested at the end of the monsoon.	They are sown before retreating monsoon and harvested before summer.	They are sown and harvested in summer
The period of growth is from June to Oct-Nov	The period of growth is from Oct to March	The period of growth is from March to June
They require warm, wet weather at major period of crop growth	Crops grow well in cold and dry weather	They require warm dry weather for major growth period.
E.g. Cotton, Rice, Jowar, Bajara etc.	E.g. Wheat, gram, sunflower etc.	E.g. Groundnuts, Watermelon, Pumpkin, Gourds etc.

Classification based on Irrigation requirement

Wet crops require ample water and are grown in wet farming systems, like rice in monsoon regions. Dry crops are grown in drier conditions using minimal water, relying on stored soil moisture or specific drought-resistant seeds, such as wheat or millet. Garden crops are typically vegetables and aromatic plants grown on a smaller, more intensive scale in a garden setting.

1. **Wet crops**

   Definition: Grown in areas with abundant water, either from high rainfall or irrigation.

   Examples: Rice, jute, and sugarcane are common examples, especially in areas with monsoon rains.

   Growing method: Wet farming depends on a consistent water supply.

2. **Dry crops**

   Definition: Grown in areas with low rainfall, where they must rely on stored soil moisture.

   Examples: Wheat, corn, and millet are examples of dryland grain crops.

   Growing method: Dry farming involves selecting drought-resistant crops and managing soil to maximize moisture retention.

3. **Garden crops**

   Definition: A broad category that includes vegetables, herbs, and fruits, typically grown in smaller-scale, intensive plots.

   Examples: Carrots, tomatoes, beans, and various herbs are common garden crops.

   Growing method: These are cultivated on a smaller scale and are often planted for personal use or smaller-scale commercial markets.

Quality of irrigation water

→ A **good irrigation water** is the one which performs the functions of irrigation water without any side effects which retard the plant growth.

→ Irrigation water may be said to be **unsatisfactory for its intended use** if it contains:
(i) Chemicals toxic to plants or the persons using plants as food.
(ii) Chemicals which react with the soil to produce unsatisfactory moisture characteristics.
(iii) Bacteria injurious to persons or animals eating plants irrigated with the water.

Classification of irrigation water

Irrigation water may be classified on the following basis:

(i) Classification based on total concentration of soluble salts.
(ii) Classification based on sodium concentration.
(iii) Classification based on electrical conductivity (EC), total salt content (TDS), sodium concentration, boron concentration.

Soil–water relationship

Water present in the soil may be classified under three heads:

1. **Hygroscopic water**
2. **Capillary water**
3. **Gravitational water**

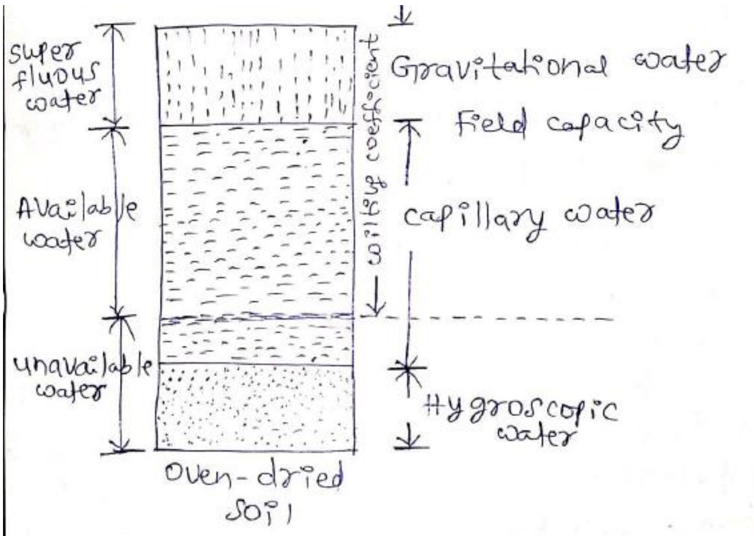


Figure: classes of soil water.

→ When an oven-dried sample is kept open in the atmosphere, it absorbs some amount of water from the atmosphere. This is known as **hygroscopic water**, and it is not capable of movement by the gravity or capillary forces.

→ **Capillary water** is that part in excess of hygroscopic water which exists in the pore space of the soil by molecular attraction.

→ **Gravitational water** is that part in excess of hygroscopic water and capillary water which will move out of the soil if favourable drainage is provided.

→ Water may also be classified as **unavailable water, available water,** and **superfluous water.** This classification is based on the availability of soil water to plants.

Duty and Delta

Duty * (D)

→ The Irrigation caplcity of a unit of water.

→ It's relation b/w vol. of available water to area which can irrigated for particular crop.

→ Unit = hect./cumec

Delta * (Δ)

→ Depth of water reqd. by crops during entire Base period of crop.

→ Unit = meter

RELATION BETWEEN DUTY AND DELTA

$$Delta = 8.64\frac{B}{D}$$

Where, B= Base period in days
D= Duty in hectares/cumce

Crop Period *

→ Time (in days), that a crops takes from the instant of its sowing to harvesting.

→ Unit = days

Base Period *

→ whole period of cultivation from first watering to its last watering.

→ Unit = days

Crop Ratio *

→ It's ratio of purposed area to be irrigated in kharif season to that in Rabi season.

FACTORS AFFECTING DUTY OF WATER

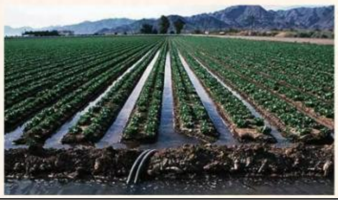
1. Method of irrigation
2. System of irrigation.
3. Method of cultivation.
4. Type of crop.
5. Base period of crop.
6. Climatic condition of the area.
7. Time of irrigation and frequency of cultivation.
8. Quality of irrigation water.
9. Methods of assessment of irrigation water rate.
10. Canal conditions.
11. Type of soil and sub-soil of the canal.
12. Type of soil and sub-soil of the irrigated field.

1. Method of irrigation

- The method of irrigation or the mode of applying water to the fields affects the duty of water because the water application efficiency varies with the method of irrigation.
- The drip irrigation method is most efficient method and duty is very high.
- In the sprinkler irrigation method duty is also high because seepage losses is reduced.
- Surface irrigation method duty is low.

2. System of Irrigation

- In the perennial irrigation system the soil of the irrigated area remains continuously wet hence less quantity of water is required for initial saturation of the soil.
- For the perennial irrigation system the duty of water is higher than for the inundation irrigation system.
- In case of tank irrigation duty is high while in canal irrigation the duty is low.



3. Method of cultivation

- If the land is properly ploughed upto the required depth and made quite loose before irrigating.
- This will reduce the number of waterings and hence result in a higher duty of water.

4. Type of crop

- Different crop require varying quantities of water and therefore duty of water varies from crop to crop.
- The crops which require large quantity of water have lower duty water than for the crops which requires less quantity of water.

5. Base period of crop

- In general when the base period of a crop is long, more water may be required thus resulting in a lower duty of water

Crop	Base period B (days)	Duty at the field D (ha/cumec)	Delta Δ = (8.64 B)/D	Area (ha)	Volume = (Δ x A) (ha-m)
Wheat	120	1800	0.576	4800	2765.0
Sugar-cane	360	800	3.890	5600	21800.0
Cotton	200	1400	1.235	2400	2965.0
Rice	120	900	1.152	3200	3690.0
Vegetables	120	700	1.480	1400	2070.0
			Total		33290

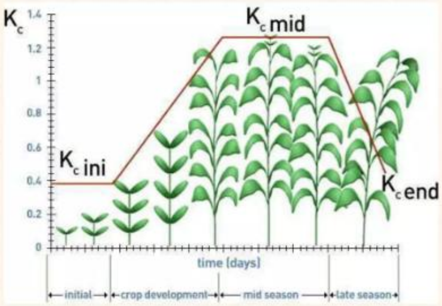
Therefore, capacity of the reservoir = 33290 / (0.8 x 0.88) = 47,300 ha-m

6. Climatic condition of the area

- The water requirement of a crop varies with the climatic condition of the area and hence it also affects the duty of water.
- The climatic conditions which affects the duty of water are
  - I. temperature,
  - II. wind velocity,
  - III. Humidity,
  - IV. Rainfall.

7. Time of irrigation and frequency of cultivation.

- In the initial stages the land to be cultivated may not be properly levelled and hence more than the required quantity of water may be applied, which will result in a lower duty of water.



8. Quality of irrigation water

- If the irrigation water contains an appreciable amount of harmful salts and alkalis dissolved in it then it is required to be applied in large quantity so that the salts are leached off.

9. Methods of assessment of irrigation water rate

- The assessment of irrigation water on volumetric basis prevents wastage of water and irrigation by the farmers, thus leading to economy of water and a higher duty of water.

10. Canal conditions

- In an earthen canal, the percolation loss will be high which will result in a low duty of water.
- The canals with good maintenance will have a higher duty of water than those having poor maintenance.

11. Type of soil and sub-soil of the canal

- If the canal is unlined and it passes through coarse grained soil then since there will be greater percolation loss the duty of water will be low.

12. Type of soil and sub-soil of the irrigated field

- If the soil and sub-soil of the field to be irrigated is coarse grained then due to high percolation loss the duty of water will be low.

* Find the delta for a crop if the duty for a base period of 110 days is 1400 hectares/cumec.

Given: $B = 110$ days

$D = 1400$ hectre/cumec

$$\Delta = 8.64 \times \frac{B}{D}$$

$$\Delta = 8.64 \times \frac{110}{1400} = 0.68m$$

$$\boxed{\Delta = 68cm}$$

Topic :

* A crop requires a total depth of 92cm of water for a base period of 120 days. Find the duty of water.

Soln :-

Given: $B = 120$ days

$\Delta = 92cm = 0.92m$

$$D = \frac{8.64B}{\Delta} \text{ hectare/cumec}$$

$$D = \frac{8.64 \times 120}{0.92}$$

$$\boxed{D = 1127 \text{ hectares/cumec}}$$

* The left branch canal carrying a discharge of 20 cumecs has cultural commanded area of 20,000 hectares. The intensity of Rabi crop is 80% and the base period is 120 days. The Right branch canal carrying discharge of 8 cumecs has cultural commanded area of 12,000 hectares, intensity of irrigation of rabi crop is 50% and the base period is 120 days. compare the efficiencies of the two canal systems.

Soln :-

(a) For the left canal

Area under rabi crop $= 20{,}000 \times \frac{80}{100}$

$= 16000$ hectares.

Discharge $= 20$ cumecs

$\therefore$ Duty $= \frac{16000}{20}$

Duty $= 800$ hectares/cumec

(b) For the right canal

Area under rabi crop $= 12000 \times \frac{50}{100}$

$= 6000$ hectares

Discharge $= 8$ cumecs

Duty $= \frac{6000}{8}$

$= 750$ hectares/cumec

Since the left canal system has higher duty, it is more efficient.

* A water course has a cultural commanded area of 1200 hectares. The intensity of irrigation for crop 'A' is 40% and for 'B' is 35%, both the crops being rabi crops. Crop 'A' has kor period of 20 days & crop 'B' has kor period of 15 days. Calculate the discharge of the water course if the kor depth for crop 'A' is 10cm and for 'B' it is 16cm.

Soln :- (a) For crop 'A'

Area under irrigation $= 1200 \times \frac{40}{100} = 480$ hectares

Kor period $= B = 20$ days

Kor depth $= \Delta = 10\text{ cm} = 0.1\text{ m}$

Duty $= D = 8.64 \times \frac{B}{\Delta}$

$= \frac{8.64 \times 20}{0.1}$

$D = 1728$ hectares/cumec

Hence Discharge required $= \frac{\text{Area under irrigation}}{\text{outlet factor}}$

$= \frac{480}{1728}$

$= 0.278$ cumecs

(b) For crop 'B'

Area under irrigation $= 1200 \times \frac{35}{100}$

$= 420$ hectares

Kor period $= B = 15$ days

Kor depth $= \Delta = 16\text{ cm} = 0.16\text{ m}$

$\therefore$ Duty (or) outlet factor $= 8.64 \times \frac{B}{\Delta}$ hectares/cumec

$= \frac{8.64 \times 15}{0.16}$

$= 810$ hectares/cumec

Hence discharge required $= \frac{420}{810} = 0.519$ cumec

$\therefore$ The design of water course =

$0.278 + 0.519$

$= 0.797$

$\boxed{= 0.8 \text{ cumec}}$