



Department of Civil Engineering

Study Material

Unit-IV

Introduction

Civil Engineers build structures and the soil beneath these structures is loaded. This results in increase of stresses resulting in strain leading to settlement of stratum. The settlement is due to decrease in volume of soil mass. When water in the voids and soil particles are assumed as incompressible in a completely saturated soil system then - reduction in volume takes place due to expulsion of water from the voids. There will be rearrangement of soil particles in air voids created by the outflow of water from the voids. This rearrangement reflects as a volume change leading to compression of saturated fine grained soil resulting in settlement. The rate of volume change is related to the rate at which pore water moves out which in turn depends on the permeability of soil. Therefore the deformation due to increase of stress depends on the “Compressibility of soils”

As Civil Engineers we need to provide answers for

1. Total settlement (volume change)
2. Time required for the settlement of compressible layer

The total settlement consists of three components

1. Immediate settlement.
2. Primary consolidation settlement
3. Secondary consolidation settlement (Creep settlement)

$$S_t = S_i + S_c + S_{sc}$$

Elastic Settlement or Immediate Settlement

This settlement occurs immediately after the load is applied. This is due to distortion (change in shape) at constant volume. There is negligible flow of water in less pervious soils. In case of pervious soils the flow of water is quick at constant volume. This is determined by elastic theory.

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Primary Consolidation Settlement

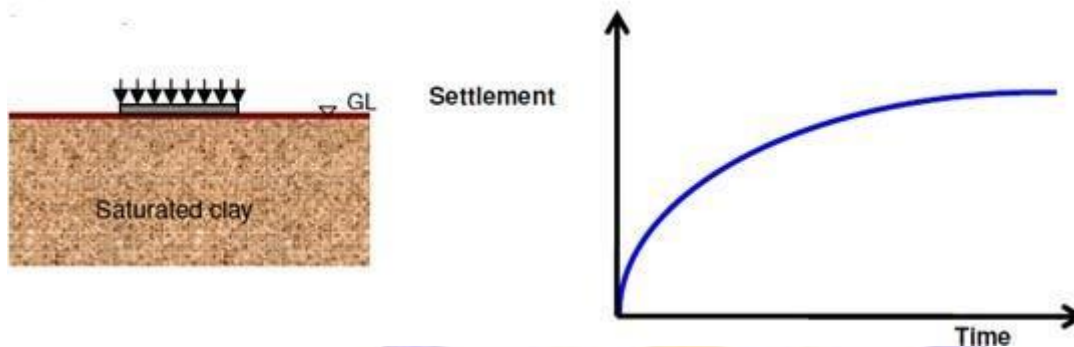


Figure Settlement versus Time

It occurs due to expulsion of pore water from the voids of a saturated soil. In case of saturated fine grained soils, the deformation is due to squeezing of water from the pores leading to rearrangement of soil particles. The movement of pore water depends on the permeability and dissipation of pore water pressure. With the passage of time the pore water pressure dissipates, the rate of flow decreases and finally the flow of water ceases. During this process there is gradual dissipation of pore water pressure and a simultaneous increase of effective stress as shown in the above Figure. The consolidation settlement occurs from the time water begins move out from the pores to the time at which flow ceases from the voids. This is also the time from which the excess pore water pressure starts reducing (effective stress increase) to the time at which complete dissipation of excess pore water pressure (total stress equal to effective stress). This time dependent compression is called “Consolidation settlement”.

Primary consolidation is a major component of settlement of fine grained saturated soils and this can be estimated from the theory of consolidation.

In case of saturated soil mass the applied stress is borne by pore water alone in the initial stages

$$\therefore \text{At } t = 0 \quad \Delta\sigma = \Delta u \quad \Delta\sigma' = 0$$

With passage of time water starts flowing out from the voids as a result the excess pore water pressure decreases and simultaneous increase in effective stress will takes place. The volume change is basically due to the change in effective stress After considerable amount of time ($t = \infty$) flow from the voids ceases the effective stress stabilizes and will be is equal to external applied total stress and this stage signifies the end of primary consolidation.

$$\text{At } t = t_1 \quad \Delta\sigma = \Delta\sigma' + \Delta u$$

$$\text{At } t = \infty \quad \Delta\sigma = \Delta\sigma' \quad \Delta u = 0 \text{ (End of primary consolidation)}$$

Secondary Consolidation Settlement:-

This is also called Secondary compression (Creep). “It is the change in volume of a fine grained soil due to

rearrangement of soil particles (fabric) at constant effective stress". The rate of secondary consolidation is very slow when compared with primary consolidation.

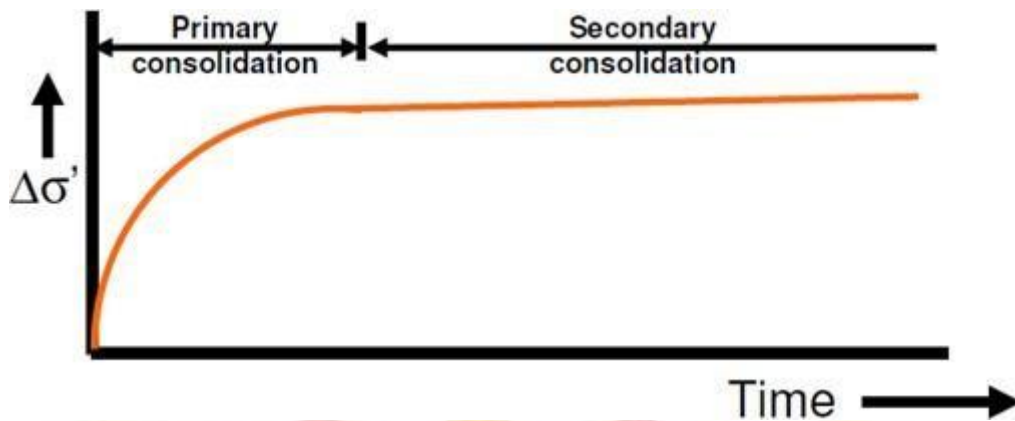


Figure Effective Stress versus Time

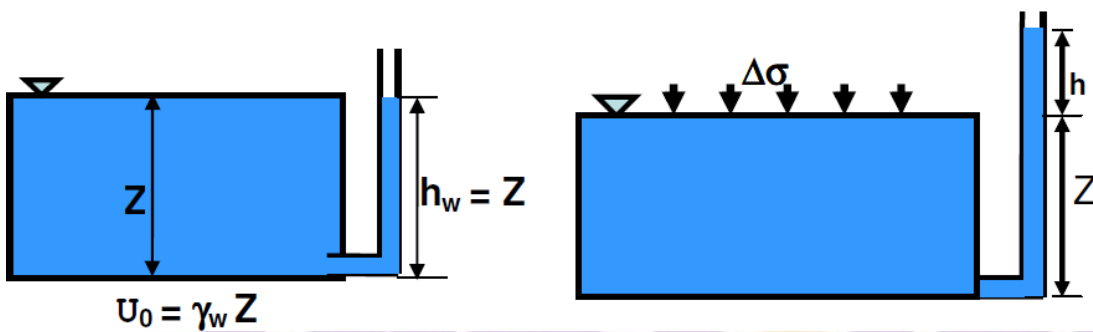
Excess Pore water Pressure (Δu)

"It is the pressure in excess of the equilibrium pore water pressure". It is represented as Δu .

$$\Delta u = h \gamma_w$$

Where h --- Piezometric head

γ_w --- Unit weight of water



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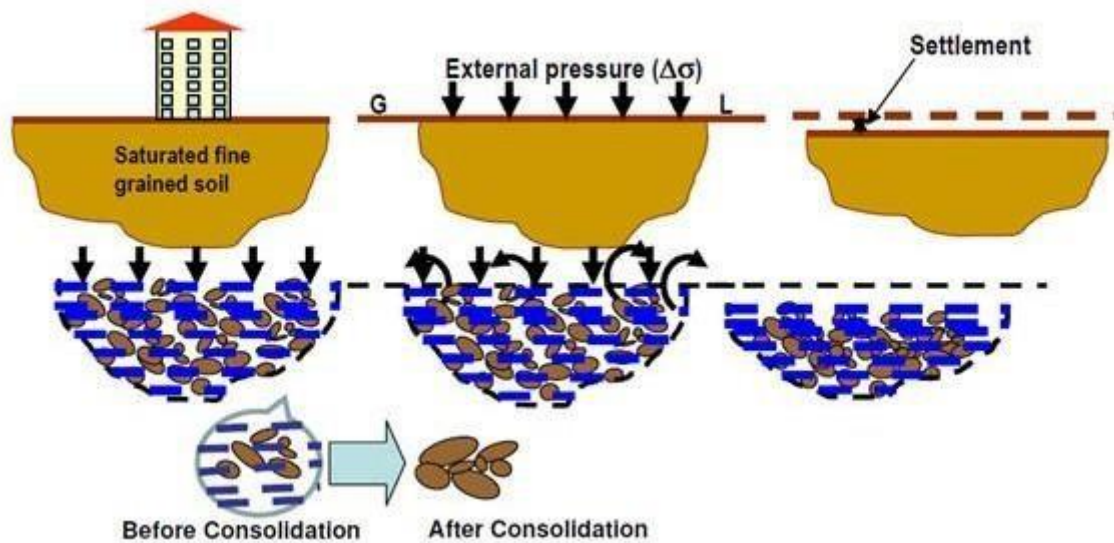


Figure Mechanism of volume change in saturated fine grained soil under external loading

When saturated soil mass is subjected to external load decrease in volume takes place due to rearrangement of soil particles. Reduction in volume is due to expulsion of water from the voids. The volume change depends on the rate at which water is expelled and it is a function of permeability.

The total vertical deformation (Consolidation settlement) depends on

1. Magnitude of applied pressure
2. Thickness of the saturated deposit

We are concerned with

- _ Measurement of volume change
- _ The time duration required for the volume change

Spring Analogy

The consolidation process is often explained with an idealized system composed of a [spring](#), a container with a hole in its cover, and water. In this system, the spring represents the compressibility or the structure itself of the soil, and the water which fills the container represents the pore water in the soil.

On figure , the tube on the left of the container shows the water pressure in the container.

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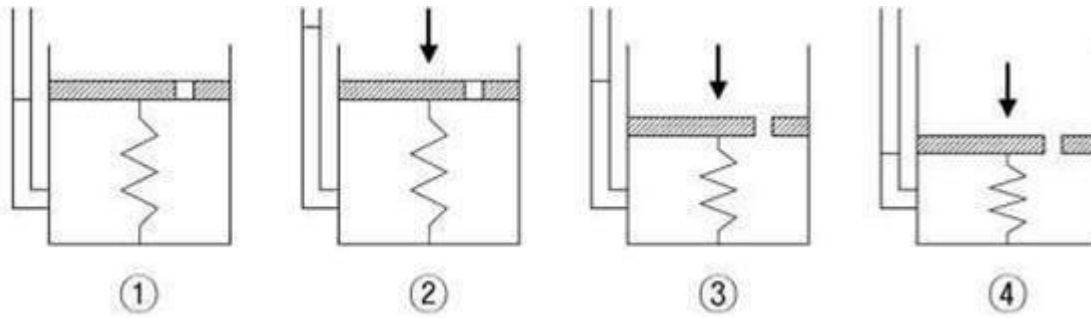


Figure: process of consolidation

1. The container is completely filled with water, and the hole is closed. (Fully saturated soil)
2. A load is applied onto the cover, while the hole is still unopened. At this stage, only the water resists the applied load. (Development of excessive pore water pressure)
3. As soon as the hole is opened, water starts to drain out through the hole and the spring shortens. (Drainage of excessive pore water)
4. After some time, the drainage of water no longer occurs. Now, the spring alone resists the applied load. (Full dissipation of excessive pore water pressure. End of consolidation)



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Terzaghi's Spring Mass Analogy-

Terzaghi's model consists of a cylindrical vessel with a series of piston separated by springs. The space between springs is filled with water the pistons are perforated to allow for passage of water. Piezometers are inserted at the centers of different compartment to measure the pressure head due to excess pore water pressure.

Terzaghi has correlated the spring mass compression process with the consolidation of saturated clay subjected to external load. The springs and the surrounding water represent the saturated soil. The springs represent the soil skeleton networks of soil grains and water in the vessels represents the water in the voids. In this arrangement the compression is one dimensional and flow will be in the vertical direction. When pressure is applied this will be borne by water surrounding the spring

$$\Delta\sigma = \Delta u \quad \text{at time } t = 0$$

Δu is called excess hydrostatic pressure due to this water level in all the

Piezometer reach the same height 'h' given by $h = \frac{\Delta u}{\gamma_w}$

$$\Delta\sigma = \Delta u \text{ and } \Delta\sigma' = 0 \text{ ----- } t=0$$

There will be no volume change. After some time 't' there will be flow of water through perforation beginning from upper compartment. In the lower compartment the volume of water remains constant since the flow is in upward direction.

Due to flow of water in the upper segment there will be reduction in volume due to this spring's get compressed and they being to carry a portion of the applied load. This signifies a reduction in excess hydrostatic pressure or pore water pressure and increase in effective stress in the upper segments. Whereas there will be no dissipation of excess hydrostatic pressure in lower compartments. The isochrones indicate that with passage of time there is flow of water from the lower compartments leading to gradual dissipation of excess hydrostatic pressure. At time $t = 0$ when no more pore water flows out the excess hydrostatic pressure will be zero in all compartments and the entire load is carried by springs.

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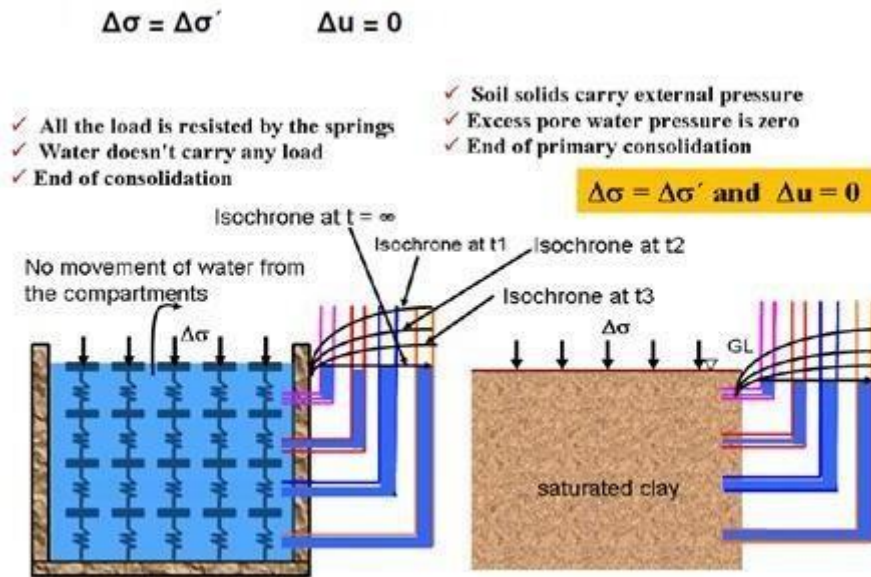


Figure Compression of Spring mass

The compression of a spring mass system is analogous to the consolidation of a saturated fine grained soil deposit subjected to external pressure.

Soil Compressibility

Compression of Sand

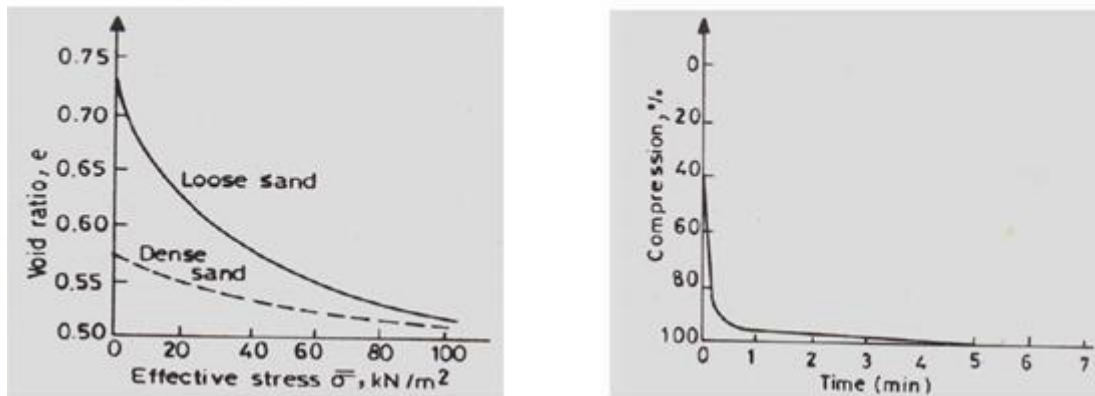


Figure Void ratio-effective stress and compression-time plots for sand

Sand deposit compresses immediately on load application. Loose sand compresses more than dense sand. Loose and dense sand deposits tend towards the same void ratio.

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Compression of fine grained soil (clay)

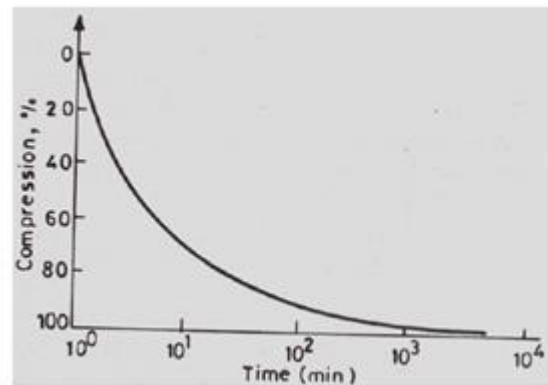
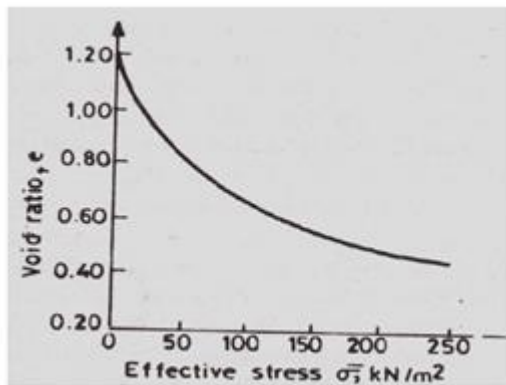


Figure Void ratio-effective stress and compression-time plots for clay

Time dependent compression takes longer time compared to sand. The magnitude of compression is also large.

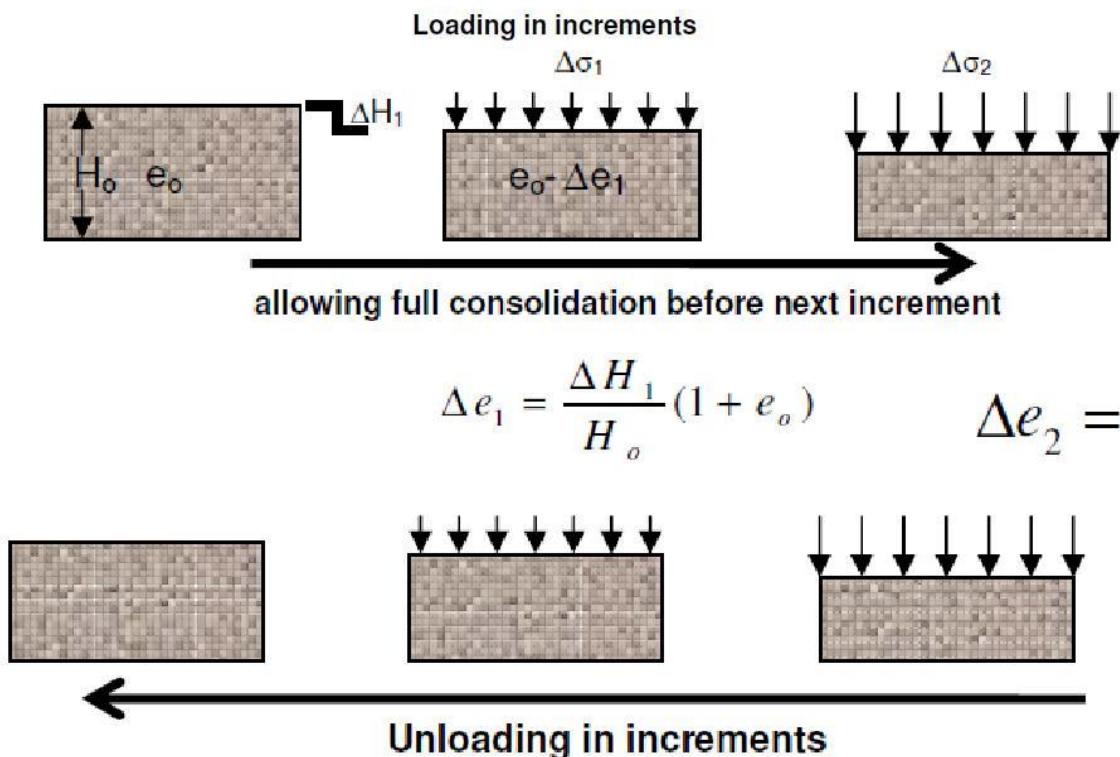
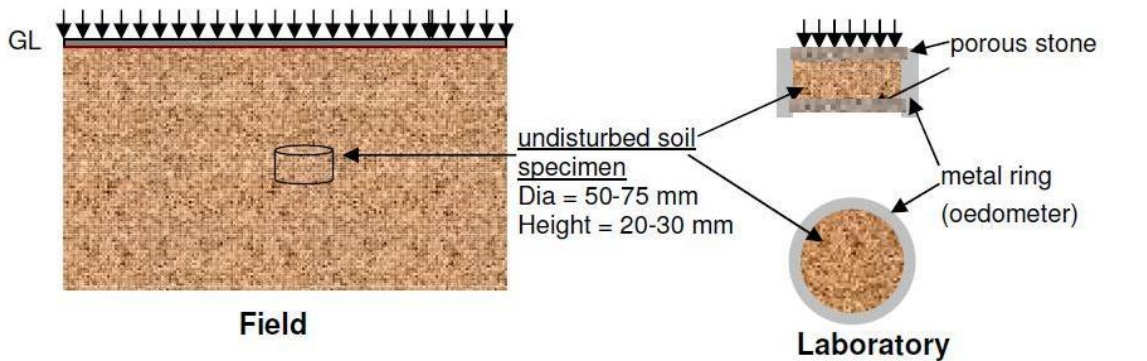


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Compression of Fine Grained Soil

The compressibility of fine grained soils can be described in terms of voids ratio versus effective stress

~ simulation of 1-D field consolidation in lab.



A laboratory soil specimen of dia 60mm and height 20mm is extracted from the undisturbed soil sample obtained from the field. This sample is subjected to 1D consolidation in the lab under various pressure increments. Each pressure increment is maintained for 24 hrs and equilibrium void ratio is recorded before the application of the next pressure increment. Then a plot of void ratio versus effective stress is made as shown in above figure. When the sample is recompressed from point D it follows DE and beyond C it merges along BCF and it compresses as it

moves along BCF

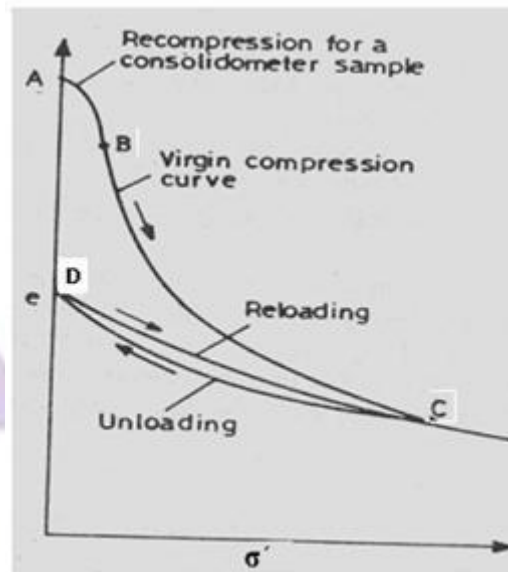


Figure Void ratio versus effective stress(on arithmetic plot)

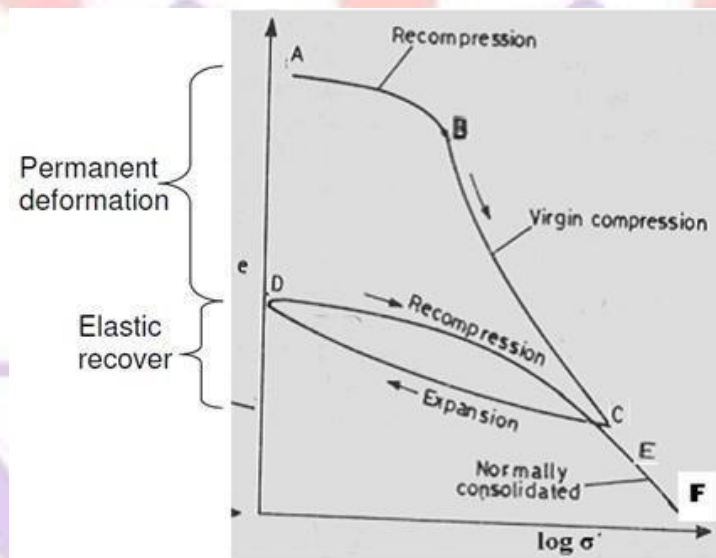


Figure Void ratio versus logarithm of effective stress (semi-log plot)

During the initial stages (at low effective stress) sample follows recompression path (portion AB) and undergoes less compression. Beyond this is the virgin compression line (portion BC) also called the normal compression line and the sample undergoes large compression.

1. BC – Virgin compression curve also called normal consolidation line
2. From 'C' when the sample is unloaded, sample expands and traces path CD (expansion curve unloading)
3. Sample undergoes Permanent strain due to irreversible soil structure and there is a small elastic recovery.
4. The deformation recovered is due to elastic rebound
5. When the sample is reloaded-reloading curve lies above the rebound curve and makes an hysteresis loop between



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expansion and reloading curves.

6. The reloaded soils shows less compression.

7. Loading beyond 'C' makes the curve to merge smoothly into portion EF as if the soil is not unloaded.



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Terzaghi's 1D Consolidation Equation

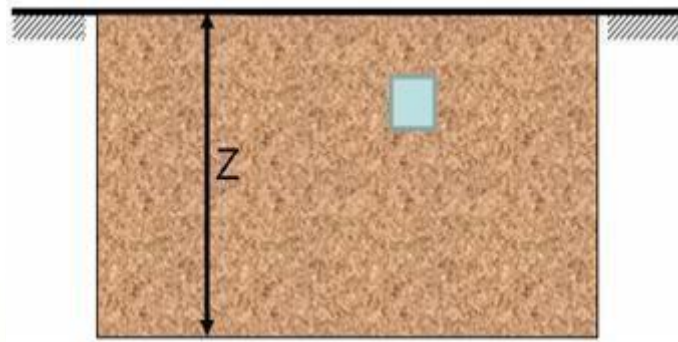


Figure Saturated soil Strata

Assumptions:

- ✓ The soil medium is completely saturated
- ✓ The soil medium is isotropic and homogeneous
- ✓ Darcy's law is valid for flow of water
- ✓ Flow is one dimensional in the vertical direction
- ✓ The coefficient of permeability is constant
- ✓ The coefficient of volume compressibility is constant
- ✓ The increase in stress on the compressible soil deposit is constant
- ✓ Soil particles and water are incompressible

One dimensional theory is based on the following hypothesis

1. The change in volume of soil is equal to volume of pore water expelled.
2. The volume of pore water expelled is equal to change in volume of voids.
3. Since compression is in one direction the change in volume is equal to change in height.

The increase in vertical stress at any depth is equal to the decrease in excess pore water pressure at the depth

$$\Delta \sigma' = \Delta u$$

This is Terzaghi's one dimensional consolidation equation

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}$$

This equation describes the variation of excess pore water pressure with time and depth

Limitation of 1D consolidation

1. In the deviation of 1D equation the permeability (Kz) and coefficient of volume compressibility (mv) are assumed constant, but as consolidation progresses void spaces decrease and this results in decrease of permeability and

therefore permeability is not constant. The coefficient of volume compressibility also changes with stress level. Therefore C_v is not constant

2. The flow is assumed to be 1D but in reality flow is three dimensional
3. The application of external load is assumed to produce excess pore water pressure over the entire soil stratum but in some cases the excess pore water pressure does not develop over the entire clay stratum.

Solution of 1D consolidation

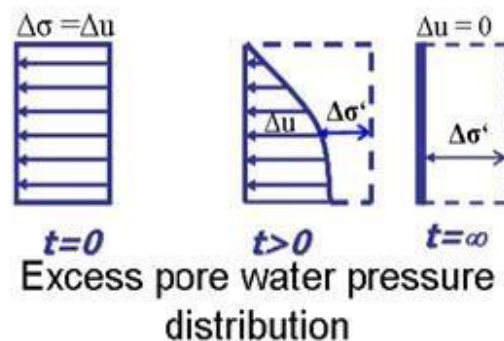
$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2} \text{ ----- 1}$$

The solution of variation of excess pore water pressure with depth and time can be obtained for various initial conditions.

Uniform excess pore water pressure with depth

1. Single Drainage (Drainage at top and bottom impervious)
2. Double Drainage (Drainage at top and bottom)

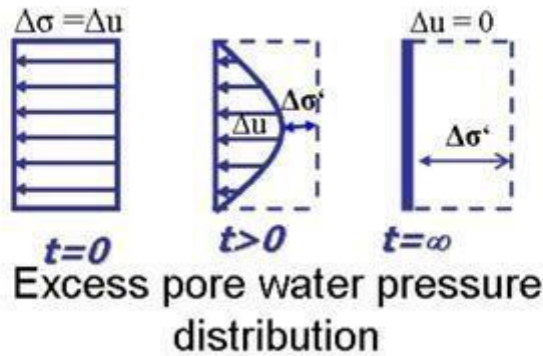
Single Drainage (drainage at top and bottom impervious)



Excess porewater pressure distribution of single drainage

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Double Drainage



Excess porewater pressure distribution of double drainage

Boundary Conditions are

- At $t = 0$ $\Delta u = \Delta \sigma$ and $\Delta \sigma' = 0$
- At the top $z = 0$ $\Delta u = 0$ $\Delta \sigma = \Delta \sigma'$
- At the bottom $z = 2H_{dr}$ $\Delta u = 0$ $\Delta \sigma = \Delta \sigma'$

A solution of equation (1) for the above boundary conditions using Fourier series is given by

$$\Delta u_{(z,t)} = \sum_{m=0}^{\infty} \frac{2\Delta u_0}{M} \sin\left(\frac{MZ}{H_{dr}}\right) e^{-M^2 T_v}$$

$$M = \frac{\pi}{2}(2m+1) \text{ Where } m = +ve \text{ integer with values from } 0 \text{ to } \infty$$

$$T_v = \frac{c_v t}{H_{dr}^2} \text{ Where } T_v = \text{Time factor (dimensionless)}$$

Graphical solution of 1D consolidation equation

$$\Delta u_{(z,t)} = \sum_{m=0}^{\infty} \frac{2\Delta u_0}{M} \sin\left(\frac{MZ}{H}\right) e^{-M^2 T_v}$$

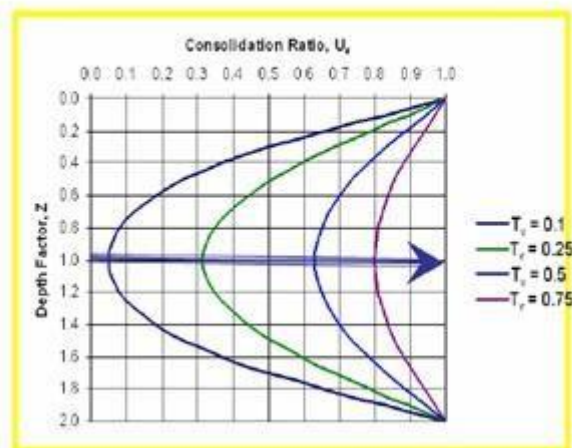
The solution of consolidation equation consists of the following three variables

1. The depth z
2. The excess pore water pressure Δu
3. The time (t) after application of loading

The above variables are expressed in the form of the following non-dimensional terms as

Sl. No	Variables	Non-dimensional terms
1	Depth (z)	$Z = z/H$ (Drainage path ratio)
2	Excess pore pressure (Δu)	U_z --- consolidation ratio This represents the dissipated pore water pressure to initial excess pore water pressure
3	Time (t)	T_v (Time factor)

The graphical solution of the above equation is as shown below



Terzaghi's solution for one-dimensional consolidation

This indicates the progress of consolidation with time and depth for a given set of boundary conditions.

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Degree of Consolidation (U_z)

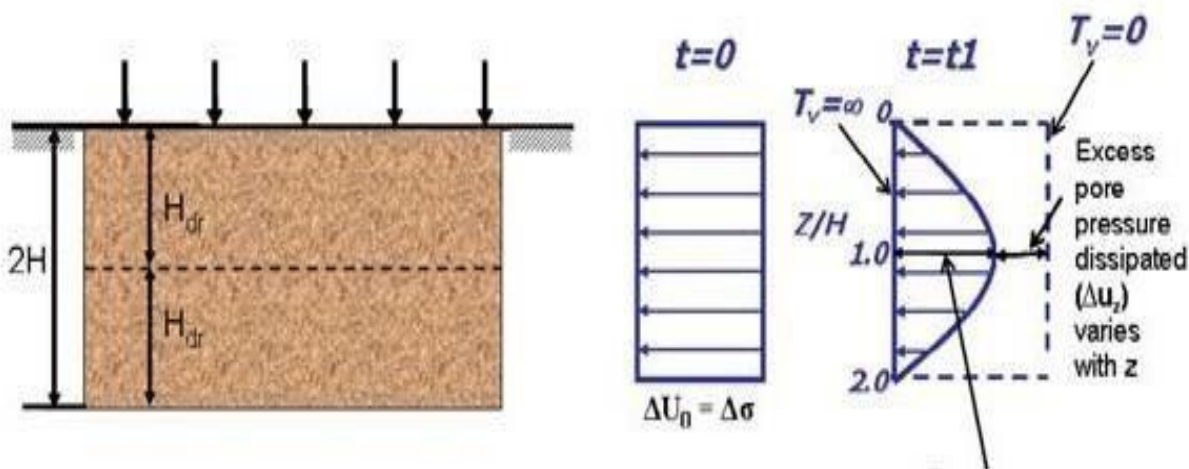


Figure: Selection of clay layer and Excess pore water pressure distribution

The degree of consolidation at any depth is given by

$$U_z = \frac{\Delta u_0 - \Delta u_z}{\Delta u_0}$$

$$1 - \frac{\Delta u_z}{\Delta u_0} = \frac{\Delta \sigma'_z}{\Delta u_0}$$

Δu_0 = Initial excess pore water pressure at that depth

Δu_z = Excess pore water pressure at that depth

$$u_z = 1 - \frac{\Delta u_z}{\Delta u_0}$$

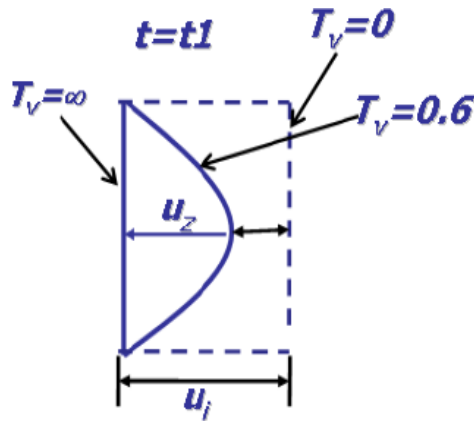
$$u_z = 1 - \sum_{m=0}^{\infty} \frac{2}{M} \sin\left(\frac{MZ}{H}\right) e^{-M^2 T_v}$$

u_z = Degree of consolidation at a particular depth at any given time

From practical point of view, the average degree of consolidation over the entire depth at any given time is desirable.

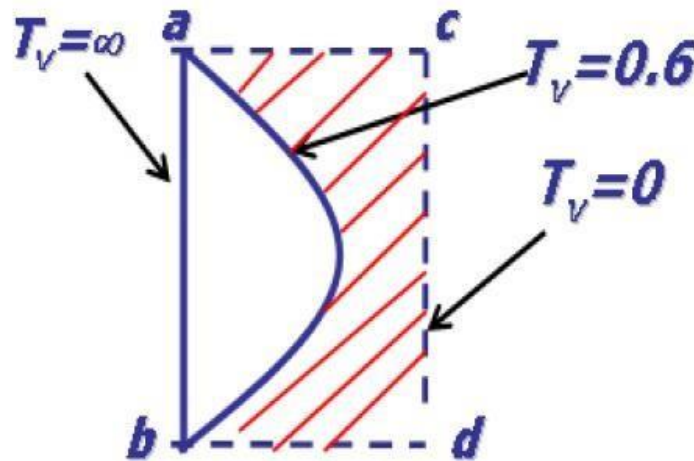
At any given time u_z varies with location and hence the degree of consolidation also varies.

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u_z = Degree of consolidation at a certain level

Average Degree of Consolidation (U)



The average degree of consolidation for the whole soil deposit at any time is given by

$$U = \frac{\text{Area of the diagram of excess pore water pressure dissipated at any time}}{\text{Area of the diagram of initial excess pore water pressure}}$$

$$U = \frac{\text{Area Shaded}}{\text{Area of abcd}}$$

Mathematically $U = f(T_v)$

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Consolidation of Soils

$$u = 1 - \sum_{m=0}^{\infty} \frac{2}{M} e^{-M^2 T_v}$$

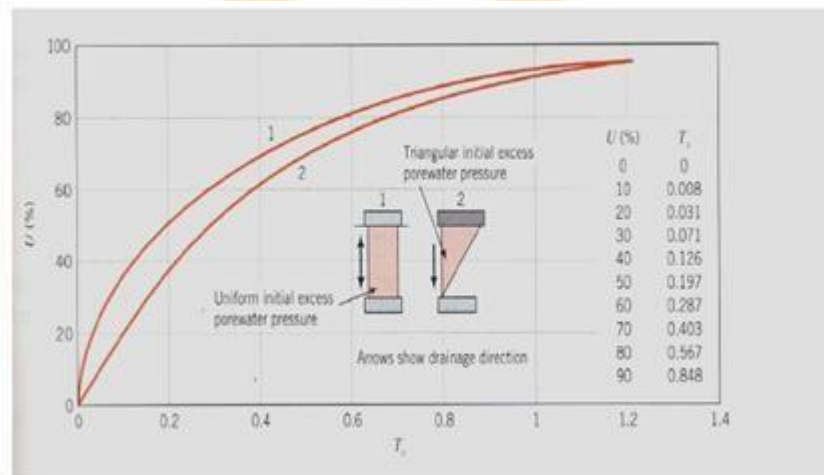


Figure: Degree of Consolidation versus time factor (T_v)

As per Taylor (1948) solution, the following approximation is possible

when $U \leq 60\%$ $T_v = \frac{\pi}{4} U^2$

For $U > 60\%$ $T_v = 1.781 - 0.933 \log(100 - U\%)$

Typical values of T_v

$U = 50\%$	$T_v = 0.197$
$U = 60\%$	$T_v = 0.287$
$U = 90\%$	$T_v = 0.848$



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Compressibility Properties-

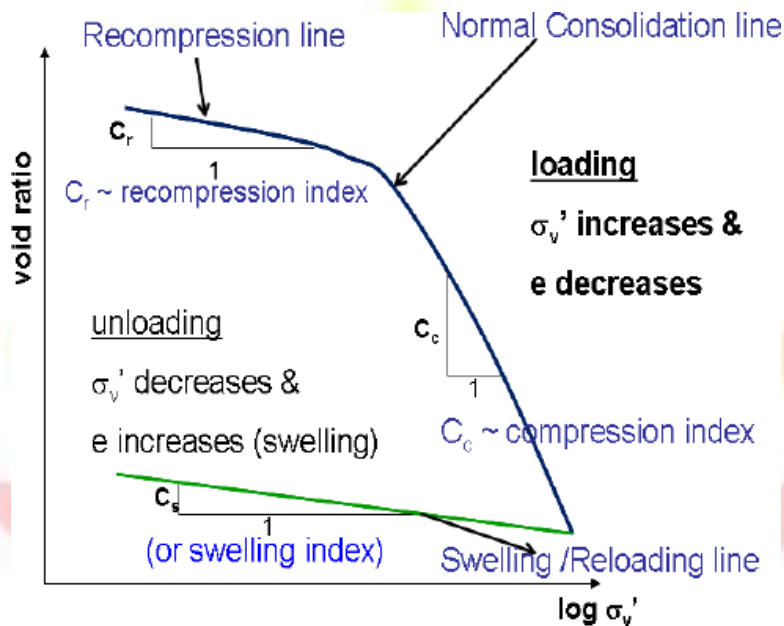


Fig. $e - \log \sigma_v'$ plot

Coefficient of compression/compression index (C_c)

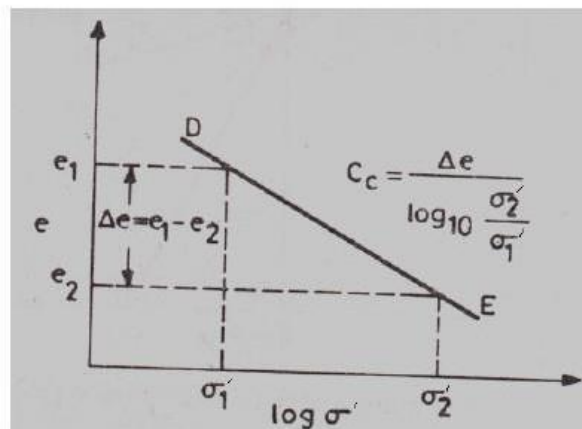


Fig. : $e - \log \sigma'$ plot

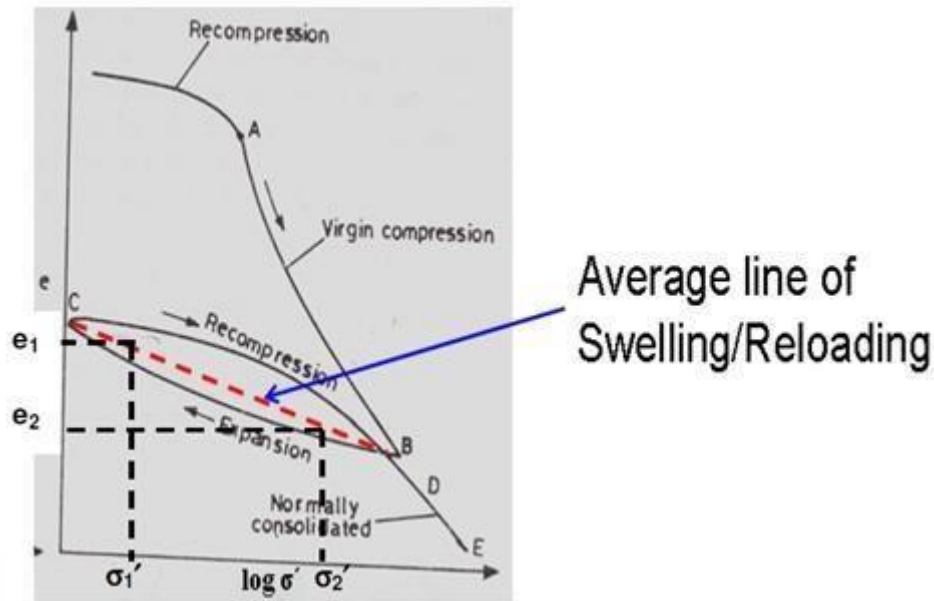


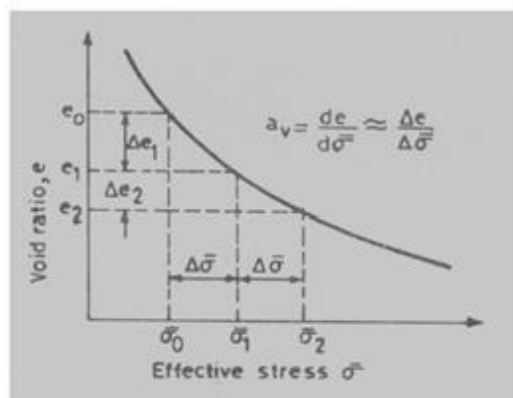
Figure: $e - \log \sigma'_v$ plot

Swelling Index(C_s)

It is the average slope of the unloading/reloading curves in $e - \log \sigma'$ plot given by

$$C_s = \frac{e_1 - e_2}{\log 10 \frac{\sigma'_2}{\sigma'_1}}$$

Co-efficient of compressibility (a_v)



Void ratio versus effective stress plot



cess...

It is the slope of the void ratio versus effective stress for a given stress increase

$\Delta\sigma'$ in void ratio versus effective stress plot as shown

$$a_v = \frac{\Delta e}{\Delta\sigma'} = \frac{e_1 - e_2}{\sigma'_2 - \sigma'_0}$$

a_v decrease with increase in effective stress

Co-efficient of volume compressibility (m_v)

It is the ratio of change in volume of a soil per unit initial volume due to unit increase in effective stress and is given by

$$m_v = \frac{\Delta e}{(1 + e_0)} \frac{1}{\Delta\sigma'}$$

Δe = Change in void ratio

e_0 = Initial void ratio

$\Delta\sigma'$ = increase in effective stress



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Preconsolidation Pressure

It is the maximum effective stress experienced by a soil in its stress history (past existence)

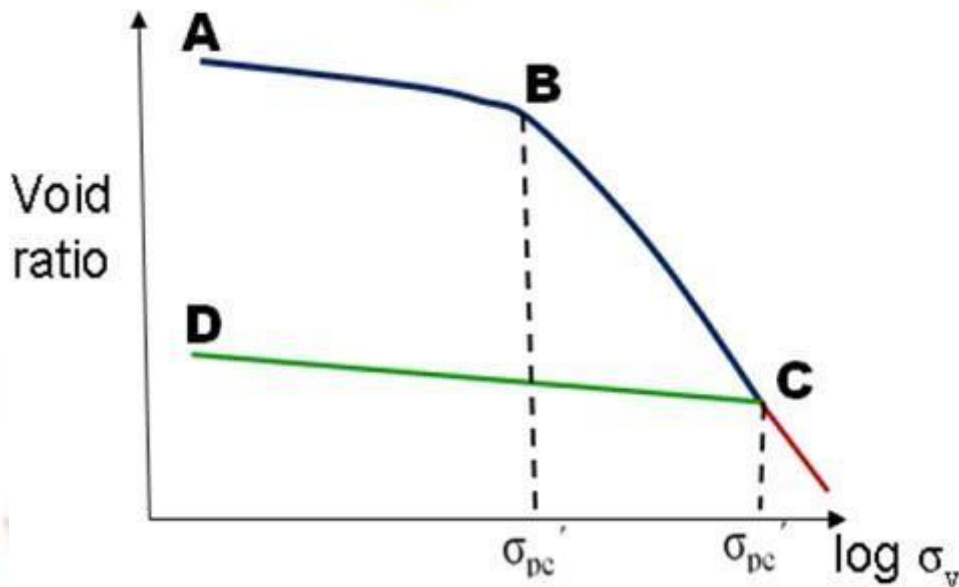


Figure: Void ratio versus effective stress (log scale)

For the soil loaded along the recompression curve AB the effective stress close to point B will be the preconsolidation pressure.

If the soil is compressed along BC and unloaded along CD and then reloaded along DC the effective stress close to point C will be the new preconsolidation pressure.

Effect of Stress History

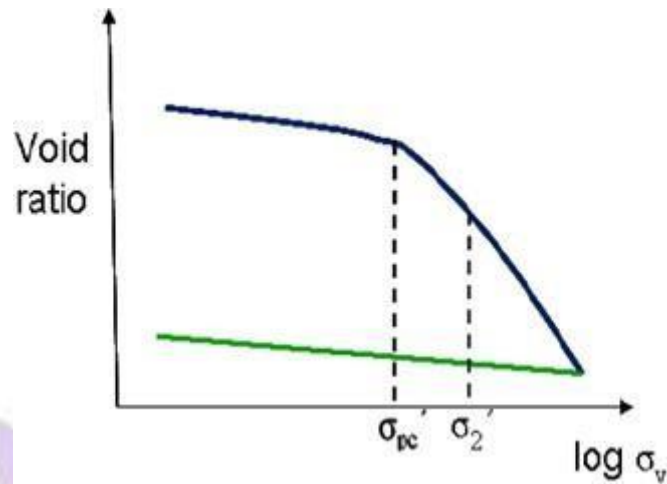
Based on the stress history (preconsolidation pressure) soils are classified as

1. Normally Consolidated Soils
2. Over Consolidated Soils
3. Under Consolidated Soils

Normally Consolidated Soils

It is a soil deposit that has never subjected to a vertical effective stress greater than the present vertical stress.

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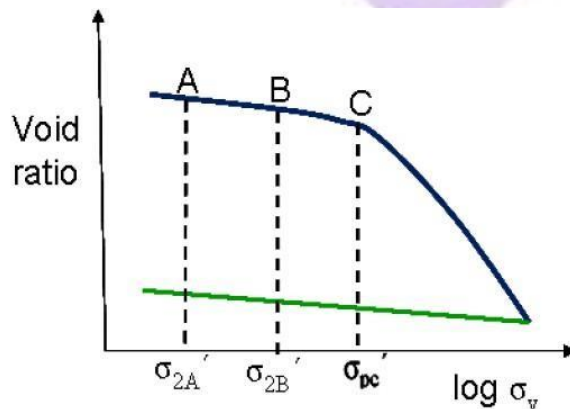
Void ratio versus effective stress (log scale)

Under Consolidated Soils

A soil deposit that has not consolidated under the present overburden pressure (effective stress) is called Under Consolidated Soil. These soils are susceptible to larger deformation and cause distress in buildings built on these deposits.

Over Consolidated Soils

It is a soil deposit that has been subjected to vertical effective stress greater than the present vertical effective stress.



Void ratio versus effective stress (log scale)

The stress state σ_{2A}' and σ_{2B}' represent over consolidated soil (well with in preconsolidation pressure) Over consolidated soil deposits are less compressible and therefore structures built on these soils undergo less settlement.

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Over Consolidation Ratio (OCR)

It is defined as the ratio of preconsolidation pressure to the present vertical effective stress

$$OCR = \frac{\sigma_{pc}'}{\sigma_z'}$$

This is indicative of the position of soil away from the normal consolidated line

OCR = 1 Normally consolidated Soils

Note: -- Soils having higher OCR are less compressible

-- They show elastic behavior to certain extent



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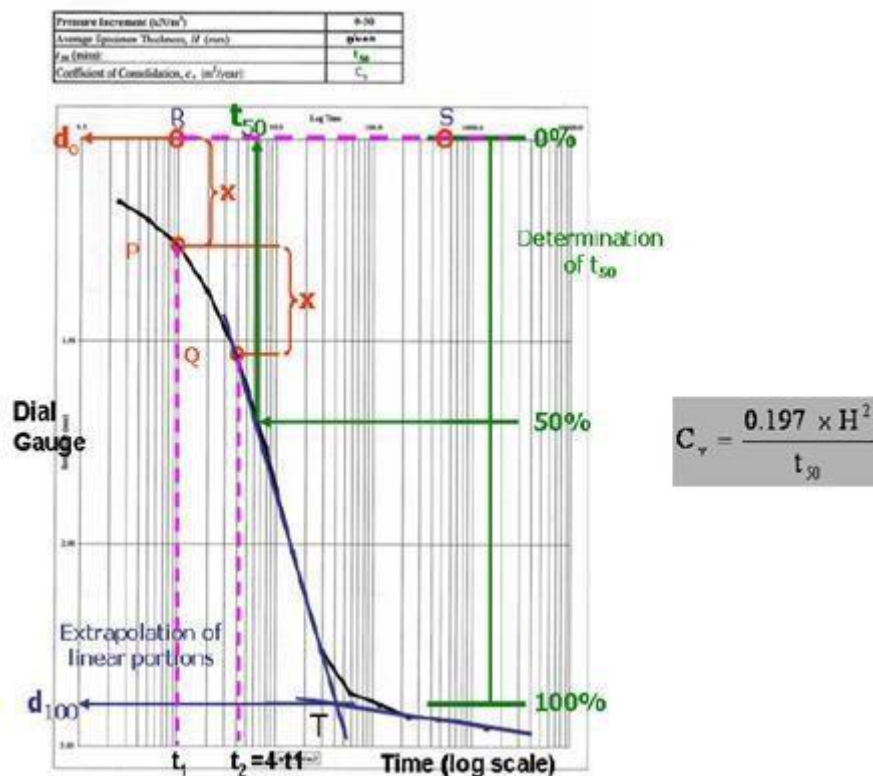
Determination of coefficient of consolidation (Cv) from laboratory data-

The coefficient of three graphical procedure are used

1. Logarithm of time method
2. Square root of time method
3. Hyperbola method

Log – time curve fitting method

The basis for this method is the theoretical (Uz) versus log Tv curve and experimental dial gauge reading and log t curves are similar.



Log-time curve fitting method

Steps

1. Plot the dial reading of compression for a given pressure increment versus time to log scale
2. Plot two points P and Q on the upper portion of the consolidation curve (say compression line) corresponding to time t₁ and t₂ such that t₂=4t₁
3. Let x be the difference in dial reading between P and Q. locate R at a vertical distance x above point P
4. Draw a horizontal line RS the dial reading corresponding to this line is d₀ which corresponds with 0% consolidation.



5. Project the straight line portion of primary and secondary consolidation to intersect at point T. The dial reading corresponding to T is d_{100} and this corresponds to 100% consolidation.

6. Determine the point V on the consolidation curve which corresponds to the dial reading of $\frac{d_0 + d_{100}}{2} = d_{50}$. The time corresponding to point V is t_{50} i.e time for 50% consolidation.

7. Determine C_v from $C_v = \frac{T_v H^2}{t}$

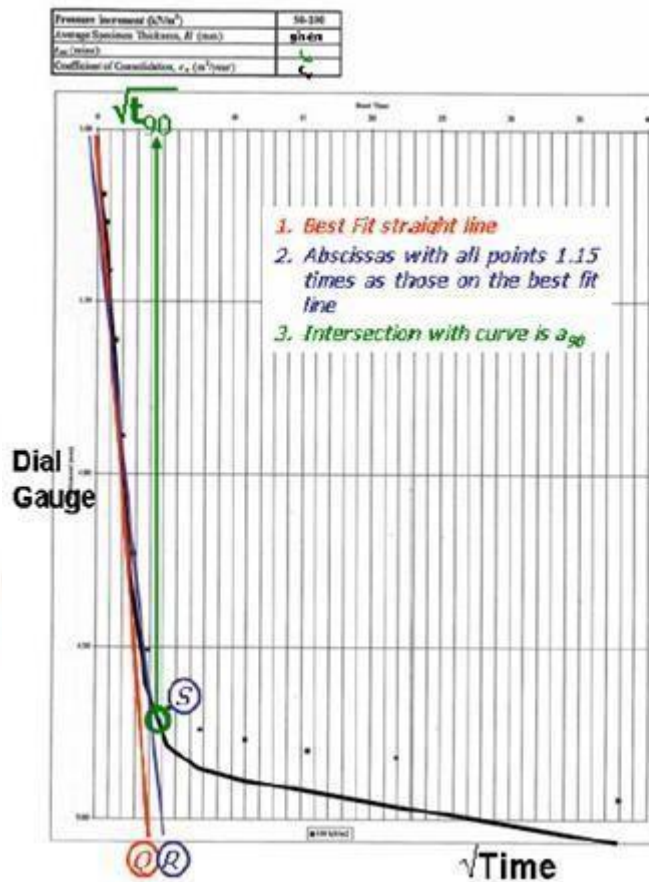
For 50% U_z $T_v = 0.197 \left(T_v = \frac{\pi}{4} \left(\frac{U_z}{100} \right)^2 \right)$

$$C_v = \frac{0.197 H^2}{t_{50}}$$



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Square-root – time curve fitting method



$$C_v = \frac{0.848 \times H^2}{t_{90}}$$

Log-time curve fitting method

1. Plot the dial reading and square root of time i.e $\sqrt{T}$ for a pressure increment as shown in figure.
2. Draw a tangent PQ to the initial portion of the plot as shown in fig.
3. Draw a line PR such that $OR=1.15OQ$.
4. The intersection of the line PR with the second portion of the curve i.e point S is marked.

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5. The time corresponding to point S represent $\sqrt{t_{90}}$ (Square root of time for 90% consolidation)

$$T_v = \frac{C_v t}{H^2}$$

$$C_v = \frac{T_v H^2}{t}$$

For $U_z > 60\%$ $T_v = 1.781 - 0.933 \log_{10} (100 - U \%)$

$$T_v = 0.848$$

$$C_v = \frac{0.848 H^2}{t_{90}}$$

Time Rate of consolidation-

We know that

$$T_v = \frac{C_v t}{H^2}$$

$$t = \frac{T_v H^2}{C_v}$$

For a given degree of consolidation (U) --- T_v is Constant

$$t \propto \frac{H^2}{C_v}$$

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Therefore the time required for a given degree of consolidation is proportional to the length of the drainage path

If the time required to reach a certain degree of consolidation is measured in the laboratory on a sample obtained from the field



The time taken by the field deposit of known thickness can be predicted by using

$$t_f = \frac{H_f^2}{H_L^2} \times t_L$$

t_f = Time required for field consolidation

t_L = Time required for laboratory consolidation

H_F = Thickness of soil in the site

H_L = Thickness of laboratory sample



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Settlement Calculations

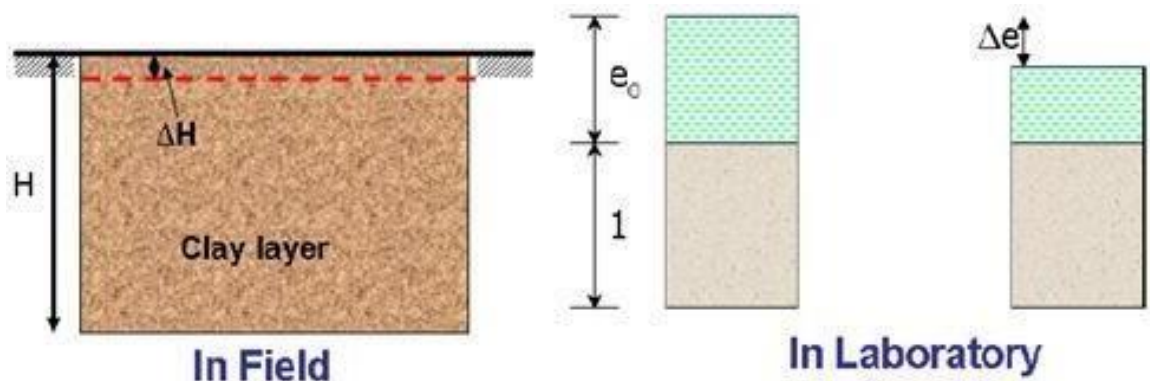


Figure: Compression of field deposit

If the clay layer of thickness H when subjected to an increase in average effective overburden pressure from σ'_0 to σ'_1 ($\sigma'_0 + \Delta\sigma'$) there will be consolidation settlement of ΔH .

The strain

$$\varepsilon = \frac{\Delta H}{H} \quad \text{in field}$$

$$\varepsilon = \frac{\Delta e}{1 + e_0} \quad \text{in lab}$$

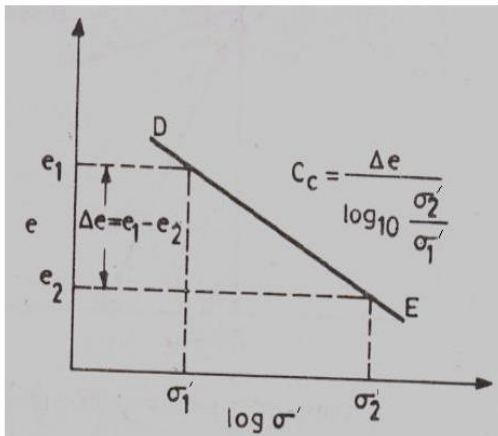
Equating

$$\frac{\Delta H}{H} = \frac{\Delta e}{1 + e_0}$$

$$\Delta H = \frac{\Delta e H}{1 + e_0}$$

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Case-1 For normally consolidated soils



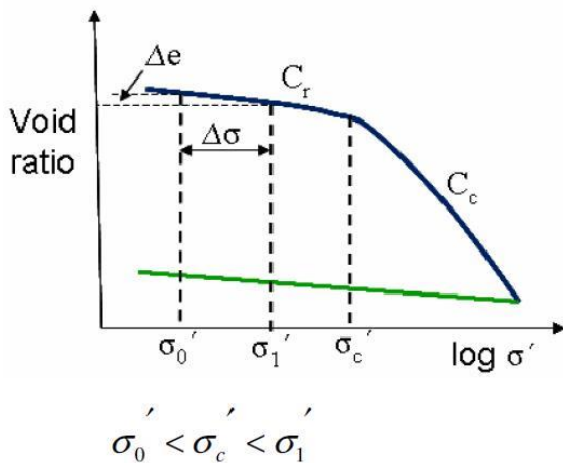
$$C_c = \frac{\Delta e}{\log_{10} \frac{\sigma'_1}{\sigma'_0}} = \frac{\Delta e}{\log_{10} \left(\frac{\sigma'_0 + \Delta \sigma}{\sigma'_0} \right)}$$

$$\Delta e = C_c \log_{10} \left(\frac{\sigma'_0 + \Delta \sigma}{\sigma'_0} \right)$$

$$\Delta H = \frac{C_c}{1 + e_0} H \log_{10} \left(\frac{\sigma'_0 + \Delta \sigma}{\sigma'_0} \right)$$

Case-2 Over consolidated soils

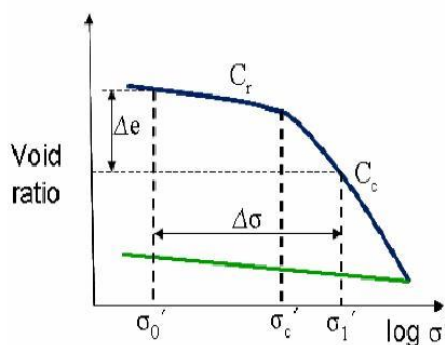
Case 1: $\sigma'_1 < \sigma'_c$



$$\Delta e = C_r \log_{10} \left(\frac{\sigma'_1}{\sigma'_0} \right)$$

$$\Delta H = \frac{C_r}{1 + e_0} H \log_{10} \left(\frac{\sigma'_0 + \Delta \sigma}{\sigma'_0} \right)$$

Case 2:



$$\Delta e = \Delta e_1 + \Delta e_2$$

$$\Delta e = C_r \log_{10} \left(\frac{\sigma'_c}{\sigma'_0} \right) + C_c \log_{10} \left(\frac{\sigma'_1}{\sigma'_c} \right)$$

$$\Delta H = C_r \log_{10} \left(\frac{\sigma'_c}{\sigma'_0} \right) + C_c \log_{10} \left(\frac{\sigma'_0 + \Delta \sigma}{\sigma'_c} \right)$$

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