

Department of Civil Engineering

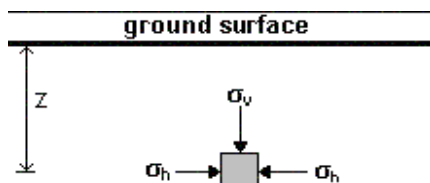
Study Material Unit-II

Stresses in the Ground:

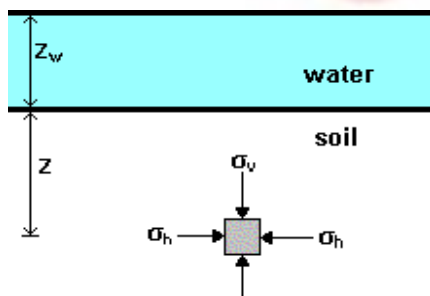
Total Stress

When a load is applied to soil, it is carried by the solid grains and the water in the pores. The **total vertical stress** acting at a point below the ground surface is due to the weight of everything that lies above, including soil, water, and surface loading. Total stress thus increases with depth and with unit weight.

Vertical total stress at depth z , $s_v = g \cdot Z$



Below a water body, the total stress is the sum of the weight of the soil up to the surface and the weight of water above this. $s_v = g \cdot Z + g_w \cdot Z_w$



The total stress may also be denoted by s_z or just s . It varies with changes in water level and with excavation.

Pore Water Pressure

The pressure of water in the pores of the soil is called **pore water pressure (u)**. The magnitude of pore water pressure depends on:

- the depth below the water table.
- the conditions of seepage flow.

Under hydrostatic conditions, no water flow takes place, and the pore pressure at a given point is given by

$$u = \gamma_w \cdot h$$

where h = depth below water table or overlying water surface

It is convenient to think of pore water pressure as the pressure exerted by a column of water in an imaginary standpipe inserted at the given point.

The natural level of ground water is called the **water table** or the **phreatic surface**. Under conditions of no seepage flow, the water table is horizontal. The magnitude of the pore water pressure at the water table is zero. Below the water table, pore water pressures are positive.

The **principle of effective stress** was enunciated by **Karl Terzaghi** in the year 1936. This principle is valid only for saturated soils, and consists of two parts:

1. At any point in a soil mass, the effective stress (represented by $\bar{\sigma}$ or σ') is related to total stress (σ) and pore water pressure (u) as

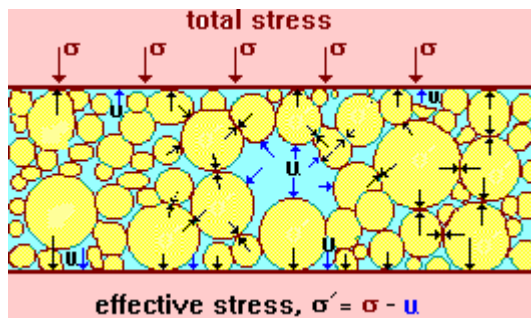
$$\bar{\sigma} = \sigma - u$$

Both the total stress and pore water pressure can be measured at any point.

2. All measurable effects of a change of stress, such as compression and a change of shearing resistance, are exclusively due to changes in effective stress.

$$\text{Compression} = f_1(\bar{\sigma})$$

$$\text{Shear Strength} = f_2(\bar{\sigma})$$

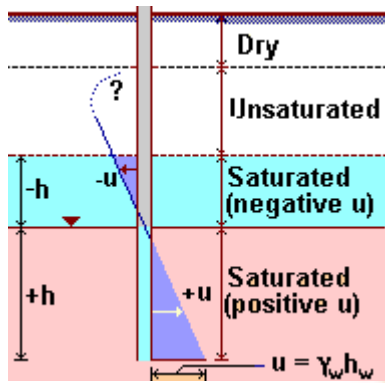


In a saturated soil system, as the voids are completely filled with water, the pore water pressure acts equally in all directions.

The effective stress is not the exact contact stress between particles but the distribution of load carried by the soil particles over the area considered. It cannot be measured and can only be computed.

If the total stress is increased due to additional load applied to the soil, the pore water pressure initially increases to counteract the additional stress. This increase in pressure within the pores might cause water to drain out of the soil mass, and the load is transferred to the solid grains. This will lead to the increase of effective stress

Above the water table, when the soil is saturated, pore pressure will be negative (less than atmospheric). The height above the water table to which the soil is saturated is called the **capillary rise**, and this depends on the grain size and the size of pores. In coarse soils, the capillary rise is very small.



Between the top of the saturated zone and the ground surface, the soil is partially saturated, with a consequent reduction in unit weight. The pore pressure in a partially saturated soil consists of two components:

Pore water pressure = u_w

Pore air pressure = u_a

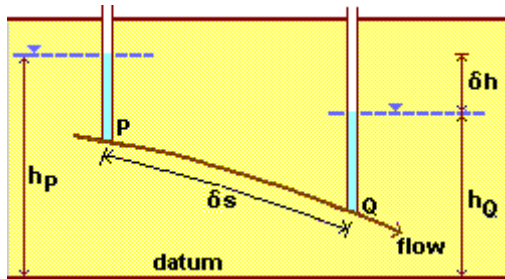
Water is incompressible, whereas air is compressible. The combined effect is a complex relationship involving partial pressures and the degree of saturation of the soil.

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Effective stress under Hydrodynamic Conditions:

There is a change in pore water pressure in conditions of **seepage flow** within the ground. Consider seepage occurring between two points **P** and **Q**. The potential driving the water flow is the hydraulic gradient between the two points, which is equal to the head drop per unit length. In steady state seepage, the gradient remains constant.

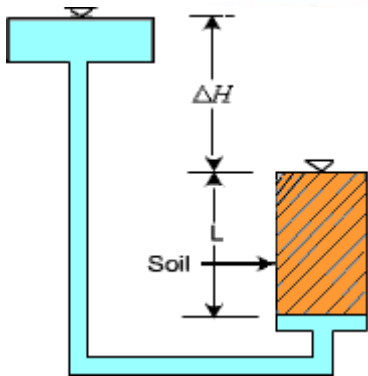


Hydraulic gradient from P to Q, $i = \Delta h / \Delta s$

As water percolates through soil, it exerts a drag on soil particles it comes in contact with. Depending on the flow direction, either downward or upward, the drag either increases or decreases inter-particle contact forces.

A downward flow increases effective stress.

In contrast, an upward flow opposes the force of gravity and can even cause to counteract completely the contact forces. In such a situation, effective stress is reduced to zero and the soil behaves like a very viscous liquid. Such a state is known as **quick sand condition**. In nature, this condition is usually observed in coarse silt or fine sand subject to artesian conditions.



At the bottom of the soil column,

$$\sigma = \gamma \cdot L$$

$$u = \gamma_w (L + \Delta H)$$

During quick sand condition, the effective stress is reduced to zero.

$$\gamma \cdot L = \gamma_w (L + \Delta H)$$

$$L(\gamma - \gamma_w) = \gamma_w \Delta H$$

$$L \gamma_s = \gamma_w \Delta H$$

$$\frac{\Delta H}{L} = \frac{\gamma_s}{\gamma_w} = i_{cr} \approx 1$$

where i_{cr} = **critical hydraulic gradient**

This shows that when water flows upward under a hydraulic gradient of about 1, it completely neutralizes the force on account of the weight of particles, and thus leaves the particles suspended in water.

Importance of Effective stress:

At any point within the soil mass, the magnitudes of both total stress and pore water pressure are dependent on the ground water position. With a shift in the water table due to seasonal fluctuations, there is a resulting change in the distribution in pore water pressure with depth.

Changes in water level **below ground** result in changes in effective stresses below the water table. A rise increases the pore water pressure at all elevations thus causing a decrease in effective stress. In contrast, a fall in the water table produces an increase in the effective stress.

Changes in water level **above ground** do not cause changes in effective stresses in the ground below. A rise above ground surface increases both the total stress and the pore water pressure by the same amount, and consequently effective stress is not altered.

In some analyses it is better to work with the *changes* of quantity, rather than in absolute quantities. The effective stress expression then becomes:

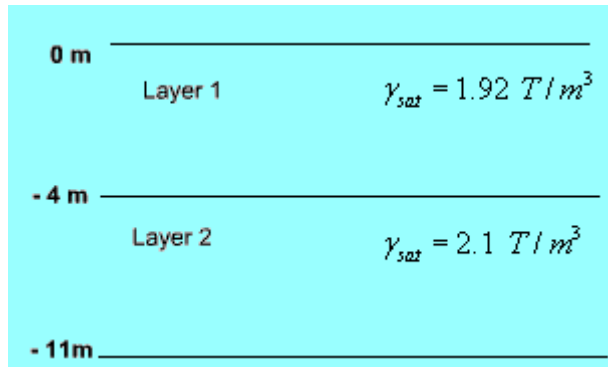
$$\sigma' = \sigma - u$$

If both total stress and pore water pressure change by the same amount, the effective stress remains constant.

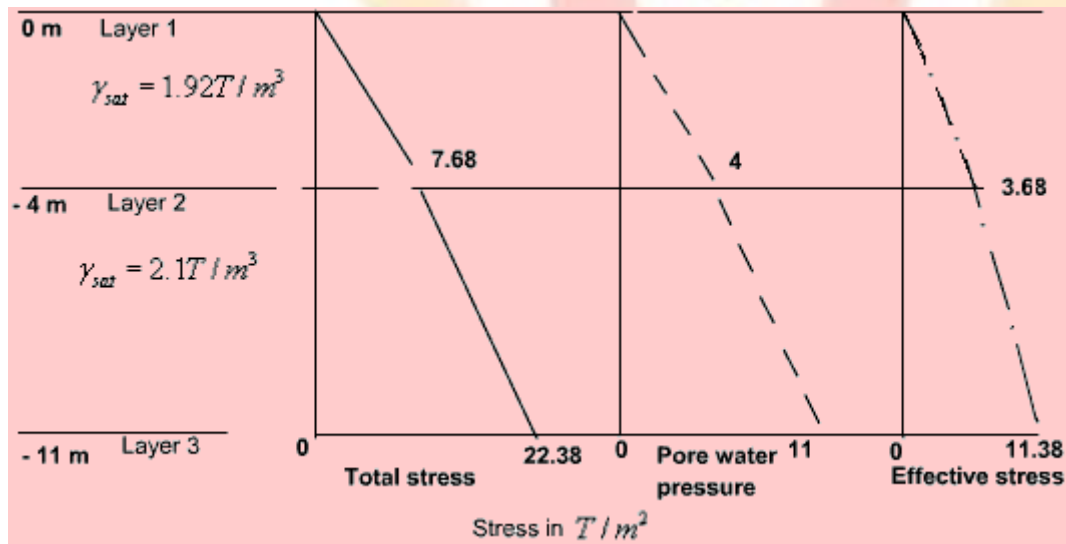
Total and effective stresses must be distinguishable in all calculations. Ground movements and instabilities can be caused by changes in total stress, such as caused by loading by foundations and unloading due to excavations. They can also be caused by changes in pore water pressures, such as failure of slopes after rainfall.

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Example 1: For the soil deposit shown below, draw the total stress, pore water pressure and effective stress diagrams. The water table is at ground level.



Solution:



Total stress

At - 4m, $\sigma = 1.92 \times 4 = 7.68 \text{ T/m}^2$

At -11m, $\sigma = 7.68 + 2.1 \times 7 = 22.38 \text{ T/m}^2$

Pore water pressure

At - 4 m, $u = 1 \times 4 = 4 \text{ T/m}^2$

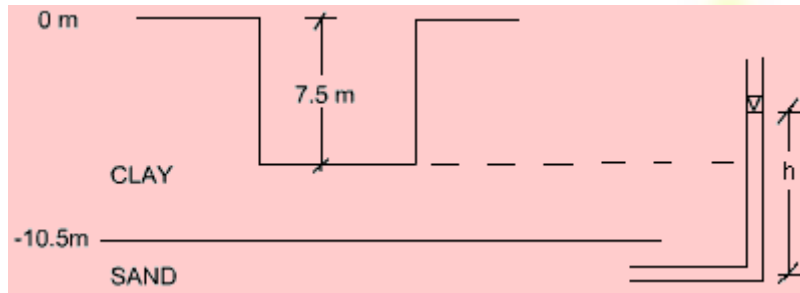
At -11 m, $u = 1 \times 11 = 11 \text{ T/m}^2$

Effective stress

At - 4 m, $\bar{\sigma} = 7.68 - 4 = 3.68 \text{ T/m}^2$

At -11m, $\bar{\sigma} = 22.38 - 11 = 11.38 \text{ T/m}^2$

Example 2: An excavation was made in a clay stratum having $\gamma_t = 2 \text{ T/m}^3$. When the depth was 7.5 m, the bottom of the excavation cracked and the pit was filled by a mixture of sand and water. The thickness of the clay layer was 10.5 m, and below it was a layer of pervious water-bearing sand. How much was the artesian pressure in the sand layer?



Solution:

When the depth of excavation was 7.5 m, at the interface of the CLAY and SAND layers, the effective stress was equal to zero.

Downward pressure due to weight of clay = Upward pressure due to artesian pressure

$$(10.5 - 7.5) \gamma_t = \gamma_w h, \text{ where } h = \text{artesian pressure head}$$

$$3 \times 2 = 1 \times h$$

$$\therefore h = 6 \text{ m} = 0.6 \text{ kg/cm}^2 \text{ or } 6 \text{ T/m}^2 \text{ artesian pressure}$$

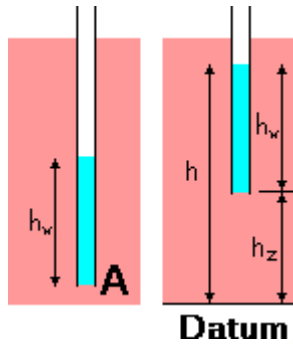
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Permeability Of Soil:

Pressure, Elevation and Total Heads

In soils, the interconnected pores provide passage for water. A large number of such flow paths act together, and the average rate of flow is termed the coefficient of permeability, or just permeability. It is a measure of the ease that the soil provides to the flow of water through its pores.



At point **A**, the pore water pressure (**u**) can be measured from the height of water in a standpipe located at that point.

The height of the water column is the **pressure head (h_w)**.

$$h_w = u / \gamma_w$$

To identify any difference in pore water pressure at different points, it is necessary to eliminate the effect of the points of measurement. With this in view, a datum is required from which locations are measured.

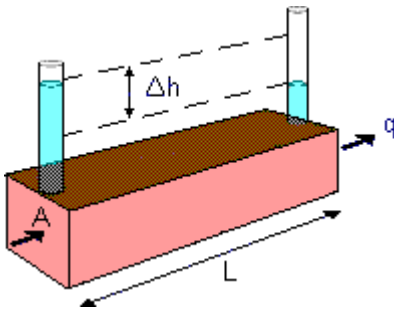
The **elevation head (h_z)** of any point is its height above the datum line. The height of water level in the standpipe above the datum is the **piezometric head (h)**.

$$h = h_z + h_w$$

Total head consists of **three components**: elevation head, pressure head, and velocity head. As seepage velocity in soils is normally low, velocity head is ignored, and total head becomes equal to the piezometric head. Due to the low seepage velocity and small size of pores, the flow of water in the pores is steady and laminar in most cases. Water flow takes place between two points in soil due to the difference in total heads.

Darcy's law states that there is a linear relationship between flow velocity (**v**) and hydraulic gradient (**i**) for any given saturated soil under steady laminar flow conditions.

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If the rate of flow is **q** (volume/time) through cross-sectional area (**A**) of the soil mass, Darcy's Law can be expressed as

$$v = q/A = k.i$$

where **k** = permeability of the soil

$$i = \Delta h/L$$

Δh = difference in total heads

L = length of the soil mass

The flow velocity (**v**) is also called the Darcian velocity or the **superficial velocity**. It is different from the actual velocity inside the soil pores, which is known as the **seepage velocity**, **v_s**. At the particulate level, the water follows a tortuous path through the pores. Seepage velocity is always greater than the superficial velocity, and it is expressed as:

$$v_s = \frac{q}{A_v} = \frac{q}{A_v} \cdot \frac{A}{A} \approx \frac{v}{n}$$

where **A_v** = Area of voids on a cross section normal to the direction of flow

n = porosity of the soil

Permeability of Different soils:

Permeability (**k**) is an engineering property of soils and is a function of the soil type. Its value depends on the average size of the pores and is related to the distribution of particle sizes, particle shape and soil structure. The ratio of permeabilities of typical sands/gravels to those of typical clays is of the order of **10⁶**. A small proportion of fine material in a coarse-grained soil can lead to a significant reduction in permeability.

For different soil types as per grain size, the orders of magnitude for permeability are as follows:

Soil	k (cm/sec)
Gravel	10 ⁰
Coarse sand	10 ⁰ to 10 ⁻¹
Medium sand	10 ⁻¹ to 10 ⁻²

Fine sand	10^{-2} to 10^{-3}
Silty sand	10^{-3} to 10^{-4}
Silt	1×10^{-5}
Clay	10^{-7} to 10^{-9}

In soils, the permeant or pore fluid is mostly water whose variation in property is generally very less. Permeability of all soils is strongly influenced by the density of packing of the soil particles, which can be represented by void ratio (**e**) or porosity (**n**).

For Sands

In sands, permeability can be empirically related to the square of some representative grain size from its grain-size distribution. For filter sands, Allen Hazen in 1911 found that $k \propto 100 (D_{10})^2$ cm/s where D_{10} = effective grain size in cm.

Different relationships have been attempted relating void ratio and permeability, such as $k \propto e^3/(1+e)$, and $k \propto e^2$. They have been obtained from the Kozeny-Carman equation for laminar flow in saturated soils.

$$k = \frac{1}{k_0 k_T S_s^2} \cdot \frac{e^3}{1+e} \cdot \frac{\gamma_w}{\eta}$$

where k_0 and k_T are factors depending on the shape and tortuosity of the pores respectively, S_s is the surface area of the solid particles per unit volume of solid material, and γ_w and η are unit weight and viscosity of the pore water. The equation can be reduced to a simpler form as

$$k = C \cdot \frac{e^3}{1+e} \approx C \cdot e^2$$

For Silts and Clays

For silts and clays, the Kozeny-Carman equation does not work well, and $\log k$ versus e plot has been found to indicate a linear relationship.

For clays, it is typically found that

$$\log_{10} k = \frac{e - e_k}{C_k}$$

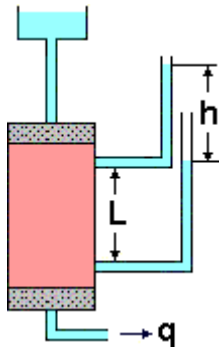
where C_k is the permeability change index and e_k is a reference void ratio.

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Laboratory Measurement of Permeability:

Constant Head Flow

Constant head permeameter is recommended for coarse-grained soils only since for such soils, flow rate is measurable with adequate precision. As water flows through a sample of cross-section area **A**, steady total head drop **h** is measured across length **L**.

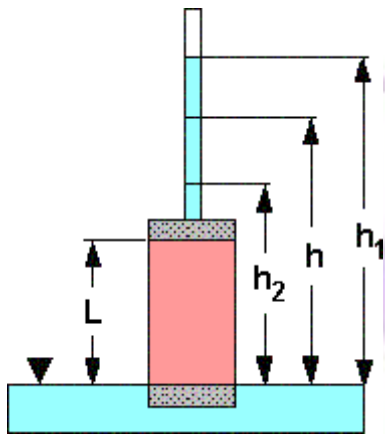


Permeability **k** is obtained from:

$$k = \frac{qL}{Ah}$$

Falling Head Flow:

Falling head permeameter is recommended for fine-grained soils.



Total head **h** in standpipe of area **a** is allowed to fall. Hydraulic gradient varies with time. Heads **h₁** and **h₂** are measured at times **t₁** and **t₂**. At any time **t**, flow through the soil sample of cross-sectional area **A** is

$$q = k \cdot h \cdot \frac{A}{L} \text{ ----- (1)}$$

Flow in unit time through the standpipe of cross-sectional area **a** is

$$= a \times \left(-\frac{dh}{dt} \right) \text{----- (2)}$$

Equating (1) and (2) ,

$$-a \cdot \frac{dh}{dt} = k \cdot h \frac{A}{L}$$

$$\text{or } -\frac{dh}{h} = \left(\frac{kA}{La} \right) dt$$

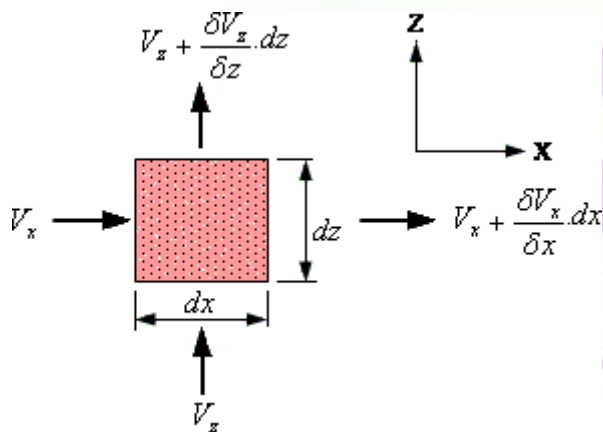
Integrating between the limits,

$$\log_e \left(\frac{h_1}{h_2} \right) = \frac{k \cdot A}{L \cdot a} (t_2 - t_1)$$

$$k = \frac{L \cdot a \log_e \left(\frac{h_1}{h_2} \right)}{A(t_2 - t_1)}$$

$$= \frac{2.3 L \cdot a \log_{10} \left(\frac{h_1}{h_2} \right)}{A(t_2 - t_1)}$$

Seepage in Soils:



A rectangular soil element is shown with dimensions **dx** and **dz** in the plane, and thickness **dy** perpendicular to this plane. Consider planar flow into the rectangular soil element.

In the **x-direction**, the net amount of the water entering and leaving the element is

$$\frac{\delta V_x}{\delta x} . dx . dy . dz$$

Similarly in the **z-direction**, the difference between the water inflow and outflow is

$$\frac{\delta V_z}{\delta z} . dz . dx . dy$$

For a two-dimensional steady flow of pore water, any imbalance in flows into and out of an element in the z-direction must be compensated by a corresponding opposite imbalance in the x-direction. Combining the above, and dividing by $dx . dy . dz$, the **continuity equation** is expressed as

$$\frac{\delta V_x}{\delta x} + \frac{\delta V_z}{\delta z} = 0$$

From Darcy's law, $V_x = k_x . \frac{\delta h}{\delta x}$, $V_z = k_z . \frac{\delta h}{\delta z}$, where **h** is the head causing flow.

When the continuity equation is combined with Darcy's law, the equation for flow is expressed as:

$$k_x . \frac{\delta^2 h}{\delta x^2} + k_z . \frac{\delta^2 h}{\delta z^2} = 0$$

For an isotropic material in which the permeability is the same in all directions (i.e. $k_x = k_z$), the **flow equation** is

$$\frac{\delta^2 h}{\delta x^2} + \frac{\delta^2 h}{\delta z^2} = 0$$

This is the **Laplace equation** governing two-dimensional steady state flow. It can be solved *graphically, analytically, numerically, or analogically*.

For the more general situation involving **three-dimensional** steady flow, Laplace equation becomes:

$$\frac{\delta^2 h}{\delta x^2} + \frac{\delta^2 h}{\delta y^2} + \frac{\delta^2 h}{\delta z^2} = 0$$

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One-dimensional Flow:

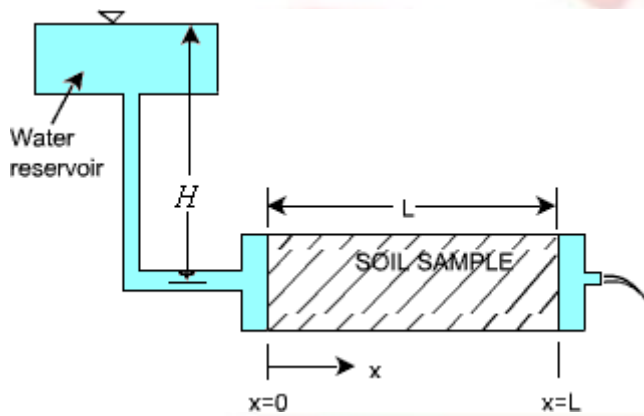
For this, the **Laplace Equation** is $\frac{\partial^2 h}{\partial x^2} = 0$

Integrating twice, a general solution is obtained.

$$\frac{\partial h}{\partial x} = c_1$$

$$h = c_2 + c_1 x$$

The values of constants can be determined from the specific boundary conditions.



As shown, at $x = 0$, $h = H$, and at $x = L$, $h = 0$

Substituting and solving,

$$c_2 = H, \quad c_1 = -\frac{H}{L}$$

The specific solution for flow in the above permeameter is

$$h = H - \frac{H}{L} x$$

which states that head is dissipated in a linearly uniform manner over the entire length of the permeameter.

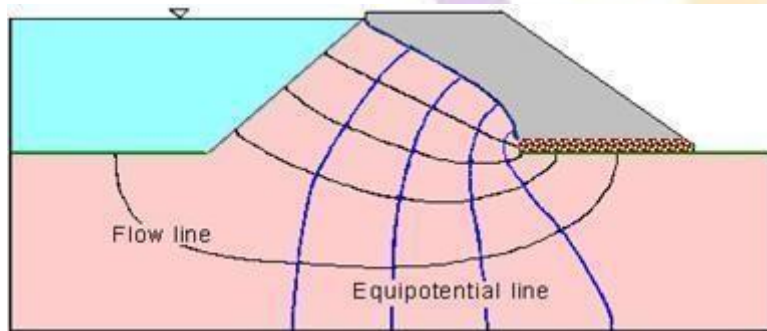
Two-dimensional Flow:

Flow Nets

Graphical form of solutions to **Laplace equation** for two-dimensional seepage can be presented as flow nets. Two orthogonal sets of curves form a flow net:

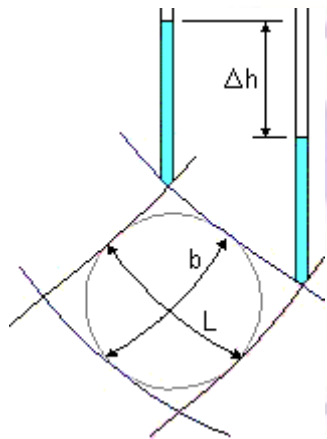
- **Equipotential lines** connecting points of equal total head **h**
- **Flow lines** indicating the direction of seepage down a hydraulic gradient

Two flow lines can never meet and similarly, two equipotential lines can never meet. The space between two adjacent flow lines is known as a **flow channel**, and the figure formed on the flownet between any two adjacent flow lines and two adjacent equipotential lines is referred to as a **field**. Seepage through an embankment dam is shown.



Calculation of flow in a channel

If standpipe piezometers were inserted into the ground with their tips on a single equipotential line, then the water would rise to the same level in each standpipe. The pore pressures would be different because of their different elevations. There can be no flow along an equipotential line as there is no hydraulic gradient.



Consider a field of length **L** within a flow channel. There is a fall of total head Δh . The average hydraulic gradient is

$$i = \frac{\Delta h}{L}$$

As the flow lines are **b** apart and considering unit length perpendicular to field, the flow rate is

$$\Delta q = kb \frac{\Delta h}{L}$$

There is an advantage in sketching flow nets in the form of **curvilinear 'squares'** so that a circle can be inscribed within each four-sided figure bounded by two equipotential lines and two flow lines.

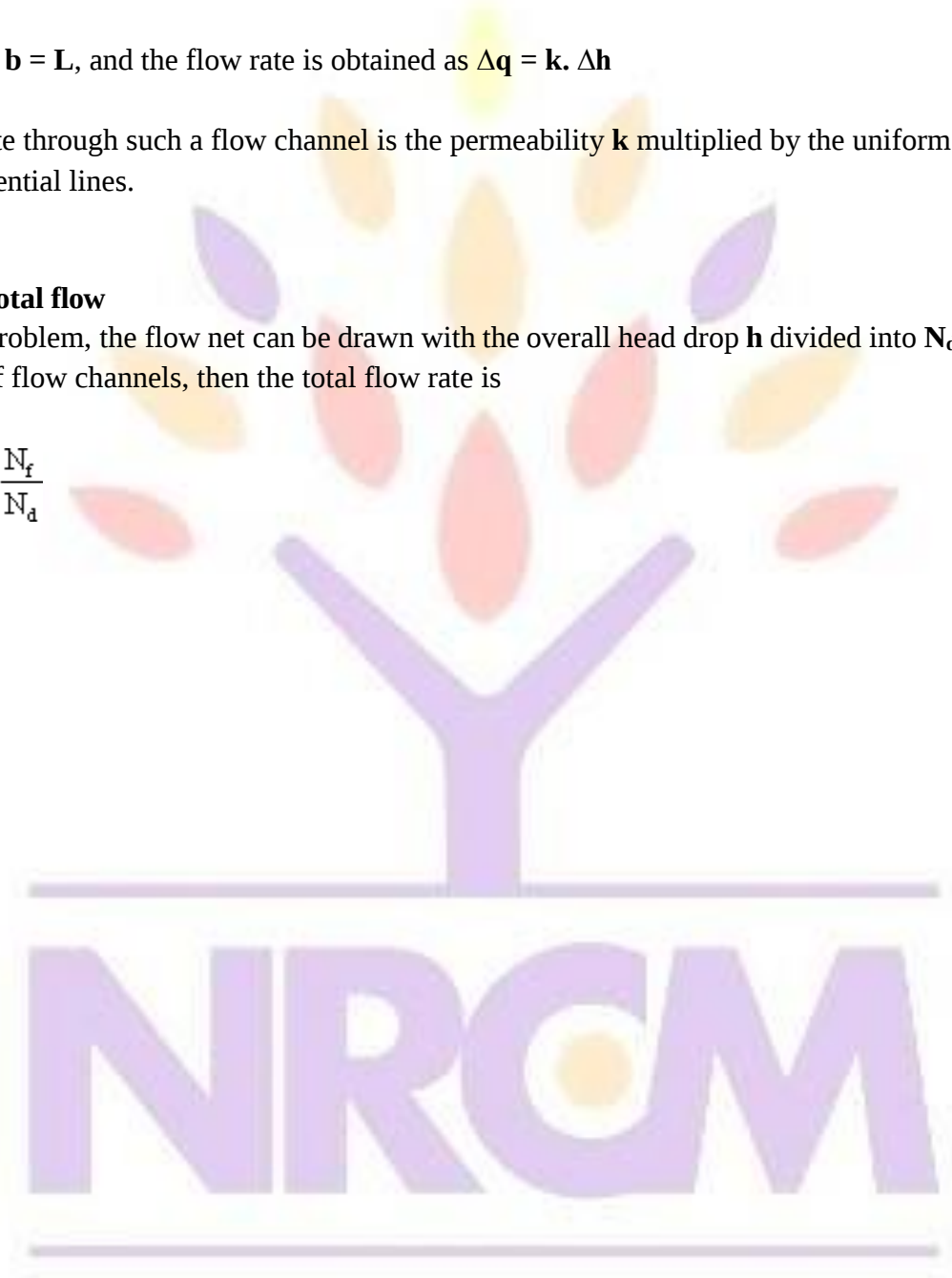
In such a square, **b = L**, and the flow rate is obtained as $\Delta q = k \cdot \Delta h$

Thus the flow rate through such a flow channel is the permeability **k** multiplied by the uniform interval Δh between adjacent equipotential lines.

Calculation of total flow

For a complete problem, the flow net can be drawn with the overall head drop **h** divided into N_d so that $\Delta h = h / N_d$. If N_f is the no. of flow channels, then the total flow rate is

$$q = \Delta q \cdot N_f = k h \cdot \frac{N_f}{N_d}$$



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Procedure for Drawing Flow Nets:

At every **point (x,z)** where there is flow, there will be a value of head **$h(x,z)$** . In order to represent these values, contours of equal head are drawn.

A flow net is to be drawn by trial and error. For a given set of boundary conditions, the flow net will remain the same even if the direction of flow is reversed. Flow nets are constructed such that the head lost between successive **equipotential lines** is the same, say Δh . It is useful in visualising the flow in a soil to plot the flow lines, as these are lines that are tangential to the flow at any given point. The steps of construction are:

1. Mark all boundary conditions, and draw the flow cross section to some convenient scale.
2. Draw a coarse net which is consistent with the boundary conditions and which has orthogonal equipotential and flow lines. As it is usually easier to visualise the pattern of flow, start by drawing the flow lines first.
3. Modify the mesh such that it meets the conditions outlined above and the fields between adjacent flow lines and equipotential lines are 'square'.
4. Refine the flow net by repeating **step 3**.

The most common **boundary conditions** are:

- (a) A submerged permeable soil boundary is an equipotential line. This could have been determined by considering imaginary standpipes placed at the soil boundary, as for every point the water level in the standpipe would be the same as the water level. (Such a boundary is marked as CD and EF in the following figure.)
- (b) The boundary between permeable and impermeable soil materials is a flow line (This is marked as AB in the same figure).
- (c) Equipotential lines intersecting a phreatic surface do so at equal vertical intervals.

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