

CHAPTER DC

CIRCUITS

DEFINITIONS

Linear elements:

In an electric circuit, a linear element is an electrical element with a linear relationship between current and voltage. Resistors are the most common example of a linear element; other examples include capacitors, inductors, and transformers.

Nonlinear Elements:

A nonlinear element is one which does not have a linear input/output relation. In a diode, for example, the current is a non-linear function of the voltage. Most semiconductor devices have non-linear characteristics.

Active Elements:

The elements which generate or produce electrical energy are called active elements. Some of the examples are batteries, generators, transistors, operational amplifiers, vacuum tubes etc.

Passive Elements:

All elements which consume rather than produce energy are called passive elements, like resistors, inductors and capacitors.

In unilateral element, voltage – current relation is not same for both the direction.

Example: Diode, Transistors.

In bilateral element, voltage – current relation is same for both the direction. Example: Resistor

The voltage generated by the source does not vary with any circuit quantity. It is only a function of time. Such a source is called an ideal voltage source.

The current generated by the source does not vary with any circuit quantity. It is only a function of time. Such a source is called an ideal current source.

Resistance: It is the property of a substance which opposes the flow of current through it. The resistance of an element is denoted by the symbol “R”. It is measured in Ohms.

$$R = P / I^2$$

INTRODUCTION TO ELECTRICAL CIRCUITS

Network theory is the study of solving the problems of electric circuits or electric networks. In this introductory chapter, let us first discuss the basic terminology of electric circuits and the types of network elements.

Basic Terminology

In Network Theory, we will frequently come across the following terms – Electric Circuit

Electric

Network Current

Voltage

Power

So, it is imperative that we gather some basic knowledge on these terms before proceeding further. Let's start with Electric Circuit.

Electric Circuit

An electric circuit contains a closed path for providing a flow of electrons from a voltage source or current source. The elements present in an electric circuit will be in series connection, parallel connection, or in any combination of series and parallel connections.

Electric Network

An electric network need not contain a closed path for providing a flow of electrons from a voltage source or current source. Hence, we can conclude that "all electric circuits are electric networks" but the converse need not be true.

Current

The current "I" flowing through a conductor is nothing but the time rate of flow of charge. Mathematically, it can be written as

$$I = d(Q)/dt$$

Where,

Q is the charge and its unit is Coulomb. t is the time and its unit is second.

As an analogy, electric current can be thought of as the flow of water through a pipe. Current is measured in terms of Ampere. In general, Electron current flows from negative terminal of source to positive terminal, whereas, Conventional current flows from positive terminal of source to negative terminal.

Electron current is obtained due to the movement of free electrons, whereas, Conventional current is obtained due to the movement of free positive charges. Both of these are called as electric current.

Voltage

Voltage "V" is nothing but an electromotive force that causes the charge (electrons) to flow. Mathematically, it can be written as

$$V =$$

$$d(W)/dQ$$

Where,

- W is the potential energy and its unit is Joule.

- QisthechargeanditsunitisColoumb.

As an analogy, Voltage can be thought of as the pressure of water that causes the water to flow through a pipe. It is measured in terms of Volt.

Power

The power "P" is nothing but the time rate of flow of electrical energy. Mathematically, it can be written as

$$P = d(W)/dt$$

Where,

W is the electrical energy and it is measured in terms of Joule.

t is the time and it is measured in seconds. We can re-

write the above equation as $P = d(W)/dt = d(W)/dQ * d(Q)/dt = VI$

Therefore, power is nothing but the product of voltage V and current I. Its unit is Watt.

Types of Network Elements

We can classify the Network elements into various types based on some parameters. Following are the types of Network elements—

- Active Elements and Passive Elements
- Linear Elements and Non-linear Elements
- Bilateral Elements and Unilateral Elements
- Lumped Elements and Distributed Elements

Active Elements and Passive Elements

We can classify the Network elements into either active or passive based on the ability of delivering power.

Active Elements deliver power to other elements, which are present in an electric circuit. Sometimes, they may absorb the power like passive elements. That means active elements have the capability of both delivering and absorbing power. Examples: Voltage sources and current sources.

Passive Elements can't deliver power (energy) to other elements, however they can absorb power. That means these elements either dissipate power in the form of heat or store energy in the form of either magnetic field or electric field. Examples: Resistors, Inductors, and capacitors.

Linear Elements and Non-Linear Elements

We can classify the network elements as linear or non-linear based on their characteristic to obey the property of linearity.

Linear Elements are the elements that show a linear relationship between voltage and current. Examples: Resistors, Inductors, and capacitors.

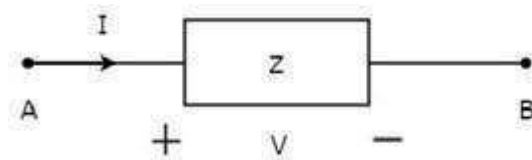
Non-Linear Elements are those that do not show a linear relation between voltage and current. Examples: Voltage sources and current sources.

Bilateral Elements and Unilateral Elements

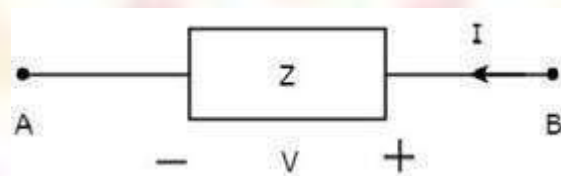
Network elements can also be classified as either bilateral or unilateral based on the direction of current flows through the network elements.

Bilateral Elements are the elements that allow the current in both directions and offer the same impedance in either direction of current flow. Examples: Resistors, Inductors and capacitors.

The concept of bilateral elements is illustrated in the following figures.



In the above figure, the current (I) is flowing from terminals A to B through a passive element having impedance of $Z\Omega$. It is the ratio of voltage (V) across that element between terminals A & B and current (I).



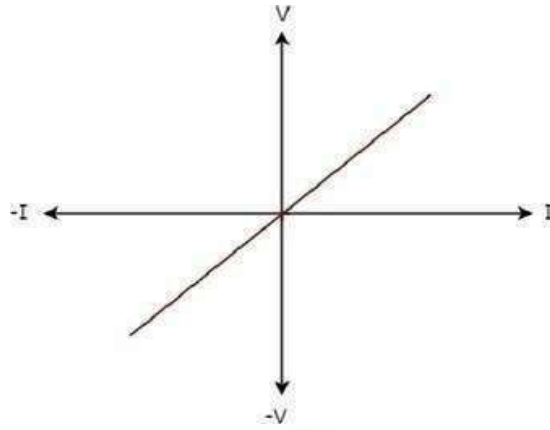
In the above figure, the current (I) is flowing from terminals B to A through a passive element having impedance of $Z\Omega$. That means the current ($-I$) is flowing from terminals A to B. In this case too, we will get the same impedance value, since both the current and voltage having negative signs with respect to terminals A & B.

Unilateral Elements are those that allow the current in only one direction. Hence, they offer different impedances in both directions.

We discussed the types of network elements in the previous chapter. Now, let us identify the nature of network elements from the V-I characteristics given in the following examples.

Example 1

The V-I characteristics of a network element is shown below.



Step1–Verifyingthenetworkelementaslinearornon-linear.

Fromtheabovefigure,theV-

Icharacteristicsofanetworkelementisastraightlinepassingthroughtheorigin.Hence,itislinearelement.

Step2–Verifyingthenetworkelementasactiveorpassive.

ThegivenV-Icharacteristicsofanetworkelementliesinthefirstandthirdquadrants.

Inthefirstquadrant,thevaluesofbothvoltage(V)andcurrent(I) arepositive.So,theratiosofvoltage (V)andcurrent(I)givespositiveimpedancevalues.

Similarly,inthethirdquadrant,thevaluesofbothvoltage(V)andcurrent(I)havenegativevalues.So,theratios ofvoltage(V)andcurrent(I)producepositiveimpedancevalues.

Since,thegivenV-Icharacteristicsofferpositiveimpedancevalues,thenetworkelementisaPassiveelement.

Step3–Verifyingthenetworkelementasbilateralorunilateral.

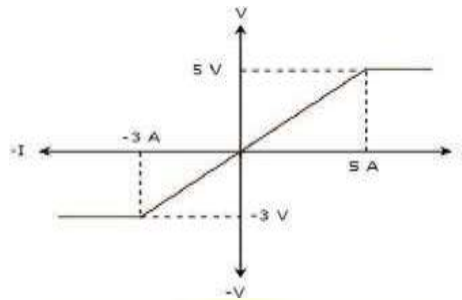
Foreverypoint(I,V)onthecharacteristics,thereexistsacorrespondingpoint(-I,- V)onthecharacteristics.Hence,thenetworkelementis aBilateralelement.

Therefore,thegivenV-

IcharacteristicsshowthatthenetworkelementisaLinear,Passive,andBilateralelement.

Example 2

The V-I characteristics of a network element is shown below.



Step 1—Verifying the network element as linear or non-linear.

From the above figure, the V-I characteristics of a network element is a straight line only between the points $(-3\text{A}, -3\text{V})$ and $(5\text{A}, 5\text{V})$. Beyond these points, the V-I characteristics are not following the linear relation. Hence, it is a Non-linear element.

Step 2—Verifying the network element as active or passive.

The given V-I characteristics of a network element lies in the first and third quadrants. In these two quadrants, the ratios of voltage (V) and current (I) produce positive impedance values. Hence, the network element is a Passive element.

Step 3—Verifying the network element as bilateral or unilateral.

Consider the point $(5\text{A}, 5\text{V})$ on the characteristics. The corresponding point $(-5\text{A}, -3\text{V})$ exists on the given characteristics instead of $(-5\text{A}, -5\text{V})$. Hence, the network element is a Unilateral element.

Therefore, the given V-I characteristics show that the network element is a Non-linear, Passive, and Unilateral element. The circuits containing them are called unilateral circuits.

Lumped and Distributed Elements

Lumped elements are those elements which are very small in size & in which simultaneous action takes place. Typically lumped elements are capacitors, resistors, inductors.

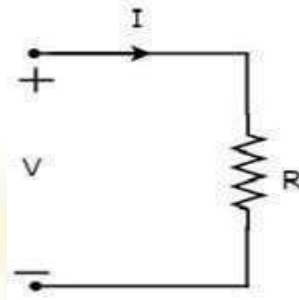
Distributed elements are those which are not electrically separable for analytical purposes.

For example, a transmission line has distributed parameters along its length and may extend for hundreds of miles.

R-L-C Parameters Resistor

The main functionality of Resistor is either opposes or restricts the flow of electric current.

Hence, the resistors are used in order to limit the amount of current flow and / or dividing (sharing) voltage. Let the current flowing through the resistor is I amperes and the voltage across it is V volts. The symbol of resistor along with current, I and voltage, V are shown in the following figure.



According to Ohm's law, the voltage across resistor is the product of current flowing through it and the resistance of that resistor. Mathematically, it can be represented as

$$V = IR \quad \text{Equation 1}$$

$$\Rightarrow I = \frac{V}{R} \quad \text{Equation 2}$$

Where, R is the resistance of a resistor.

From Equation 2, we can conclude that the current flowing through the resistor is directly proportional to the applied voltage across resistor and inversely proportional to the resistance of resistor.

Power in an electric circuit element can be represented as

$$P = VI \quad \text{Equation 3}$$

Substitute, Equation 1 in Equation 3.

$$\begin{aligned} P &= (IR)I \\ \Rightarrow P &= I^2 R \end{aligned} \quad \text{Equation 4}$$

Substitute, Equation 2 in Equation 3.

$$\begin{aligned} P &= V\left(\frac{V}{R}\right) \\ \Rightarrow P &= \frac{V^2}{R} \end{aligned} \quad \text{Equation 5}$$

So, we can calculate the amount of power dissipated in the resistor by using one of the formulae mentioned in Equations 3 to 5.

Inductor

In general, inductors will have number of turns. Hence, they produce magnetic flux when current flows through it. So, the amount of total magnetic flux produced by an inductor depends on the current, if flowing through it and they have a linear relationship.

Mathematically, it can be written as

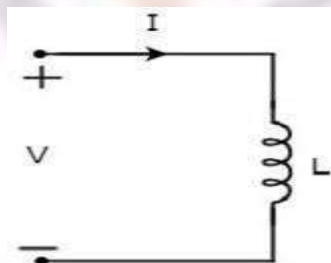
$$\Psi \propto I$$

$$\Rightarrow \Psi = LI$$

Where,

- Ψ is the total magnetic flux
- L is the inductance of an inductor

Let the current flowing through the inductor is I amperes and the voltage across it is V volts. The symbol of inductor along with current I and voltage V are shown in the following figure.



According to Faraday's law, the voltage across the inductor can be written as

$$V = \frac{d\Psi}{dt}$$

Substitute $\Psi = LI$ in the above equation.

$$V = \frac{d(LI)}{dt}$$

$$\Rightarrow V = L \frac{dI}{dt}$$

$$\Rightarrow I = \frac{1}{L} \int V dt$$

From the above equations, we can conclude that there exists a linear relationship between voltage across inductor and current flowing through it.

We know that power in an electric circuit element can be represented as

$$P = VI$$

Substitute $V = L \frac{dI}{dt}$ in the above equation.

$$P = (L \frac{dI}{dt}) I$$

$$\Rightarrow P = LI \frac{dI}{dt}$$

By integrating the above equation, we will get the energy stored in an inductor as

$$W = \frac{1}{2} LI^2$$

So, the inductor stores the energy in the form of magnetic field.

Capacitor

In general, a capacitor has two conducting plates, separated by a dielectric medium. If positive voltage is applied across the capacitor, then it stores positive charge. Similarly, if negative voltage is applied across the capacitor, then it stores negative charge.

So, the amount of charge stored in the capacitor depends on the applied voltage V across it and they

$$Q \propto V$$

$$\Rightarrow Q = CV$$

have linear relationship. Mathematically, it can be written as Where,

- Q is the charge stored in the capacitor.
- C is the capacitance of a capacitor.

Let the current flowing through the capacitor is I amperes and the voltage across it is V volts. The symbol of capacitor along with current I and voltage V are shown in the following figure.

We know that the **current** is nothing but the **time rate of flow of charge**. Mathematically, it can be represented as

I

$$I = \frac{dQ}{dt}$$

Substitute $Q = CV$ in the above equation.

$$I = \frac{d(CV)}{dt}$$

$$\Rightarrow I = C \frac{dV}{dt}$$

$$\Rightarrow V = \frac{1}{C} \int I dt$$

From the above equations, we can conclude that there exists a linear relationship between voltage across capacitor and current flowing through it.

We know that power in an electric circuit element can be represented as

$$P = VI$$

Substitute $I = C \frac{dV}{dt}$ in the above equation.

$$P = V \left(C \frac{dV}{dt} \right)$$

$$\Rightarrow P = CV \frac{dV}{dt}$$

By integrating the above equation, we will get the energy stored in the capacitor as

$$W = \frac{1}{2} CV^2$$

So, the capacitor stores the energy in the form of electric field.

Types of Sources

Active Elements are the network elements that deliver power to other elements present in an electric circuit. So, active elements are also called

as sources of voltage or current type. We can classify these sources into the following two categories—

- IndependentSources

- Dependent Sources

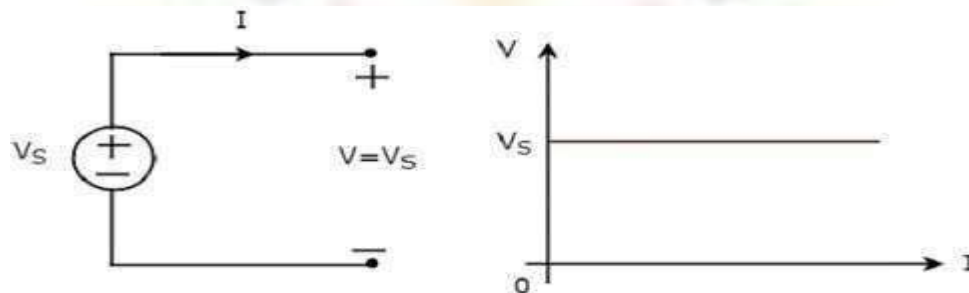
Independent Sources

As the name suggests, independent sources produce fixed values of voltage or current and these are not dependent on any other parameter. Independent sources can be further divided into the following two categories—

- Independent Voltage Sources
- Independent Current Sources

Independent Voltage Sources

An independent voltage source produces a constant voltage across its two terminals. This voltage is independent of the amount of current that is flowing through the two terminals of voltage source.



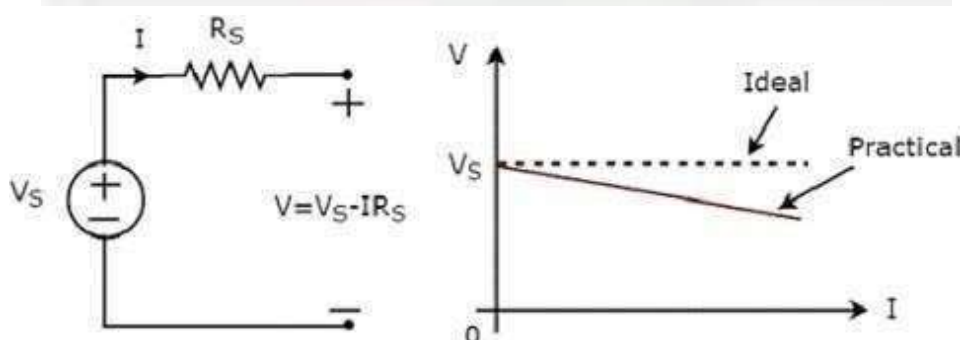
Independent ideal voltage source and its V-I characteristics are shown in the following figure.

The V-I characteristics of an independent ideal voltage source is a constant line, which is always equal to the source voltage (V_S) irrespective of the current value (I). So, the internal resistance of an independent ideal voltage source is zero Ohms.

Hence, the independent ideal voltage sources do not exist practically, because there will be some internal resistance.

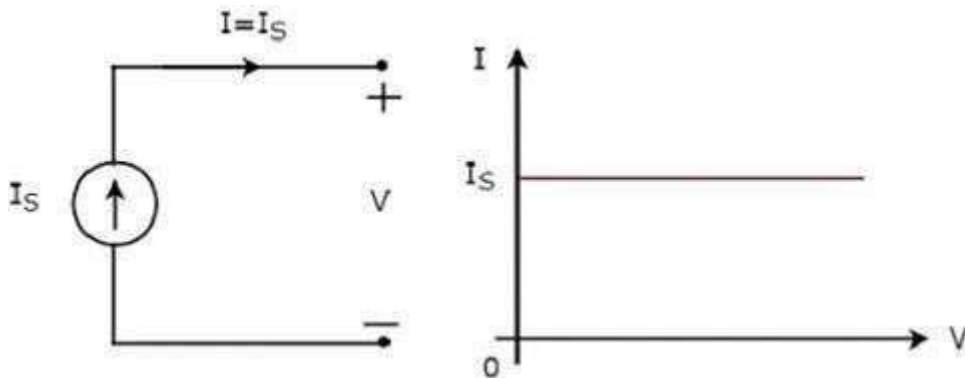
Independent practical voltage source and its V-I characteristics are shown in the following figure.

There is a deviation in the V-I characteristics of an independent practical voltage source from the V-I characteristics of an independent ideal voltage source. This is due to the voltage drop across the internal resistance (R_S) of an independent practical voltage source.



Independent Current Sources

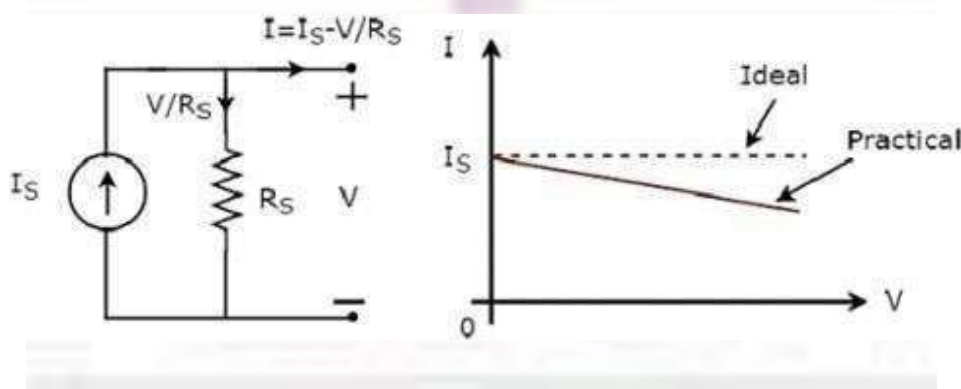
An independent current source produces a constant current. This current is independent of the voltage across its two terminals. Independent ideal current source and its V-I characteristics are shown in the following figure.



The V-I characteristics of an independent ideal current source is a constant line, which is always equal to the source current (I_S) irrespective of the voltage value (V). So, the internal resistance of an independent ideal current source is infinite ohms.

Hence, the independent ideal current sources do not exist practically, because there will be some internal resistance.

Independent practical current source and its V-I characteristics are shown in the following figure.



There is a deviation in the V-I characteristics of an independent practical current source from the V-I characteristics of an independent ideal current source. This is due to the amount of current flow through the internal shunt resistance (R_S) of an independent practical current source.

Dependent Sources

As the name suggests, dependent sources produce the amount of voltage or current that is dependent on some other voltage or current. Dependent sources are also called as controlled sources. Dependent sources can be further divided into the following two categories—

- Dependent Voltage Sources
- Dependent Current Sources

Dependent Voltage Sources

A dependent voltage source produces a voltage across its two terminals. The amount of this voltage is dependent on some other voltage or current. Hence, dependent voltage sources can be further classified into the following two categories—

- Voltage Dependent Voltage Source (VDVS)
- Current Dependent Voltage Source (CDVS)

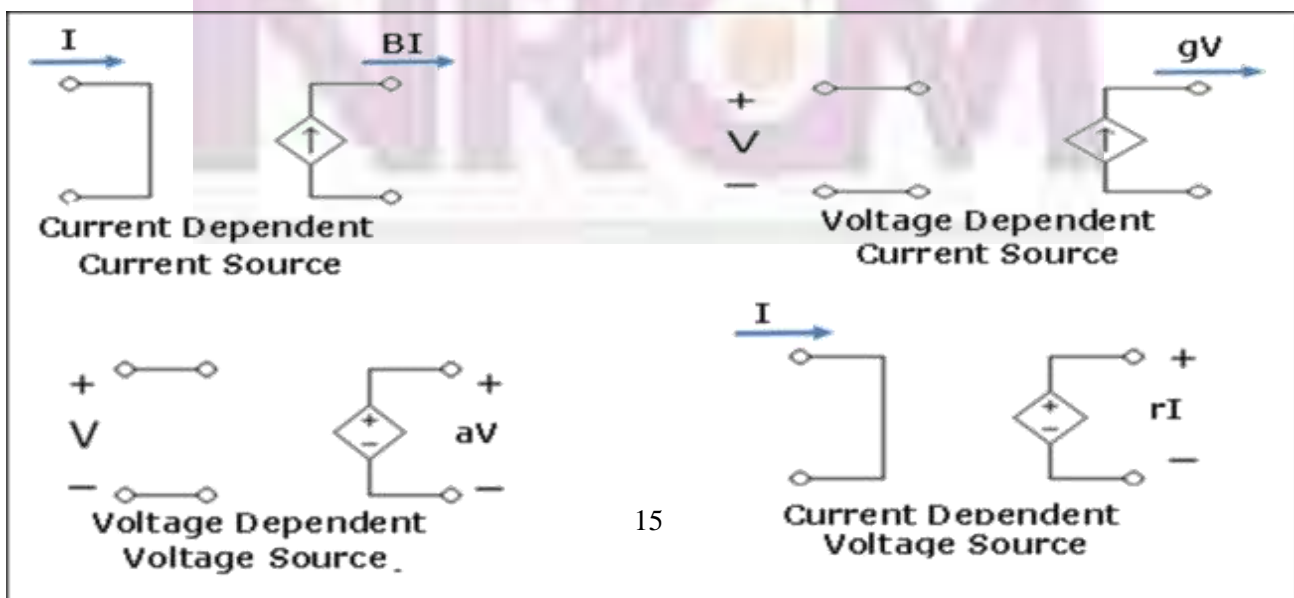
Dependent voltage sources are represented with the signs '+' and '-' inside a diamond shape. The magnitude of the voltage source can be represented outside the diamond shape.

Dependent Current Sources

A dependent current source produces a current. The amount of this current is dependent on some other voltage or current. Hence, dependent current sources can be further classified into the following two categories

- Voltage Dependent Current Source (VDCS)
- Current Dependent Current Source (CDCS)

Dependent current sources are represented with an arrow inside a diamond shape. The magnitude of the current source can be represented outside the diamond shape. We can observe these dependent or controlled sources in equivalent models of transistors.



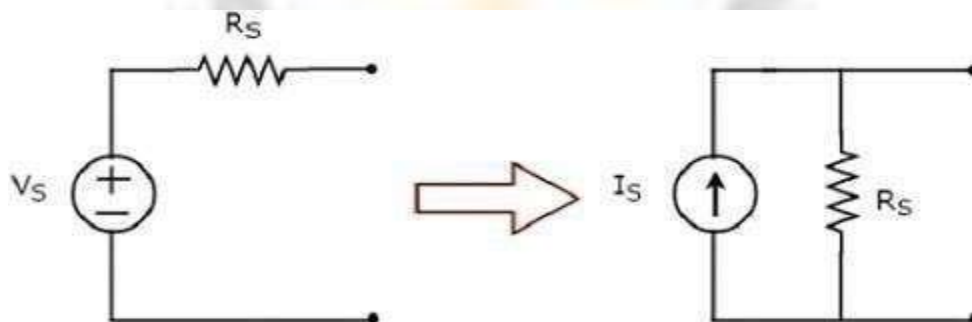
Source Transformation Technique

We know that there are two practical sources, namely, voltage source and current source. We can transform (convert) one source into the other based on the requirement, while solving network problems. The technique of transforming one source into the other is called as source transformation technique. Following are the two possible source transformations—

- Practical voltage source into a practical current source
- Practical current source into a practical voltage source

Practical voltage source into a practical current source

The transformation of practical voltage source into a practical current source is shown in the following figure



Practical voltage source consists of a voltage source (V_S) in series with a resistor (R_S). This can be converted into a practical current source as shown in the figure. It consists of a current source (I_S) in parallel with a resistor (R_S).

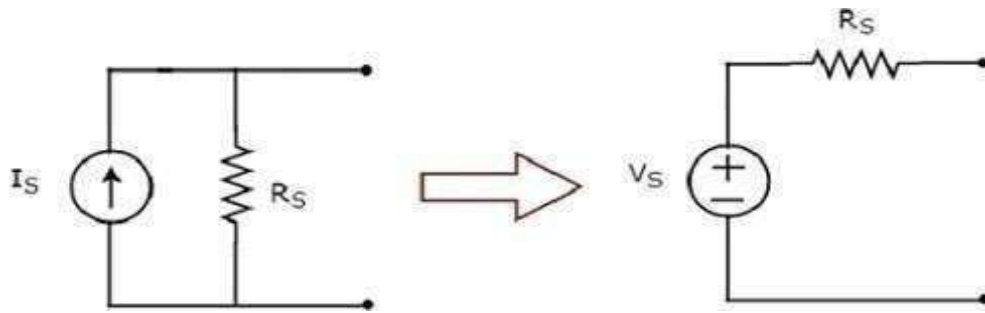
The value of I_S will be equal to the ratio of V_S and R_S . Mathematically, it can be represented as

$$I_S = \frac{V_S}{R_S}$$

Practical current source into a practical voltage source

The transformation of practical current source into a practical voltage source is shown in the following figure.

Practical current source consists of a current source (I_S) in parallel with a resistor (R_S). This can be converted into a practical voltage source as shown in the figure. It consists of a voltage source (V_S) in series with a resistor (R_S).



The value of V_S will be equal to the product of I_S and R_S . Mathematically, it can be represented as

$$V_S = I_S R_S$$

In this chapter, we will discuss in detail about the passive elements such as Resistor, Inductor, and Capacitor. Let us start with Resistors.

Kirchhoff's Laws

Network elements can be either of active or passive type. Any electrical circuit or network contains one of these two types of network elements or a combination of both.

Now, let us discuss about the following two laws, which are popularly known as Kirchhoff's laws.

- Kirchhoff's Current Law
- Kirchhoff's Voltage Law

Kirchhoff's Current Law

Kirchhoff's Current Law (KCL) states that the algebraic sum of currents leaving (or entering) a node is equal to zero.

A Node is a point where two or more circuit elements are connected to it. If only two circuit elements are connected to a node, then it is said to be a simple node. If three or more circuit elements are connected to a node, then it is said to be a Principal Node.

Mathematically, KCL can be represented as

$$\sum_{m=1}^M I_m = 0$$

Where,

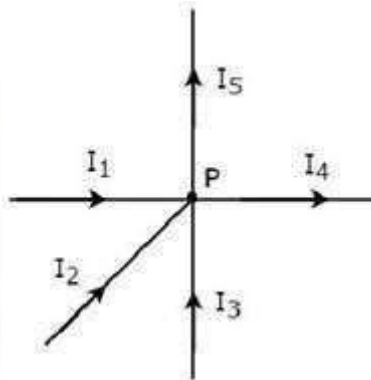
- I_m is the m^{th} branch current leaving the node.

- **M**isthenumberofbranchesthatareconnectedtoanode.

The above statement of KCL can also be expressed as "the algebraic sum of currents entering a node is equal to the algebraic sum of currents leaving a node". Let us verify this statement through the following example.

Example

Write KCL equation at node P of the following figure.



In the above figure, the branch currents I_1 , I_2 and I_3 are entering at node P. So, consider negative signs for these three currents.

In the above figure, the branch currents I_4 and I_5 are leaving from node P. So, consider positive signs for these two currents.

The KCL equation at node P will be

$$-I_1 - I_2 - I_3 + I_4 + I_5 = 0$$

$$\Rightarrow I_1 + I_2 + I_3 = I_4 + I_5$$

In the above equation, the left-hand side represents the sum of entering currents, whereas the right-hand side represents the sum of leaving currents.

In this tutorial, we will consider positive sign when the current leaves a node and negative sign when it enters a node. Similarly, you can consider negative sign when the current leaves a node and positive sign when it enters a node. In both cases, the result will be the same.

Note—KCL is independent of the nature of network elements that are connected to a node.

Kirchhoff's Voltage Law

Kirchhoff's Voltage Law (KVL) states that the algebraic sum of voltages around a loop or mesh is equal to zero.

A Loop is a path that terminates at the same node where it started from. In contrast, a Mesh is a loop that doesn't contain any other loops inside it.

Mathematically, KVL can be represented as

$$\sum_{n=1}^N V_n = 0$$

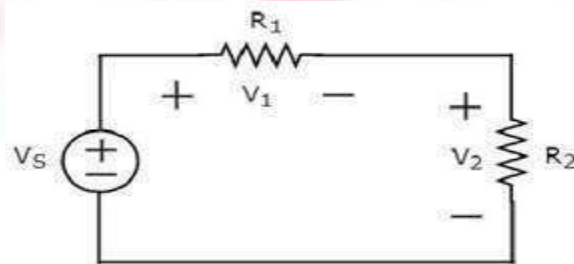
Where,

- V_n is the n^{th} element's voltage in a loop (mesh).
- N is the number of network elements in the loop (mesh).

The above statement of KVL can also be expressed as "the algebraic sum of voltage sources is equal to the algebraic sum of voltage drops that are present in a loop." Let us verify this statement with the help of the following example.

Example

Write KVL equation around the loop of the following circuit.



The above circuit diagram consists of a voltage source, V_S in series with two resistors R_1 and R_2 . The voltage drops across the resistors R_1 and R_2 are V_1 and V_2 respectively.

Apply KVL around the loop.

$$V_S - V_1 - V_2 = 0$$

$$\Rightarrow V_S = V_1 + V_2$$

In the above equation, the left-hand side term represents single voltage source V_S . Whereas, the right-hand side represents the sum of voltage drops. In this example, we considered only one voltage source. That's why the left-hand side contains only one term. If we consider multiple voltage sources, then the left side contains sum of voltages.

agesources.

In this tutorial, we consider the sign of each element's voltage as the polarity of the second terminal that is present while travelling around the loop. Similarly, you can consider the sign of each voltage as the polarity of the first terminal that is present while travelling around the loop. In both cases, the result will be the same.

Note—KVL is independent of the nature of network elements that are present in a loop.

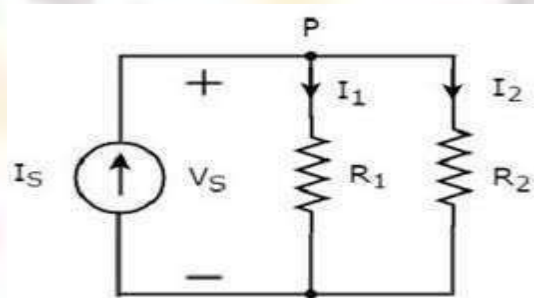
In this chapter, let us discuss about the following two division principles of electrical quantities.

- Current Division Principle
- Voltage Division Principle

Current Division Principle

When two or more passive elements are connected in parallel, the amount of current that flows through each element gets divided (shared) among themselves from the current that is entering the node.

Consider the following circuit diagram.



The above circuit diagram consists of an input current source I_S in parallel with two resistors R_1 and R_2 . The voltage across each element is V_S . The currents flowing through the resistors R_1 and R_2 are I_1 and I_2 respectively.

The KCL equation at node P will be



$$I_S = I_1 + I_2$$

- Substitute $I_1 = \frac{V_S}{R_1}$ and $I_2 = \frac{V_S}{R_2}$ in the above equation.

$$I_S = \frac{V_S}{R_1} + \frac{V_S}{R_2} = V_S \left(\frac{R_2 + R_1}{R_1 R_2} \right)$$

$$\Rightarrow V_S = I_S \left(\frac{R_1 R_2}{R_1 + R_2} \right)$$

- Substitute the value of V_S in $I_1 = \frac{V_S}{R_1}$.

$$I_1 = \frac{I_S}{R_1} \left(\frac{R_1 R_2}{R_1 + R_2} \right)$$

$$\Rightarrow I_1 = I_S \left(\frac{R_2}{R_1 + R_2} \right)$$

- Substitute the value of V_S in $I_2 = \frac{V_S}{R_2}$.

$$I_2 = \frac{I_S}{R_2} \left(\frac{R_1 R_2}{R_1 + R_2} \right)$$

$$\Rightarrow I_2 = I_S \left(\frac{R_1}{R_1 + R_2} \right)$$

From equations of I_1 and I_2 , we can generalize that the current flowing through any passive element can be found by using the following formula.

$$I_N = I_S \left(\frac{Z_1 || Z_2 || \dots || Z_{N-1}}{Z_1 + Z_2 + \dots + Z_N} \right)$$

This is known as current division principle and it is applicable, when two or more passive elements are connected in parallel and only one current enters the node.

Where,

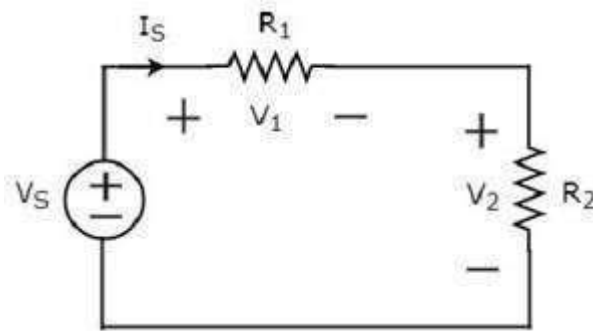
- I_N is the current flowing through the passive element of N^{th} branch.
- I_S is the input current, which enters the node.

Z_1, Z_2, \dots, Z_N are the impedances of 1st branch, 2nd branch, ..., N^{th} branch respectively.

Voltage Division Principle

When two or more passive elements are connected in series, the amount of voltage present across each element gets divided (shared) among themselves from the voltage that is available across that entire combination.

Consider the following circuit diagram.



The above circuit diagram consists of a voltage source, V_S in series with two resistors R_1 and R_2 . The current flowing through these elements is I_S . The voltage drops across the resistors R_1 and R_2 are V_1 and V_2 respectively.

The KVL equation around the loop will be

$$V_S = V_1 + V_2$$

Substitute $V_1 = I_S R_1$ and $V_2 = I_S R_2$ in the above equation

$$V_S = I_S R_1 + I_S R_2 = I_S (R_1 + R_2)$$

$$I_S = \frac{V_S}{R_1 + R_2}$$

Substitute the value of I_S in $V_1 = I_S R_1$.

$$V_1 = \left(\frac{V_S}{R_1 + R_2} \right) R_1$$

$$\Rightarrow V_1 = V_S \left(\frac{R_1}{R_1 + R_2} \right)$$

Substitute the value of I_S in $V_2 = I_S R_2$.

$$V_2 = \left(\frac{V_S}{R_1 + R_2} \right) R_2$$

$$\Rightarrow V_2 = V_S \left(\frac{R_2}{R_1 + R_2} \right)$$

From equations of V_1 and V_2 , we can generalize that the voltage across any passive element can be found by using the following formula.

$$V_N = V_S \left(\frac{Z_N}{Z_1 + Z_2 + \dots + Z_N} \right)$$

This is known as voltage division principle and it is applicable, when two or more passive elements are connected in series and only one voltage is available across the entire combination.

Where,

- V_N is the voltage across N^{th} passive element.
- V_S is the input voltage, which is present across the entire combination of series passive elements.
- Z_1, Z_2, \dots, Z_N are the impedances of 1st passive element, 2nd passive element, ..., N^{th} passive element respectively.

Network Reduction Techniques:

There are two basic methods that are used for solving any electrical network: Nodal analysis and Mesh analysis. In this chapter, let us discuss about the Mesh analysis method.

Series and parallel connections of resistive networks:

If a circuit consists of two or more similar passive elements and are connected in exclusively of series type or parallel type, then we can replace them with a single equivalent passive element. Hence, this circuit is called as an equivalent circuit.

In this chapter, let us discuss about the following two equivalent circuits.

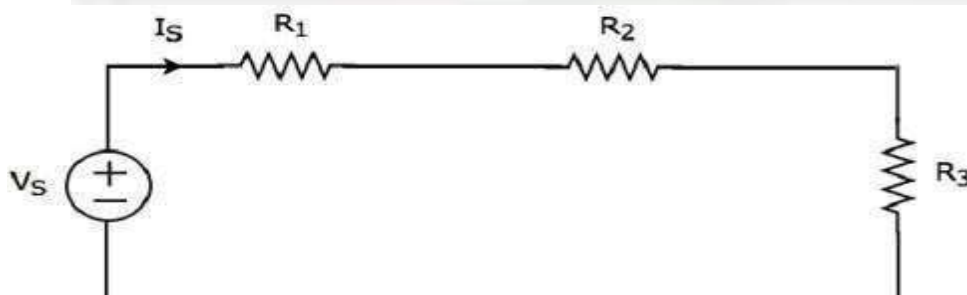
- Series Equivalent Circuit
- Parallel Equivalent Circuit

Series Equivalent Circuit

If similar passive elements are reconnected in series, then the same current will flow through all these elements. But, the voltage gets divided across each element.

Consider the following circuit diagram.

It has a single voltage source (V_S) and three resistors having resistances of R_1, R_2 and R_3 . All these



elements are connected in series. The current I_S flows through all these elements. The above circuit has only one mesh. The KVL equation around this mesh is

$$V_S = V_1 + V_2 + V_3$$

Substitute $V_1 = I_S R_1$, $V_2 = I_S R_2$ and $V_3 = I_S R_3$ in the above equation.

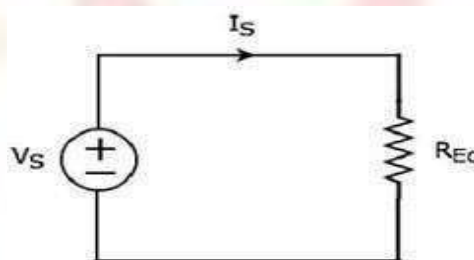
$$\begin{aligned} V_S &= I_S R_1 + I_S R_2 + I_S R_3 \\ \Rightarrow V_S &= I_S (R_1 + R_2 + R_3) \end{aligned}$$

The above equation is in the form of $V_S = I_S R_{Eq}$ where,

$$R_{Eq} = R_1 + R_2 + R_3$$

The equivalent circuit diagram of the given circuit is shown in the following figure.

That means, if multiple resistors are connected in series, then we can replace them with an equivalent resistor.



The resistance of this equivalent resistor is equal to the sum of the resistances of all those multiple resistors. **Note 1** – If ‘N’ inductors having inductances of L_1, L_2, \dots, L_N are connected in series, then the equivalent inductance will be

$$L_{Eq} = L_1 + L_2 + \dots + L_N$$

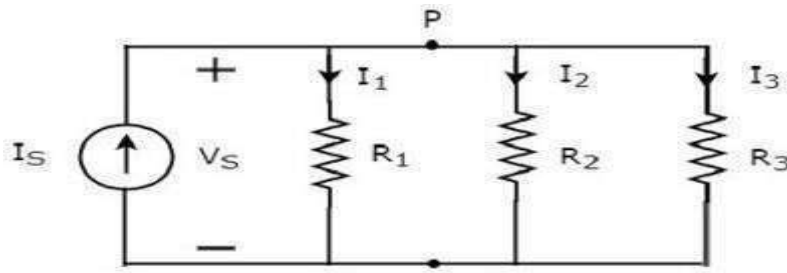
Note 2 – If ‘N’ capacitors having capacitances of C_1, C_2, \dots, C_N are connected in series, then the equivalent capacitance will be

$$\frac{1}{C_{Eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}$$

Parallel Equivalent Circuit

If similar passive elements are connected in parallel, then the same voltage will be maintained across each element. But, the current flowing through each element gets divided.

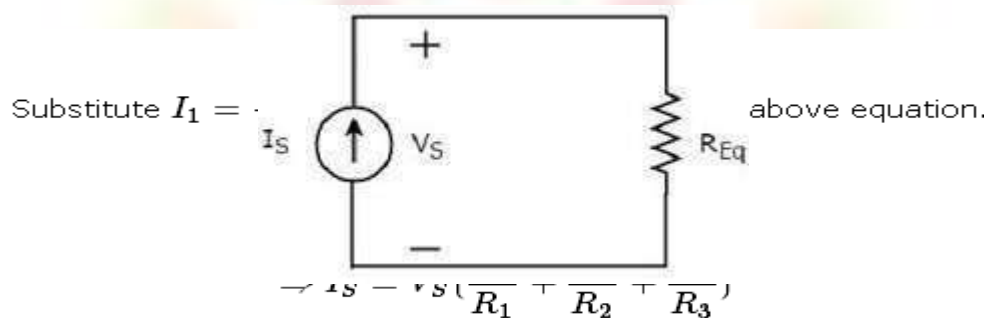
Consider the following circuit diagram.



It has a single current source (I_S) and three resistors having resistances of R_1 , R_2 , and R_3 . All these elements are connected in parallel. The voltage (V_S) is available across all these elements.

The above circuit has only one principal node (P) except the Ground node. The KCL equation at this principal node (P) is

The equivalent circuit diagram of the given circuit is shown in the following figure.



$$\Rightarrow V_S = I_S \left[\frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)} \right]$$

The above equation is in the form of $V_S = I_S R_{Eq}$ where,

$$R_{Eq} = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)}$$

$$\frac{1}{R_{Eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

That means, if multiple resistors are connected in parallel, then we can replace them with an equivalent resistor. The resistance of this equivalent resistor is equal to the reciprocal of the sum of the reciprocal of each resistance of all those multiple resistors.

Note 1—If N inductors having inductances of L_1, L_2, \dots, L_N are connected in parallel, then the

$$\frac{1}{L_{Eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_N}$$

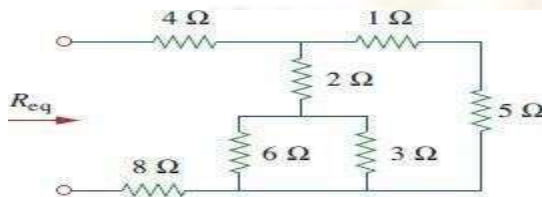
equivalent inductance will be

Note 2–If 'N' capacitors having capacitances of C_1, C_2, \dots, C_N are connected in parallel, then the equivalent capacitance will be

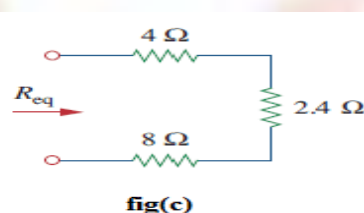
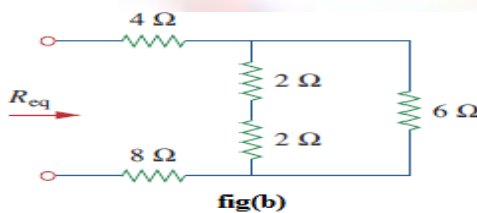
$$C_{Eq} = C_1 + C_2 + \dots + C_N$$

Example Problems:

Find the R_{eq} for the circuits shown in below figure.



Solution:



To get R_{eq} we combine

resistors in series and in parallel. The 6 ohms and 3 ohms resistors are in parallel, so their equivalent resistance is

$$6 \Omega \parallel 3 \Omega = \frac{6 \times 3}{6 + 3} = 2 \Omega$$

Also, the 1 ohm and 5 ohms resistors are in series; hence their equivalent resistance is

$$1 \Omega + 5 \Omega = 6 \Omega$$

Thus the circuit in Fig.(b) is reduced to that in Fig.(c). In Fig.(b), we notice that the two 2 ohms resistors are in series, so the equivalent resistance is

$$2 \Omega + 2 \Omega = 4 \Omega$$

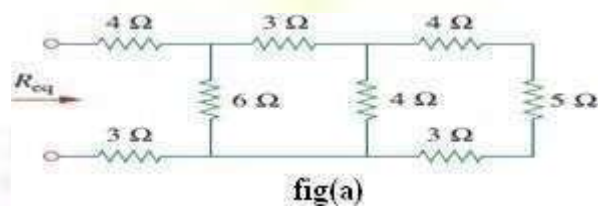
This 4 ohms resistor is now in parallel with the 6 ohms resistor in Fig.(b); their equivalent resistance is

$$4\ \Omega \parallel 6\ \Omega = \frac{4 \times 6}{4 + 6} = 2.4\ \Omega$$

The circuit in Fig.(b) is now replaced with that in Fig.(c). In Fig.(c), the three resistors are in series. Hence, the equivalent resistance for the circuit is

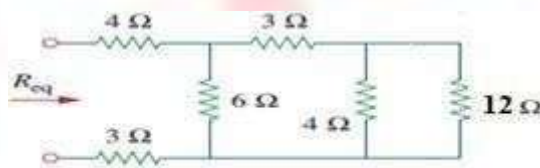
$$R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega$$

Find the R_{eq} for the circuit shown in below figure.

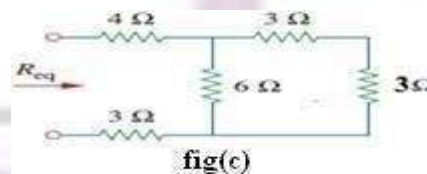


Solution:

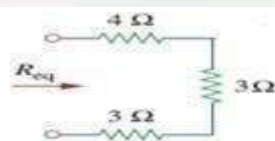
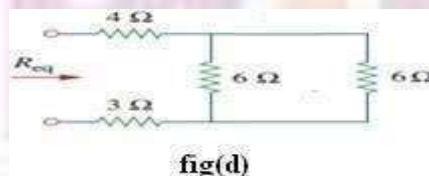
In the given network 4ohms, 5ohms and 3ohms come in series then the equivalent resistance is $4+5+3=12$ ohms



From fig(b), 4ohms and 12ohms are in parallel, equivalent is 3ohms



From fig(c), 3ohms and 3ohms are in series, equivalent resistance is 6ohms



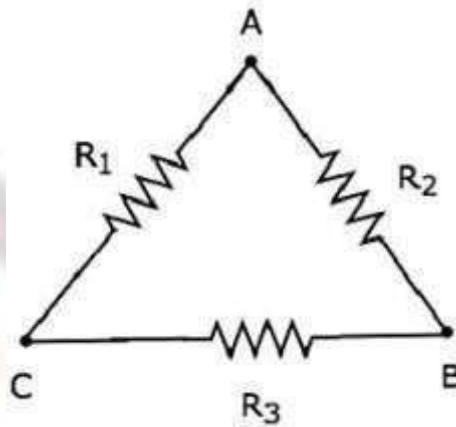
From fig(d), 6ohms and 6ohms are in parallel, equivalent resistance is 3ohms

From fig(e), 4ohms, 3ohms and 3ohms are in series. Hence $R_{eq} = 4 + 3 + 3 = 10\text{ohms}$.

Star-to-Delta and Delta-to-Star Transformations for Resistive Networks: Delta to Star Transformation

Delta Network

Consider the following delta network as shown in the following figure.

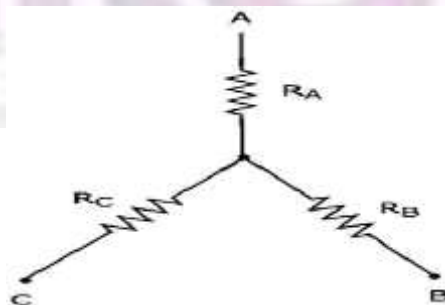


The following equations represent the equivalent resistance between two terminals of a delta network, when the third terminal is kept open.

$$R_{AB} = \frac{(R_1 + R_3)R_2}{R_1 + R_2 + R_3}$$

$$R_{BC} = \frac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3}$$

$$R_{CA} = \frac{(R_2 + R_3)R_1}{R_1 + R_2 + R_3}$$



The following figure shows the equivalent star network corresponding to the above delta

The following equations represent the equivalent resistance between two terminals of a star network, when the third terminal is kept open.

$$R_{AB} = R_A + R_B$$

$$R_{BC} = R_B + R_C$$

$$R_{CA} = R_C + R_A$$

Star Network Resistances in terms of Delta Network Resistances

We will get the following equations by equating the right-

hand side terms of the above equations for which the left-hand side terms are the same.

$$R_A + R_B = \frac{(R_1 + R_3)R_2}{R_1 + R_2 + R_3} \quad \text{Equation 1}$$

$$R_B + R_C = \frac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3} \quad \text{Equation 2}$$

$$R_C + R_A = \frac{(R_2 + R_3)R_1}{R_1 + R_2 + R_3} \quad \text{Equation 3}$$

By adding the above three equations, we will get

$$\begin{aligned} 2(R_A + R_B + R_C) &= \frac{2(R_1R_2 + R_2R_3 + R_3R_1)}{R_1 + R_2 + R_3} \\ \Rightarrow R_A + R_B + R_C &= \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1 + R_2 + R_3} \quad \text{Equation 4} \end{aligned}$$

Subtract Equation 2 from Equation 4.

$$R_A + R_B + R_C - (R_B + R_C) = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1 + R_2 + R_3} - \frac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3}$$

$$R_A = \frac{R_1R_2}{R_1 + R_2 + R_3}$$

By subtracting Equation 3 from Equation 4, we will get

$$R_B = \frac{R_2R_3}{R_1 + R_2 + R_3}$$

By subtracting Equation 1 from Equation 4, we will get

$$R_C = \frac{R_3R_1}{R_1 + R_2 + R_3}$$

By using the above relations, we can find the resistances of star network from the resistances of deltanetwork. In this way, we can convert a delta network into a star network.

Star to Delta Transformation

In the previous chapter, we discussed about the conversion of delta network into an equivalent star network. Now, let us discuss about the conversion of star network into an equivalent delta network. This conversion is called as Star to Delta Conversion.

In the previous chapter, we got the resistances of star network from delta network as

$$R_A = \frac{R_1 R_2}{R_1 + R_2 + R_3} \quad \text{Equation 1}$$

$$R_B = \frac{R_2 R_3}{R_1 + R_2 + R_3} \quad \text{Equation 2}$$

$$R_C = \frac{R_3 R_1}{R_1 + R_2 + R_3} \quad \text{Equation 3}$$

Delta Network Resistances in terms of Star Network Resistances

Let us manipulate the above equations in order to get the resistances of delta network in terms of resistances of star network.

- Multiply each set of two equations and then add.

By using the above relations, we can find the resistances of delta network from the resistances of star network.

In this way, we can convert star network into delta network.

$$\begin{aligned} R_A R_B + R_B R_C + R_C R_A &= \frac{R_1 R_2^2 R_3 + R_2 R_3^2 R_1 + R_3 R_1^2 R_2}{(R_1 + R_2 + R_3)^2} \\ \Rightarrow R_A R_B + R_B R_C + R_C R_A &= \frac{R_1 R_2 R_3 (R_1 + R_2 + R_3)}{(R_1 + R_2 + R_3)^2} \\ \Rightarrow R_A R_B + R_B R_C + R_C R_A &= \frac{R_1 R_2 R_3}{R_1 + R_2 + R_3} \end{aligned} \quad \text{Equation 4}$$

- By dividing Equation 4 with Equation 2, we will get

$$\begin{aligned} \frac{R_A R_B + R_B R_C + R_C R_A}{R_B} &= R_1 \\ \Rightarrow R_1 &= R_C + R_A + \frac{R_C R_A}{R_B} \end{aligned}$$

- By dividing Equation 4 with Equation 3, we will get

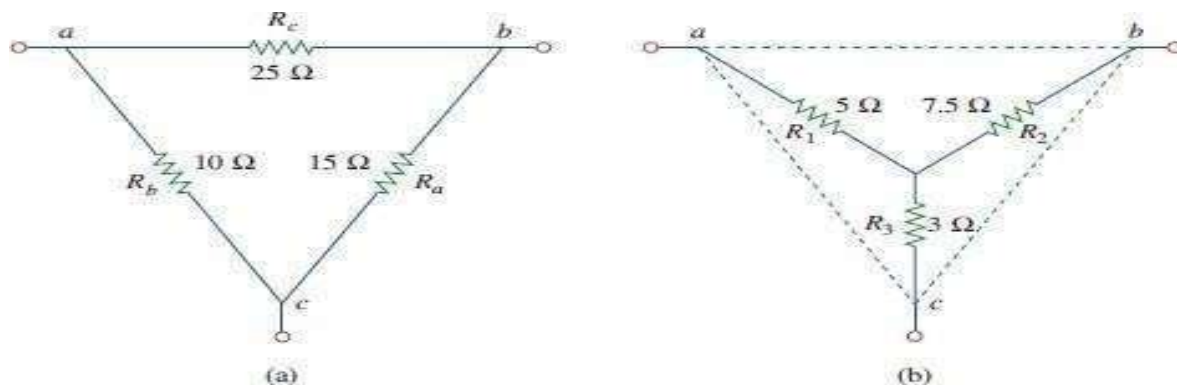
$$R_2 = R_A + R_B + \frac{R_A R_B}{R_C}$$

- By dividing Equation 4 with Equation 1, we will get

$$R_3 = R_B + R_C + \frac{R_B R_C}{R_A}$$

Example problems:

1) Convert the Delta network in Fig. (a) to an equivalent star network Solution:



1) Convert the star network in fig(a) to delta network



Solution: The equivalent delta for the given star is shown in fig(b), where

$$R_{12} = 1.67 + 5 + \frac{1.67 \times 5}{2.5} = 1.67 + 5 + 3.33 = 10\ \Omega$$

$$R_{23} = 5 + 2.5 + \frac{5 \times 2.5}{1.67} = 5 + 2.5 + 7.5 = 15\ \Omega$$

$$R_{31} = 2.5 + 1.67 + \frac{2.5 \times 1.67}{5} = 2.5 + 1.67 + 0.833 = 5\ \Omega$$

Mesh Analysis:

Mesh analysis provides general procedure for analyzing circuits using mesh currents as the circuit variables. Mesh Analysis is applicable only for planar networks. It is preferably useful for the circuits that have many loops. This analysis is done by using KVL and Ohm's law.

In Mesh analysis, we will consider the currents flowing through each mesh. Hence, Mesh analysis is also called as Mesh-current method.

A branch is a path that joins two nodes and it contains a circuit element. If a branch belongs to only one mesh, then the branch current will be equal to mesh current.

If a branch is common to two meshes, then the branch current will be equal to the sum (or difference) of two mesh currents, when they are in same (or opposite) direction.

Procedure of Mesh Analysis

Follow these steps while solving any electrical network or circuit using Mesh analysis.

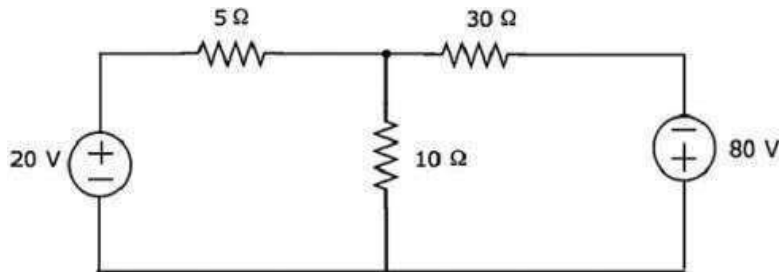
- **Step1**—Identify the meshes and label the mesh currents in either clockwise or anti-clockwise direction.
- **Step2**—Observe the amount of current that flows through each element in terms of mesh currents.
- **Step3**—Write mesh equations to all meshes. Mesh equation is obtained by applying KVL first and then Ohm's law.
- **Step4**—Solve the mesh equations obtained in Step3 in order to get the mesh currents.

Now, we can find the current flowing through any element and the voltage across any element that is present in the given network by using mesh currents.



Example

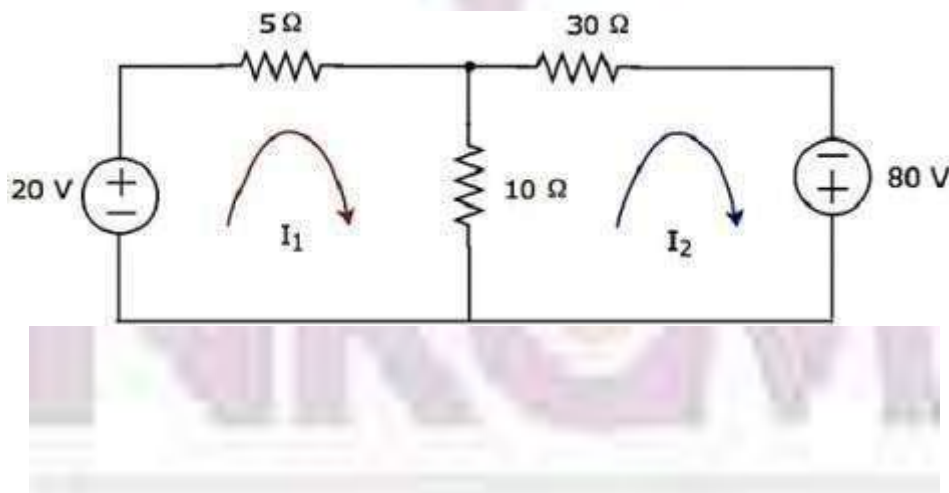
Find the voltage across 30Ω resistor using Mesh analysis.



Step 1—There are two meshes in the above circuit. The mesh currents I_1 and I_2 are considered in clockwise direction. These mesh currents are shown in the following figure.

Step 2—The mesh current I_1 flows through 20V voltage source and 5Ω resistor. Similarly, the mesh current I_2 flows through 30Ω resistor and -80 V voltage source. But, the difference of two mesh currents, I_1 and I_2 , flows through 10Ω resistor, since it is the common branch of two meshes.

Step 3 – In this case, we will get two mesh equations since there are two meshes in the given circuit. When we write the mesh equations, assume the mesh current of that particular mesh as greater than all other mesh currents of the circuit. The mesh equation of first mesh is



$$20 - 5I_1 - 10(I_1 - I_2) = 0$$

$$\Rightarrow 20 - 15I_1 + 10I_2 = 0$$

$$\Rightarrow 10I_2 = 15I_1 - 20$$

Divide the above equation with 5.

$$2I_2 = 3I_1 - 4$$

Multiply the above equation with 2.

$$4I_2 = 6I_1 - 8 \quad \text{Equation 1}$$

The **mesh equation** of second mesh is

$$-10(I_2 - I_1) - 30I_2 + 80 = 0$$

Divide the above equation with 10.

$$-(I_2 - I_1) - 3I_2 + 8 = 0$$

$$\Rightarrow -4I_2 + I_1 + 8 = 0$$

$$4I_2 = I_1 + 8 \quad \text{Equation 2}$$

Step 4–Finding mesh currents I_1 and I_2 by solving Equation 1 and Equation 2.

The left-hand side terms of Equation 1 and Equation 2 are the same. Hence, equate the right-hand side terms of Equation 1 and Equation 2 in order to find the value of I_1 .

Therefore, the voltage across $30 \, \Omega$ resistor of the given circuit is 84 V .

$$6I_1 - 8 = I_1 + 8$$

$$\Rightarrow 5I_1 = 16$$

$$\Rightarrow I_1 = \frac{16}{5} \text{ A}$$

Substitute I_1 value in Equation 2.

$$4I_2 = \frac{16}{5} + 8$$

$$\Rightarrow 4I_2 = \frac{56}{5}$$

$$\Rightarrow I_2 = \frac{14}{5} \text{ A}$$

Note 1 – From the above example, we can conclude that we have to solve ‘m’ mesh equations, if the electric circuit is having ‘m’ meshes. That’s why we can choose Mesh analysis when the number of meshes is less than the number of principal nodes (except the reference node) of any electrical circuit.

Note 2 – We can choose either Nodal analysis or Mesh analysis, when the number of meshes is equal to the number of principal nodes (except the reference node) in any electric circuit.

So, we got the mesh currents I_1 and I_2 as $\frac{16}{5}$ A and $\frac{14}{5}$ A respectively.

Step 5 – The current flowing through $30\ \Omega$ resistor is nothing but the mesh current I_2 and it is equal to $\frac{14}{5}$ A. Now, we can find the voltage across $30\ \Omega$ resistor by using Ohm’s law.

$$V_{30\Omega} = I_2 R$$

Substitute the values of I_2 and R in the above equation.

$$V_{30\Omega} = \left(\frac{14}{5}\right)30$$

$$\Rightarrow V_{30\Omega} = 84V$$

NODAL ANALYSIS

Nodal analysis is used for solving any electrical network, and it is defined as

The mathematical method for calculating the voltage distribution between the circuit nodes.

This method is also known as the node-voltage method since the node voltages are with respect to the ground. The following are the three laws that define the equation related to the voltage that is measured between each circuit node:

- ☐ Ohm's law
- ☐ Kirchhoff's voltage law
- ☐ Kirchhoff's current law

Procedure of Nodal Analysis

The following steps are to be followed while solving any electrical circuit using nodal analysis:

Step 1:

To identify the principal nodes and select one of them as a reference node. This reference node will be treated as the ground.

Step 2:

All the node voltages with respect to the ground from all the principal nodes should be labelled except the reference node.

Step 3:

The node equations at all the principal nodes except the reference nodes should have a node equation. The node equation is obtained from Kirchhoff's current law and then from Ohm's law.

Step 4:

To obtain the node voltages, the node equations can be determined by following Step 3.

Hence, for a given electrical circuit, the current flowing through any element and the voltage across any element can be determined using the node voltages.

Example 1: Using Nodal method, find the current through resistor r_2 (Figure 1).

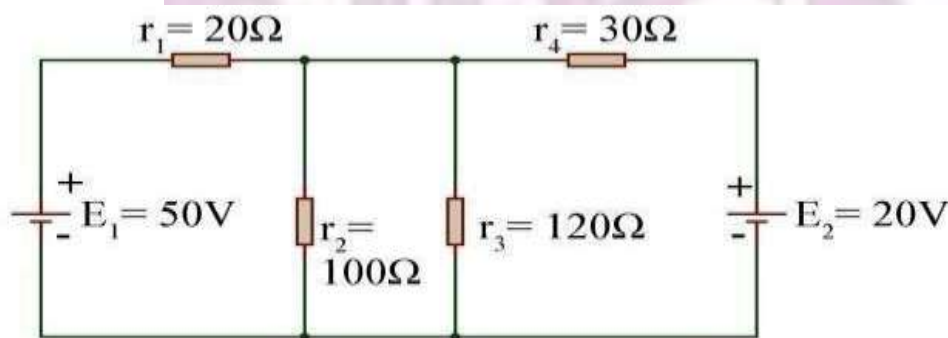


Figure: 1

Solution: Let us redraw the circuit with naming of the nodes and branch current as shown in figure 2.

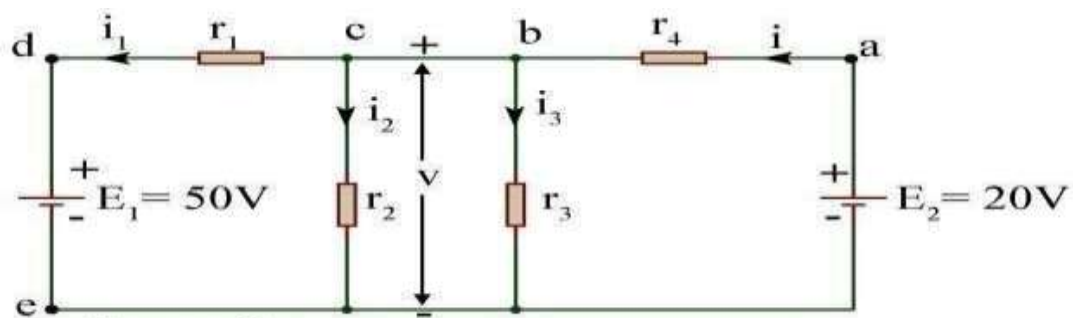


Figure: 2

At node "b", $i = i_1 + i_2 + i_3$

(electrically nodes b and c are same) Assuming the polarity of the voltage v

at node c, we thus get $\frac{v}{r_4} = \frac{50}{r_1} + \frac{v}{r_2} + \frac{v}{r_3}$

$$\frac{v - 20}{30} + \frac{v - 50}{20} + \frac{v}{100} + \frac{v}{120} = 0$$

or,
 $\therefore v = 31.18V$

$$\therefore i_2 = \frac{v}{r_2} = \frac{31.18}{100} A = 0.3118 A$$

i.e. current through $r_2 = 311.8 \text{ mA}$.

Example 2: Using Nodal method find the current through the resistors in the circuit configuration of figure.

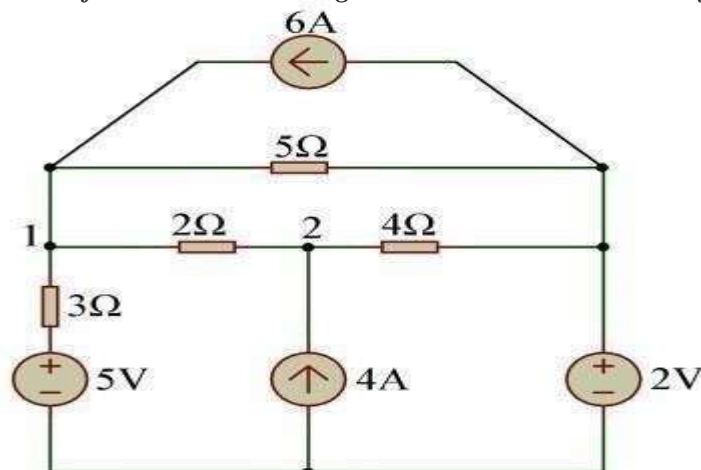


Figure: 3

Solution:

Naming the respective nodes of the circuit as (1) and (2) and assuming the voltages to be v_1 (+ve) and v_2 (+ve) respectively at these nodes, nodal equation at nodes (1) and (2) are as follows:

For node (1),

$$\frac{v_1 - 5}{3} + \frac{v_1 - v_2}{2} + \frac{v_1 - 2}{5} = 6$$

$$v_1 \left(\frac{1}{3} + \frac{1}{2} + \frac{1}{5} \right) - \frac{v_2}{2} - \frac{2}{5} - \frac{5}{3} - 6 = 0$$

$$\text{or, } \frac{31}{30}v_1 - \frac{v_2}{2} - \frac{121}{15} = 0$$

For node (2),

$$\frac{v_2 - v_1}{2} + \frac{v_2 - 2}{4} = 4$$

$$\text{or, } v_2 \left(\frac{1}{2} + \frac{1}{4} \right) - \frac{v_1}{2} - \frac{1}{2} = 4$$

$$\text{or, } \frac{3}{4}v_2 - \frac{v_1}{2} - \frac{9}{2} = 0$$

.....(b)

Solving (a) and

(b), $v_1 = 15.76\text{V}$ while $v_2 = 16.51$

V

$$= \frac{v_1 - 5}{3} = \frac{15.76 - 5}{3} \approx 3.6\text{A}$$

Current through 2Ω resistor

$$= \frac{v_1 - v_2}{2} = \frac{15.76 - 16.51}{2} = -0.375\text{A}$$

[i.e., following from node (2) to node (1)].

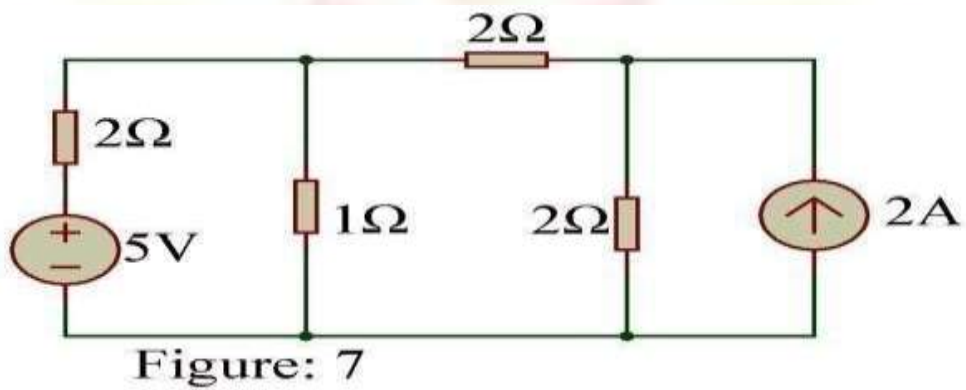
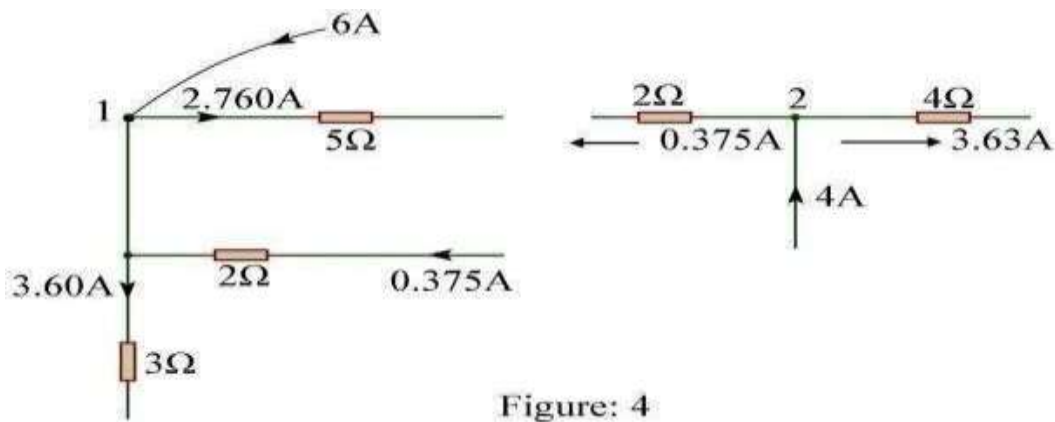
Current through 5Ω resistor

$$= \frac{v_1 - 2}{5} = \frac{15.76 - 2}{5} = 2.76\text{A}$$

Current through 4Ω resistor

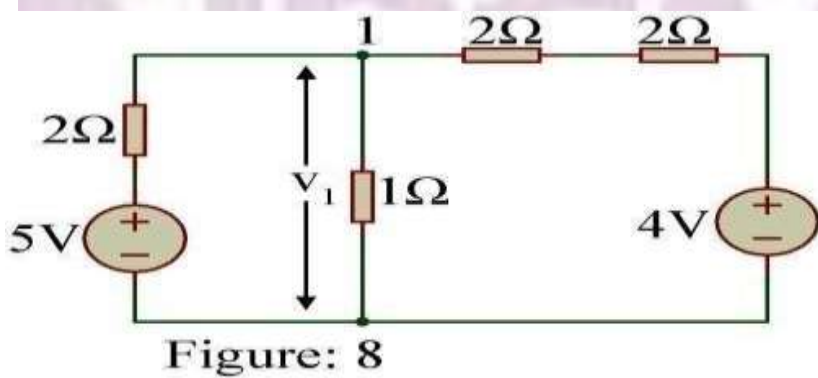
$$= \frac{v_2 - 2}{4} = \frac{16.51 - 2}{4} = 3.63\text{A}$$

Figure 4 confirms the KCL both at nodes (1) and nodes (2)



Solution:

Let us first convert the current source of figure 7 to voltage source and draw the equivalent network (figure 8). Let the voltage at node (1) be V_1 .



∴ Using nodal analysis,

$$\frac{v_1}{1} + \frac{v_1 - 5}{2} + \frac{v_1 - 4}{4} = 0$$

or, $v_1 = 2V$

Hence, the current through 1Ω resistor is

$$\frac{v_1}{1} = 2A$$

Example 5: Find V_1 and V_2 in figure 9.

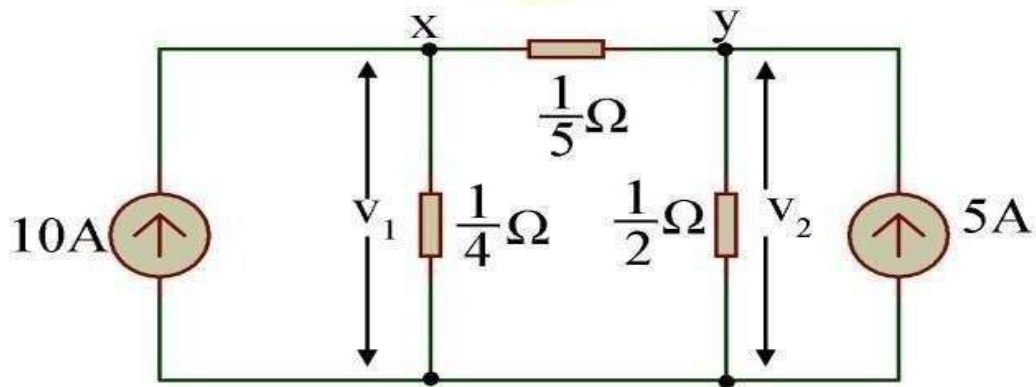


Figure: 9

Solution:

At node "x",

$$10 = \frac{V_1}{1/4} + \frac{V_1 - V_2}{1/5} = 4V_1 + 5V_1 - 5V_2$$

or, $9V_1 - 5V_2 = 10$ (1)

At node "y",

$$5 = \frac{V_2}{1/2} + \frac{V_2 - V_1}{1/5} = 2V_2 + 5V_2 - 5V_1$$

or, $-5V_1 + 7V_2 = 5$ (2)

Solving (1) and (2),

$$V_2 = 2.5V; V_1 = 2.5V$$

Time-domain analysis of first-order RL and RC circuits.

The solution of the differential equation represents the response of the circuit. Now we will find out the response

of the basic RL and RC circuits with DC excitation.

RL CIRCUIT with external DC excitation:

Let us take a simple RL network subjected to external DC excitation as shown in the figure. The circuit consists of a battery whose voltage is V in series with a switch, a resistor R , and an inductor L .

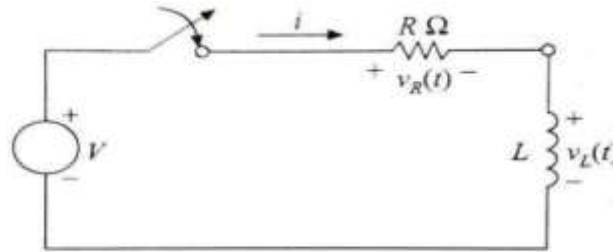


Fig: RL Circuit with external DC excitation

The switch is closed at $t = 0$. Fig: RL Circuit with external DC excitation When the switch is closed current tries to change in the inductor and hence a voltage $v_L(t)$ is induced across the terminals of the inductor in opposition to the applied voltage. The rate of change of current decreases with time which allows current to build up to its maximum value.

It is evident that the current $i(t)$ is zero before $t = 0$ and we have to find out current $i(t)$ for time $t > 0$. We will find $i(t)$ for time $t > 0$ by writing the appropriate circuit equation and then solving it by separation of the variables and integration. Applying Kirchhoff's voltage law to the above circuit we get:

$$V = v_R(t) + v_L(t) \quad i(t) = 0 \text{ for } t = 0$$

One direct method of solving such a differential equation consists of writing the equation in such a way that the variables are separated, and then integrating each side of the equation. The variables in the above equation are i and t . This equation is multiplied by dt and arranged with the variables separated as shown below:

$$R i \cdot dt + L di = V \cdot dt$$

$$L di = (V - Ri) dt$$

$$i \cdot L di / (V - Ri) = dt$$

$$\text{Next each side is integrated directly to get: } -(L/R) \ln(V - Ri) = t + k$$

Where k is the integration constant. In order to evaluate k , an initial condition must be invoked. Prior to $t = 0$, $i(t)$ is zero, and thus $i(0^-) = 0$. Since the current in an inductor cannot change by a finite amount in zero time without being associated with an infinite voltage, we have $i(0^+) = 0$. Setting $i = 0$ at $t = 0$, in the above equation we obtain

$-(L/R)\ln(V)=k$ and, hence,

$$-L/R[\ln(V-Ri)-\ln V]=t \text{ Rearranging}$$

$$\ln[(V-Ri)/V]=-(R/L)t$$

Taking antilogarithm on both sides we get $(V-Ri)/V=e^{-Rt/L}$ From which we can see that

$$i(t)=(V/R)-(V/R)e^{-Rt/L} \text{ for } t>0$$

Thus, an expression for the response valid for all time would be $i(t)=V/R[1-e^{-Rt/L}]$ This is normally written as:

$$i(t)=V/R[1-e^{-t/\tau}]$$

where ' τ ' is called the time constant of the circuit and it's unit is seconds. The voltage across the resistance and the Inductor for $t>0$ can be written as: $v_R(t)=i(t).R=V[1-e^{-t/\tau}]$

$$v_L(t)=V-v_R(t)=V-V[1-e^{-t/\tau}]=V(e^{-t/\tau})$$

A plot of the current $i(t)$ and the voltages $v_R(t)$ & $v_L(t)$ is shown in the figure below.

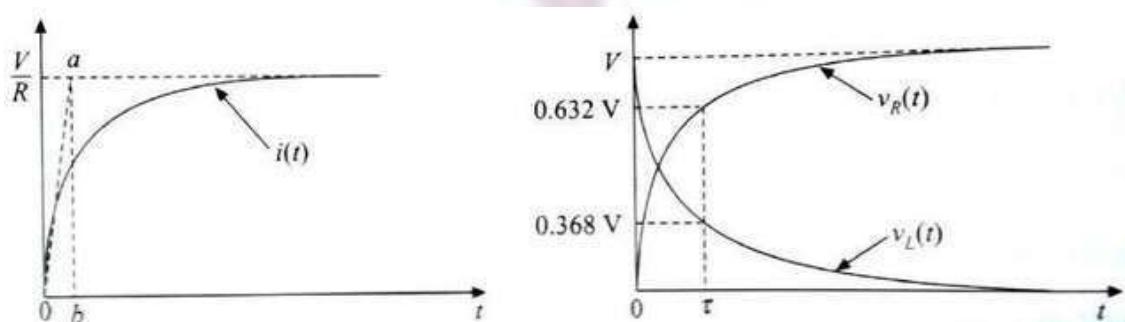


Fig: Transient current and voltages in the Series RL circuit.

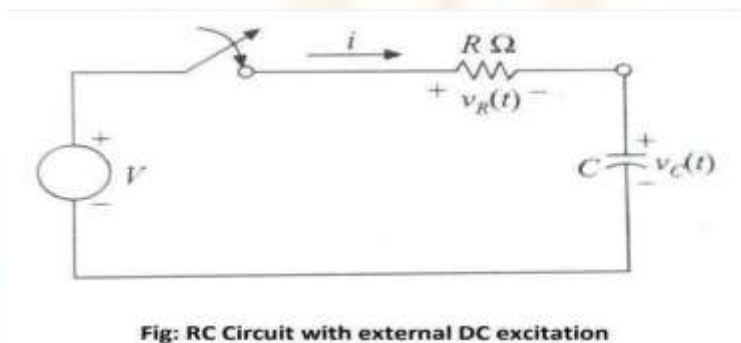
At $t = \tau$ the voltage across the inductor will be $v_L(\tau)=V(e^{-\tau/\tau})=V/e=$

$0.36788V$ and the voltage across the Resistor will be $v_R(\tau)=V[1-e^{-\tau/\tau}]=0.63212V$

The plots of current $i(t)$ and the voltage across the Resistor $v_R(t)$ are called exponential growth curves and the voltage across the inductor $v_L(t)$ is called exponential decay curve.

RCCIRCUITwithexternalDCexcitation:

A series RC circuit with external DC excitation V volts connected through a switch is shown in the figure below. If the capacitor is not charged initially i.e. its voltage is zero, then after the switch S is closed at time $t=0$, the capacitor voltage builds up gradually and reaches its steady state value of V volts after a finite time. The charging current will be maximum initially (since initially capacitor voltage is zero and voltage across a capacitor cannot change instantaneously) and then it will gradually come down as the capacitor voltage starts building up. The current and the voltage during such charging periods are called Transient Current and Transient Voltage.



Applying KVL around the loop in the above circuit we can write $V = v_R(t) + v_C(t)$

Using the standard relationships of voltage and current for an Ideal Capacitor we get $v_C(t) = \frac{1}{C} \int i(t) dt$ or $i(t) = C \cdot [dv_C(t)/dt]$

$$i(t) = C \cdot [dv_C(t)/dt]$$

and using this relation, $v_R(t)$ can be written as $v_R(t) = Ri(t) = R \cdot C \cdot [dv_C(t)/dt]$

Using the above two expressions for $v_R(t)$ and $v_C(t)$ the above expression for V can be rewritten as:

$$V = R \cdot C \cdot [dv_C(t)/dt] + v_C(t)$$

Or finally $dv_C(t)/dt + (1/RC) \cdot v_C(t) = V/RC$

The inverse coefficient of $v_C(t)$ is known as the time constant of the circuit τ and is given by $\tau = RC$ and its units are seconds.

The above equation is a first order differential equation and can be solved by using the same method of separation of variables as we adopted for the LC circuit.

Multiplying the above equation $dv_C(t)/dt + (1/RC). v_C(t) = V/RC$ both sides by 'dt' and rearranging the terms so as to separate the variables $v_C(t)$ and t we get: $dv_C(t) + (1/RC).v_C(t).dt = (V/RC).dt$
 $dv_C(t) = [(V/RC) - (1/RC).v_C(t)].Dt$

$$dv_C(t) / [(V/RC) - (1/RC).v_C(t)] = dt$$

$$R.C.dv_C(t) / [V - v_C(t)] = dt$$

Now integrating both sides w.r.t their variables i.e. ' $v_C(t)$ ' on the LHS and ' t ' on the RHS we get

$$-RC \ln[V - v_C(t)] = t + k$$

where 'k' is the constant of integration. In order to evaluate k, an initial condition must be invoked. Prior to $t = 0$, $v_C(t)$ is zero, and thus $v_C(t)(0-) = 0$. Since the voltage across a capacitor cannot change by a finite amount in zero time, we have $v_C(t)(0+) =$

0. Setting $v_C(t) = 0$ at $t = 0$, in the above equation we obtain: $-RC \ln[V] = k$ and substituting this value of $k = -RC \ln[V]$ in the above simplified equation

$$-RC \ln[V - v_C(t)] = t + k \text{ we get:}$$

$$-RC \ln[V - v_C(t)] = t - RC \ln[V]$$

$$\text{i.e. } -RC \ln[V - v_C(t)] + RC \ln[V] = t \text{ i.e. } -RC [\ln\{V - v_C(t)\} - \ln(V)] = t$$

$$\text{i.e. } \ln[\{V - v_C(t)\} / (V)] = -t/RC$$

$$\text{Taking antilogarithm we get } [\{V - v_C(t)\} / (V)] = e^{-t/RC}$$

$$\text{i.e. } v_C(t) = V(1 - e^{-t/RC})$$

which is the voltage across the capacitor as a function of time.

$$\text{The voltage across the Resistor is given by: } v_R(t) = V - v_C(t) = V - V(1 - e^{-t/RC}) = V.e^{-t/RC}$$

And the current through the circuit is given by: $i(t) = C \cdot [dv_C(t)/dt] = (CV/RC)e^{-t/RC} = (V/R)e^{-t/RC}$ Or the other way: $i(t) = v_R(t)/R = (V \cdot e^{-t/RC})/R = (V/R)e^{-t/RC}$

In terms of the time constant τ the expressions for $v_C(t)$, $v_R(t)$ and $i(t)$ are given by: $v_C(t) = V(1 - e^{-t/RC})$

$$v_R(t) = V \cdot e^{-t/RC} \quad i(t) = (V/R)e^{-t/RC}$$

The plots of current $i(t)$ and the voltages across the resistor $v_R(t)$ and capacitor $v_C(t)$ are shown in the figure below.

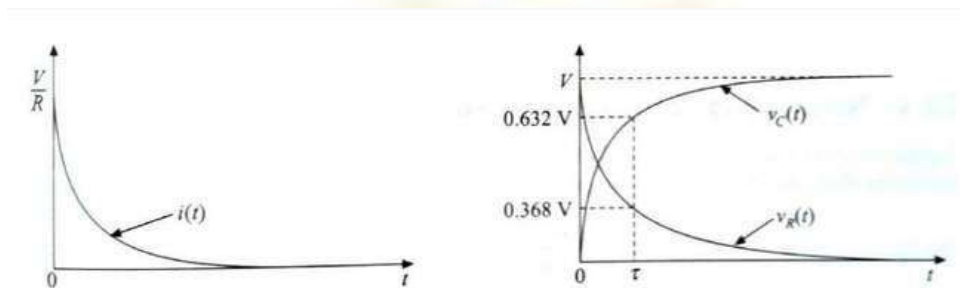


Fig : Transient current and voltages in RC circuit with DC excitation.

At $t = \tau$ the voltage across the capacitor will be:

$$v_C(\tau) = V[1 - e^{-\tau/\tau}] = 0.63212V$$

the voltage across the Resistor will be: $v_R(\tau) = V(e^{-\tau/\tau}) = V/e = 0.36788V$

and the current through the circuit will be: $i(\tau) = (V/R)(e^{-\tau/\tau}) = V/R \cdot e = 0.36788(V/R)$

Thus it can be seen that after one time constant the charging current has decayed to approximately 36.8% of its value at $t=0$.

At $t = 5\tau$ charging current will be

$$i(5\tau) = (V/R)(e^{-5\tau/\tau}) = V/R \cdot e^{-5} = 0.0067(V/R)$$

This value is very small compared to the maximum value of (V/R) at $t=0$. Thus it can be assumed that the capacitor is fully charged after 5 time constants.

The following similarities may be noted between the equations for the transients in the LC and RC circuits:

- The transient voltage across the Inductor in a LC circuit and the transient current in the RC circuit have the same form $k(e^{-t/\tau})$
- The transient current in a LC circuit and the transient voltage across the capacitor in the RC circuit have the same form $k(1 - e^{-t/\tau})$
- But the main difference between the RC and RL circuits is the effect of resistance on the duration of the transients.
- In a RL circuit a larger resistance shortens the transients since the time constant $\tau = L/R$ becomes small.
- Whereas in a RC circuit a larger resistance prolongs the transients since the time constant $\tau = RC$ becomes large.

Example 1: Find the current in a series RL circuit having $R = 2\Omega$ and $L = 10H$ when a DC voltage V of 100V is applied. Find the value of the current 5 secs. after the application of the DC voltage.

Solution: This is a straightforward problem which can be solved by applying the formula.

First let us find out the Time constant of the series LR circuit which is given by $\tau = L/R$ secs.

$$\therefore \tau = 10/2 = 5 \text{ secs}$$

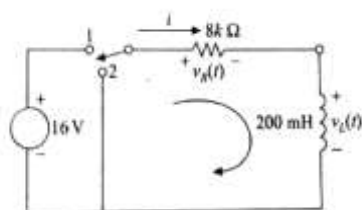
The current in a series LR circuit after the sudden application of a DC voltage is given by $i(t) = \frac{V}{R}(1 - e^{-t/\tau})$

$$\therefore i(t) \text{ at } 5 \text{ secs} = 100/2(1 - e^{-5/5}) = 50(1 - e^{-1}) = 50(1 - 1/e) = 31.48$$

$$\therefore i(t) \text{ at } 5 \text{ secs} = 31.48 \text{ Amps}$$

Example : In the circuit shown below the switch is kept in position 1 upto 250 μ secs and then moved to position 2. Find (a) The current and voltage across the resistor at $t = 100 \mu$ secs (b) The current and voltage across the resistor at $t = 350 \mu$ secs

Solution: The time constant of the circuit is given by $\tau = L/R = 200\text{mH}/8K\Omega = 25\mu\text{sec}$ and is same in both the switch positions.



a) The current in the circuit up to 250 μsec (till switch is in position 1) is given by:

$$i(t)_{\text{growing}} = V/R(1 - e^{-t/\tau}) = (16/8) \times 10^{-3} (1 - e^{-t/25 \times 10^{-6}}) = 2 \times (1 - e^{-t/25 \times 10^{-6}}) \text{ mA}$$

The current in the circuit @ 100 μsec is given by

$$i(t) @ 100 \mu\text{sec} = 2 \times (1 - e^{-100 \mu\text{sec}/25 \mu\text{sec}}) \text{ mA} = 2 \times (1 - e^{-4}) \text{ mA} = 1.9633 \text{ mA}$$

$$i(t) @ 100 \mu\text{sec} = 1.9633 \text{ mA}$$

Voltage across the resistor is given by $v_R @ 100 \mu\text{sec} = R \times$

$$i(t) @ 100 \mu\text{sec} \quad v_R @ 100 \mu\text{sec} = 8 \text{ K}\Omega \times 1.9633 \text{ mA} = 15.707 \text{ V}$$

$$v_R @ 100 \mu\text{sec} = 15.707 \text{ V}$$

(a) The current in the circuit @ 350 μsec is the decaying current and is given by:

$$i(t)_{\text{Decaying}} = I(0) \cdot e^{-t/\tau}$$

where $I(0)$ is the initial current and in this case it is the growing current @ 250 μsec . (Since the switch is changed @ 250 μsec) The time t is to be reckoned from this time of

$$250 \mu\text{sec}. \text{Hence } t = (350 - 250) =$$

100 μsec . So we have to calculate first $i(t)_{\text{growing}} @ 250 \mu\text{sec}$ which is given by:

$$i(t)_{\text{growing}} @ 250 \mu\text{sec} = V/R(1 - e^{-t/\tau}) = (16/8) \times 10^{-3} (1 - e^{-t/25 \mu\text{sec}}) = 2 \times (1 - e^{-250/25 \mu\text{sec}}) \text{ mA} = 2 \times (1 - e^{-10}) \text{ mA} = 1.999 \text{ mA}$$

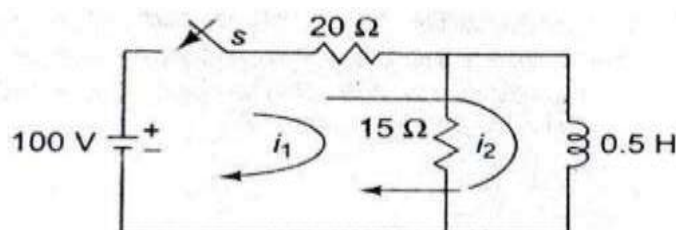
$$i(t)_{\text{growing}} @ 250 \mu\text{sec} = 1.999 \text{ mA} = I(0)$$

$$\text{Hence } i(t) @ 350 \mu\text{sec} = I(0) \cdot e^{-t/\tau} = 1.99 \times e^{-100 \mu\text{sec}/25 \mu\text{sec}} \text{ mA} = 1.99 \times e^{-4} \text{ mA} = 0.03663 \text{ mA}$$

$$i(t) @ 350 \mu\text{sec} = 0.03663 \text{ mA}$$

$$\text{The voltage across the resistor } v_R @ 350 \mu\text{sec} = R \times i(t @ 350 \mu\text{sec}) = 8 \text{ K}\Omega \times 0.03663 \text{ mA} \quad v_R @ 350 \mu\text{sec} = 0.293 \text{ V}$$

Example: In the circuit shown below find out the expressions for the current i_1 and i_2 when the switch is closed at time $t=0$



Solution: It is to be noted that in this circuit there are two current loops 1 and 2.

Current i_1 alone flows through the resistor 15Ω and the current i_2 alone flows through the inductance $0.5H$ whereas both the currents i_1 and i_2 flow through the resistor 20Ω .

Applying KVL to the two loops taking care of this point we get $20(i_1 + i_2) + 15i_1 = 100$. $e^{35i_1 + 20i_2 = 100}$ (1)

and $20(i_1 + i_2) + 0.5 \frac{di_2}{dt} = 100$; $20i_1 + 20i_2 + 0.5 \frac{di_2}{dt} = 100$ (2)

Substituting the value of $i_1 = [100/35 - (20/35)i_2] = 2.86 - 0.57i_2$ obtained from the above equation (1) into equation (2) we get:

$$20[2.86 - 0.57i_2] + 20i_2 + 0.5 \frac{di_2}{dt} = 100$$

$$57.14 - 11.4i_2 + 20i_2 + 0.5 \frac{di_2}{dt} = 100$$

$$\frac{di_2}{dt} i_2 + 17.14 i_2 = 85.72$$

The solution for this equation is given by $i_2(t) = K \cdot e^{-17.14t} + 85.72/17.14$ and the constant K can be evaluated by invoking the initial condition. The initial current through the inductor = 0 at time $t = 0$. Hence $K = -85.72/17.14 = -5$

Therefore $i_2(t) = 5(1 - e^{-17.14t})$ Amps

And current $i_1(t) = 2.86 - 0.57i_2 = 2.86 - 0.57[5(1 - e^{-17.14t})] = 0.01 + 2.85e^{-17.14t}$ Amps

And current $i_1(t) = 0.01 + 2.85e^{-17.14t}$ Amps

AC CIRCUITS

The alternating quantity is one whose value varies with time. This alternating quantity may be periodic and non-periodic. Periodic quantity is one whose value will be repeated for every specified interval. Generally to represent alternating voltage or current we prefer sinusoidal wave form, because below listed properties

1. Derivative of sine is an sine function only.
2. Integral of sine is an sine function only.
1. It is easy to generate sine function using generators.
2. Most of the 2nd order system response is always sinusoidal.

1.2 Alternating quantity:

defined with degree or radians as reference.

At, 0 degrees --- 0

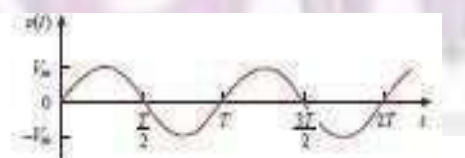
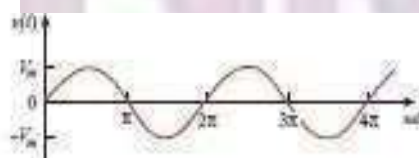
90 degrees --- maximum

180 degrees --- 0

270 degrees --- maximum

360 degrees --- 0

i.e value of sine function varies with time, firstly increases from zero and reaches maximum and again falls to zero, there after tends to increase in opposite direction and reaches maximum value and falls to zero. This the variation of sine in 1st cycle is called as positive half cycle and other negative half cycle.(i.e during direction is required one and during 2nd half cycle direction actual required direction.). Therefore one positive and negative cycle combinely forms one complete cycle



Sine equation , voltage , $V(t) = V_m \sin \omega t$.

Where, V_m = peak value or maximum value

ω – angular frequency.

Definitions:

Peak to peak value: It is total value from positive peak to the negative peak. ($2V_m$)

Instantaneous value: It is the magnitude of wave form at any specified time. $V(t)$

Average value : It is ratio of area covered by wave form to its length. (V_d)

$$V_d = (1/T) \int V(t) dt$$

$$V_d = (1/2\pi) \int V_m \sin \omega t d\omega t$$

$$= -V_m / 2\pi \cos \omega t \text{---with limits of } 2\pi \text{ and } 0$$

$$= 0. \text{ (i.e. average value of sine wave over a full cycle is zero)}$$

Hence it is defined for half cycle.

$$V_d = (1/\pi) \int V_m \sin \omega t d\omega t$$

$$= -V_m / \pi \cos \omega t \text{ with limits of } \pi \text{ and } 0$$

Peak factor:

$$V_{rms} = \sqrt{(1/T) \int V(t)^2 dt}$$

RMS value:

It is the root mean square value of the function, which gives

$$= \sqrt{(1/2\pi) \int V_m^2 \sin^2 \omega t d\omega t}$$

$$= V_m / \sqrt{2} = \text{effective value.}$$

It is the ratio of peak value to the rms value.

$$P_p = V_p / V_{rms} = \sqrt{2}$$

Form factor:

It is the ratio of average value to the rms value.

$$F_p = V_d / V_{rms} = 2\sqrt{2} / \pi = 1.11$$

Eg: Find the peak, peak to peak, average, rms, peak factor and form factor of given current function ,

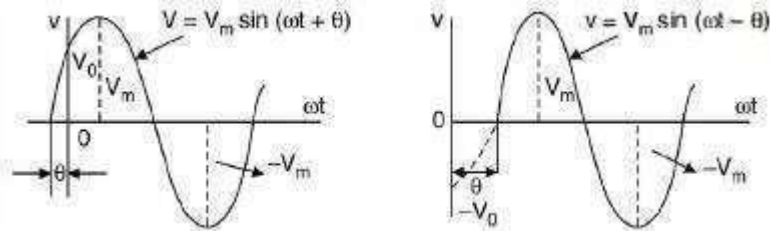
$$i(t) = 5 \sin \omega t.$$

1.4 Phase and phase difference:

Phase of the sine indicates starting phase of the sine wave. i.e

Let , $V(t) = V_m \sin \omega t$, here we can say that phase is zero as function starts from origin.

$V(t) = V_m \sin(\omega t - \theta)$, here we can say that phase of function is θ degrees to right shift.
 $V(t) = V_m \sin(\omega t + \theta)$, here we can say that phase of function is θ degrees to left shift.



Phase difference is the difference of phase between two wave forms taking one as reference.

Eg: If wave form A is $V_m \sin(\omega t + 15)$, B is $V_m \sin(\omega t - 30)$ and C is $V_m \sin(\omega t + 45)$.

Determine the phase difference between every pair if wave forms.

When A and B are compared , phase difference is 45 degrees.

When C and B are compared , phase difference is 75 degrees.

When A and C are compared , phase difference is 30 degrees.

1.5 Phasor diagram:

Phasor diagram is the pictorial representation of sine wave. Here magnitude and phase of the wave function are represented in four quadrant axis. We assume positive phases in anti-clock wise direction and negative phases in clock wise direction. From the phasor diagram we can easily identify the phase difference between different wave forms. We can also identify whether function is right shift or left shift.

1.6 Phase relations of network parameters:

Resistor:

Let us consider resistor allowing alternating current $i(t)$. Then the voltage drop across resistor is given as,

$$\begin{aligned} \text{If, } V(t) &= V_m \sin \omega t & = V_m / I_m \\ V(t) &= i(t).R & i(t) = I_m \sin \omega t. \end{aligned}$$

$$i(t) = V(t) / R$$

$$Z = V_m \sin \omega t / I_m \sin \omega t.$$

$$= V_m \sin \omega t / R$$

Hence we can say that $V(t)$ and $i(t)$ in resistor element are in phase

The ratio of $V(t) / i(t) = Z = \text{impedance offered by resistor. (ohms).}$

Inductor:

Let us consider an coil of N turns allowing current $i(t)$. ($I_m \sin \omega t$)

Hence emf induced in the coil is ,

$$\begin{aligned} V(t) &= L \, di(t) / dt \\ &= L \, d(I_m \sin \omega t) / dt \\ &= L \, \omega I_m \cos \omega t \\ &= V_m \cos \omega t = V_m \sin(\omega t + 90^\circ). \end{aligned}$$

Where, $V_m = L \, \omega I_m = I_m \cdot X_L$

$X_L = \text{reactance offered by coil.}$

Impedance offered by coil is , $Z = V(t) / i(t)$

$$= V_m \sin(\omega t + 90^\circ) / I_m \sin \omega t$$

The function $V_m \sin \omega t = V_m \angle 0$.

$$Z = V_m \angle 90^\circ / I_m \angle 0$$

$$Z = V_m \angle 90^\circ / I_m$$

$$= j \, \omega L = j \, X_L. \quad (j = 1 \angle 90^\circ)$$

As there is left shift in $V(t)$, we can say that $i(t)$ lags $V(t)$ by 90 degrees.

Capacitor:

Let us consider an capacitor allowing current $i(t)$. ($I_m \sin \omega t$)

Hence voltage across it is , Where, $V_m = \quad = V_m \sin(\omega t - 90^\circ).$

$$\begin{aligned} V(t) &= 1 / C \int i(t) \, dt \\ &= 1 / C \int I_m \sin \omega t \, dt \\ &= - \cos \omega t \cdot I_m / \omega C \end{aligned}$$

$$I_m / \omega C = I_m \cdot X_C$$

$$= V_m \sin(\omega t - 90) / I_m \sin \omega t$$

X_C = reactance offered by capacitor

Impedance offered by capacitor is , $Z = V(t) / i(t)$

1.7 Power in Ac circuits $Z = V_m \angle -90 / I_m \angle 0$

The function $V_m \sin \omega t = V_m \angle 0$.

In the case of DC circuits power is given as product of voltage and current in that element.

$$P = V \cdot I \text{ (W)}$$

$$\text{Let } V(t) = V_m \sin \omega t$$

$$i(t) = I_m \sin(\omega t + 90)$$

Instantaneous power, $P(t) = V(t) \cdot i(t)$

$$= V_m \sin \omega t \cdot I_m \sin(\omega t + \Phi)$$

$$= V_m \cdot I_m \sin \omega t \sin (\omega t + \Phi).$$

$$\frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} 2 \sin(\omega t + \Phi)$$

$$\frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} [\cos \phi - \cos (2\omega t + \Phi)]$$

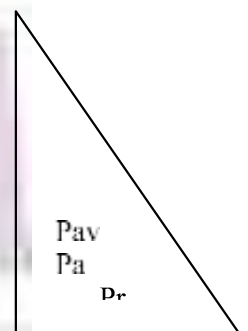
$$\text{Average power, } P_{av} = \frac{1}{2\pi} \int_0^{2\pi} p(t) d\omega t.$$

$$= V_{rms} I_{rms} \cos \Phi [\omega t] \text{ ----- with limits } 2\pi \text{ and } 2\pi$$

$$P_{av} = V_{rms} \cdot I_{rms} \cos \Phi \text{ (W) = true power = active power.}$$

$\cos \Phi = P_{av} / V_{rms} \cdot I_{rms}$ = defined as power factor of the circuit.

$$\cos \Phi = P_{av} / P_a$$



$$= 1/2\pi \int V_m \text{Im}[\cos \Phi - \cos(2\omega t + \Phi)] d\omega t$$

$$\sqrt{2} \cdot \sqrt{2}$$

$P_{av} = P_a \cos \Phi = V_{rms} I_{rms} \cos \Phi = \text{active power} = W$
 As average value over full cycle is equal to zero, hence second term can be neglected.

$$P_r = P_a \sin \Phi = V_{rms} I_{rms} \sin \Phi = \text{reactive power} = \text{VAR}$$

$$P_{av} = 1/2\pi \int V_m \text{Im}[\cos \Phi] d\omega t$$

Let us consider commercial capacitor,

$$\bar{Z} = \text{true power} / \text{apparent power}$$

$$Z = R - jX_C$$

Where, $Z = \text{impedance of the capacitor}$
 $= \text{actual power utilized by load} / \text{total generated power.}$

$$P_a = \text{apparent power} = V_{rms} \cdot I_{rms} = V \cdot A$$

Let us consider commercial inductor $Z = R + jX_L$

Where, $Z = \text{impedance of the coil}$

$X_L = \text{reactance offered by the coil.}$

$R = \text{internal resistance of the coil}$

$$I(t) Z = I(t) R + I(t) jX_L$$

$$I^2 Z = I^2 R + j I^2 X_L$$

$$P_a = P_{av} + j P_r$$

power triangle with phase Φ

$R = \text{internal resistance of the capacitor}$

$X_C = \text{reactance offered by the capacitor.}$

$$I(t) Z = I(t) R - I(t) jX_C$$

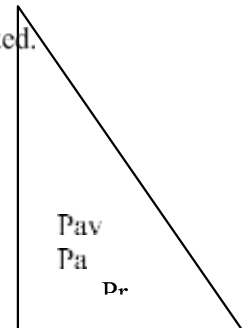
$$I^2 Z = I^2 R - j I^2 X_C$$

$$P_a = P_{av} - j P_r$$

power triangle with phase Φ

$$P_{av} = P_a \cos \Phi = V_{rms} I_{rms} \cos \Phi = \text{active power} = W$$

$$P_r = P_a \sin \Phi = V_{rms} I_{rms} \sin \Phi = \text{reactive power} = \text{VAR}$$



1.8 Complex power:

Complex power is represented with S.

$$S = V(t).i(t)^*$$

$$= P + jQ \text{ or } P - jQ$$

Where, P = active power

Q = reactive power

Here only useful power is true power where as net reactive power over an cycle will be zero.

Complex numbers:

Complex numbers can be represented in two ways, either rectangle form or polar form

Rectangular form = $a + j b$

Polar form = $\sqrt{a^2 + b^2} \angle \tan^{-1}(b/a)$

Here j operator plays major role in complex number, which is define MODULE vector rotating in anti-clock wise direction with phase 90.

$$j = 1 \angle 90 = \sqrt{-1}$$

$$j^2 = -1$$

$$j^3 = -\sqrt{-1}$$

RMS VALUE:

The **RMS (Root Mean Square)** value (also known as **effective** or **virtual** value) of of an alternating current (AC) is the value of direct current (DC) when flowing through a circuit or [resistor](#) for the specific time period and produces same amount of heat which produced by the alternating current (AC) when flowing through the same [circuit](#) or resistor for a specific time.

The value of an AC which will produce the same amount of heat while passing through in a heating element (such as resistor) as DC produce through the element is called R.M.S Value.

- ☐ In short,
- ☐ The RMS Value of an Alternating Current is that when it compares to the Direct Current, then both AC and DC current produce the same amount of heat when flowing through the same circuit for a specific time period.

$$I_{RMS} = \frac{I_M}{\sqrt{2}}, V_{RMS} = \frac{V_M}{\sqrt{2}}$$

For a sinusoidal wave

$$I = 0.707 \times I_M, V = 0.707 \times V_M$$

or

$$I_{RMS} = 0.707 \times I_M, E_{RMS} = 0.707 E_M$$

Actually, the RMS value of a sine wave is the measurement of heating effect of sine wave. For example, when a resistor is connected across an AC voltage source, it produces a specific amount of heat (Fig 2)

– a). When the same resistor is connected across the DC voltage source as shown in (fig 2 – b). By adjusting the value of DC voltage to get the same amount of heat generated before in AC voltage source in fig a. It means the RMS value of a sine wave is equal to the DC Voltage source producing the same amount of heat generated by AC Voltage source.

- In more clear words, the domestic voltage level in US is 110V, while 220V AC in UK. This voltage level shows the effective value of (110V or 220V R.M.S) and it shows that the home wall socket is capable to provide the same amount of average positive power as 110V or 220V DC Voltage.
- *Keep in mind that the ammeter and voltmeter connected in AC circuits always showing the RMS values (of current and voltage).*

- For AC sine wave, RMS values of current and voltage are:

$$I_{RMS} = 0.707 \times I_M, V_{RMS} = 0.707 V_M$$

- Let's see how to find the R.M.S values of a sine wave.
- We know that the value of sinusoidal alternating current (AC) =

$$I_M \sin \omega t = I_M \sin \theta$$

- While the mean of square of instantaneous values of current in half a cycle is: The

Square root of this value is:

$$= \sqrt{\frac{\int_0^{2\pi} i^2 d\theta}{2\pi}}$$

Hence, the RMS value of the current is (while putting $I = I_M \sin \theta$):

$$I = \sqrt{\left(\int_0^{2\pi} \frac{i^2 d\theta}{2\pi} \right)} = \sqrt{\left(\frac{I_m^2}{2\pi} \int_0^{2\pi} \sin^2 \theta d\theta \right)}$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta \quad \therefore \sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\begin{aligned} I &= \sqrt{\left(\frac{I_m^2}{4\pi} \int_0^{2\pi} (1 - \cos 2\theta) d\theta \right)} = \sqrt{\left(\frac{I_m^2}{4\pi} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{2\pi} \right)} \\ &= \sqrt{\frac{I_m^2}{4} \cdot 2} = \sqrt{\frac{I_m^2}{2}} \quad \therefore I = \frac{I_m}{\sqrt{2}} = 0.707 I_m \end{aligned}$$

Now,

Therefore, We may find that for a symmetrical sinusoidal current:

IRMS = Max Value of Current \times 0.707 Average Value:

If we convert the alternating current (AC) sine wave into direct current (DC) sine wave through rectifiers, then the converted value to the DC is known as the average value of that alternating current sine wave.

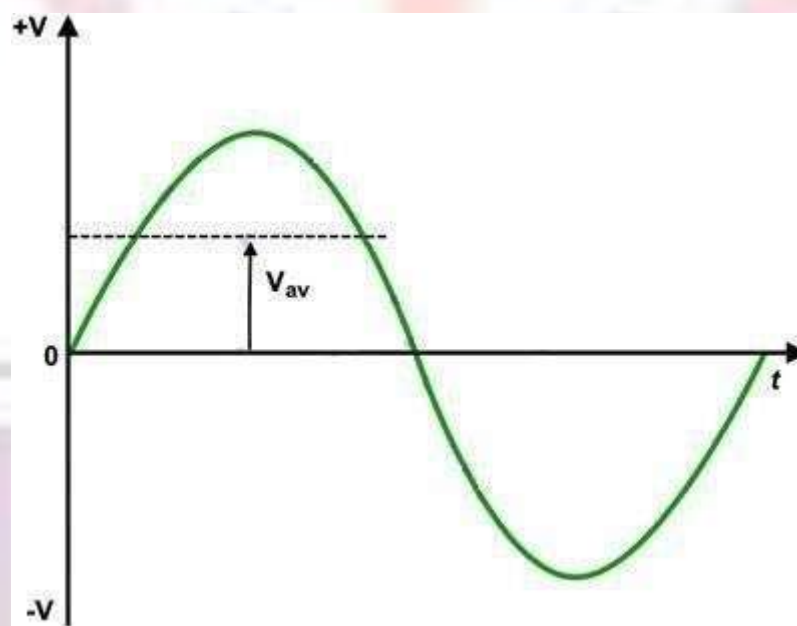


Fig4–Average Value of Voltage

If the maximum value of alternating current is “ I_{MAX} “, then the value of converted DC current through rectifier would be “ $0.637 I_M$ ” which is known as average value of the AC sine wave (I_{AV}).

$$\text{Average Value of Current} = I_{AV} = 0.637 I_M \quad \text{Average Value of Voltage} = E_{AV} = 0.637 E_M$$

The Average Value (also known as Mean Value) of an Alternating Current (AC) is expressed by that Direct

Current (DC) which transfers across any circuit the same amount of charge as is transferred by that Alternating Current (AC) during the same time.

Keep in mind that the average or mean value of a full sinusoidal wave is “Zero” the value of current in first half (Positive) is equal to the the next half cycle (Negative) in the opposite direction. In other words, There are same amount of current in the positive and negative half cycles which flows in the opposite direction, so the average value for a complete sine wave would be “0”. That’s the reason that’s why we don’t use average value for plating and battery charging. If an AC wave is converted into DC through a rectifier, It can be used for electrochemical works.

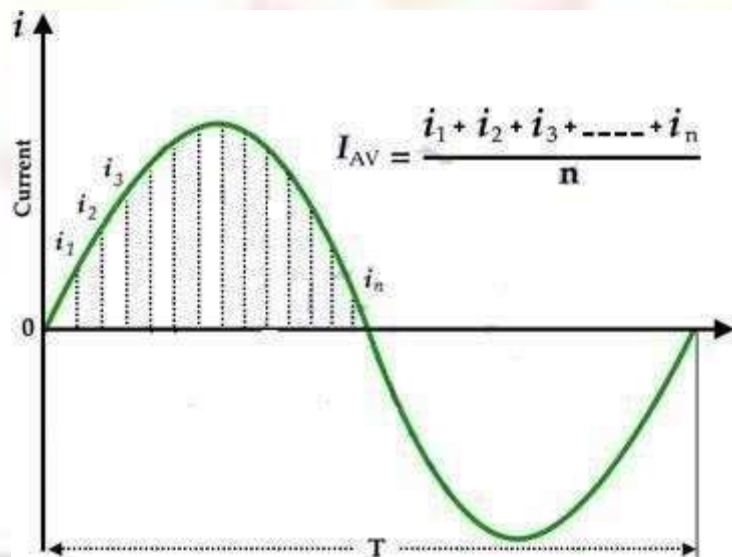


Fig5-Average Value of Current

In short, the average value of a sine wave taken over a complete cycle is always zero, because the positive values (above the zero crossing) offset or neutralize the negative values (below the zero crossing.)

We know that the standard equation of alternating current is

$$i = \sin \omega \theta = I_m \sin \theta$$

- Maximum value of current on sine wave = I_m
- Average value of current on sine wave = I_{AV}
- Instantaneous value of current on sine wave = i
- The angle specified for “ i ” after zero position of current = θ
- Angle of half cycle = π radians
- Angle of full cycle = 2π radians

(a) Average value of complete cycle:

$$\begin{aligned}
 I_{AV} &= \frac{1}{2\pi} \int_0^{2\pi} i \, d\theta = \frac{1}{2\pi} \int_0^{2\pi} i_m \sin \theta \, d\theta \\
 &= \frac{i_m}{2\pi} [-\cos \theta]_0^{2\pi} = \frac{i_m}{2\pi} (\cos 2\pi - \cos 0) \\
 &= \frac{i_m}{2\pi} (1 - 1) = 0 ; I_{AV} = 0
 \end{aligned}$$

$$I = i_m \sin \theta$$

Thus, the average value of a sinusoidal wave over a complete cycle is zero.

(b) Average value of current over a half cycle

$$\begin{aligned}
 I_{AV} &= \frac{1}{\pi} \int_0^{\pi} i \, d\theta \\
 &= \frac{1}{\pi} \int_0^{\pi} i_m \sin \theta \, d\theta \quad [\because i = i_m \sin \theta] \\
 &= \frac{i_m}{\pi} [-\cos \theta]_0^{\pi} = \frac{i_m}{\pi} [-\cos \pi - (-\cos 0)] \\
 &= \frac{i_m}{\pi} [(+1) - (-1)] = \frac{i_m}{\pi} (+2) \\
 I_{AV} &= \frac{2}{\pi} i_m = 0.637 I_m
 \end{aligned}$$

Average Value of Current (Half Cycle)

$$I_{AV} = 0.637 I_m$$

Similarly, the average value of voltage over a half cycle

$$V_{AV} = 0.637 V_m$$

★ Average Voltage Value

$$V_{AV} = \frac{2V_P}{\pi} = 0.637 \times V_P$$

$$V_{AV} = \frac{2V_M}{\pi} = 0.637 \times V_M$$

★ Average Current Value

$$I_{AV} = \frac{2I_M}{\pi} = 0.637 \times I_M$$

What is Peak Voltage or Maximum Voltage Value?

Peak value is also known as **Maximum Value**, **Crest Value** or **Amplitude**. It is the maximum value of alternating current or voltage from the “0” position no matter positive or negative half cycle in a sinusoidal wave as shown in fig 8. It is expressed as **IM** and **EM** or **VP** and **IM**.

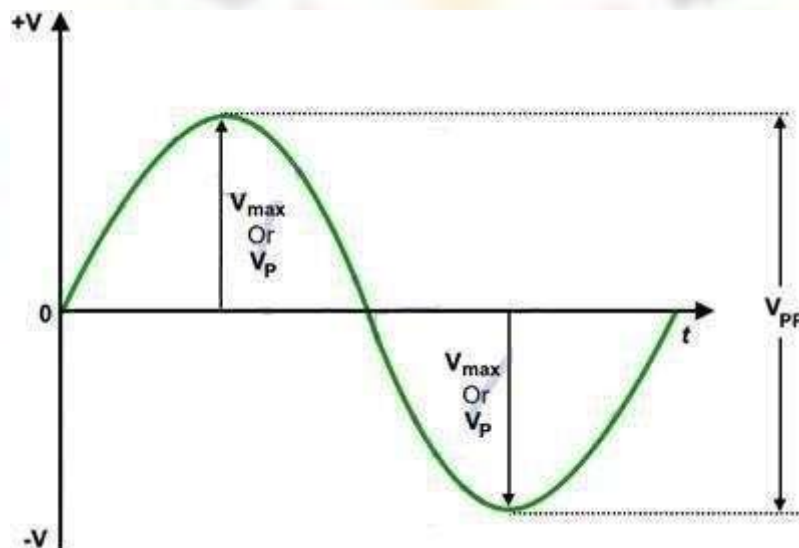
Equations of Peak Voltage Value is:

$$V_P = \sqrt{2} \times V_{RMS} = 1.414 V_{RMS}$$

$$V_P = V_{P-P} / 2 = 0.5 V_{P-P}$$

$$V_P = \pi / 2 \times V_{AV} = 1.571 \times V_{AV}$$

In other words, It is the value of voltage or current at the positive or the negative maximum



(peaks) with respect to zero. In simple words, it is the instantaneous value with maximum intensity. Fig 8–Peak or Maximum Values of Voltages

Peak to Peak Value:

The sum of positive and negative peak values is known as peak to peak value. It is expressed as **IPP** or **VPP**. Equations and formulas for Peak to Peak Voltage are as follow:

$$V_{P-P} = 2\sqrt{2} \times V_{RMS} = 2.828 \times V_{RMS} \quad V_{P-P} = 2 \times V_P$$

$$P = \pi \times V_{AV} = 3.141 \times V_{AV}$$

In other words, the peak to peak value of a sine wave, is the voltage or current from positive peak to the negative peak and its value is double as compared to peak value or maximum value

as shown in fig 8 above.

Peak Factor:

Peak Factor is also known as Crest Factor or Amplitude Factor.

$$\text{Peak Factor} = \frac{\text{Maximum Value}}{\text{R.M.S Value}}$$

It is the ratio between maximum value and RMS value of an alternating wave. For a sinusoidal alternating voltage:

$$\frac{E_M}{0.707 E_M} = 1.414$$

$$\frac{I_M}{0.707 I_M} = 1.414$$

For a sinusoidal alternating current:

Form Factor: The ratio between RMS value and Average value of an alternating quantity (Current or Voltage) is known as Form Factor.

$$\text{Form Factor} = \frac{\text{RMS Value}}{\text{Average Value}}$$

Apparent Power, True Power, Reactive Power and Power Factor

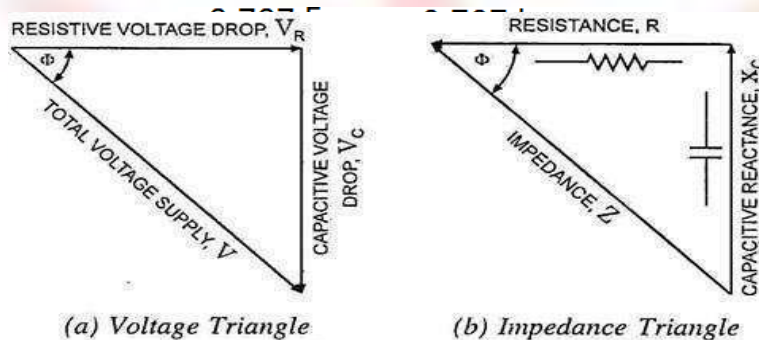


Fig. 4.19

The product of rms values of current and voltage, VI is called the apparent power and is measured in volt-amperes or kilo-voltamperes (kVA).

The true power in an ac circuit is obtained by multiplying the apparent power by the power factor and is expressed in watts or kilo-watts (kW).

The product of apparent power, VI and the sine of the angle between voltage and current, $\sin \phi$ is called the reactive power. This is also known as wattless power and is expressed in reactive volt-amperes or kilo-voltamperes reactive (kVAR).

i.e. Apparent power, $S = VI$ volt-amperes or $\frac{VI}{1,000}$ kVA

True power, $P = VI \cos \Phi$ watts or $\frac{VI \cos \Phi}{1,000}$ kW

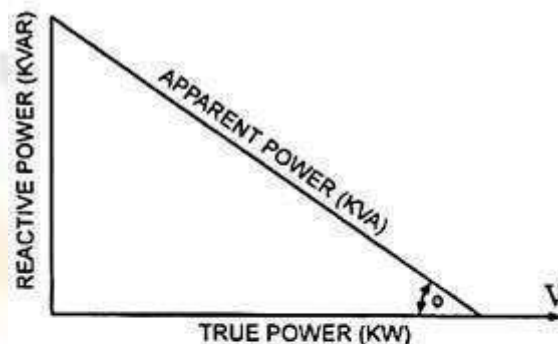
Reactive power, $Q = VI \sin \Phi$ VAR or $\frac{VI \sin \Phi}{1,000}$ kVAR

$$\text{and kVA} = \sqrt{(\text{kW})^2 + (\text{kVAR})^2}$$

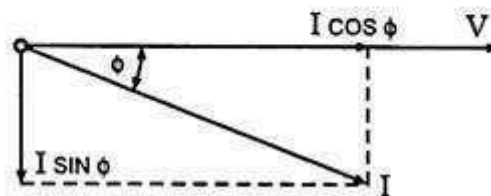
The above relations can easily be followed by referring to the power diagram shown in Fig. 4.7(a).

Power factor may be defined as:

- (i) Cosine of the phase angle between voltage and current,
- (ii) The ratio of the resistance to impedance, or



(a) Power Triangle



(b)

Fig. 4.7

- (iii) The ratio of true power to apparent power.

The power factor can never be greater than unity. The power factor is expressed either as a fraction or as a percentage. It is usual practice to attach the word 'lagging' or 'leading' with the numerical value of power factor to signify whether the current lags behind or leads the voltage.

CONCEPT OF REACTANCE, IMPEDANCE, SUSCEPTANCE AND ADMITTANCE:

Reactance is essentially inertia against the motion of electrons. It is present anywhere electric or magnetic fields are developed in proportion to applied voltage or current, respectively; but most notably in capacitors and inductors. When alternating current goes through a pure reactance, a voltage drop is produced that is 90° out of phase with the current. Reactance is mathematically symbolized by the letter "X" and is measured in the unit of ohms (Ω).

Impedance is a comprehensive expression of any and all forms of opposition to electron flow, including both resistance and reactance. It is present in all circuits, and in all components. When alternating current goes through an impedance, a voltage drop is produced that is somewhere between 0° and 90° out of phase with the current. Impedance is mathematically symbolized by the letter “Z” and is measured in the unit of ohms (Ω), in complex form

Admittance is also a complex number as impedance which is having a real part, Conductance (G)

and imaginary part, Susceptance (B). (it is negative for capacitive susceptance and positive for inductive susceptance)

$$Y = G + jB$$

$Y \rightarrow$ Admittance in Siemens

$$G \rightarrow \text{Conductance in Siemens} = \frac{R}{R^2 + X^2}$$

$$B \rightarrow \text{Susceptance in Siemens} = -\frac{X}{R^2 + X^2}$$

$$j^2 = -1$$

$$|Y| = \sqrt{G^2 + B^2} = \frac{1}{\sqrt{R^2 + X^2}}$$

$$\angle Y = \arctan\left(\frac{B}{G}\right) = \arctan\left(-\frac{X}{R}\right)$$

Susceptance (symbolized B) is an expression of the ease with which alternating current (AC) passes through a capacitance or inductance

Other Terms Related To AC Circuits Waveform

- The path traced by a quantity (such as voltage or current) plotted as a function of some variable (such as time, degree, radians, temperature etc.) is called waveform.

Cycle

- One complete set of positive and negative values of alternating quantity (such as voltage and current) is known as cycle.
- The portion of a waveform contained in one period of time is called cycle.
- A distance between two same points related to value and direction is known as cycle.
- A cycle is a complete alternation.

Period

- The time taken by an alternating quantity (such as current or voltage) to complete one cycle is called its time period “T”.
- It is inversely proportional to the Frequency “f” and denoted by “T” where the unit of time period is second.
- Mathematically;

$$T = 1/f$$

Frequency

- Frequency is the number of cycles passed through per second. It is denoted by “f” and has the unit cycle per second i.e. Hz (Hertz).
- The number of completed cycles in 1 second is called frequency.
- It is the number of cycles of alternating quantity per second in hertz.
- Frequency is the number of cycles that a sine wave completes in one second or the number of cycles that occurs in one second.

$$f = 1/T$$

Amplitude

- The maximum value, positive or negative, of an alternating quantity such as voltage or current is known as its amplitude. It is denoted by V_P , I_P or E_{MAX} and I_{MAX} .
- Alternation
- One half cycle of a sine wave (Negative or Positive) is known as an alternation which spans 180° degree.

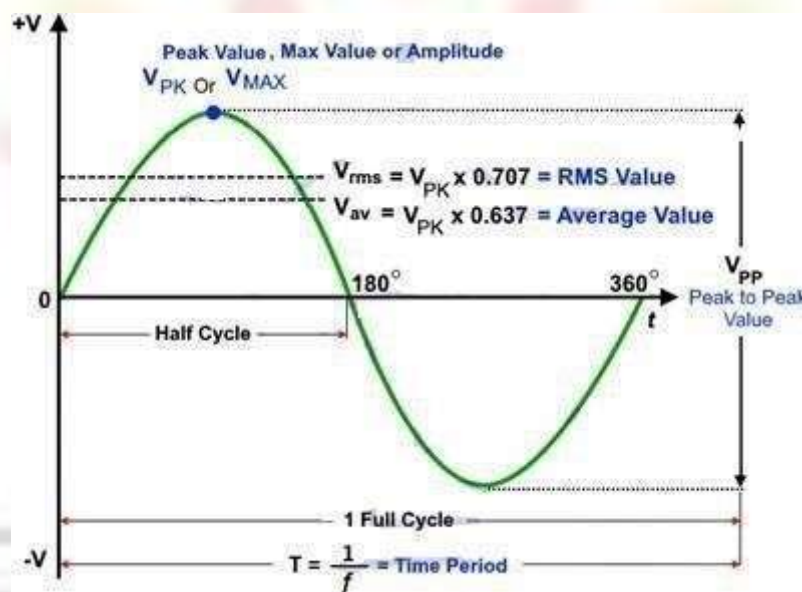


Fig9–Different Terms used in AC Circuits and Sine Wave

Introduction to Single Phase AC Circuit:

- In a dc circuit the relationship between the applied voltage V and current flowing through the circuit I is a simple one and is given by the expression $I = V/R$ but in an ac circuit this simple relationship does not hold good. Variations in current and applied voltage setup magnetic and electrostatic effects respectively and these must be taken into account with the resistance of the circuit while determining the quantitative relations between current and applied voltage.
- With comparatively low-voltage, heavy-current circuits magnetic effects may be very large, but electrostatic effects are usually negligible. On the other hand with high-voltage circuits electrostatic effects may be of appreciable magnitude, and magnetic effects are also present.

- Here it has been discussed how the magnetic effects due to variations in current do and electrostatic effects due to variations in the applied voltage affect the relationship between the applied voltage and current.

Purely Resistive Circuit:

- A purely resistive or a non-inductive circuit is a circuit which has inductance so small that at normal frequency its reactance is negligible as compared to its resistance. Ordinary filament lamps, water resistances etc., are the examples of non-inductive resistances. If the circuit is purely non-inductive, no reactance emf (i.e., self-induced or back emf) is set up and whole of the applied voltage is utilized in overcoming the ohmic resistance of the circuit.
- Consider an ac circuit containing a non-inductive resistance of R ohms connected across a sinusoidal voltage represented by $v = V \sin \omega t$, as shown in Fig.

As already said, when the current flowing through a pure resistance changes, no back emf is set up,

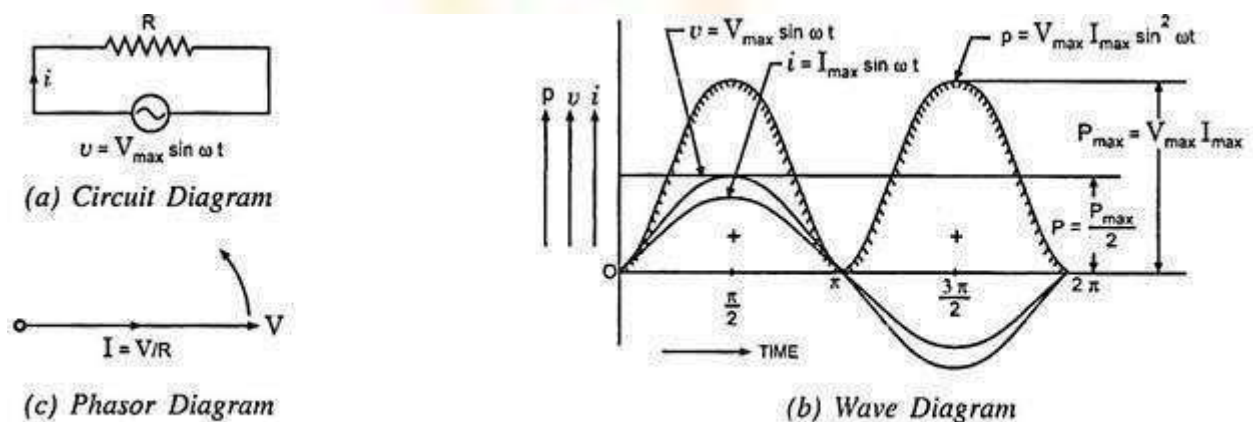


Fig. 4.1 Purely Resistive Circuit

i.e. $iR = v$

$$\text{or } i = \frac{v}{R} = \frac{V_{\max}}{R} \sin \omega t$$

Current will be maximum when $\omega t = \frac{\pi}{2}$ or $\sin \omega t = 1$

$$\therefore I_{\max} = \frac{V_{\max}}{R}$$

therefore, applied voltage has to overcome the ohmic drop of iR only:

And instantaneous current may be expressed as:

$$i = I_{\max} \sin \omega t$$

From the expressions of instantaneous applied voltage and instantaneous current, it is evident that in a pure resistive circuit, the applied voltage and current are in phase with each other, as shown by wave and phasor diagrams in Figs. 4.1(b) and (c) respectively.

Power in Purely Resistive Circuit:

The instantaneous power delivered to the circuit in question is the product of the instantaneous values of applied voltage and current.

$$\text{i.e. } p = v i = V_{\max} \sin \omega t I_{\max} \sin \omega t = V_{\max} I_{\max} \sin^2 \omega t$$

$$\begin{aligned} \text{or } p &= \frac{V_{\max} I_{\max}}{2} (1 - \cos 2 \omega t) & \text{Since } \sin^2 \omega t &= \frac{1 - \cos 2 \omega t}{2} \\ &= \frac{V_{\max} I_{\max}}{2} - \frac{V_{\max} I_{\max}}{2} \cos 2 \omega t \end{aligned}$$

$$\text{Average power, } P = \text{Average of } \frac{V_{\max} I_{\max}}{2} - \text{average of } \frac{V_{\max} I_{\max}}{2} \cos 2 \omega t$$

Since average of $\frac{V_{\max} I_{\max}}{2} \cos 2 \omega t$ over a complete cycle is zero,

$$P = \frac{V_{\max} I_{\max}}{2} = \frac{V_{\max}}{\sqrt{2}} \cdot \frac{I_{\max}}{\sqrt{2}} = V I \text{ watts}$$

Where V and I are the rms values of applied voltage and current respectively.

Thus for purely resistive circuits, the expression for power is the same as for dc circuits. From the power curve for a purely resistive circuit shown in Fig. 4.1 (b) it is evident that power consumed in a pure resistive circuit is not constant, it is fluctuating.

However, it is always positive. This is so because the instantaneous values of voltage and current are always either positive or negative and, therefore, the product is always positive. This means that the voltage source constantly delivers power to the circuit and the circuit consumes it.

Purely Inductive Circuit:

An inductive circuit is a coil with or without an iron core having negligible resistance. Practically pure inductance can never be had as the inductive coil has always small resistance. However, a coil of thick copper wire wound on a laminated iron core has negligible resistance and is known as an choke coil.

When an alternating voltage is applied to a purely inductive coil, an emf, known as self-induced emf, is induced in the coil which opposes the applied voltage. Since coil has no resistance, at every instant applied voltage has to overcome this self-induced emf only.

Let the applied voltage $v = V_{\max} \sin \omega t$
and self inductance of coil = L henry

Self induced emf in the coil, $e_L = -L \frac{di}{dt}$

Since applied voltage at every instant is equal and opposite to the self induced emf i.e. $v = -e_L$

$$\therefore V_{\max} \sin \omega t = - \left(-L \frac{di}{dt} \right)$$

$$\text{or } di = \frac{V_{\max}}{L} \sin \omega t dt$$

Integrating both sides we get

$$i = \frac{V_{\max}}{L} \int \sin \omega t dt = \frac{V_{\max}}{\omega L} (-\cos \omega t) + A$$

where A is a constant of integration, which is found to be zero from initial conditions

$$\text{i.e. } i = \frac{-V_{\max}}{\omega L} \cos \omega t = \frac{V_{\max}}{\omega L} \sin \left(\omega t - \frac{\pi}{2} \right)$$

Current will be maximum when $\sin \left(\omega t - \frac{\pi}{2} \right) = 1$, hence, maximum value of current,

$$I_{\max} = \frac{V_{\max}}{\omega L}$$

and instantaneous current may be expressed as $i = I_{\max} \sin \left(\omega t - \frac{\pi}{2} \right)$

From the expressions of instantaneous applied voltage and instantaneous current flowing through a purely inductive coil it is observed that the current lags behind the applied voltage by $\pi/2$ as shown in Fig. 4.2(b) by wave diagram and in Fig 4.2(c) by phasor diagram.



Inductive Reactance:

ωL in the expression $I_{\max} = V_{\max} / \omega L$ is known as inductive reactance and is denoted by X_L i.e., $X_L = \omega L$

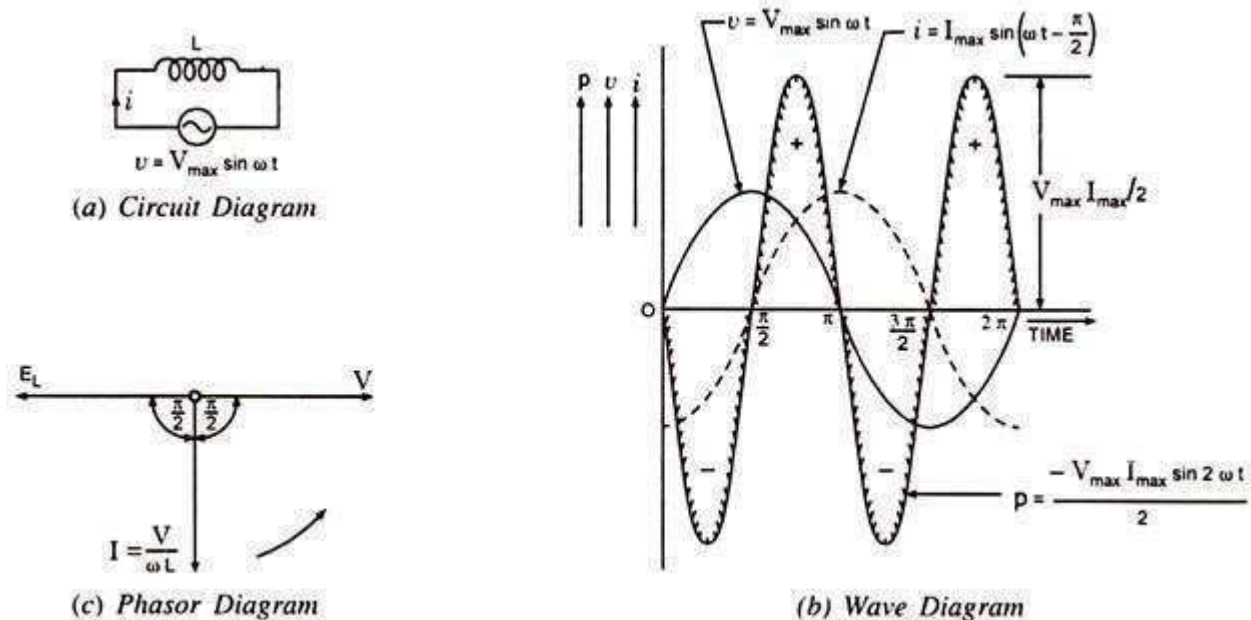


Fig. 4.2 Purely Inductive Circuit

If L is in henry and ω is in radians per second then X_L will be in ohms.

Power in Purely Inductive Circuit:

Instantaneous power, $p = v \times i = V_{\max} \sin \omega t I_{\max} \sin(\omega t - \pi/2)$ Or $p = -V_{\max} I_{\max} \sin \omega t \cos \omega t = -V_{\max} I_{\max} / 2 \sin 2\omega t$

The power measured by wattmeter is the average value of p which is zero since average of a sinusoidal quantity of double frequency over a complete cycle is zero. Hence in a purely inductive circuit power absorbed is zero.

Physically the above fact can be explained as below:

During the second quarter of a cycle the current and the magnetic flux of the coil increases and the coil draws power from the supply source to build up the magnetic field (the power drawn is positive and the energy drawn by the coil from the supply source is represented by the area between the curve p and the time axis). The energy stored in the magnetic field during buildup is given as $W_{\max} = 1/2 L I_{\max}^2$.

In the next quarter the current decreases. The emf of self-induction will, however, tend to oppose its

decrease. The coil acts as a generator of electrical energy, returning the stored energy in the magnetic field to the supply source (now the power drawn by the coil is negative and the curve p lies below the time axis). The chain of events repeats itself during the next half cycles. Thus, a proportion of power is continually exchanged between the field and the inductive circuit and the power consumed by a purely inductive coil is zero.

Purely Capacitive Circuit:

When a dc voltage is impressed across the plates of a perfect condenser, it will become charged to full voltage almost instantaneously. The charging current will flow only during the period of “build up” and will cease to flow as soon as the capacitor has attained the steady voltage of the source. This implies that for a direct current, a capacitor is a break in the circuit or an infinitely high resistance.

In Fig. 4.4 a sinusoidal voltage is applied to a capacitor. During the first quarter-cycle, the applied voltage increases to the peak value, and the capacitor is charged to that value. The current is maximum in the beginning of the cycle and becomes zero at the maximum value of the applied voltage, so there is a phase difference of 90° between the applied voltage and current. During the first quarter-cycle the current flows in the normal direction through the circuit; hence the current is positive.

In the second quarter-cycle, the voltage applied across the capacitor falls, the capacitor loses its charge, and current flows through it against the applied voltage because the capacitor discharges into the circuit. Thus, the current is negative during the second quarter-cycle and attains a maximum value when the applied voltage is zero.

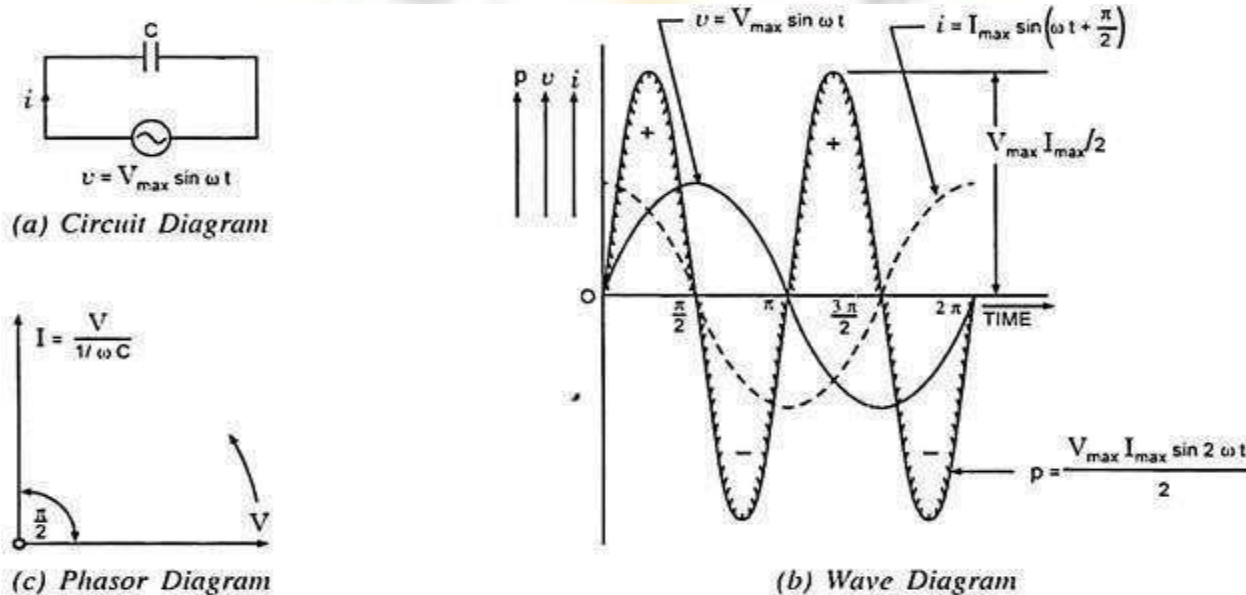


Fig. 4.4 Purely Capacitive Circuit

The third and fourth quarter-cycles repeat the events of the first and second, respectively, with the difference that the polarity of the applied voltage is reversed, and there are corresponding current changes.

In other words, an alternating current flow in the circuit because of the charging and discharging of the capacitor. As illustrated in Figs. 4.4 (b) and (c) the current begins its cycle 90° ahead of the voltage, so the current in a capacitor leads the applied voltage by 90° – the opposite of the inductance current-voltage relationship.

Let an alternating voltage represented by $v = V_{\max} \sin \omega t$ be applied across a capacitor of capacitance C farads.

The expression for instantaneous charge is given as:

$$q = CV_{\max} \sin \omega t$$

Since the capacitor current is equal to the rate of change of charge, the capacitor current may be obtained by differentiating the above equation:

From the equations of instantaneous applied voltage and instantaneous current flowing through capacitance, it is observed that the current leads the applied voltage by $\pi/2$, as shown in Figs. 4.4 (b) and (c) by wave and phasor diagrams respectively.

$$i = \frac{dq}{dt} = [C V_{\max} \sin \omega t] = \omega C V_{\max} \cos \omega t = \frac{V_{\max}}{1/\omega C} \sin \left(\omega t + \frac{\pi}{2} \right)$$

Current is maximum when $t = 0$

$$\therefore I_{\max} = \frac{V_{\max}}{1/\omega C}$$

Substituting $\frac{V_{\max}}{1/\omega C} = I_{\max}$ in the above equation for instantaneous current, we get

$$i = I_{\max} \sin \left(\omega t + \frac{\pi}{2} \right)$$

Capacitive Reactance:

$1/\omega C$ in the expression $I_{\max} = V_{\max}/1/\omega C$ is known as capacitive reactance and is denoted by X_C . i.e., $X_C = 1/\omega C$ if C is in farads and ω is in radians/s, then X_C will be in ohms.

Power in Purely Capacitive Circuit:

$$\begin{aligned} p = v i &= V_{\max} \sin \omega t \cdot I_{\max} \sin \left(\omega t + \frac{\pi}{2} \right) = V_{\max} I_{\max} \sin \omega t \cos \omega t \\ &= \frac{V_{\max} I_{\max}}{2} \sin 2\omega t \end{aligned}$$

Average power, $P = \frac{V_{\max} I_{\max}}{2} \times \text{average of } \sin 2\omega t \text{ over a complete cycle} = 0.$

Hence power absorbed in a purely capacitive circuit is zero. The same is shown graphically in Fig. 4.4(b). The energy taken from the supply circuit is stored in the capacitor during the first quarter-cycle and returned during the next.

The energy stored by a capacitor at maximum voltage across its plates is given by the expression:

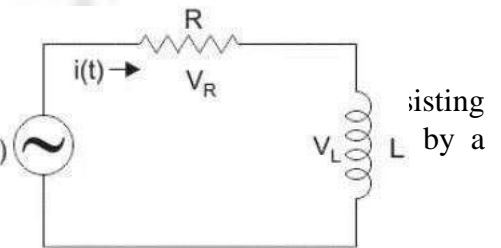
$$W_C = \frac{1}{2} C V_{\max}^2$$

This can be realized when it is recalled that no heat is produced and no work is done while current is flowing through a capacitor. As a matter of fact, in commercial capacitors, there is a slight energy loss in the dielectric in addition to a minute $I^2 R$ loss due to flow of current over the plates having definite ohmic resistance.

The power curve is a sine wave of double the supply frequency. Although it raises the power factor from zero to 0.002 or even a little more, but for ordinary purposes the power factor is taken to be zero. Obviously the phase angle due to dielectric and ohmic losses decreases slightly.

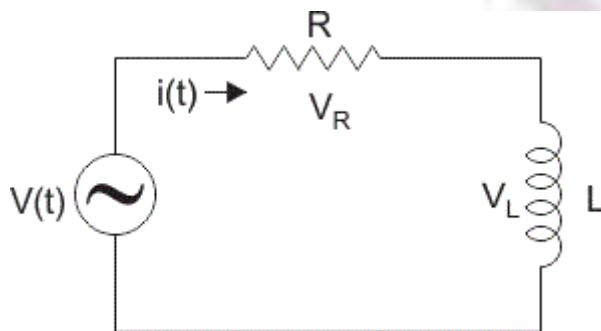
RL Series Circuit Analysis (Phasor Diagram, Examples & Derivation)

An RL circuit (also known as an RL filter or RL network) is composed of the passive circuit elements of a resistor (R) and an inductor (L) by a voltage source or current source.



Due to the presence of a resistor in the ideal form of the circuit, an RL circuit will consume energy, akin to an RC circuit or RLC circuit.

This is unlike the ideal form of an LC circuit, which will consume no energy due to the absence of a resistor. Although this is only in the ideal form of the circuit, and in practice, even an LC circuit will consume some energy because of the non-zero resistance of the components and connecting wires.



Consider a simple RL circuit in which resistor, R and inductor, L are connected in series with a voltage supply of V volts. Let us think the current flowing in the circuit is I (amp) and current through resistor and inductor is IR and IL respectively. Since both resistance and inductor are connected in series, so the current in both the elements and the circuit remains the same. i.e. $IR = IL = I$. Let V_R and V_L be the voltage drop across resistor and inductor.

Applying Kirchhoff's voltage law (i.e. sum of voltage drop must be equal to applied voltage) to this circuit

$$V = V_R + V_L$$

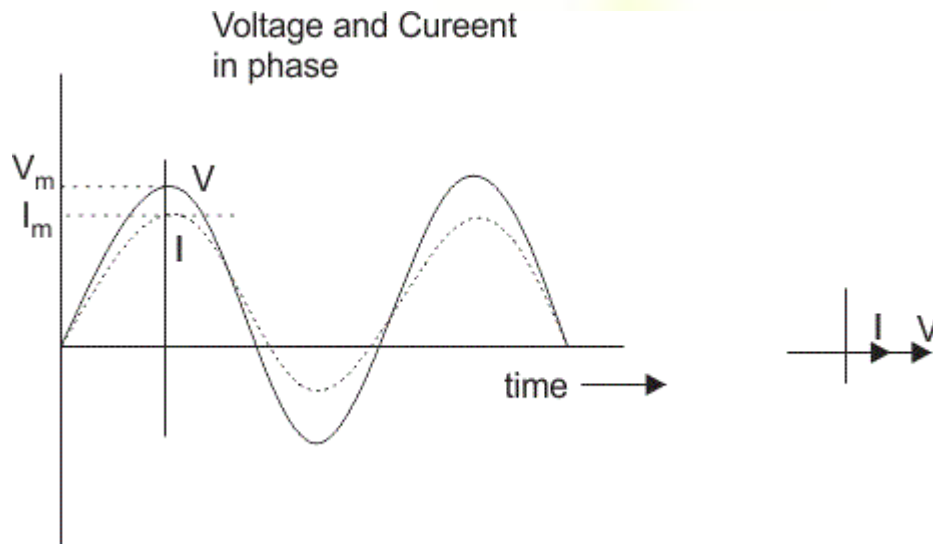
we get,

Phasor Diagram for RL Circuit

Before drawing the **phasor diagram of series RL circuit**, one should know the relationship between voltage and current in case of resistor and inductor.

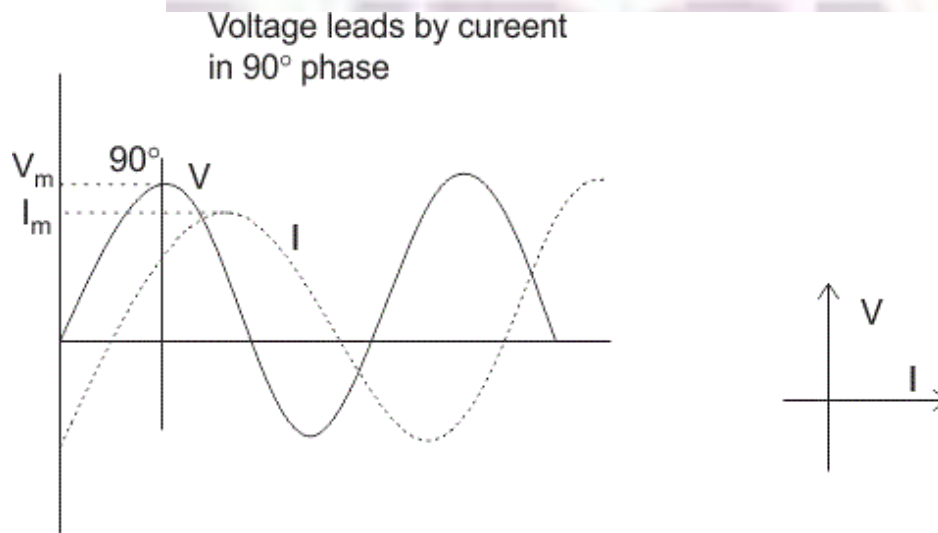
Resistor

In case of resistor, the voltage and the current are in same phase or we can say that the phase angle difference between voltage and current is zero.



Inductor

In inductor, the voltage and the current are not in phase. The voltage leads that of current by 90° or in other words, voltage attains its maximum and zero value 90° before the current attains it.



RL Circuit

For drawing the phasor diagram of series RL circuit; follow the following steps:

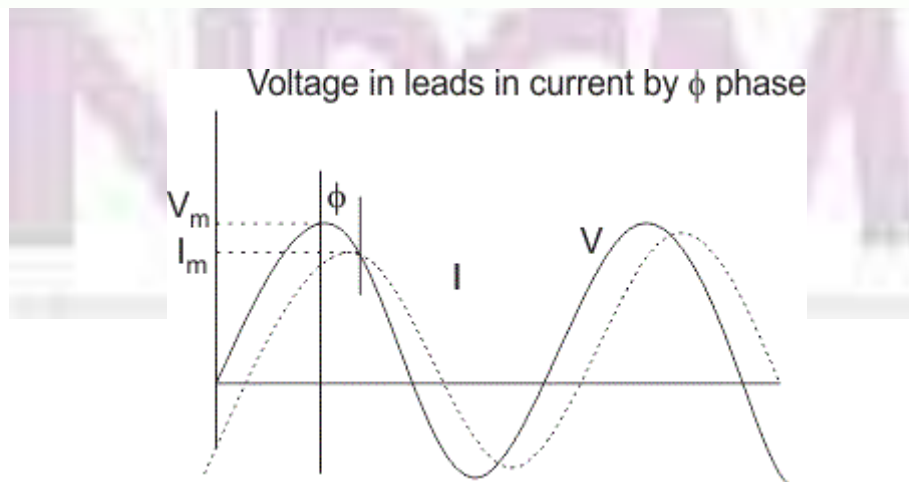
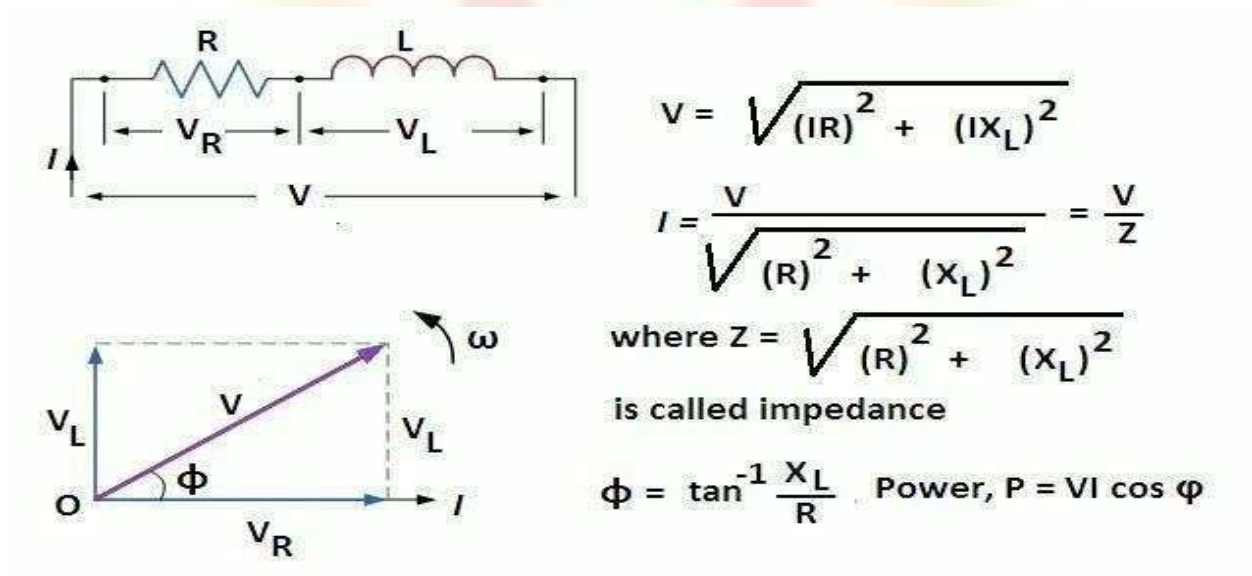
Step- I. In case of series RL circuit, resistor and inductor are connected in series, so current flowing in both the elements are same i.e. $I_R = I_L = I$. So, take current phasor as reference and draw it on horizontal axis as shown in diagram.

Step- II. In case of resistor, both voltage and current are in same phase. So draw the voltage phasor, V_R along same axis or direction as that of current phasor. i.e. V_R is in phase with I . **Step- III.** We know that in inductor, voltage leads current by 90° , so draw V_L (voltage drop across inductor) perpendicular to current phasor.

Step- IV. Now we have two voltages V_R and V_L . Draw the resultant vector (V) of these two voltages. Such as,

$$V_R^2 + V_L^2 = V^2 \text{ (Pythagorean theorem)}$$

$$\theta = \tan^{-1}(V_L/V_R)$$



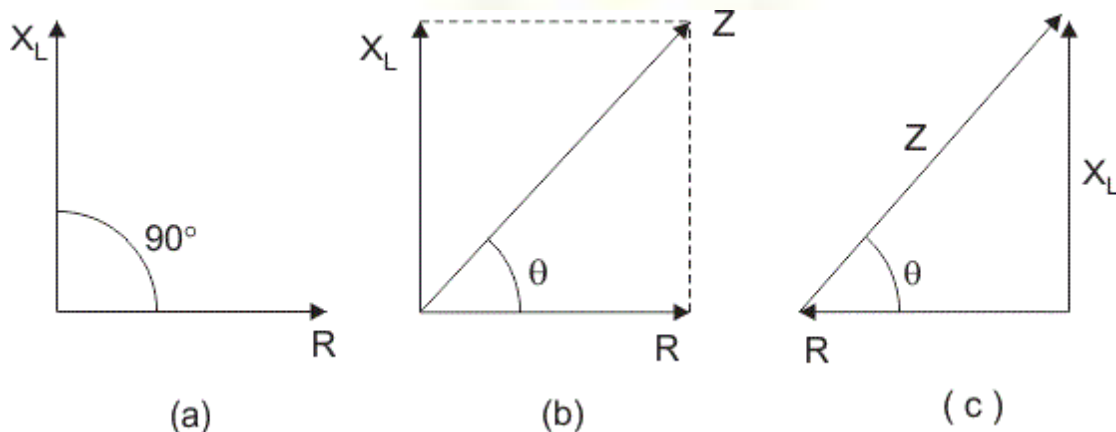
CONCLUSION: In case of pure resistive circuit, the phase angle between voltage and current is zero and in case of pure inductive circuit, phase angle is 90° but when we combine both resistance and inductor, the phase angle of a series RL circuit is between 0° to 90° .

Impedance of Series RL Circuit

The impedance of series RL circuit opposes the flow of alternating current.

The impedance of series RL circuit is nothing but the combined effect of resistance (R) and inductive reactance (X_L) of the circuit as a whole. The impedance Z in ohms is given by,

$Z = \sqrt{R^2 + X_L^2}$ and from right angle triangle, phase angle $\theta = \tan^{-1}(X_L/R)$.



Series RL Circuit Analysis

In series RL circuit, the values of frequency f , voltage V , resistance R and inductance L are known and there is no instrument for directly measuring the value of inductive reactance and impedance; so, for complete analysis of series RL circuit, follow these simple steps:

Step 1. Since the value of frequency and inductor are known, so firstly calculate the value of inductive reactance X_L : $X_L = 2\pi fL$ ohms.

Step 2. From the value of X_L and R , calculate the total impedance of the circuit which is given by

$$Z = \sqrt{R^2 + X_L^2}$$

Step 3. Calculate the total phase angle for the circuit $\theta = \tan^{-1}(X_L/R)$.

Step 4. Use Ohm's Law and find the value of the total current: $I = V/Z$ amp.

Step 5. Calculate the voltages across resistor R and inductor L by using Ohm's Law. Since the

$$V_R = RI \text{ Volts and } V_L = X_L I \text{ Volts}$$

resistor and the inductor are reconnected in series, so current in them remains the same.

Power in an RL Circuit

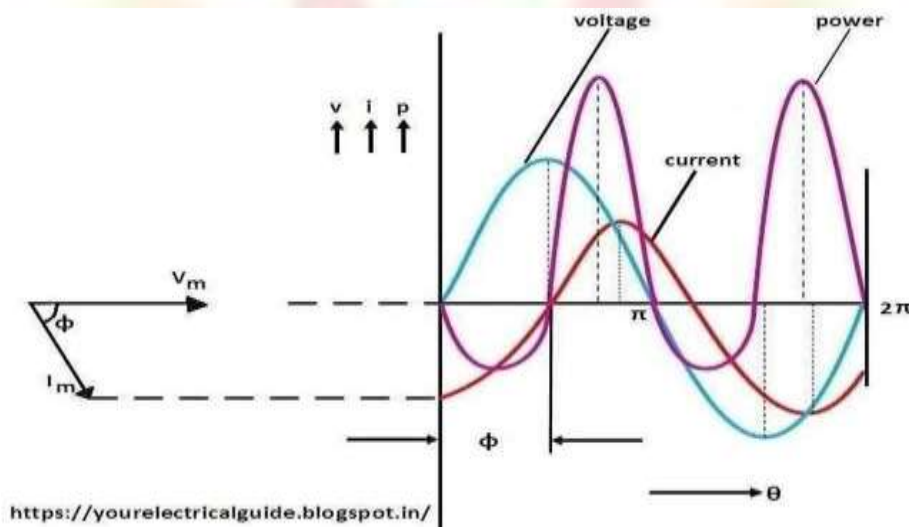
In series RL circuit, some energy is dissipated by the resistor and some energy is alternately stored and returned by the inductor-

1. The instantaneous power delivered by voltage source is $P = VI$ (watts).
2. Power dissipated by the resistor in the form of heat, $P = I^2 R$ (watts).
3. The rate at which energy is stored in inductor,

$$P = V_L I = LI \frac{dI}{dt} \text{ (watts)}$$

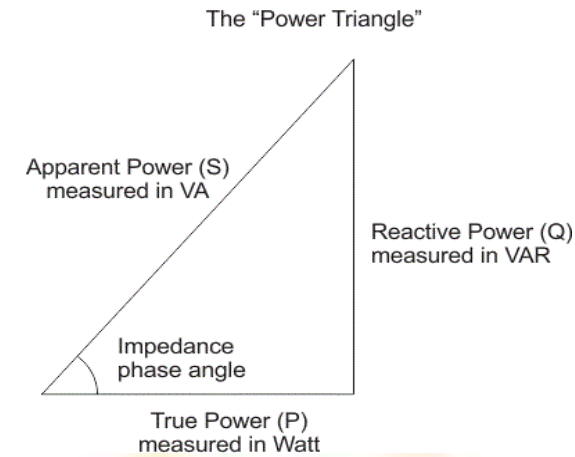
Power in series RL circuit is given by adding the power dissipated by the resistor and the power by the inductor.

$$P = I^2 R + LI \frac{dI}{dt}$$



The power waveform for RL series circuit is shown in the figure. In this figure, voltage wave is considered as a reference. The points for the power waveform are obtained from the product of the corresponding instantaneous values of voltage and current.

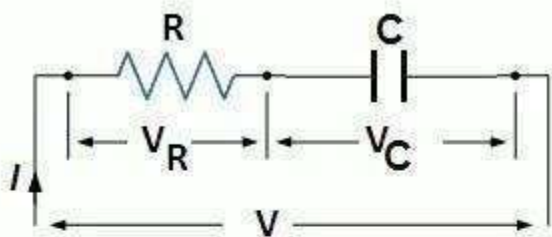
It is clear from the power waveform that power is negative between 0 and ϕ and between 180° and $(180^\circ + \phi)$. The power is positive during the rest of the cycle.



Power triangle for series RL circuit is shown below,

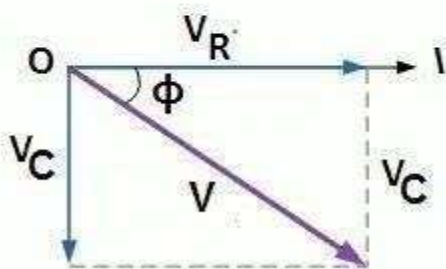
The electrical power factor $\cos\theta$ is defined as a ratio of the true power to apparent power.

RC Series Circuit



$$V = \sqrt{(IR)^2 + (IX_C)^2}$$

$$I = \frac{V}{\sqrt{(R)^2 + (X_C)^2}} = \frac{V}{Z}$$



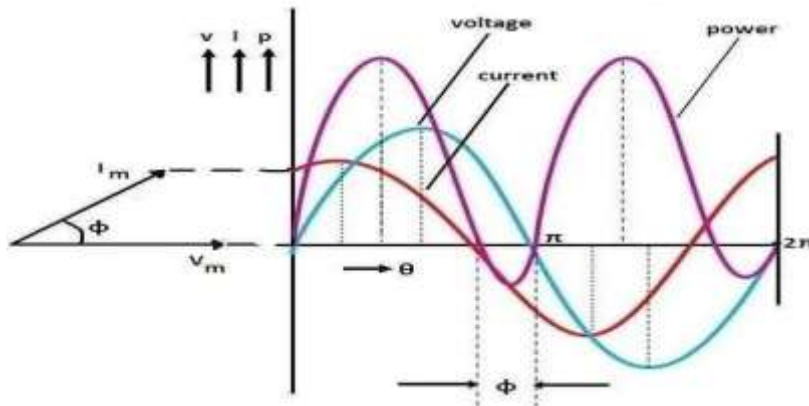
$$\text{where } Z = \sqrt{(R)^2 + (X_C)^2}$$

is called impedance

$$\phi = \tan^{-1} \frac{X_C}{R} \quad \text{Power, } P = VI \cos \phi$$

In an RC series circuit, a pure resistance (R) is connected in series with a pure capacitor (C). To draw the phasor diagram of RC series circuit, the current I (RMS value) is taken as reference vector. Voltage drop V_R is in phase with current vector, whereas, the voltage drop in capacitive reactance V_C lags behind the current vector by 90° , since

current leads the voltage by 90° in the pure capacitive circuit. The vector sum of these two voltage drops is equal to the applied voltage V (RMS value).



The power waveform for RC series circuit is shown in the figure. In this figure, voltage wave is considered as a reference. The points for the power waveform are obtained from the product of the corresponding instantaneous values of voltage and current. It is clear from the power waveform that power is negative between $(180^\circ - \phi)$ and 180° and between $(360^\circ - \phi)$ and 360° . The power is positive during the rest of the cycle.

Since the area under the positive loops is greater than that under the negative loops, the net power over a complete cycle is positive. Hence a definite quantity of power is consumed by the RC series circuit. But power is consumed in resistance only; capacitor does not consume any power.

RLC CIRCUIT:

In an **RLC circuit**, the most fundamental elements of a resistor, inductor, and capacitor are connected across a voltage supply. All of these elements are linear and passive in nature. Passive components are ones that consume energy rather than producing it; linear elements are those which have a linear relationship between voltage and current.

There are number of ways of connecting these elements across voltage supply, but the most common method is to connect these elements either in series or in parallel. The **RLC circuit** exhibits the property of resonance in same way as LC circuit exhibits, but in this circuit the oscillation dies out quickly as compared to LC circuit due to the presence of resistor in the circuit.

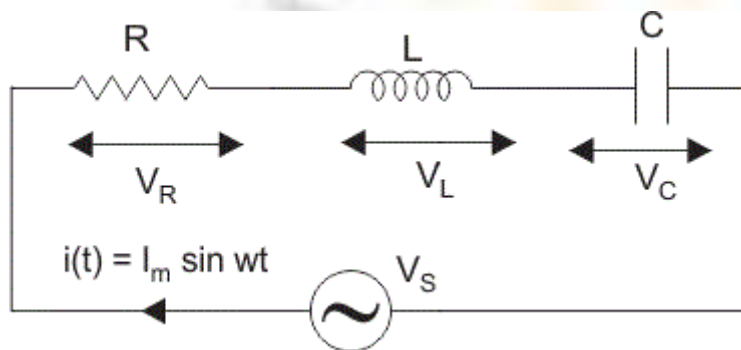
Series RLCCircuit

When a resistor, inductor and capacitor are connected in series with the voltage supply, the circuit so formed is called series RLC circuit.

Since all these components are connected in series, the current in each element remains the same,

$$I_R = I_L = I_C = I(t) \text{ where } I(t) = I_M \sin \omega t$$

Let V_R be the voltage across resistor, R . V_L be the voltage across inductor, L . V_C be the voltage across capacitor, C . X_L be the inductive reactance.



X_C be the capacitive reactance.

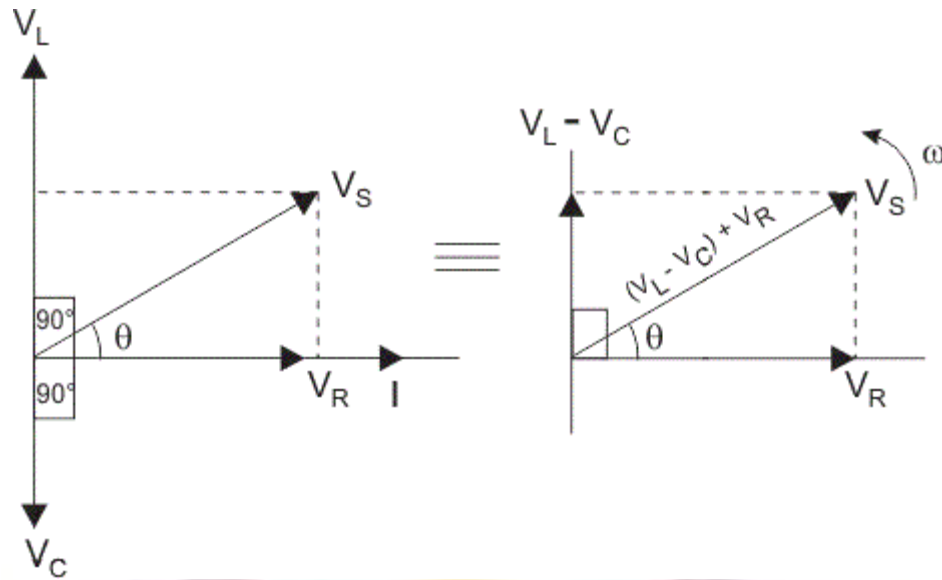
The total voltage in the RLC circuit is not equal to the algebraic sum of voltages across the resistor, the inductor, and the capacitor; but it is a vector sum because, in the case of the resistor the voltage is in phase with the current, for inductor the voltage leads the current by 90° and for capacitor, the voltage lags behind the current by 90° .

So, voltages in each component are not in phase with each other; so they cannot be added arithmetically. The figure below shows the phasor diagram of the series RLC circuit. For drawing the phasor diagram for RLC series circuit, the current is taken as reference because, in series circuit the current in each element remains the same and the corresponding voltage vectors for each component are drawn in reference to common current vector.

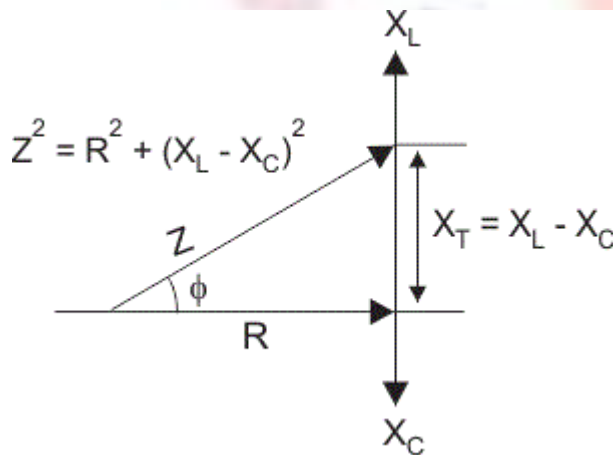
$$V_S^2 = V_R^2 + (V_L - V_C)^2 \text{ (if } V_L > V_C \text{)}$$

$$V_S^2 = V_R^2 + (V_L - V_C)^2 \text{ (if } V_L < V_C \text{)}$$

Where $V_R = IR$, $V_L = IX_L$, $V_C = IX_C$



The Impedance for a Series RLC Circuit



The impedance Z of a series RLC circuit is defined as opposition to the flow of current due to circuit resistance R , inductive reactance, X_L and capacitive reactance, X_C . If the inductive reactance is greater than the capacitive reactance i.e. $X_L > X_C$, then the RLC circuit has a lagging phase angle and if the capacitive reactance is greater than the inductive reactance i.e. $X_C > X_L$ then, the RLC circuit has a leading phase angle and if both inductive and capacitive are same i.e. $X_L = X_C$ then the circuit will behave as a purely resistive circuit.

We know that

$$V_S^2 = V_R^2 + (V_L - V_C)^2$$

Where,

$$V_R = IR, V_L = I X_L, V_C = I X_C$$

Substituting the values

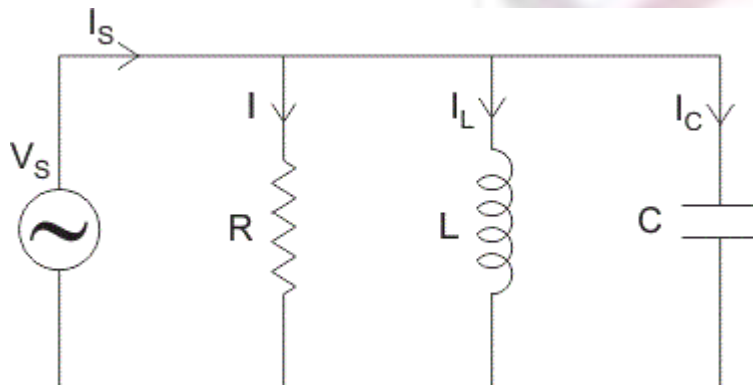
$$V_S^2 = IR^2 + (I X_L - I X_C)^2$$

$$V_S = I \sqrt{R^2 + (X_L - X_C)^2} \text{ or impedance } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Parallel RLC Circuit

In parallel RLC Circuit the resistor, inductor and capacitor are connected in parallel across a voltage supply. The parallel RLC circuit is exactly opposite to the series RLC circuit. The applied voltage remains the same across all components and the supply current gets divided.

The total current drawn from the supply is not equal to mathematical sum of the current flowing in the individual component, but it is equal to its vector sum of all the currents, as the current flowing in resistor, inductor and capacitor are not in the same phase with each other; so they cannot be added arithmetically.



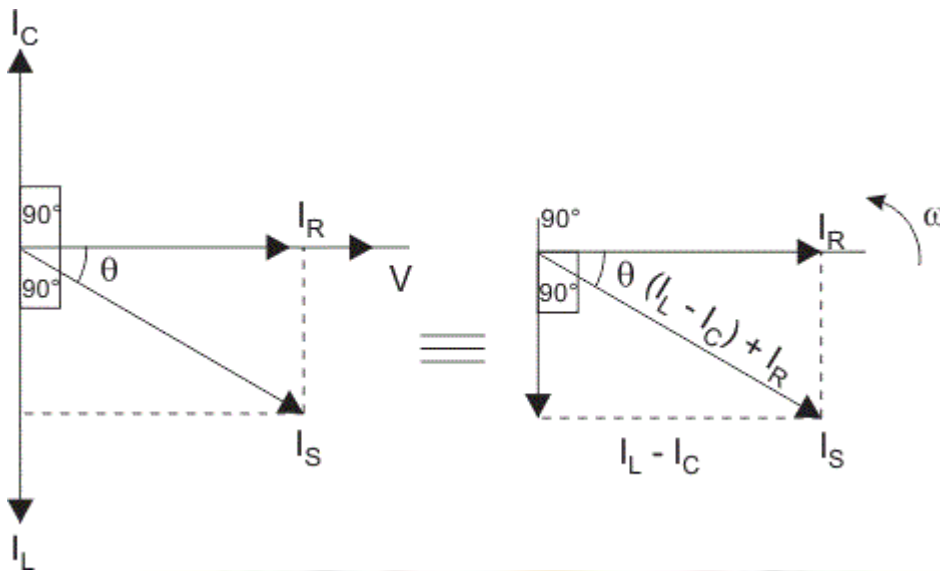
Phasor diagram of parallel RLC circuit, I_R is the current flowing in the resistor, I_R in amps. I_C is the current flowing in the capacitor, I_C in amps.

I_L is the current flowing in the inductor, I_L in amps.

I_S is the supply current in amps. In the parallel RLC circuit, all the components are connected in parallel; so the voltage across each element is same. Therefore, for drawing phasor diagram, take voltage as reference vector and all the other currents i.e. I_R , I_C , I_L are drawn relative to this voltage vector. The current

through each element can be found using Kirchhoff's Current Law,

which states that the sum of currents entering a junction or node is equal to the sum of current leaving that node.



$$I_S^2 = I_R^2 + (I_L - I_C)^2$$

$$\text{Now, } I_R = \frac{V}{R}, I_C = \frac{V}{X_C} \text{ and } I_L = \frac{V}{X_L}$$

$$I_S = \sqrt{\frac{V^2}{R^2} + \left(\frac{V}{X_L} - \frac{V}{X_C} \right)^2}$$

$$\text{So, admittance, } \frac{1}{Z} = \frac{I_S}{V} = Y = \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C} \right)^2}$$

As shown above in the equation of impedance, Z of a parallel RLC circuit; each element has reciprocal of impedance (1 / Z) i.e. admittance, Y. So in parallel RLC circuit, it is convenient to use admittance instead of impedance.

Resonance in RLCCircuit

In a circuit containing inductor and capacitor, the energy is stored in two different ways.

1. When a current flows in an inductor, energy is stored in magnetic field.

When a capacitor is charged, energy is stored in static electric field. The magnetic field in the inductor is built by the current, which gets provided by the discharging capacitor. Similarly, the capacitor is charged by the current produced by collapsing magnetic field of inductor and this process continues on and on, causing electrical energy to oscillate between the magnetic field and the electric field.

In some cases at certain a certain frequency known as the resonant frequency, the inductive reactance of the circuit becomes equal to capacitive reactance which causes the electrical energy to oscillate between the electric field of the capacitor and magnetic field of the inductor. This forms a harmonic oscillator for current. In **RLC circuit**, the presence of resistor causes these oscillations to die out over period of time and it is called as the damping effect of resistor.

FORMULA FOR RESONANT FREQUENCY

$$X_L = X_C$$

We know that, $X_L = 2\pi fL$ and $X_C = \frac{1}{2\pi fC}$

Therefore at resonant frequency, f_r : $2\pi f_r L = \frac{1}{2\pi f_r C}$

or $f = \frac{1}{2\pi\sqrt{LC}}$

During resonance, at certain frequency called resonant frequency, f_r .

When resonance occurs, the inductive reactance of the circuit becomes equal to capacitive reactance, which causes the circuit impedance to be minimum in case of series RLC circuit; but when resistor, inductor and capacitor are connected in parallel, the circuit impedance becomes maximum, so the parallel RLC circuit is sometimes called as anti-resonator. Note

that the lowest resonant frequency of a vibrating object is known as its fundamental frequency

DIFFERENCE BETWEEN SERIES RLC CIRCUIT AND PARALLEL RLC CIRCUIT

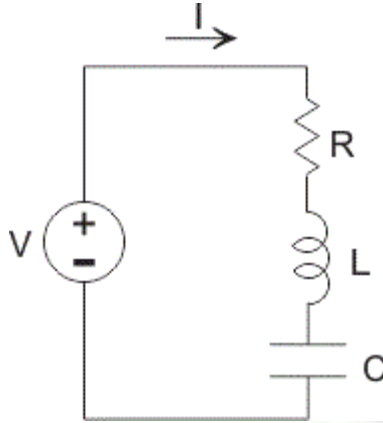
RLC SERIES CIRCUIT	RLC PARALLEL CIRCUIT
1. Resistor, inductor and capacitor are connected in series	Resistor, inductor and capacitor are connected in parallel
2. Current is same in each element	Current is different in all elements and the total current is equal to vector sum of each branch of current. i.e. $I^2 = I_R^2 + (I_L - I_C)^2$

3	<p>Voltage across all the elements is different and the total voltage is equal to the vector sum of</p> $V_s = \sqrt{V_R^2 + (V_L - V_C)^2}$ <p>For drawing phasor diagram, current is taken as reference vector</p> <p>Voltage across each element is given by : $V_R = IR, V_L = I X_L, V_C = I X_C$</p> <p>It's more convenient to use impedance for calculations</p> <p>At resonance, when $X_L = X_C$, the circuit has minimum impedance</p>	<p>Voltage across each element remains the same</p> <p>For drawing phasor diagram, voltage is taken as reference vector</p> <p>Current in each element is given by : $I_R = V/R, I_C = V/X_C, I_L = V/X_L$</p> <p>It's more convenient to use admittance for calculations</p> <p>At resonance, when $X_L = X_C$, the circuit has maximum impedance</p>
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Equation of RLCCircuit

Consider a **RLC circuit** having resistor R , inductor L , and capacitor C connected in series and are driven by a voltage source V . Let Q be the charge on the capacitor and the current flowing in the circuit



$$L.I'(t) + Q.I(t) + \frac{1}{C}Q(t) = V(t)$$

is I. Apply Kirchhoff's voltage law

In this equation; resistance, inductance, capacitance and voltage are known quantities but current and charge are unknown quantities. We know that an current is a rate of electric charge flowing, so it is given by

Differentiating again $I'(t) = Q''(t)$

$$L.Q''(t) + R.Q'(t) + \frac{1}{C}Q(t) = V(t)$$

Differentiating the above equation with respect to 't' we get,

$$L.I''(t) + R.I'(t) + \frac{1}{C}I(t) = V'(t)$$

Now at time $t = 0$, $V(0) = 0$ and at time $t = t$, $V(t) = E_o \sin \omega t$ Differentiating with respect to 't' we get $V'(t) = \omega E_o \cos \omega t$ Substitute the value of $V'(t)$ in above equation

$$L.I''(t) + R.I'(t) + \frac{1}{C}I(t) = \omega E_o \cos \omega t$$

Let us say that the solution of this equation is $I_P(t) = A \sin(\omega t - \phi)$ and if $I_P(t)$ is a solution of above equation then it must satisfy this equation,

$$L.I_P(t) + R.I_P(t) + \frac{1}{C}I_P(t) = \omega E_o \cos \omega t$$

Now substitute the value of $I_P(t)$ and differentiate it we get,

$$\frac{dQ}{dt}(t) = I(t) \text{ or } I(t) = Q(t)$$

$$-L\omega^2 A \sin(\omega t - \phi) + R\omega A \cos(\omega t - \phi) + \frac{1}{C} A \sin(\omega t - \phi) = \omega E_o \cos \omega t$$

$$-L\omega^2 A \sin(\omega t - \phi) + R\omega A \cos(\omega t - \phi) + \frac{1}{C} A \sin(\omega t - \phi) = \omega E_o \cos(\omega t - \phi + \phi)$$

Apply the formula of $\cos(A+B)$ and combine similar terms we get,

$$\left(\frac{1}{C} - L\omega^2\right) A \sin(\omega t - \phi) + R\omega A \cos(\omega t - \phi) = \omega E_o \cos \phi \cos(\omega t - \phi) - \omega E_o \sin \phi \sin(\omega t - \phi)$$

Match the coefficient of $\sin(\omega t - \phi)$ and $\cos(\omega t - \phi)$ on both sides we get,

$$\left(-\frac{1}{C} + 2L\omega\right) A = \omega E_o \sin \phi \text{ and } R\omega A = \omega E_o \cos \phi$$

Now we have two equations and two unknowns i.e ϕ and A , and by dividing the above two equations we get,

$$\tan \phi = \frac{-\frac{1}{C} + 2L\omega}{R\omega}$$

Squaring and adding above equation, we get

$$A \sqrt{\left(-\frac{1}{C} + 2L\omega\right)^2 + (R\omega)^2} = \omega E_o$$

$$\text{or } A = \frac{\omega E_o}{\sqrt{\left(-\frac{1}{C} + 2L\omega\right)^2 + (R\omega)^2}}$$

Analysis of RLCC Circuit Using

Laplace Transformation Step 1: Draw a phasor diagram for given circuit.

Step 2: Use Kirchhoff's voltage law in RLC series circuit and current law in RLC parallel circuit to form differential equations in the time-domain.

Step 3 : Use [Laplace transformation](#) to convert these differential equations from time-domain into the s-domain.

Step 4: For finding unknown variables, solve these equations.

Step 5: Apply inverse Laplace transformation to convert back equations from s-domain into time domain.

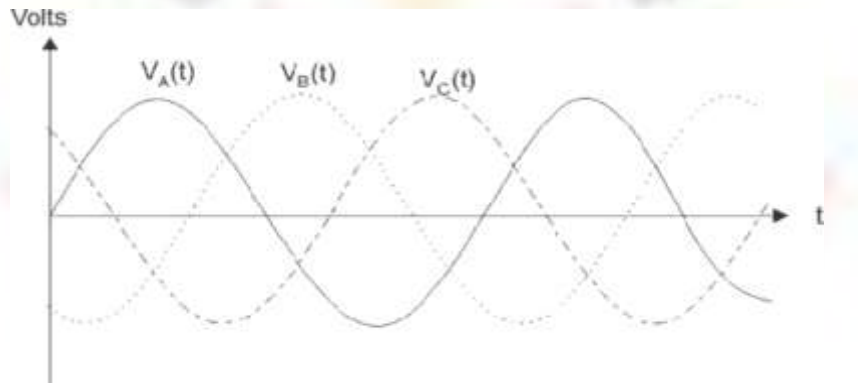
Applications of RLCC Circuit

It is used as [low pass filter](#), [high pass filter](#), [band-pass filter](#), [band-stop filter](#), [voltage multiplier](#) and [oscillator](#) circuit. It is used for tuning radio or audio receiver.

BALANCED THREE PHASE CIRCUITS:

There are two types of systems available in electrical circuits, single phase and three phase. In single phase circuits, there will be only one phase, i.e. the current will flow through only one wire and there will be one return path called neutral line to complete the circuit. So in single phase minimum amount of power can be transported. Here the generating station and load station will also be single phase.

Three phase circuit is the polyphase system where the three phases are sent together from generator to the load. Each phase is having a phase difference of 120° , i.e. 120° angle electrically. So from the total of 360° , three phases are equally divided into 120° each. The power in three phase system is continuous as all the three phases are involved in generating the total power. The sinusoidal waves for 3 phase system are shown below.



The three phase can be used as single phase each. So if the load is single phase, then one phase can be taken from the three phase circuit and the neutral can be used as ground to complete the circuit.

Why three phase is preferred over single phase?

There are various reasons for this question because there are numbers of advantages over single phase circuit. The three phase system can be used as three single phase line so it can act as three single phase system. The three phase generation and single phase generation is same in the generator except the arrangement of coil in the generator to get 120° phase difference. The conductor needed in three phase circuit is 75% that of conductor needed in single phase circuit.

And also the instantaneous power in single phase system falls down to zero as in single phase we can see from the sinusoidal curve but in three phase system the net power from all the phases gives a continuous power to the load.

Till now we can say that there are three voltage sources connected together to form a three phase circuit and actually it is inside generator. The generator is having three voltage sources which are acting together in 120° phase difference. If we can arrange three single phase circuit with 120° phase difference, then it will become a three phase circuit. So 120° phase difference is must otherwise the circuit will not work, the three phase load will not be able to get active and it may also cause damage to the system.

The size or metal quantity of three phase devices is not having much difference. Now if we consider the transformer, it will be almost same size from both single phase and three phase because transformer will make only the linkage of flux. So the three phase system will have higher efficiency compared to single phase for the same or little difference in mass of transformer, three phase line will be out whereas in single phase will be only one. And losses will be minimum in three phase circuit. So overall in conclusion the three phase system will have better and higher efficiency compared to the single phase system.

A balanced polyphase system is one in which there are two or more equal voltages of the same frequency displaced equally in time phase, which supply power to loads connected to the lines. In general, in a n -phase balanced polyphase system, there are n -equal voltages displaced in time phase by $360^\circ/n$

or $2\pi/n$ (except in the case of a 2-phase system, in which there are two equal voltages differing in phase by 90°). Systems of six or more phases are used in polyphase rectifiers to obtain rectified voltage with low ripple. But three phase system is most commonly used polyphase system for generation and transmission of power. Hence we study in detail the 3-phase voltage generation and analysis of 3-phase circuit in this unit.

A 3-phase system has the following advantages over single phase system. For a given frame size of a machine a 3-phase machine will have large capacity than a single phase machine. The torque produced in a 3-phase motor will be more uniform whereas in a 1-phase motor it is pulsating. The amount of copper required in a certain amount of power over a particular distance, is less compared to a single phase system.

Phase sequence:

It is the order in which the phase voltages will attain their maximum values. From the fig it is seen that the voltage in A phase will attain maximum value first and followed by B and C phases. Hence three phase sequence is ABC. This is also evident from phasor diagram in which the phasors with +ve direction of anti-clockwise rotation passes a fixed point in the order ABC, ABC and so on. The phase sequence depends on the direction of rotation of the coils in the magnetic field. If the

coils rotate in the opposite

direction then the phase voltages attain maximum value in the order ACB. The phase sequence gets reversed with direction of rotation. Then the voltage for this sequence can be represented as

$$e_a = E_m \sin \omega t$$

$$e_c = E_m \sin(\omega t - 120^\circ)$$

$$e_b = E_m \sin(\omega t - 240^\circ)$$

The RMS values of voltage can be expressed as

$$E_A = E \angle 0^\circ$$

$$E_C = E \angle -120^\circ$$

$$E_B = E \angle -240^\circ$$

Star and Delta connection:

The three phase windings have six terminals i.e., A,B,C are starting end of the windings and A',B' and C' are finishing ends of windings. For 3 phase system two types of common interconnections are employed.

Star connection:

The finishing ends or starting ends of the three phase windings are connected to a common point as shown in. A', B', C' are connected to a common point called neutral point. The other ends A, B, C are called line terminals and the common terminal neutral is brought outside. Then it is called a 3 phase 4 wire star connected system. If neutral point is not available, then it is called 3 phase, 3 wire star connection.

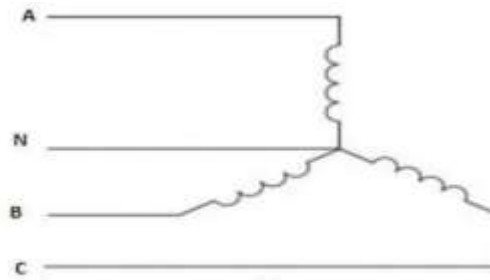
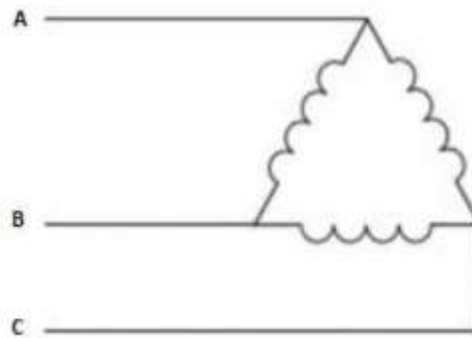


Fig. 1.2

Delta connection:

In this form of interconnection the dissimilar ends of the three coils i.e A and B', B and C', and C and A' are connected to form a closed Δ circuit (starting end of one phase is connected to finishing end of the next phase). The three junction are brought outside as line terminal A, B, C. the three phase windings are connected in series and form a closed path. The sum of the voltages in the closed path for balanced system of voltages at any instant will be zero.



The main advantage of star connection is that we can have two different 3- phase voltages. The voltage that was the line terminals between A & B, B & C, and C & A are called line voltages and form a balanced three phase voltage. Another voltage is between the terminals A & N, B & N, and C & N are called phase voltage and form another balanced three phase voltage (line to neutral voltage or wye voltage).

Relation between line and phase voltage and currents in balanced systems:

In this section we will derive the relation between line and phase values of voltages and currents of 3- phase star connected and delta connected systems.

1 Star connection:

We will employ double subscript notation to represent voltages and currents. The terminal corresponding to first subscript is assumed to be at a higher potential with respect to the terminal corresponding to second subscript.

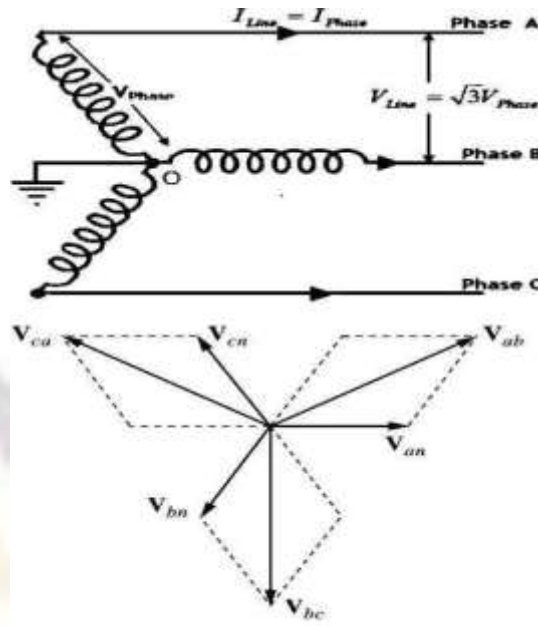


Fig.1.4

The voltage across each coil, i.e., the voltage between A & A', B & B', and C & C' are called phase voltages (acting from finishing end to starting end). V_{AA} , V_{BB} , V_{CC} , or V_{AN} , V_{BN} , V_{CN} represent phase voltages.

The voltages across line terminals A & B, B & C, C & A are called line voltages. The connection diagram and the corresponding phasor diagram of voltages is shown in fig. From the star-connected 3 phase system, it is clearly observed that whatever currents flow through the lines A, B, C also flow through the respective phase windings. Hence in star connected system, the phase currents and line currents are identical.

$$\text{Phase current (I}_{ph}\text{)} = \text{Line currents (I}_L\text{)} \quad I_{ph} = I_{Line}$$

The voltage V_{AB} between lines A and B is obtained by adding V_{AN} and V_{NB} respectively.

$$V_{AB} = V_{AN} + V_{NB} = V_{AN} - V_{BN}$$

Similarly

$$V_{BC} = V_{BN} + V_{NC} = V_{BN} - V_{CN} \quad V_{CA} = V_{CN} + V_{NA} = V_{CN} - V_{AN}$$

The line voltage V_{AB} is obtained by adding V_{AN} with reversed vector of V_{BN} . V_{AB} bisects the angle between V_{AN} and V_{BN} .

$$V_{AB} = \sqrt{3} V_{ph}$$

Line voltage = $\sqrt{3}$ phase voltage

The line voltages V_{AB} , V_{BC} , V_{CA} are equal in magnitude and differ in phase by 120° . Hence they form a balanced 3-phase voltage of magnitude $\sqrt{3} V_{ph}$. The two voltages differ in phase by 30° . When the system is balanced, the three phase currents I_A , I_B , I_C are balanced. The magnitude and phase angle of current is determined by circuit parameters.

I_A , I_B , I_C are line or phase currents. The current in the neutral wire is I_N and is by applying Kirchhoff's current law at star point, we get

$$I_N = -(I_A + I_B + I_C)$$

If the currents are balanced, then the neutral current is zero.

1.1.1 Delta connection or MESH connection:

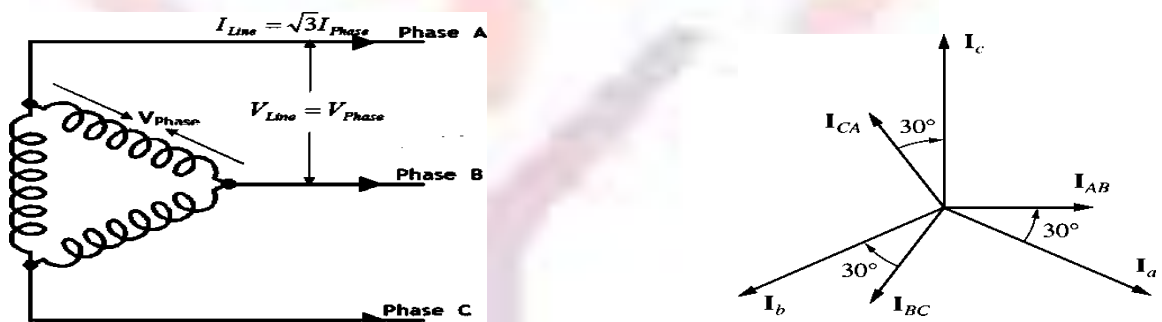


Fig.1.5.

The currents flowing through the phase windings $I_{AA'}$, $I_{BB'}$, and $I_{CC'}$ or I_{AB} , I_{BC} , and I_{CA} are called phase currents and are balanced as shown in phasor diagram Fig.1.5.

By applying KCL at node A

$$I_A + I_{CA} = I_{AB}, I_A = I_{AB} - I_{CA}$$

Similarly by applying KCL at nodes B and C

$$I_B = I_{BC} - I_{AB}, I_C = I_{CA} - I_{BC}$$

The line current I_A is obtained by adding I_{AB} and $-I_{CA}$ vectorially. I_A bisects the angle between I_{AB} and $-I_{CA}$

$$I_L = \sqrt{3} I_{ph} \text{ Line current } (I_L) = \sqrt{3} \text{ phase voltage } (I_{ph})$$

The line current I_A , I_B , I_C and also equal and differ in phase by 120° . They form a balanced system of currents. The line and phase currents differ in phase by 30° .

Analysis of balanced three-phase circuits

A set of three impedances interconnected in the form of a star or delta form a 3-phase star or delta connected load. If the three impedances are identical and equal then it is a balanced 3-phase load, otherwise it is an unbalanced 3-phase load.

The analysis of balanced 3-phase circuits is illustrated as follows

Balanced delta connected load:

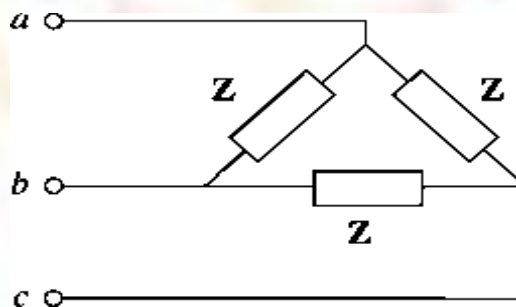


Fig.1.6

Let us consider a balanced 3-phase delta connected load. Determination of phase voltages:

$$V_{AB} = V \angle 0^\circ, V_{BC} = V \angle -120^\circ, V_{CA} = V \angle -240^\circ = V \angle 120^\circ$$

Determination of phase currents:

Phase current = Phase voltage / Load impedance

$$I_{AB} = \frac{V_{AB}}{Z}; I_{BC} = \frac{V_{BC}}{Z};$$

$$I_{CA} = \frac{V_{CA}}{Z}$$

Determination of line currents:

Line currents are calculated by applying KCL at nodes A, B, C

$$I_A = I_{AB} - I_{CA}; I_B = I_{BC} - I_{AB}; I_C = I_{CA} - I_{BC}$$

Note: Line currents are also balanced and equal to $\sqrt{3}$ phase current.

Balanced star connected load:

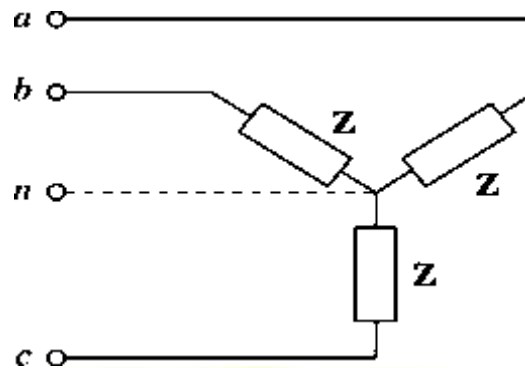


Fig.1.7

Let us consider a balanced 3-phase star connected load. For star connection, phase voltage = Line voltage / ($\sqrt{3}$)

For ABC sequence, the phase voltage is polar form are taken as

$$V_{AN} = V_{ph} \angle -90^\circ; V_{CN} = V_{ph} \angle 150^\circ; V_{BN} = V_{ph} \angle 30^\circ$$

For star connection line currents and phase currents are equal

$$I_A = \frac{V_{AN}}{Z}; I_B = \frac{V_{BN}}{Z}; I_C = \frac{V_{CN}}{Z}$$

To determine the current in the neutral wire apply KVL at star point

$$I_N + I_A + I_B + I_C = 0$$

$$I_N = -(I_A + I_B + I_C) \text{ (since they are balanced)}$$

In a balanced system the neutral current is zero. Hence if the load is balanced, the current and voltage will be same whether neutral wire is connected or not. Hence for a balanced 3-phase star connected load, whether the supply is 3-phase 3 wire or 3-phase 4 wire, it is immaterial. In case of unbalanced load, there will be neutral current.

UNIT II

ELECTRICAL INSTALLATIONS

SWITCHGEAR:

The apparatus used for switching, controlling and protecting the electrical circuits and equipment is known as switchgear.

COMPONENTS OF SWITCHGEAR

A low-tension line is a low voltage line and a high-tension line is a high voltage line. In India LT supply is of 400 Volts for three-phase connection and 230 Volts for single-phase connection.

CLASSIFICATION OF SWITCHGEAR:

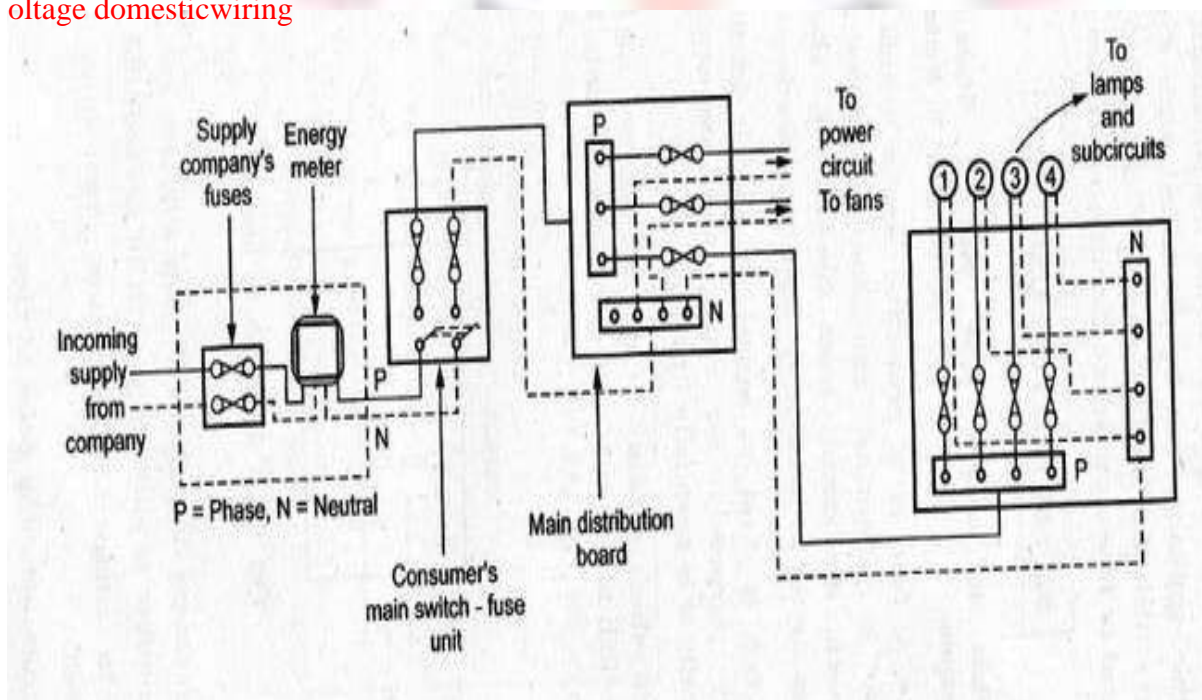
Switchgear can be classified on the basis of voltage level into the following:

1. Low voltage (LV) Switchgear: up to 1 KV
2. Medium voltage (MV) Switchgear: 3 KV to 33 KV
3. High voltage (HV) Switchgear: Above 33 KV

There are four types of components of LT switchgear

1. Switch Fuse Unit (SFU)
2. Miniature Circuit Breaker (MCB)
3. Earth Leakage Circuit Breaker (ELCB)
4. Molded Case Circuit Breaker (MCCB)

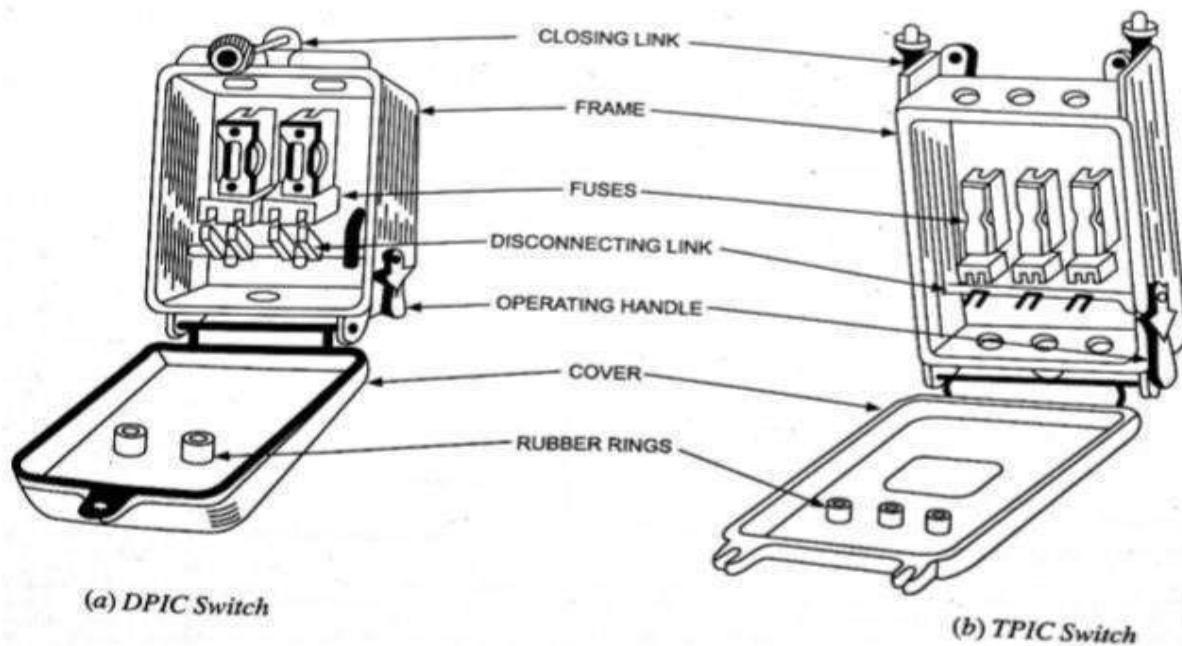
Low voltage domestic wiring



SWITCH FUSE UNIT (SFU):

Switch fuse is a combined unit and is known as an iron clad switch, being made of iron. It may be double pole for controlling single phase two-wire circuits or triple pole for controlling three-phase, 3-wire circuits or triple pole with neutral link for controlling 3-phase, 4-wire circuits. The respective switches are known as double pole iron clad (DPIC), triple pole iron clad (TPIC), and triple pole with neutral link iron clad (TPNIC) switches.

Miniature Circuit Breaker (MCB):

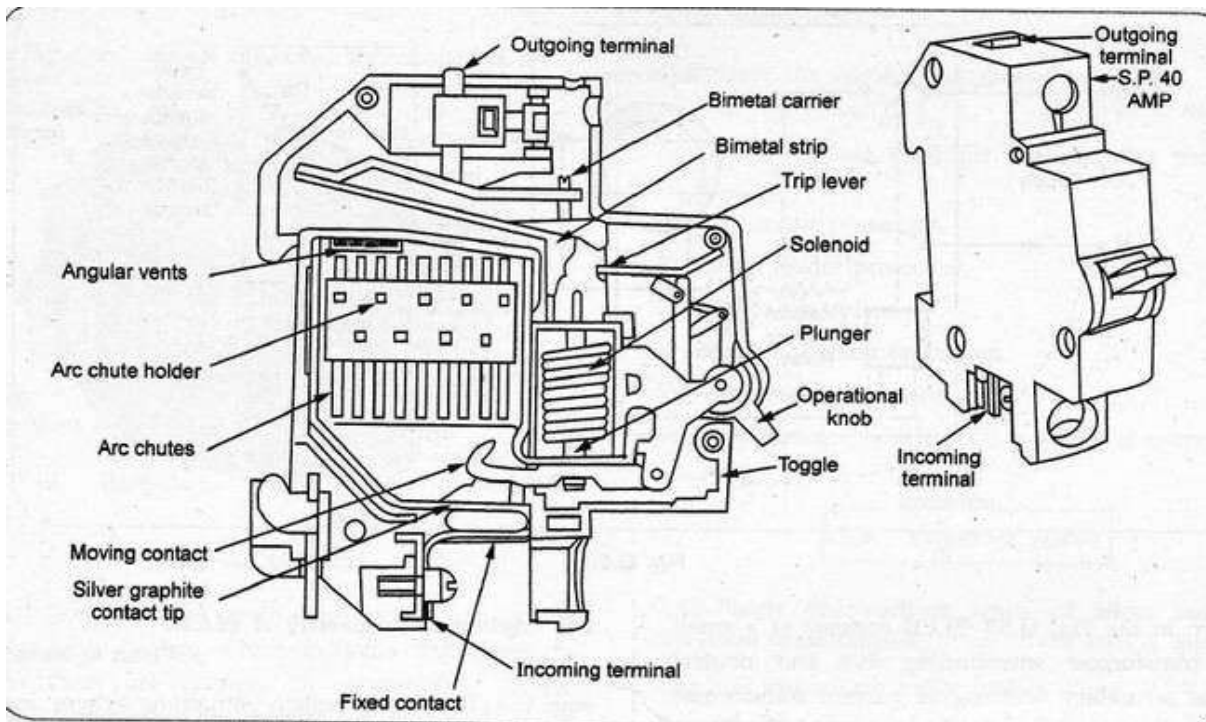


A device which provides definite protection to the wiring installations and sophisticated equipment against over-currents and short-circuit faults

Thermal operation (overload protection) is achieved with a bimetallic strip, which deflects when heated by any over-currents flowing through it. In doing so, releases the latch mechanism and causes the contact to open.

MCBs are available with different current ratings of 0.5, 1.2, 2.5, 3, 4, 5, 6, 7.5, 10, 16, 20, 25, 32, 35, 40, 63, 100, 125, 160 A and voltage rating of 240/415 VAC and up to 220 VDC. Operating time is very short (less than 5 ms).

They are suitable for the protection of important and sophisticated equipment, such as air-conditioners, refrigerators, computers etc.



Working Principle of MCB

The primary function of a miniature circuit-breaker is to protect an installation or appliance against sustained overloading and short-circuit faults,

When the current overflow occurs through MCB – Miniature Circuit Breaker, the bimetallic strip gets heated and deflects by bending. The deflection of the bi-metallic strip releases a latch. The latch causes the MCB to turn off by stopping the current flow in the circuit.

MOLDED CASE CIRCUIT BREAKER (MCCB):

It is a type of electrical protection device that can be used for a wide range of voltages, and frequencies of both 50Hz and 60Hz, the main distinctions between molded case and miniature circuit breaker are that MCCB can have current rating up to 2500 amperes, and its trip setting are normally adjustable. MCCBs are much larger than MCBs. An MCCB has three main functions:

- **PROTECTION AGAINST OVERLOAD.**
- **Protection against electrical faults.**
- **Switching a circuit ON and OFF.** This is a less common function of circuit breakers, but they can be used for that purpose if there is not an adequate manual switch.

Operating Mechanism: At its core, the protection mechanism employed by MCCBs is based on the same physical principles used by all types of thermal-magnetic circuit breakers.

- **Overload protection** is accomplished by means of a **thermal mechanism**. MCCBs have a bimetallic contact that expands and contracts in response to changes in temperature. Under normal operating conditions, the contact allows electric current through the MCCB.

However, as soon as the current exceeds the adjusted trip value, the contact will start to heat and expand until the circuit is interrupted.

- **The thermal protection** against overload is designed with a time delay to allow short duration overcurrent, which is a normal part of operation for many devices. However any overcurrent conditions, that last more than what is normally expected represent an overload, and the MCCB is tripped to protect the equipment and personnel. On the other hand, fault protection is accomplished with electromagnetic induction, and the response is instant. Fault currents should be interrupted immediately, no matter if their duration is short or long. Whenever a fault occurs, **the extremely high current induces a magnetic field in a solenoid coil located inside the breaker- this magnetic induction trips a contact and current is interrupted.** As a complement to the magnetic protection mechanism, MCCBs have internal arc dissipation measures to facilitate interruption.

EARTH LEAKAGE CIRCUIT BREAKER (ELCB):

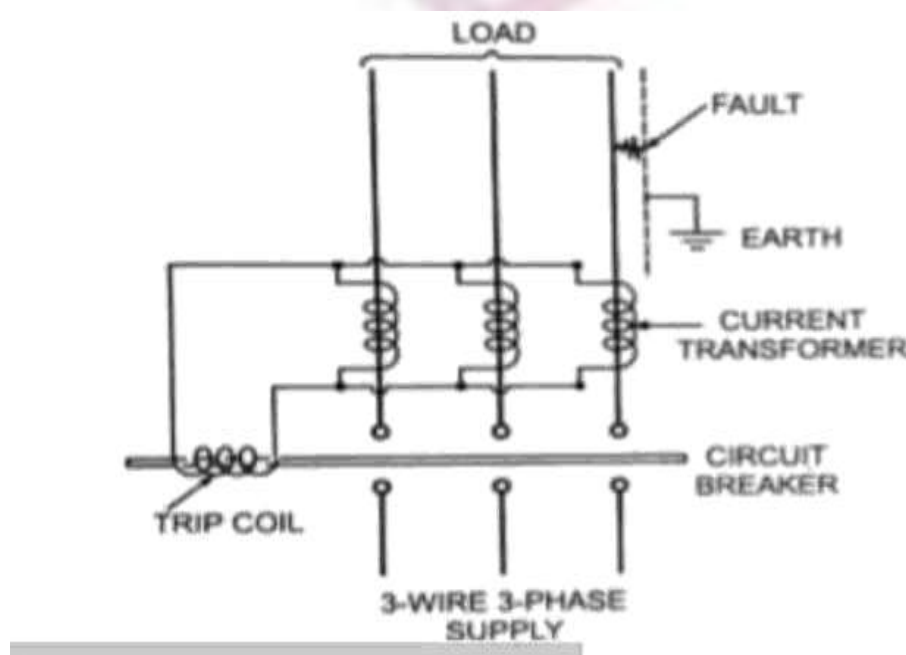
It is a device that provides **protection against earth leakage**. These are of two types.

1. CURRENT OPERATED EARTH LEAKAGE CIRCUIT BREAKER:

2. Voltage operated earth leakage circuit breaker. 1. Current operated earth leakage circuit breaker:

It is used when the **product of the operating current in amperes and the earth-loop impedance in ohms does not exceed 40**. such circuit breakers is used where consumer's earthing terminal is connected to a suitable earth electrode. A current-operated earth leakage circuit breaker is **applied to a 3-phase, 3-wire circuit**.

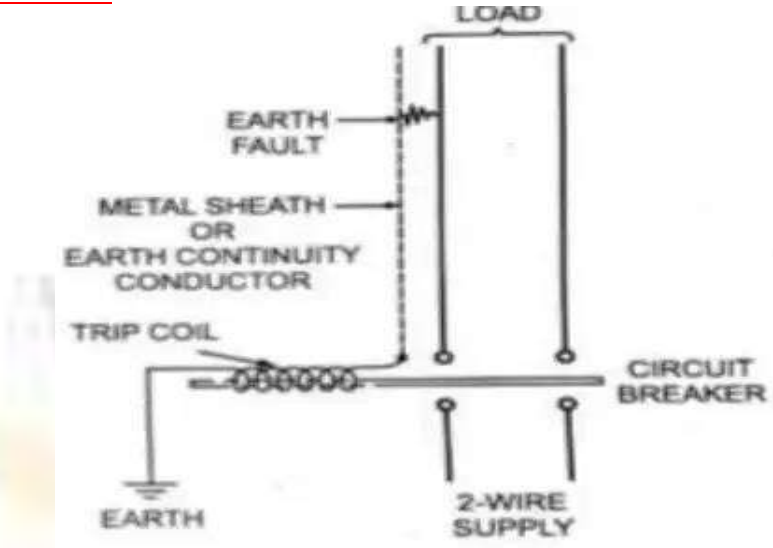
In normal condition when there is no earth leakage, the algebraic sum of the currents in the three coils of the current transformers is zero, and no current flows through the trip coil. In case of any earth leakage, the currents are unbalanced and the trip coil is energized and thus the circuit breaker is tripped.



VOLTAGE OPERATED EARTH LEAKAGE CIRCUIT BREAKER:

It is suitable for use when the earth-loop impedance exceeds the values applicable to fuses or excess-current circuit breaker or to current operated earth leakage circuit breaker. When the voltage between the earth continuity conductor (ECC) and earth electrode rises to sufficient value, the trip coil will carry the required current to trip the circuit breaker. With such a circuit breaker the earthing lead between the trip coil and the earth electrode must be insulated.

TYPES OF WIRES AND CABLES:



Wire:

A [wire](#) is defined as one electrical conductor, while a cable is defined as a group of individually insulated wires (conductors) encased together in sheathing.

- Sheathing is a non-conducting material with protective properties to shield the conducting part of the wire/cable.
- Wires and cables can be made from various materials, such as copper, gold, and aluminum.
- The two categories of single-conductor wires are solid and stranded (also called braided).
- Solid wire is rigid and conducts electricity better. Stranded wire consists of smaller wires braided together. Stranded wires are less prone to breakage when flexed repeatedly, which is why this type of wire is common in phone chargers.
- [Jumper wires](#) are pre-cut flexible stranded wires of different lengths that have stiff ends to allow the wire to be easily inserted in a breadboard.
- [Hook-up wire](#) is typically single conductor insulated wire used in low current, low voltage (<600 Volts) applications for making internal connections.
- [Magnet wire](#) is a copper or aluminum wire coated with a very thin layer of insulation. Magnet wire allows multiple layers of wire to be wound together without short circuiting. When the wire is wound into a coil and energized, it creates an electromagnetic field. Magnet wire is often used in transformers, inductors, motors, and electromagnets.

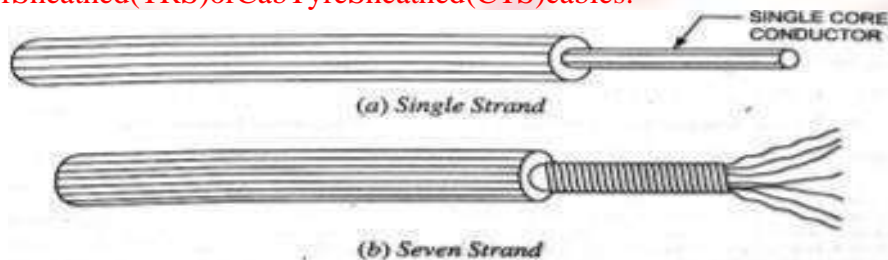
For internal wiring of any building, wires and cables may be categorized into the following groups:

1. **Conductor Used:** According to conductor material used in the cables, these may be divided into two classes known as **copper conductor** cables and **aluminium conductor** cables.
2. **Number of Cores Used:** It may be divided into different classes known as: single core cables, twin core cables, three core cables, two core with ECC (Earth Continuity Conductor) cables etc.
3. **Voltage Grading:** According to voltage grading the cables may be divided into two classes (i) 250/440 Volt cables and (ii) 650/1100 volt cables
4. **Types of Insulation Used:** According to type of insulation the cables are of following types:
 - Vulcanized Indian Rubber (VIR) insulated cables
 - Tough Rubber Sheathed (TRS) or Cab Tyre Sheathed (CTS) cables.
 - Lead Sheathed Cables.
 - Polyvinyl Chloride (PVC) Cables.
 - Weatherproof cables.
 - Flexible cords and cables.
 - XLPE cables.
 - Multi-strand cables.

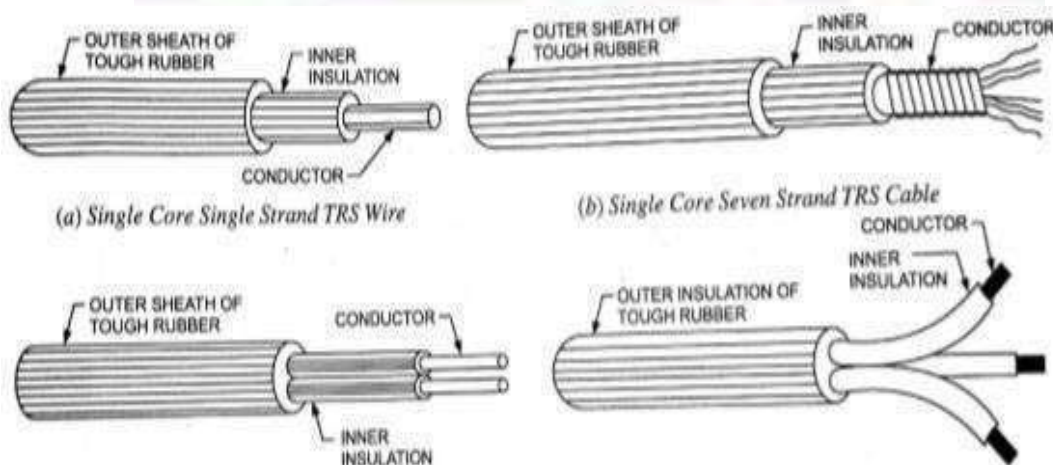
VULCANIZED INDIAN RUBBER (VIR) INSULATED CABLES:

VIR cables are available in 240/415 volts as well as in 650/1100 volt grades. VIR cables consist of either Tinned copper conductor Covered with a layer of VIR insulation. Over the rubber Insulation cotton tapes sheathed Covering is provided with Moisture resistant compound bitumen wax or some other insulating material for making the cables moisture proof.

Tough Rubber Sheathed (TRS) or Cab Tyre Sheathed (CTS) cables:

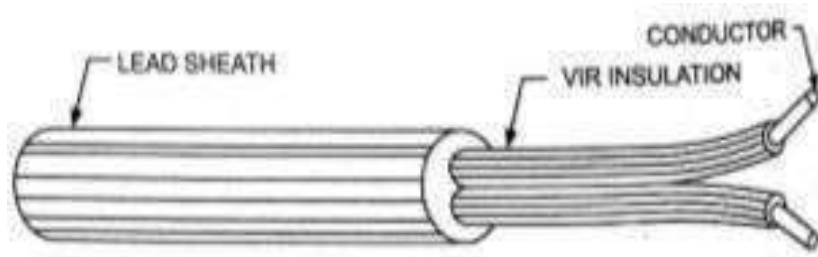


These cables are available in 250/440 volt and 650/1100 volt grades and used in CTS (or TRS) wiring. TRS cable is nothing but a VIR conductor with an outer protective covering of tough rubber, which provides additional insulation and protection against wear and tear.



LEAD SHEATHED CABLES:

These cables are available in 240/415 volt grade. The lead sheathed cable is a vulcanized rubber insulated conductor covered with a continuous sheath of lead. The lead sheath provides very good protection against the absorption of moisture and sufficient protection against mechanical injury and so can be used without casing or conduits system.



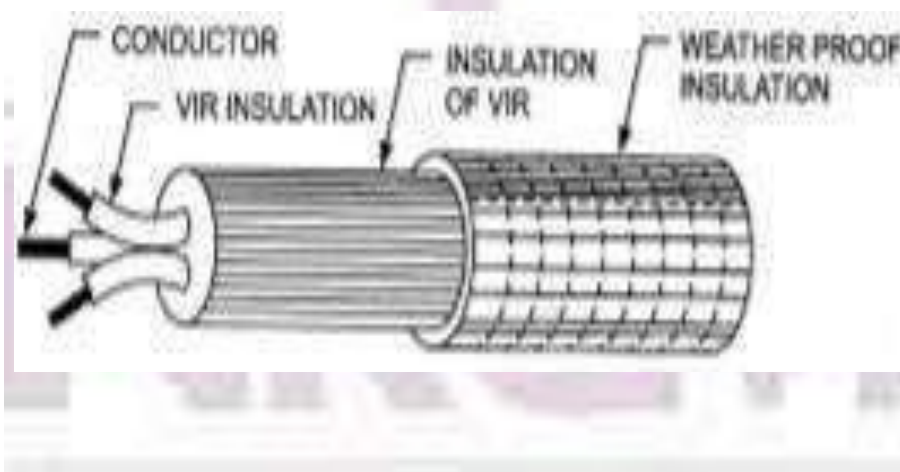
POLYVINYLCHLORIDE INSULATED CABLES:

These cables are available in 250/440 volt and 650/1100 volt grades

- PVC insulation has better insulating qualities.
- PVC insulation provides better flexibility.
- PVC insulation has no chemical effect on metal of the wire.
- Thin layer of PVC insulation will provide the desired insulation level.
- PVC coated wire gives smaller diameter of cable and, therefore, more no. of wires can be accommodated in the conduit of a given size in comparison to VIR or CTS wires.

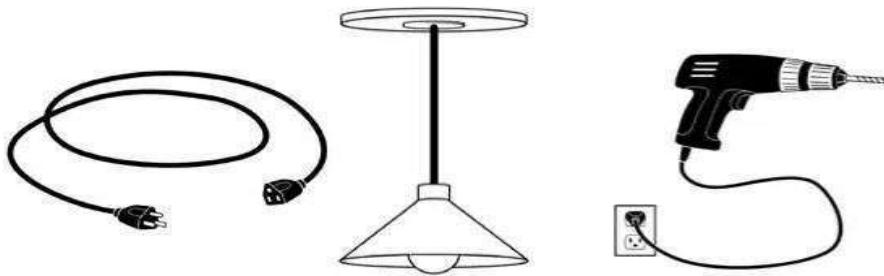
WEATHER PROOF CABLES:

- i. These cables are used for outdoor wiring and for power supply or industrial supply
- ii. These cables are either PVC insulated or vulcanized rubber insulated conductors being suitably taped braided and then compounded with weather resisting material.
- iii. These cables are available in 240/415 volt and 650/1100 volt grades



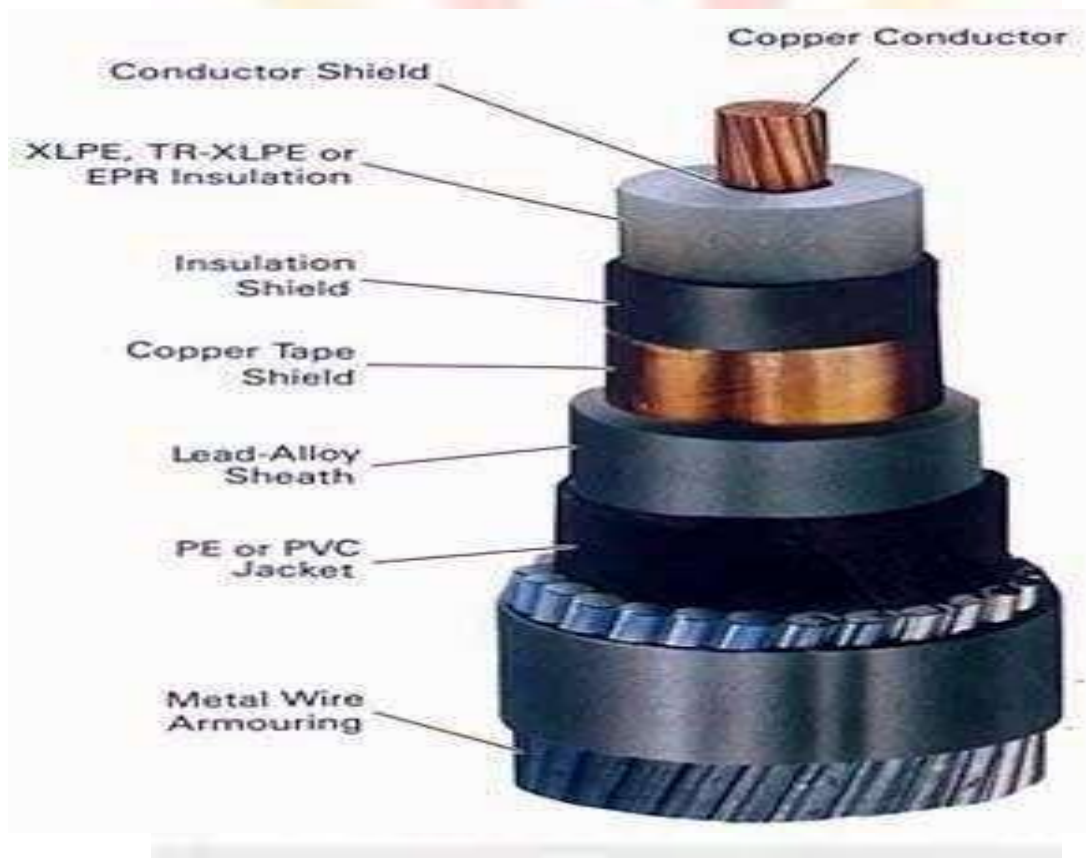
FLEXIBLE CORDS AND CABLES:

Flexible cords and cables are used to connect electrical equipment to a power source. Flexible cords may have an electrical plug that connects to a power source or they may be permanently wired into a power source. Extension cords (cord sets), cables, and electrical cords are types of flexible cords.



XLPE CABLE:

XLPE cable (Cross-linked polyethylene) is a type of electrical cable commonly used for power transmission and distribution. XLPE cable is made of a thermoset material, which means it is highly resistant to heat, moisture, and chemicals.



MULTI-STRAND CABLES:

A stranded wire is one that is made of multiple thin strands – often made of copper. They are twisted and bundled together within a PVC cable, thereby creating a multi-strand conductor.

Versatile in nature, they are available in different types.



EARTHING:

Earthing is defined as “the process in which the instantaneous discharge of the electrical energy takes place by transferring charges directly to the earth through low resistance wire.” Low resistance earthing wire is chosen to provide the least resistance path for leakage or fault current.

1. The main objective of earthing is to provide safety of operation.
2. Another objective of the earthing, though not widely used nowadays, is to save conducting material.

METHODS OF EARTHING:

Earthing should be done in a way so that on a short circuit, the earth loop impedance is low enough to pass 3 times the current if fuses are used, and 1.5 times the current if MCBs are used. The metal work should be solidly earthed without using any switch or fuse in the circuit.

There are different types of earthing methods used:

1. Strip or Wire Earthing.
2. Rod Earthing.
3. Pipe Earthing.
4. Plate Earthing.

Pipe and Plate Earthing are commonly used.

STRIP OR WIRE EARTHING: In this system of earthing, strip electrodes of cross section not less than 25 mm X 1.6 mm if of copper and 25 mm X 4 mm if of galvanized iron or steel are buried in horizontal trenches of minimum depth 0.5 metre. If round conductors are used

ROD EARTHING: In this type of earthing, 12.5 mm diameter solid rods of copper or 16 mm diameter solid rods of galvanized iron or steel or hollow section 25 mm GI pipes of length not less than 2.5 metres are driven vertically into the earth either manually or by pneumatic hammer.

PIPE EARTHING:

Earthing, is the process of connecting an electrical system to the earth. This is done to protect against electrical shocks, reduce electromagnetic interference, and provide a stable reference voltage for the system. Pipe earthing is a type of earthing system that uses a pipe made of galvanized iron or copper buried in the ground to connect the electrical system to the earth.

The pipe used for earthing is typically around 2-3 meters in length and 40-50 mm in diameter. The top end of the pipe is connected to the electrical system, while the bottom end is buried in a pit filled with charcoal and salt. The pipe is then connected to a grounding plate or rod, which is also buried in the ground.

WORKING PRINCIPLE OF PIPE EARTHING:

The principle behind pipe earthing is that the earth acts as a large conductor, which can absorb and dissipate electrical charges. When an electrical fault occurs in the system, such as a short circuit or lightning strike, the current flows through the pipe and into the ground. The charcoal and salt in the pit help to improve the conductivity of the soil, making it easier for the current to flow.

The grounding plate or rod also helps to improve the conductivity of the earth, as it provides a larger surface area for the current to flow through. The size and depth of the pit, as well as the type of soil, are important factors that can affect the effectiveness of the pipe earthing system.

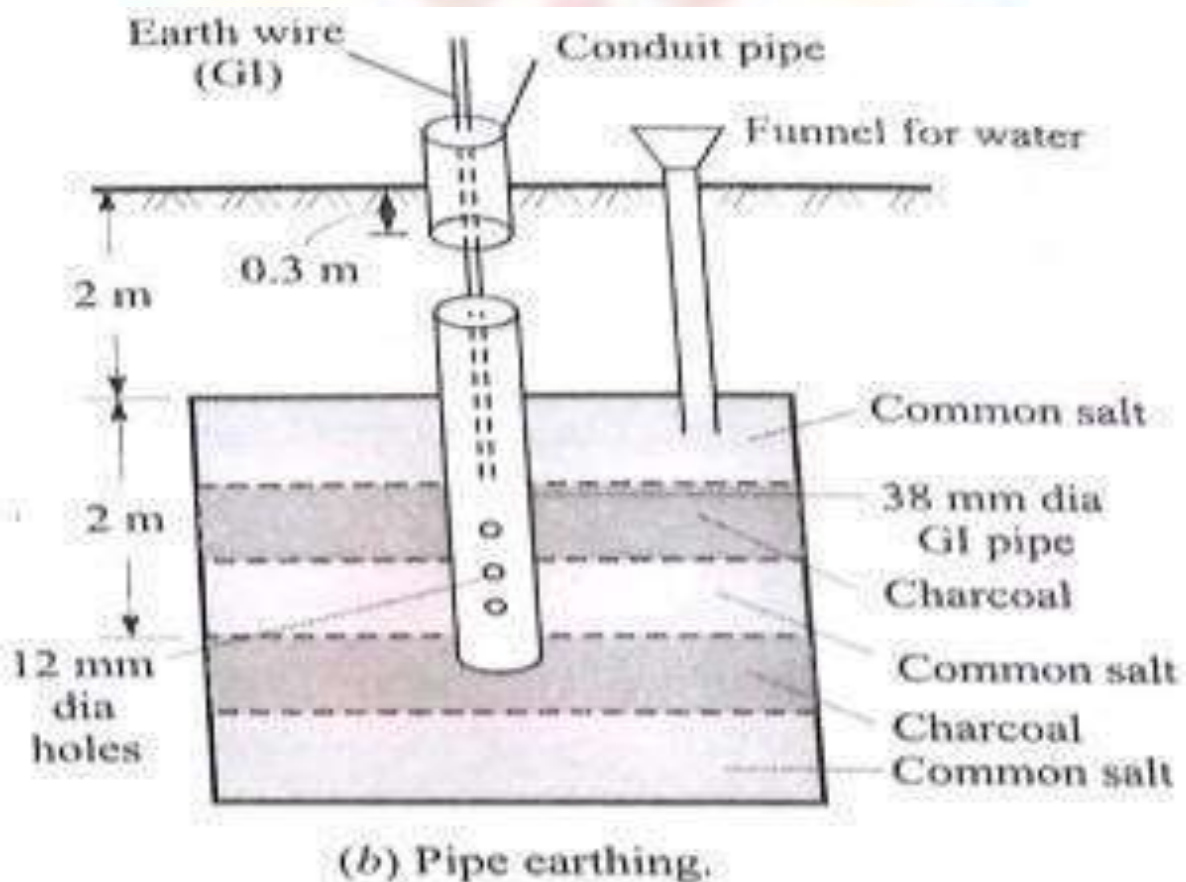


PLATE EARTHING:

Plate Earthing is a method where a plate made of galvanized copper or iron is buried vertically at least 3 meters below ground level. This plate connects all conductors to the earth, providing a path for electrical discharge. The Diagram of Plate Earthing typically illustrates this setup, showing the plate's position in relation to the ground level and the conductors it connects.

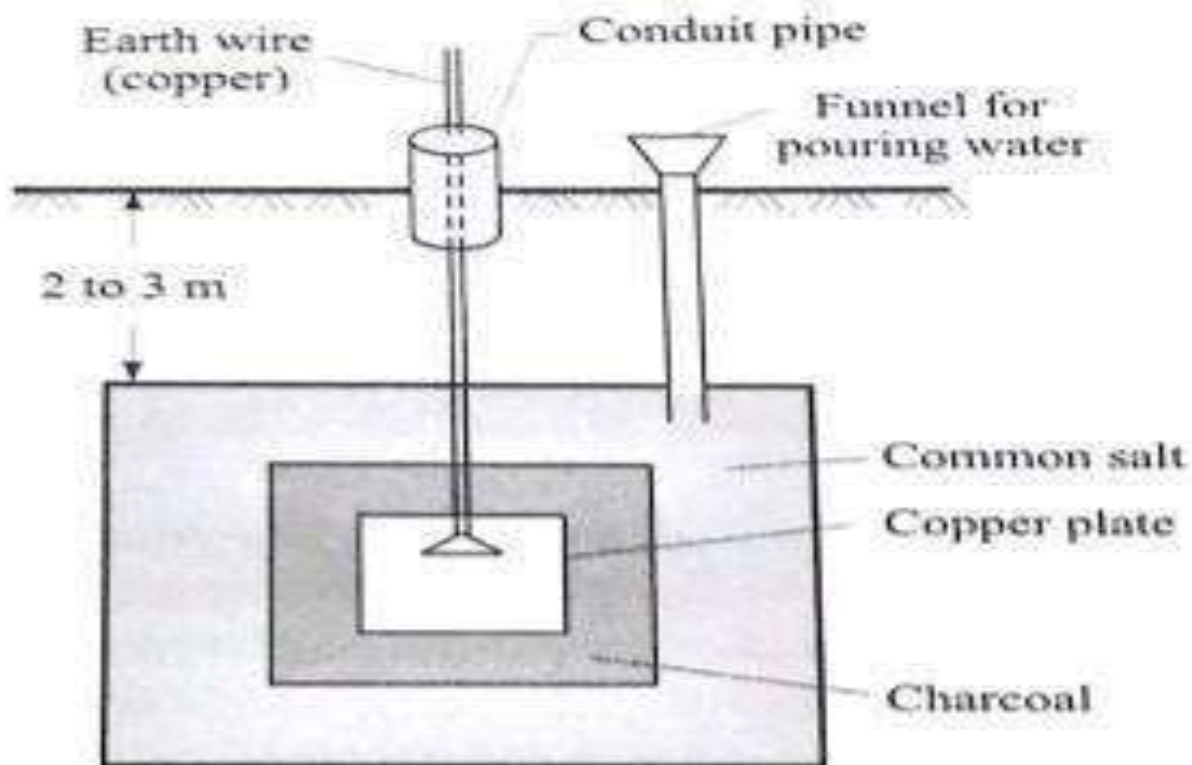
The Plate Earthing Diagram also often includes the dimensions of the plate. For instance, a copper plate used in this method typically measures 600mm x 600mm x 6.35mm. The plate's size and material can vary based on specific requirements, but the principle remains the same, to provide a safe path for fault current to the earth.

The diagram of plate earthing shows a plate electrode, which is either made of galvanized iron or steel (with a minimum thickness of 6.3 mm) or copper (with a minimum thickness of 3.15mm). The plates should be at least 60cm by 60cm in size.

The plate is buried in the earth, surrounded by alternating layers of charcoal and salt. The charcoal layer is used to retain moisture, which helps to maintain a low earth resistance.

A galvanized iron strip is connected to the plate and extends above the ground. This strip is used to connect the plate to the electrical system that is being earthed.

A pipe is also shown in the diagram, which is used for watering the earth around the plate. This helps to maintain the moisture levels around the plate, ensuring effective earthing.



(a) Plate earthing.

Finally, an inspection chamber is built around the earth pit. This chamber allows for regular inspection and maintenance of the earthing system.

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The plate is buried in the earth, surrounded by alternating layers of charcoal and salt. The charcoal layer is used to retain moisture, which helps to maintain a low earth resistance.

A galvanized iron strip is connected to the plate and extends above the ground. This strip is used to connect the plate to the electrical system that is being earthed.

A pipe is also shown in the diagram, which is used for watering the earth around the plate. This helps to maintain the moisture levels around the plate, ensuring effective earthing.

Finally, an inspection chamber is built around the earth pit. This chamber allows for regular inspection and maintenance of the earthing system.

PLATE EARTHING PROCEDURE

The procedure for Plate Earthing involves several steps, often illustrated in a Diagram of Plate Earthing:

1. **Earth Pit:** An earth pit is excavated at a suitable location in the substation, with a minimum size of 900mm x 900mm and a depth of 3m below the surface.
2. **Plate Electrode:** A GI plate of minimum size 600mm x 600mm and thickness of 6.3 mm is used. If a copper plate is used, a minimum thickness of 3.15mm is required. The plate is surrounded by alternating layers of charcoal and salt.
3. **Earthing Connection:** Galvanized Iron strips are fixed and welded to the plate at two different locations. Loose earthing can adversely affect the electrode system resistivity, so these connections are made strong.
4. **Water Connection:** A pipe is fixed at the top to maintain moist conditions around the earth plate. The pipe is covered with a wire mesh, and water is poured through it. The excavated pit is then filled with stone-free soil.
5. **Inspection Chamber:** A brick chamber is built over the earth pit on a P.C.C layer. The top cover is placed with cast iron hinges to a CI frame.

THE IMPORTANT DIFFERENCE BETWEEN EARTHING AND GROUNDING

Parameter	Earthing	Grounding
Definition	It protects human life from electrocuted.	It protects the entire power system from malfunctioning.

Potential	It contains zero potential.	It does not possess any zero potential.
Location	It is placed between the equipment body and earth and kept under the earth's surface.	It is placed between the neutral of the equipment and ground.
Types	It is of five types such as Pipe, Plate, Rod earthing, tap earthing, and strip earthing.	It is of three types such as Solid, Resistance, and Reactance grounding.
Color of wire	The earth wire is of green color.	The ground wire is black in color.
Example	It is used in transformer, generator, and motor for connecting to the	It is used as a neutral generator and power transformer and connected to the

earth.

ground.

Earthing di
scharges

Grounding pr
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rloads.

Applicatio
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theelectrical

energy to
theearth.

Itismainlyinuse

to avoid
shocking

thehumans.

Somemorefacts:

1. Theearthingisfortheconnectionofthenon-currentcarryingparttotheearth.Whereas,ingroundingthecurrent-carryingpartdirectlyconnectedtotheground.
2. Thegroundingis responsibleforloadbalancingandearthingisresponsibleforprotectionfromelectricalshock.
3. Theearthingelectrodemustnotbeplacednearthebuildingwhoseinstallationsystemiseearthed morethan1.5maway.
4. Theristanceoftheearthwireshouldbelessthan1ohm.
5. Itmustbetakencarethat thematerialofwireusedfortheelectrodeandcircuitsshouldbethesame.
6. Theelectrodeshavetobeinaverticalpositionsothatitcantouchthelayersoftheearth.
7. Thesizeofthe conductormustbemore than2.6squaremmandonlyhalfofthewireshouldbeusedforelectricalwiring.

BATTERIES

Types of Batteries: There are two types of batteries which are given below:

1) Primary Battery

2) Secondary Battery

- Primary batteries are “single use” and cannot be recharged. Dry cells and (most) alkaline batteries are examples of primary batteries.
- These second type is rechargeable and is called a secondary battery. EX: Nickel-cadmium (NiCd), lead acid, and lithium ion batteries.

IMPORTANT CHARACTERISTICS FOR BATTERIES:

Electrical Characteristics:

There are three important characteristics of an accumulator (or storage battery) namely,

- Voltage
- Capacity and
- Efficiency

Voltage: Average emf of cell is approximately 2.0 volts. The value of emf of a cell does not remain constant but varies with the change in specific gravity of electrolyte, temperature and the length of time since it was last charged.

Capacity: The quantity of electricity which a battery can deliver during single discharge until its terminal voltage falls to 1.8V/cell is called the capacity of a battery.

$$\text{Capacity of Battery or Cell} = I_d T_d$$

Efficiency: The efficiency of the cell can be given in two ways:

1. **The Quantity or Ampere – Hour (A-H) Efficiency:** The ratio of output ampere-hour during discharging to the input ampere-hour during charging of the battery is called **quantity or ampere-hour** efficiency of the battery.

$$\eta_{AH}$$

$$= \frac{I_d T_d}{I_c T_c}$$

$$I_c T_c$$

Where I_d = Discharging Current in Ampere

I_c = Charging Current in Ampere

T_d = Discharging Time of cell or battery in hours

T_c = Charging Time of cell or battery in hours

2. **Energy or Watt – Hour (W-H) Efficiency:** The ratio of output watt-hour during discharging to the input watt-hour during charging of the battery is called **energy or watt-hour** efficiency of

thebattery.

η_{WH}

$= \frac{V_d I_d T_d}{V_c I_c T_c}$

Where V_d = Average Terminal Voltage during Discharging

V_c = Average Terminal Voltage during Charging

I_d = Discharging Current in Ampere

I_c = Charging Current in Ampere

T_d = Discharging Time of cell or battery in hours

T_c = Charging Time of cell or battery in hours

TWO WATTMETER METHOD

A three-phase two-watt

meter measures the current and voltage from any of the 2 supply lines of 3 phase corresponding to the 3rd supply line of 3 phase. The 3 phase 2 wattmeter is said to be a balanced load condition if the current in every phase lags at an angle " ϕ " with phase voltage.

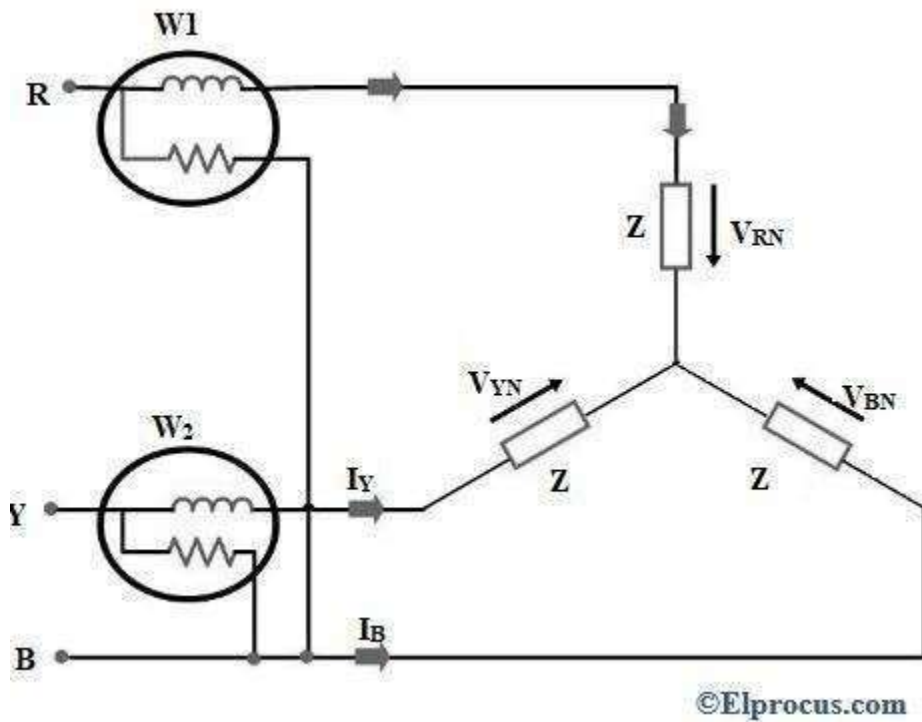
CONSTRUCTION OF TWO WATTMETER METHOD

The 3-phase power of a 3-phase circuit can be measured using 3 ways they are,

- 3 Wattmeter Method
- 2 Wattmeter Method
- 1 Wattmeter Method.

The main concept of 2 Wattmeter with 3 phase voltage is to balance the 3 phase load by satisfying the condition of current lagging at an angle ' ϕ ' with the voltage phase. The schematic diagram of 3 phase 2 wattmeter is shown below





Circuit Diagram

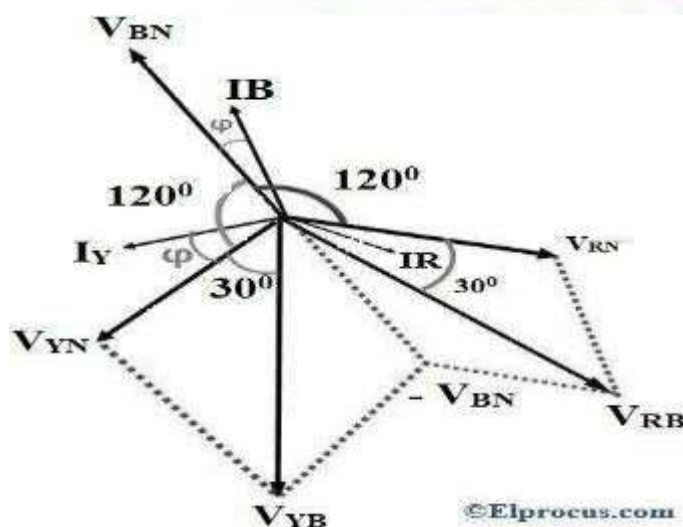
It consists of 2 wattmeters like W1 and W2, where each wattmeter has a current coil 'CC' and a pressure coil 'PC'. Here, one end of wattmeter 'W1' is connected to 'R' terminal whereas one end of wattmeter 'W2' is connected to 'Y' terminal. The circuit also consists of 3 inductors 'Z' which are constructed in a star topology. The 2 ends of inductors are connected to 2 terminals of a wattmeter whereas the third terminal of the inductor is connected to B.

DERIVATION OF TWO WATTMETER METHOD

Two Wattmeter is used to determine two main parameters they are,

- Power factor
- Reactive power.

Consider the load used as an inductive load which is represented by following the phasor diagram as shown below.



The voltages V_{RN} , V_{YN} , and V_{BN} are electrically 120° in phase with one another, we can observe that the current I_Y

hase lags at the " φ^0 " angle with voltage phase.

The current in wattmeter W_1 is represented as

$$W_1 = I_R(1)$$

where I_R is current

The potential difference across the wattmeter W_1 coil is given as

$$W_1 = \sim V_{RB} = [\sim V_{RN} - \sim V_{BN}] \dots \dots \dots (2)$$

Where V_{RN} and V_{BN} are voltages

The phase difference between the voltage ' V_{YB} ' and current ' I_Y ' is given as $(30^\circ + \phi)$. Hence the power measured by wattmeter is given as

$$W_2 = V_{YB} I_Y \cos(30^\circ + \phi) \dots (3)$$

At balanced load condition

$$I_R = I_Y = I_B = I_L \text{ and } \dots (4)$$

$$V_{RY} = V_{YB} = V_{BR} = V_L \dots (5)$$

Therefore we obtain wattmeter readings as

$$W_1 = V_L I_L \cos(30^\circ - \Phi) \text{ AND } \dots \dots \dots (6) W_2 = V_L I_L \cos(30^\circ + \phi) \dots \dots \dots (7)$$

TOTAL POWER DERIVATION

The total wattmeter reading is given as

$$W_1 + W_2 = V_L I_L \cos(30^\circ - \phi) + V_L I_L \cos(30^\circ + \phi) \dots (8)$$

$$\begin{aligned} &= V_L I_L [\cos(30^\circ - \Phi) + \cos(30^\circ + \Phi)] \\ &= V_L I_L [\cos 30^\circ \cos \phi + \sin 30^\circ \sin \phi + \cos 30^\circ \cos \phi - \sin 30^\circ \sin \phi] \end{aligned}$$

$$\begin{aligned} &= V_L I_L [2 \cos 30^\circ \cos \Phi] \\ &= V_L I_L [(2 \sqrt{3}/2) \cos 30^\circ \cos \phi] \end{aligned}$$

$$\begin{aligned} &= \sqrt{3} [V_L I_L \cos \Phi] \\ &\quad (9) W_1 + W_2 = P \\ &\quad (10) \end{aligned}$$

Where ' P ' is the total observed power in a 3-phase balanced load condition.

POWER FACTOR DERIVATION

Definition: It is the ratio between actual power observed by the load to apparent power flowing in the circuit.

The power factor of three phase balanced load condition can be determined and derived from wattmeter readings as follows

From equation 9

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos \Phi$$

$$\text{Now } W_1 - W_2 = V_L I_L [\cos(30^\circ - \phi) - \cos(30^\circ + \phi)]$$

$$= V_L I_L [\cos 30^\circ \cos \phi + \sin 30^\circ \sin \phi - \cos 30^\circ \cos \phi + \sin 30^\circ \sin \phi]$$

$$= 2 V_L I_L \sin 30^\circ \sin \phi$$

$$W_1 - W_2 = V_L I_L \sin \Phi \quad (11)$$

Dividing equations 11 and 9

$$[W_1 - W_2 / W_1 + W_2] = V_L I_L \sin \Phi / \sqrt{3} V_L I_L \cos \Phi$$

$$\tan \Phi = \sqrt{3} [W_1 - W_2 / W_1 + W_2]$$

The power factor of the load is given as

$$\cos \Phi = \cos \tan^{-1} [\sqrt{3} [W_1 - W_2 / W_1 + W_2]] \quad (12)$$

Reactive Power Derivation

Definition: It is the ratio between complex power corresponding to storage and revival of energy rather than consumption.

To obtain reactive power, we multiply equation 11 with

$$\sqrt{3} [W_1 - W_2] = \sqrt{3} [V_L I_L \sin \Phi]$$

$$\Phi = \text{PRPR} = \sqrt{3} [W_1 - W_2] \dots (13)$$

Where P is the reactive power obtained from 2 wattmeters.

ELEMENTARY CALCULATIONS FOR ENERGY CONSUMPTION AND SAVINGS:

Electrical energy is supplied to a consumer by the supplier. To charge the electrical energy consumed by a consumer, an energy meter is installed to its quantity. The reading of the energy meter is taken every month. The difference between the fresh reading and the previous reading tells about the consumption of electrical energy in that month. This quantity of energy is multiplied by the rate (tariff) fixed by the supplier to prepare an electricity bill. However, some other charges such as meter rent, GST, other taxes applicable etc. are also added in the bill.

BATTERY BACK-UP:

UP: The time (in hrs) for which a battery can deliver the desired current is called battery back-up of the bank.

EX: A consumer uses a 10 kW geyser, a 6 kW electric furnace and five 100 W bulbs for 15 hours. How many units (kWh) of electrical energy have been used?

Explanation : Given that Load –
1 = 10 kW geyser

Load – 2 = 6 kW electric furnace

Load – 3 = 500 watt (five 100 watt bulbs) Total load = 10 kW + 6 kW + 0.5 kW = 16.5 kW Time taken = 15 hours

Energy consumed = Power in kW × Time in hours

$$= 16.5 \times 15 = 247.5 \text{ kWh}$$

So, the total energy consumption = 247.5 units

If the cost per

unit is 2.5, then the total cost of energy consumption $247.5 \times 2.5 = 618.75/-$

Power Factor Improvement

The cosine of angle between voltage and current in a.c. circuit is known as **power factor**.

Most of the loads (e.g. induction motors, arc lamps) are inductive in nature and hence have low lagging power factor. The low power factor is highly undesirable as it causes an increase in current, resulting in additional losses of active power in all the elements of power system from power station generator down to the utilization devices. In order to ensure most favorable conditions for a supply system from engineering and economic standpoint, it is important to have power factor as close to unity as possible.

Power Triangle:

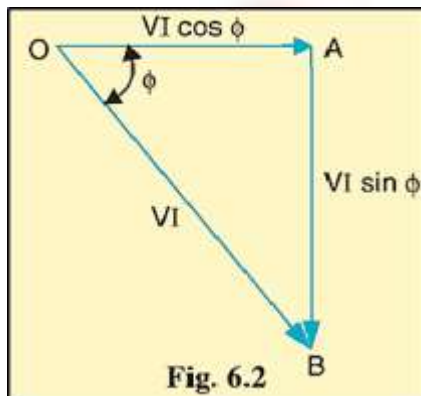
$OA = VI \cos \phi$ and represents the **active power, (P)** in watts or kW.

$AB = VI \sin \phi$ and represents the **reactive power, (Q)** in VAR or kVAR.

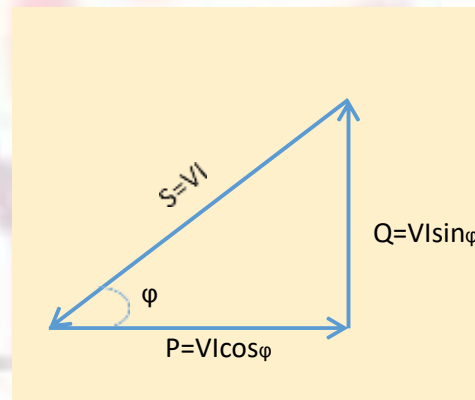
$OB = VI$ and represents the **apparent power, (S)** in VA or kVA.

- (i) The apparent power (S) in a.c. circuit has two components viz., active (P) and reactive power (Q) at right angles to each other. $OB^2 = OA^2 + AB^2$ or $S^2 = P^2 + Q^2$

(ii) Power factor, $\cos \phi = \frac{OA}{OB} = \frac{P}{S}$



lagging power factor



Leading power factor

- (iii) The lagging reactive power is responsible for the low power factor. It is clear from the power triangle that smaller the reactive power component, the higher is the power factor of the circuit.

$kVAR = kVA \sin \phi$
 $\cos \phi$

— $\sin \phi$

$$kVAR = kW \tan \phi$$

(iv) For leading power factor, the power triangle becomes reversed. This fact provides a key to the power factor improvement. If a device taking leading reactive power (e.g. capacitor) is connected in parallel with the load, then the lagging reactive power of the load will be partly neutralised, thus improving the power factor of the load.

(v) The power factor of a circuit can be defined in one of the following three ways:

(a) Power factor = $\cos \phi$ = cosine of angle between V and I.

(b) Power factor = $\frac{R}{Z}$ = $\frac{\text{Resistor}}{\text{Impedance}}$.

(c) Power factor = $\frac{VI \cos \phi}{VI}$ = $\frac{\text{Active power}}{\text{Apparent power}}$

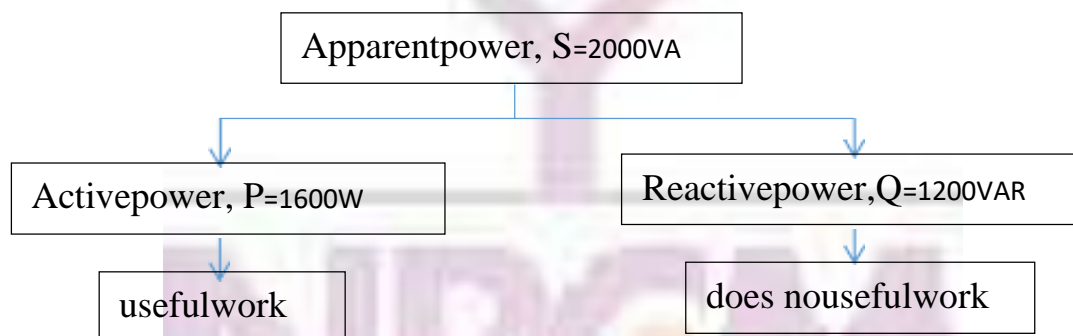
(vi) The reactive power is neither consumed in the circuit nor does it do any useful work. It merely flows back and forth in both directions in the circuit. A wattmeter does not measure reactive power.

Example:

Suppose a circuit draws a current of 10 A at a voltage of 200 V and its p.f. is 0.8 lagging. Apparent power,

$$S = VI = 200 \times 10 = 2000 \text{ VA.}$$

$$\text{Active power, } P = VI \cos \phi = 200 \times 10 \times 0.8 = 1600 \text{ W. Reactive power } Q = VI \sin \phi = 200 \times 10 \times 0.6 = 1200 \text{ VAR.}$$



The circuit receives an apparent power of 2000 VA and is able to convert only 1600 watts into active power. The reactive power is 1200 VAR and does no useful work. It merely flows into and out of the circuit periodically. In fact, reactive power is a liability on the source because the source has to supply the additional current (i.e., $I \sin \phi$).

Disadvantages of Low Power Factor:

The power factor plays an important role in a.c. circuits since power consumed depends upon this factor.

$$P = V_L I_L \cos \varphi \quad (\text{For single phase supply})$$

$$\therefore I_L = \frac{P}{V_L \cos \varphi} \quad \dots\dots\dots(i)$$

$$P = \sqrt{3} V_L I_L \cos \varphi \quad (\text{For 3 phase supply})$$

$$\therefore I_L = \frac{P}{\sqrt{3} V_L \cos \varphi} \quad \dots\dots\dots(ii)$$



It is clear from above that for fixed power and voltage, the load current is inversely proportional to the power factor. Lower the power factor, higher is the load current and vice-versa. A power factor less than unity results in the following disadvantages:

(i) **Large kVA rating of equipment.** The electrical machinery (e.g., alternators, transformers, switchgear) is always rated in kVA.

$$\text{kVA} = \frac{\text{kW}}{\cos \phi}$$

It is clear that kVA rating of the equipment is inversely proportional to power factor. The smaller the power factor, the larger is the kVA rating. Therefore, at low power factor, the kVA rating of the equipment has to be made more, making the equipment larger and expensive.

i.e. $\cos \phi \propto \frac{1}{\text{kVA}}$

(ii) **Greater conductor size.** To transmit or distribute a fixed amount of power at constant voltage, the conductor will have to carry more current at low power factor. This necessitates large conductor size.

For example, take the case of a single phase a.c. motor having an input of 10 kW on full load, the terminal voltage being 250 V. At unity p.f., the input full load current would be $10,000/250 = 40$ A. At 0.8 p.f.; the kVA input would be $10/0.8 = 12.5$ and the current input $12,500/250 = 50$ A.

If the motor is worked at a low power factor of 0.8, the cross-sectional area of the supply cables and motor conductors would have to be based upon a current of 50 A instead of 40 A which would be required at unity power factor.

Large copper losses. The large current at low power factor causes more I^2R losses in all the elements of the supply system. This results in poor efficiency.

Poor voltage regulation. The large current at low lagging power factor causes greater voltage drops in alternators, transformers, transmission lines and distributors. This results in the decreased voltage available at the supply end, thus impairing the performance of utilisation devices. In order to keep the receiving end voltage within permissible limits, extra equipment (i.e., voltage regulators) is required.



The above discussion leads to the conclusion that **low power factor is an objectionable feature in the supply system**

Causes of Low Power Factor

Low power factor is undesirable from economic point of view. Normally, the power factor of the whole load on the supply system is lower than 0.8. The following are the causes of low power factor:

(i) **Most of the a.c. motors are of induction type** (1 ϕ and 3 ϕ induction motors) which have low lagging power factor. These motors work at a power factor which is extremely small on light load (0.2 to 0.3) and rises to 0.8 or 0.9 at full load.

(iii) **Arc lamps**, electric discharge lamps and industrial heating furnaces operate at low lagging power factor.

(iv) **The load on the power system is varying**; being high during morning and evening and low at other times. During low load period, supply voltage is increased which increases the magnetization current. This results in the decreased power factor.

Power Factor Improvement:

The low power factor is mainly due to the fact that **most of the power loads are inductive and, therefore, take lagging currents**. In order to **improve** the power factor, some device taking leading power should be connected in parallel with the load. One of such devices can be a capacitor. **The capacitor draws a leading current and partly or completely neutralises the lagging reactive component of load current**. This raises the power factor of the load.

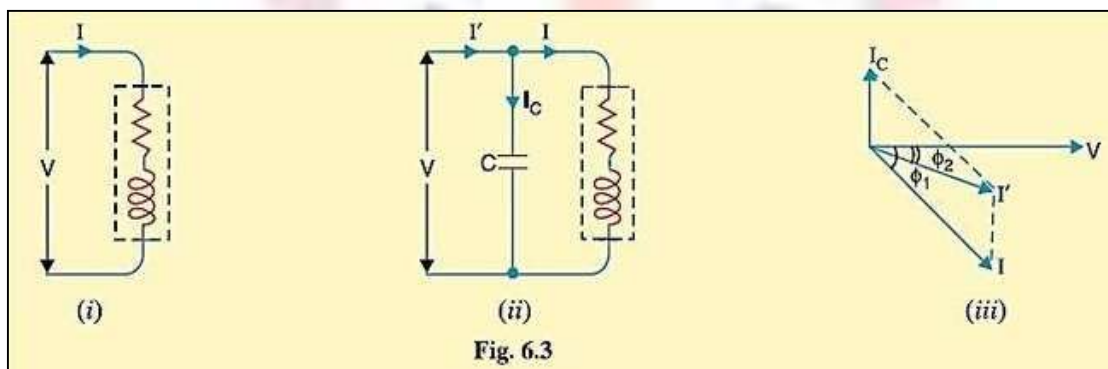


Illustration. To illustrate the power factor improvement by a capacitor, consider a single phase load taking lagging current I at a power factor $\cos \phi_1$ as shown in Fig. 6.3.

The capacitor C is connected in parallel with the load. The capacitor draws current I_c which leads the supply voltage by 90° . The resulting line current I' is the phasor sum of I and I_c and its angle of lag is ϕ_2 as shown in the phasor diagram of Fig. 6.3. (iii). It is clear that ϕ_2 is less than ϕ_1 , so that $\cos \phi_2$ is greater than $\cos \phi_1$. Hence, the power factor of the load is improved. The following points are worth noting:

- (i) The circuit current I' after p.f. correction is less than the original circuit current I .
- (ii) The active or wattful component remains the same before and after p.f. correction because only the lagging reactive component is reduced by the capacitor.

$$\therefore I \cos \phi_1 = I' \cos \phi_2$$

- (iii) The lagging reactive component is reduced after p.f. improvement and is equal to the difference between lagging reactive component of load ($I \sin \phi_1$) and capacitor current

$$(I_c) \text{ i.e.,}$$

$$I' \sin \varphi_2 = I \sin \varphi_1 - I_c$$

$$(iv) \quad As I \cos \phi_1 = I' \cos \phi_2$$

$$\therefore VI \cos \phi_1 = VI' \cos \phi_2 \text{ [Multiplying by } V]$$

Therefore, active power (kW) remains unchanged due to power factor improvement.

$$(v) \quad I' \sin \phi_2 = I \sin \phi_1 - I_C$$

$$\therefore VI' \sin \phi_2 = VI \sin \phi_1 - VI_C \text{ [Multiplying by } V] \text{ i.e., Net}$$

$$\text{kVAR after p.f. correction} =$$

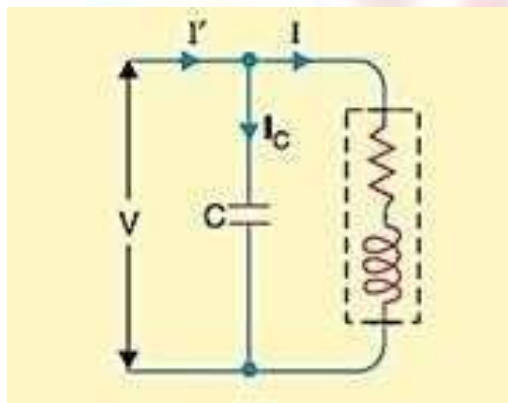
$$[\text{Lagging kVAR before p.f. correction} - \text{leading kVAR of equipment}].$$

Power Factor Improvement Equipment*

Normally, the power factor of the whole load on a large generating station is in the region of 0.8 to 0.9. However, sometimes it is lower and in such cases it is generally desirable to take special steps to improve the power factor. This can be achieved by the following equipment:

1. Static capacitors.
2. Synchronous condenser.
3. Phase advancers.

1. Static capacitor. The power factor can be improved by connecting capacitors in parallel with the equipment operating at lagging power factor. The capacitor (generally known as static capacitor) draws a leading current and partly or completely neutralises the lagging reactive component of load current. This raises the power factor of the load.



For three-phase loads, the capacitors can be connected in delta or star as shown in Fig. 6.4. Static capacitors are invariably used for power factor improvement in factories.

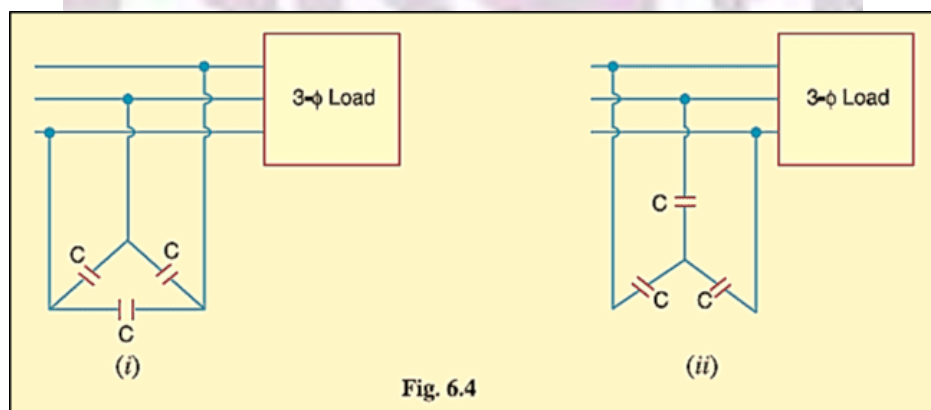
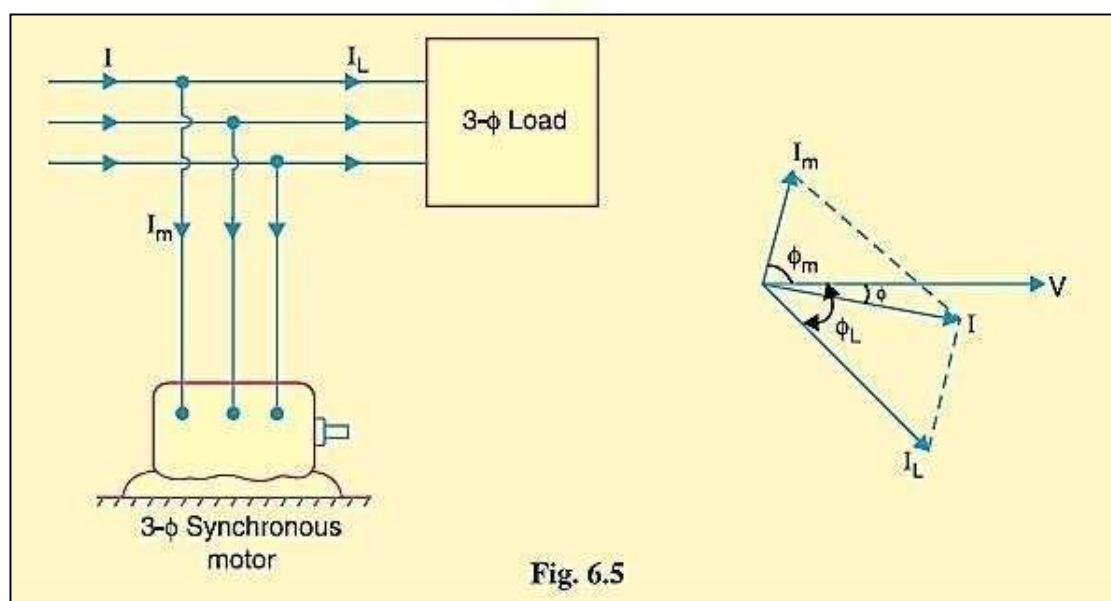


Fig. 6.4

Synchronous condenser. A synchronous motor takes a leading current when over-excited and, therefore, behaves as a capacitor. An over-excited synchronous motor running on no load is known as **synchronous condenser**. When such a machine is connected in parallel with the supply, it takes a leading current which partly neutralises the lagging reactive component of the load. Thus the power factor is improved.

Fig 6.5 shows the power factor improvement by synchronous condenser method. The 3 ϕ load takes current I_L at low lagging power factor $\cos \phi_L$. The synchronous condenser takes a current I_m which leads the voltage by an angle ϕ_m^* . The resultant current I is the phasor sum of I_m and I_L and lags behind the voltage by an angle ϕ . It is clear that ϕ is less than ϕ_L so that $\cos \phi$ is greater than $\cos \phi_L$.

Thus the power factor is increased from $\cos \phi_L$ to $\cos \phi$. Synchronous condensers are generally used at major bulk supply substations for power factor improvement.



Q. Compare between the mechanism of improving power factor with static capacitor and Synchronous motor

static capacitor	Synchronous motor
The p.f. improvement by switching on the capacitors in various groupings	The p.f. improvement by varying the field excitation

Q. What are the similarity states between a synchronous motor and capacitor?

synchronous motor takes a leading current when over-excited and, therefore, behaves as a capacitor

Q. At what conditions a synchronous motor is known as a synchronous condenser?

1. when a synchronous motor is an over-excited and takes a leading current.
2. when a synchronous motor is running on no load.

Calculations of Power Factor Correction

The power factor correction can also be illustrated from power triangle. Thus referring to Fig. 6.7, the power triangle OAB is for the power factor $\cos \phi_1$, whereas power triangle OAC is for the improved power factor $\cos \phi_2$. It may be seen that active power (OA) does not change with power factor improvement.

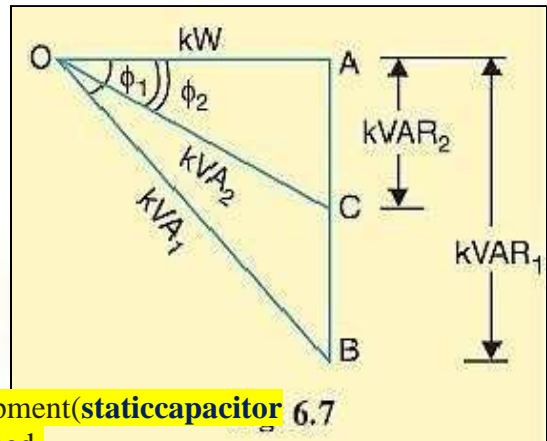
However, the lagging kVAR of the load is reduced by the p.f. correction equipment, thus improving the p.f. to $\cos \phi_2$.

$$BC = AB - AC$$

$$kVAR_{\text{final}} = kVAR_1 - kVAR_2$$

$$kVAR_{\text{final}} = OA(\tan \phi_1 - \tan \phi_2)$$

$$kVAR_{\text{final}} = kW(\tan \phi_1 - \tan \phi_2)$$



Knowing

*the lagging kVAR₁ supplied by the load

*the leading kVAR₂ supplied by the p.f. correction equipment (static capacitor or Synchronous motor), the desired results can be obtained.

Example

6.3 A single phase a.c. generator supplies the following loads:

- Lighting load of 20 kW at unity power factor.
- Induction motor load of 100 kW at p.f. 0.707 lagging.
- Synchronous motor load of 50 kW at p.f. 0.9 leading.

Calculate the total kW and kVA delivered by the generator and the power factor at which it works.

Solution: Using the suffixes 1, 2 and 3 to indicate the different loads, we have,

$$kVA_1 = \frac{kW_1}{\cos \phi_1} = \frac{20}{1} = 20 \text{ kVA}$$

$$kVA_2 = \frac{kW_2}{\cos \phi_2} = \frac{100}{0.707} = 141.4 \text{ kVA}$$

$$kVA_3 = \frac{kW_3}{\cos \phi_3} = \frac{50}{0.9} = 55.6 \text{ kVA}$$

These loads are represented in Fig. 6.10. The three kVAs are not in phase. In order to find the total kVA, we resolve each kVA into rectangular components – kW and kVAR as shown in Fig. 6.10.

The total kW and kVAR may then be combined to obtain total kVA.

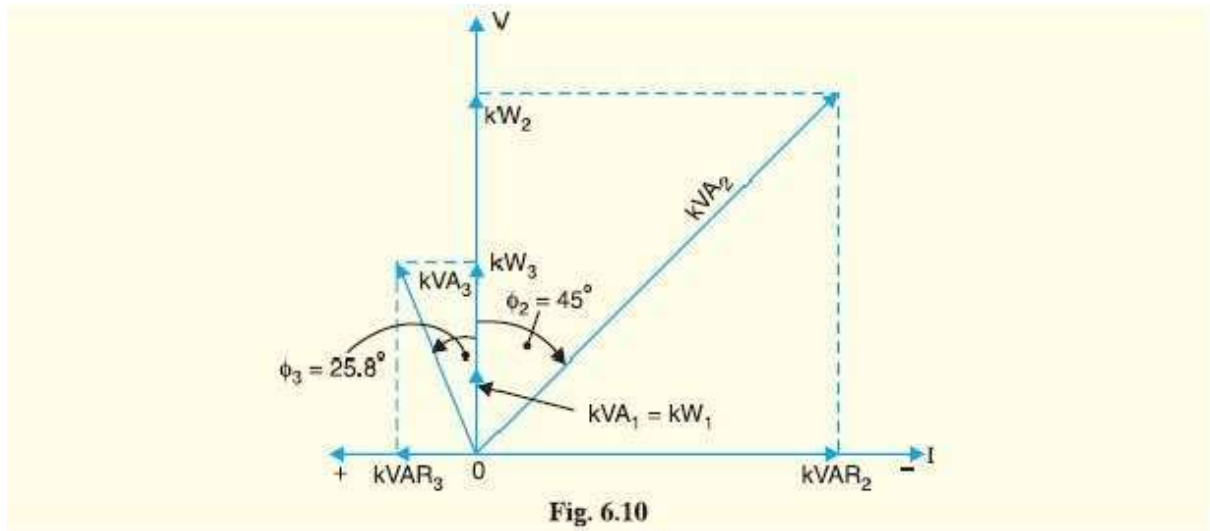


Fig. 6.10

$$\text{kVAR}_1 = \text{kVA}_1 \sin \phi_1 = 20 \times 0 = 0$$

$$\text{kVAR}_2 = \text{kVA}_2 \sin \phi_2 = -141.4 \times 0.707 = -100 \text{ kVAR}$$

$$\text{kVAR}_3 = \text{kVA}_3 \sin \phi_3 = +55.6 \times 0.436 = +24.3 \text{ kVAR}$$

Note that kVAR_2 and kVAR_3 are in opposite directions ; kVAR_2 being a lagging while kVAR_3 being a leading kVAR.

$$\text{Total kW} = 20 + 100 + 50 = \mathbf{170 \text{ kW}}$$

$$\text{Total kVAR} = 0 - 100 + 24.3 = -75.7 \text{ kVAR}$$

$$\text{Total kVA} = \sqrt{(\text{kW})^2 + (\text{kVAR})^2} = \sqrt{(170)^2 + (75.7)^2} = \mathbf{186 \text{ kVA}}$$

$$\text{Power factor} = \frac{\text{Total kW}}{\text{Total kVA}} = \frac{170}{186} = \mathbf{0.914 \text{ lagging}}$$

The power factor must be lagging since the resultant kVAR is lagging.



UNIT-III

ELECTRICAL MACHINES

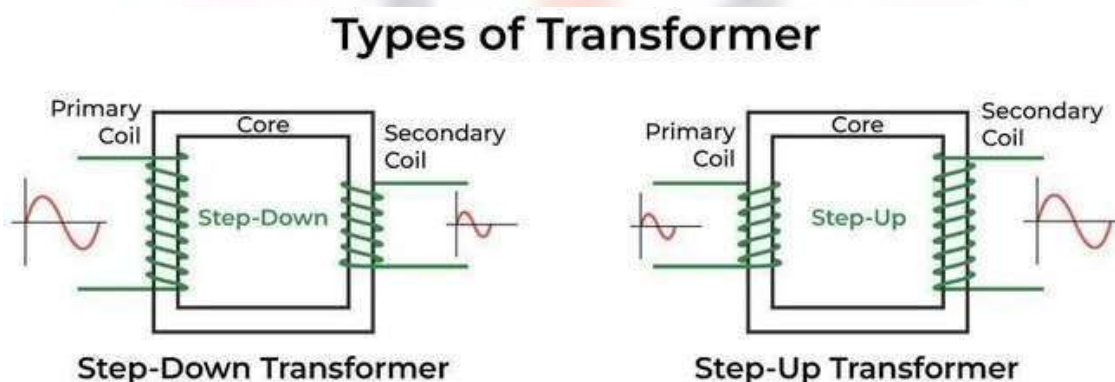
A transformer is a static electrical device that transmits AC power from one circuit to another at a constant frequency, but the voltage level may be changed, implying the voltage can be increased or decreased depending on the requirement.

Transformer is the simplest device that is used to transfer electrical energy from one alternating-current circuit to another circuit or multiple circuits, through the process of electromagnetic induction. A transformer works on the principle of **electromagnetic induction** to step up or step down voltage. Transformer either increases AC voltage (Step-up transformer) or decreases AC voltage (Step-down transformer). Transformer which is normally utilized in the transmission and distribution of alternating current power is fundamentally a voltage control device. Transformer are used for a wide range of purposes, including increasing the voltage from electric generators to enable long-distance transmission of electricity and decreasing the voltage of conventional power circuits to run low-voltage devices like doorbells and toy electric trains.

Types of Transformer

Transformer types based on Voltage Level

There are primarily two types of Transformer based on the operating voltage. The following are some of them:



- **Step-down Transformer:** The primary voltage is converted to a low voltage across the secondary output using a step-down transformer. The number of windings on the primary side of a step-down transformer is more than on the secondary side. As a result, the overall secondary-to-primary winding ratio will always be less than one. Step-down transformers are used in electrical systems that distribute electricity over long distances and operate at extremely high voltages to ensure minimum loss and economical solutions. Step-down transformers are used to change high-voltage into low-voltage supply lines.
- **Step-up Transformer:** The secondary voltage of a step-up transformer is raised from the low primary voltage. Because the primary winding has fewer turns than the secondary winding in this sort of transformer, the ratio of the primary to secondary winding will be greater than one. Step-up transformers are frequently used in electronics stabilizers, inverters, and other devices.

that convert low voltage to a significantly high voltage. A step-up transformer is also used in the

distribution of electrical power. For applications connected to power distribution, high voltage is necessary. In the grid, a step-up transformer is used to raise the voltage level prior to distribution.

Transformer Types based on Core Material

Different types of Transformer are used in the power and electronics industries, depending on the core materials, which are:

- **Iron Core Transformer:** Multiple soft iron plates are used as the core of an iron core transformer. The iron's strong magnetic properties of the iron core transformer have extremely high flux linkage. As a result, the iron core transformer has high efficiency. The soft iron core plates come in a variety of sizes and shapes. A few typical shapes include E, I, U, and L.
- **Ferrite Core Transformer:** Due to its high magnetic permeability, a ferrite core transformer uses one. In the high-frequency application, this kind of transformer provides incredibly low losses. In high-frequency applications like switch mode power supplies (SMPS), RF-related applications, etc., ferrite core transformers are used as a result.
- **Toroidal Core Transformer:** Iron core or ferrite core are two examples of toroid-shaped core materials used in transformer. For their excellent electrical performance, toroids, which have a ring- or donut-shaped core material, are frequently used. The ring form results in very low leakage inductance and extremely high inductance and Q factors.
- **Air Core transformer:** The core material of an air core transformer is not a real magnetic core. The air is used solely in the air-core transformer flux linkage. The primary coil of an air-core transformer generates an alternating current, producing an electromagnetic field all around it.

Transformer Types based on Winding Arrangement

- **Auto Winding transformer:** The primary and secondary windings have always been fixed, but with an auto-winding transformer, they can be connected in series, and the center-tapped node can be moved. The secondary voltage can be altered by changing the location of the center tap. The auto is used to alert the self or a single coil and is not the abbreviation for Automatic. This coil creates a ratio using main and secondary components. The main and secondary ratio is determined by the location of the center tap node, which changes the output voltage. The VARIAC, a device that generates variable AC from a steady AC input, is used the most frequently.

Types of Transformer based on Usage

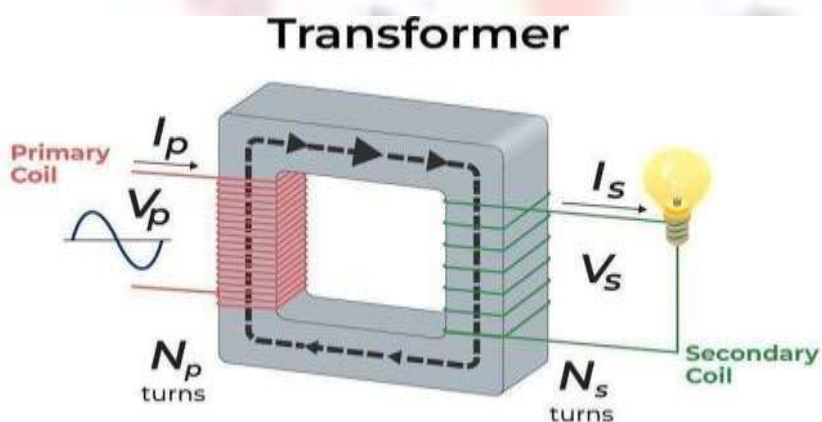
Transformer come in a wider range of variants, each of which operates in a distinct field. Thus, based on their proposed use, transformer can be categorized as follows:

- **Power Transformer:** The energy is transferred to the substation or the general electrical supply using a larger power transformer. Between the major distribution grid and the power generator, this transformer serves as a link. Power Transformer can be further divided into three groups based on their power rating and specification-
 - Small power transformer,
 - Medium power transformer, and
 - Large power transformer

- **Measurement Transformer:** Instrument transformer is another name for measurement transformer. This is yet another measurement tool that is usually utilized in the power domain. To separate the primary power and convert the current and voltage in a smaller ratio to its secondary output, a measuring transformer is used.
- **Distribution Transformer:** The distribution transformer functions as a step-down transformer, converting high grid voltage to the appropriate voltage for the end user, typically 110V or 230V. Depending on the conversion capacity or ratings, the distribution transformer might be less in size or larger.
- **Pulse Transformer:** One of the most popular PCB-mounted transformer that generates electrical pulses with a consistent amplitude are pulse transformer. It is utilized in a number of digital circuits where the demand for isolated pulse creation exists.
- **Audio Output Transformer:** Another frequent transformer in the electronics industry is the audio transformer. It is specifically used in applications involving audio where impedance matching is necessary.

WORKING PRINCIPLE OF A TRANSFORMER

The fundamental principle of how the transformer functions are mutual induction between the two coils or [Faraday's Law of Electromagnetic Induction](#). Below is a description of how the transformer operates. The laminated silicon steel core of the transformer is covered by two distinct windings. According to the diagram below, the primary winding is the one to which the AC supply is connected, and the secondary winding is the one to which the load is connected. Only alternating current can be used because mutual induction between the two windings requires an alternating flux.



The transformer primary winding produces an alternating flux, known as the mutual flux, when an alternating voltage is applied, in accordance with the mutual inductance principle.

According to Faraday's rule of electromagnetic induction, this alternating flux links the transformer primary and secondary windings magnetically and generates EMFs E_1 in the primary winding and E_2 in the secondary winding. The EMF (E_1) is referred to as the primary EMF,

while

the EMF (E_2) is the secondary EMF.

$$E_2 = -N_2 \frac{d\phi_m}{dt}$$

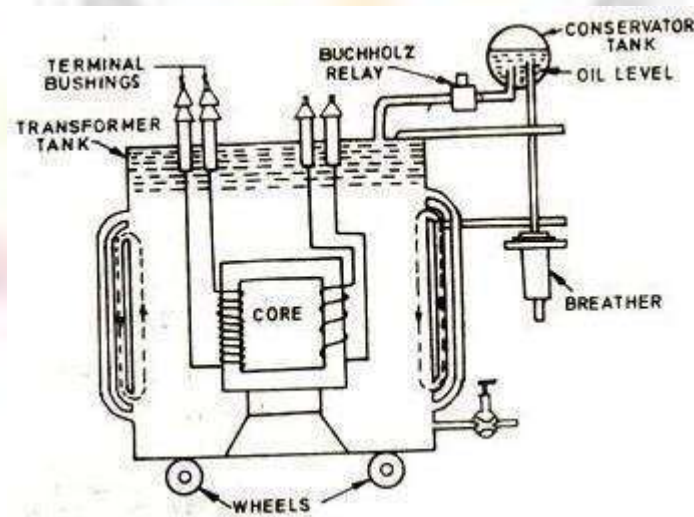
And Dividing above equations, to obtain the ratio as:

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

From the expression above, it is clear that the size of EMFs E_1 and E_2 is dependent on the number of turns in the transformer primary and secondary windings, respectively. If $N_2 > N_1$, then $E_2 > E_1$, and the transformer will be a step-up transformer; if $N_2 < N_1$, then $E_2 < E_1$, and the transformer will be a step-down transformer.

If a load is now connected across the secondary winding, the load current I_2 will flow through the load as a result of the EMF E_2 . As a result, a transformer makes it possible to transfer electricity with a change in voltage level from one electric circuit to another.

CONSTRUCTION OF A TRANSFORMER



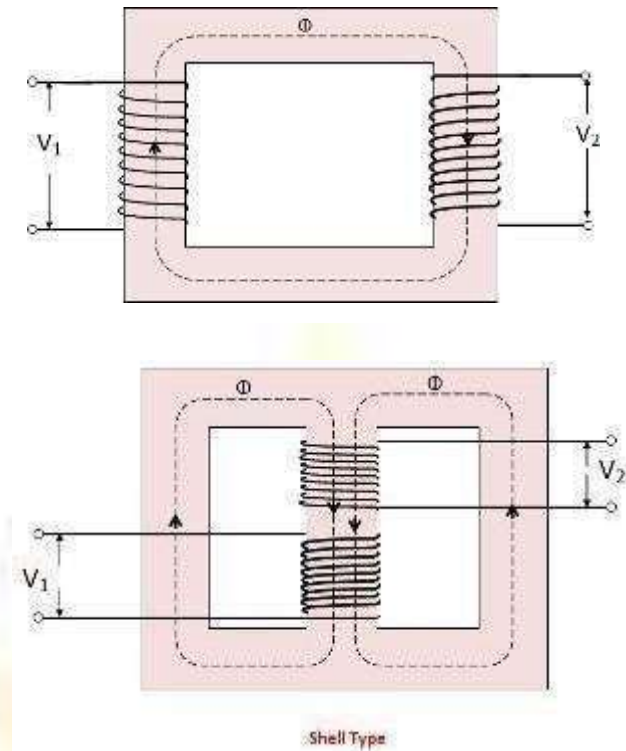
A transformer majorly consists of three parts:

Core

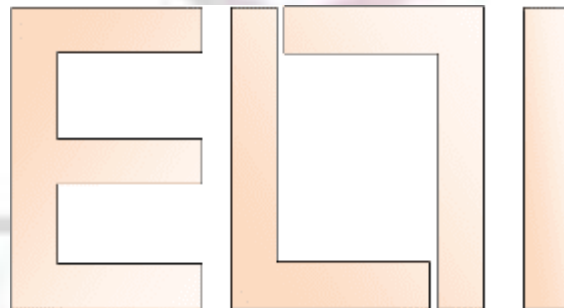
The transformer core serves as a support for the winding. Additionally, it offers a magnetic flux flow channel with minimal resistance. As seen in the image, the winding is looped around the core. To cut down on losses in a transformer, it has a laminated soft iron core. Core composition is determined by variables including operational voltage, current, and power, among others. The core diameter is negatively correlated with iron losses and directly correlated with copper losses.

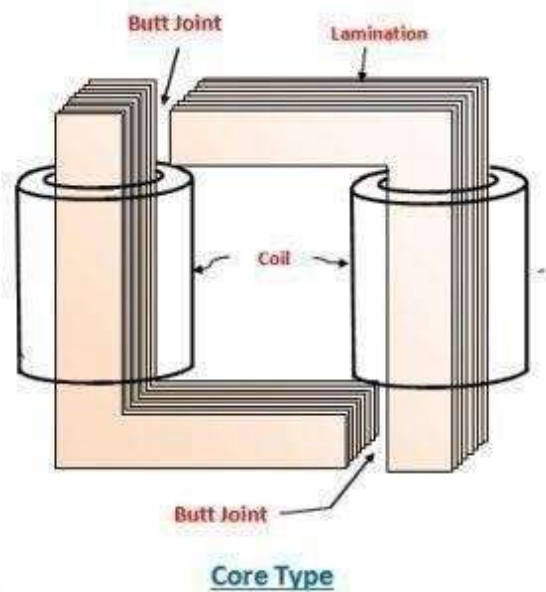
The core lamination joined in the form of a strip which is shown in fig.. For avoiding narrow gaps in between layer, the laminated layers are staggered in order. And these staggered joints are said 'imbriated' joint. Constructionally, the transformer are two types. The two types are—

1. Core type.
2. Shell type.



In both core is shell-type transformer, the laminations are in the following types. i.e L, E, & I types which are shown in fig. To avoid high reluctance at joints, the laminations are butted each other which is shown in fig.

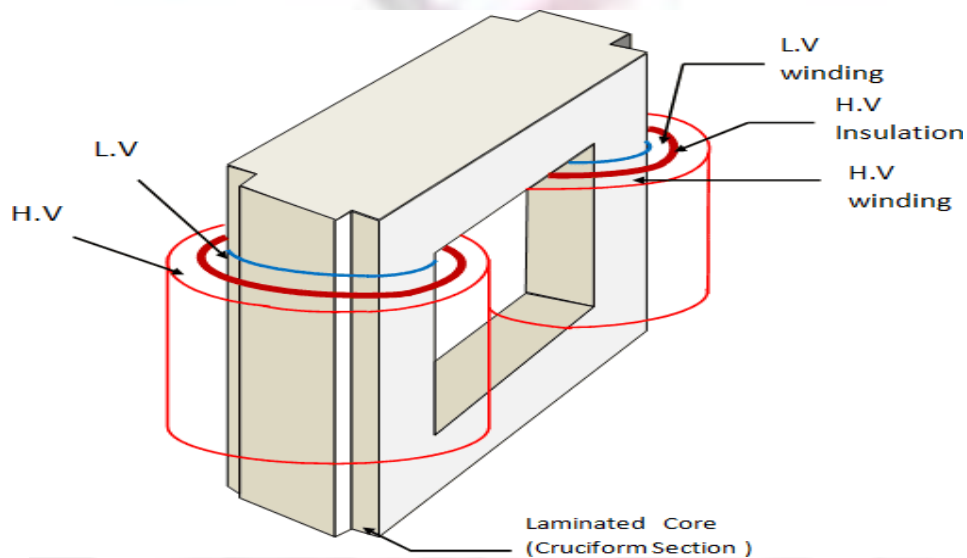




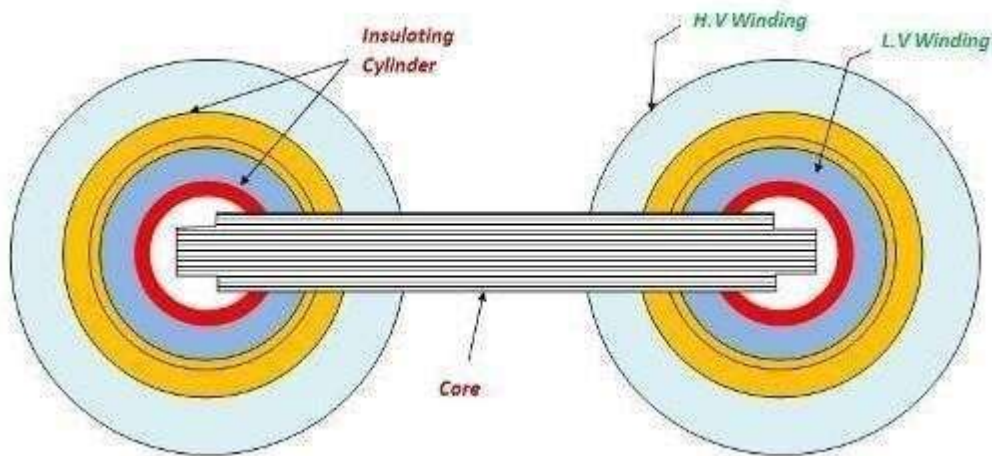
Core-type Transformer

In this Core type transformer, the windings surround a considerable part of the core. Generally, the core of the core-type transformer is a rectangular shape and the coils are both circular or rectangular in form and the windings are located on the opposite limbs of the core which is shown in fig. In most of the large-size core-type transformer, round or circular cylindrical coils are used because the mechanical strength of circular cylindrical coils is high.

And these cylindrical coils are wound in a helical layer with different layers insulated from each other by paper, cloth, micarta board or cooling ducts. For reducing leakage flux, and H.V & L.V

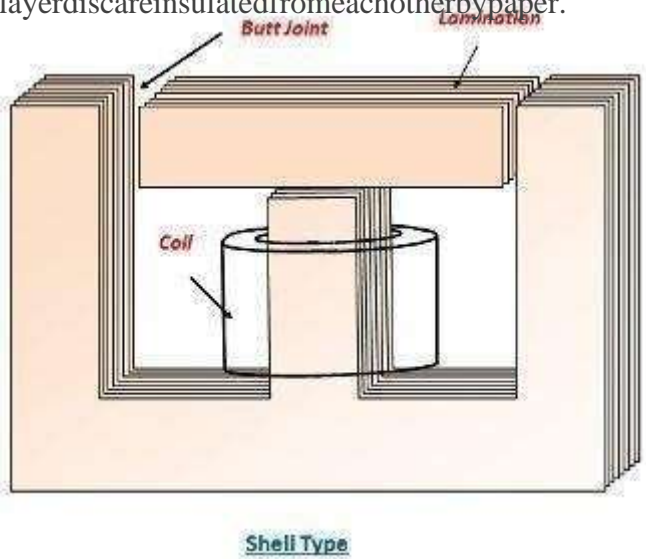


windings are placed one after another separating with high insulation cylinder on fuller board which is shown in fig.



Shelltype Transformer

The shell type transformer is a simple rectangular form and the core surrounds the considerable portion of the windings which is shown in fig. Both the primary & secondary windings are placed in the one limb. And the coils are wound in from multi-layer disc type. The different layers of the multi-layer disc are insulated from each other by paper.



Windings

The copper wires that are wound over the transformer core are known as windings. Copper cables are used because Copper's high conductivity reduces transformer loss because resistance to current flow lowers as conductivity rises. And copper's high degree of ductility makes it possible to produce incredibly thin wires out of it.

The two basic types of windings are, windings for the primary and secondary coils. The primary winding is the group of winding turns that receive supply current. The number of winding turns from which output is derived is known as secondary winding. Insulation coating agents are used to insulate the primary and secondary windings from one another.

Insulation Agents

Transformer require insulation to keep the windings apart and prevent short circuits. This makes mutual induction easier. Transformer stability and durability are influenced by insulation agents. In a transformer, the following are employed as insulating mediums: Insulating fluid, tape, Paper, and Lamination made of wood.

Tank

A transformer tank serves two purposes:

- The core and the windings are protected from the elements, such as rain and dust.
- It functions as an oil container as well as a support for all other

transformer attachments. **Transformer Oil**

The majority of the huge transformer are submerged in oil. The transformer oil adds insulation between the conductors, improves heat dissipation from the coils, and has fault-detecting capabilities. Transformer oil is typically made of hydrocarbon mineral oil.

Oil Conservators

The oil conservator is situated above the transformer tank and bushings. Some transformer oil conservators contain a rubber bladder. When a transformer is loaded, the ambient temperature rises, causing the amount of oil inside the transformer to increase. The transformer conservator tank has enough room for the increased transformer oil. It also serves as a reservoir for oil that is used to insulate buildings.

Breather

All oil-immersed transformer with conservator tank includes it. It aids in the protection of the oil against moisture.

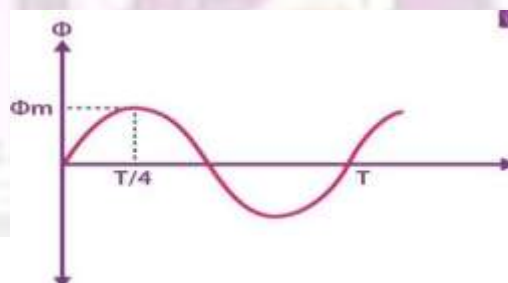
Radiators and Fans

The majority of the power lost in the transformer is dissipated as heat. Radiators and fans aid in the dissipation of heat generated by the transformer and provide protection against failure. The majority of dry transformer are cooled by natural air.

Ideal Transformer

The ideal transformer has no losses. There is no magnetic leakage flux, ohmic resistance in its windings and no iron loss in the core.

EMF EQUATION OF TRANSFORMER



N_2 – Number of turns in the

secondary Φ_m –

Maximum flux in the weber (Wb)

T – Time period. It is the time taken for 1 cycle.

The flux formed is a sinusoidal wave. It rises to a maximum value of Φ_m and decreases to a negative maximum of Φ_m . So, flux reaches a maximum in one-quarter of a cycle. The time taken is equal to $T/4$.

Average rate of change of flux = $\Phi_m / (T/4) = 4f\Phi_m$ Where, f = frequency, $T = 1/f$

Induced EMF per turn = Rate of change of flux per turn
Form factor = RMS value / average value
 $\text{RMS value} = 1.11(4f\Phi_m) = 4.44f\Phi_m$ [form factor of a sine wave is 1.11]

RMS value of EMF induced in winding = RMS value of EMF per turn \times No. of turns

Primary Winding

RMS value of induced EMF = $E_1 = 4.44f\Phi_m \times N_1$

Secondary Winding

RMS value of induced EMF = $E_2 = 4.44f\Phi_m \times N_2$

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = 4.44f\Phi_m$$

This

is the EMF equation of the transformer. For an ideal transformer at no load condition, E_1 = Supply voltage on the primary winding

E_2 = Terminal voltage (theoretical or calculated) on the secondary winding
Voltage Transformation Ratio

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = k$$

k is called the voltage transformation ratio, which is a constant. Case 1: If $N_2 > N_1$, $k > 1$, it is called a step-up transformer.

Case 2: If $N_2 < N_1$, $k < 1$, it is called a step-down transformer.

IDEAL TRANSFORMER:

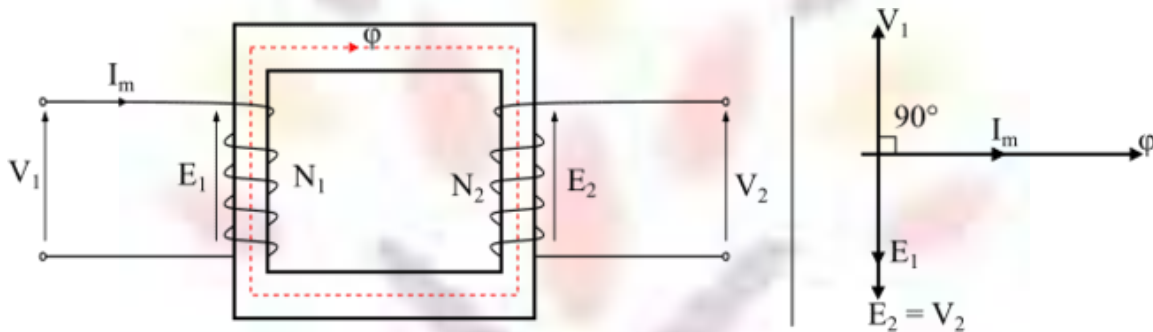
An ideal transformer is an imaginary transformer which has the following characteristics—

- The primary and secondary windings have negligible (or zero) resistance.
- No leakage flux, i.e., whole of the flux is confined to the magnetic circuit.
- The magnetic core has infinite permeability, thus negligible mmf is required to establish flux in the core.
- There are no losses due to winding resistances, hysteresis and eddy currents. Hence, the efficiency is 100%.

Working of Ideal Transformer

Ideal Transformer on No Load

Consider an ideal transformer on no-load, i.e., its secondary winding is open circuited (see the figure). Thus, the primary winding is a coil of pure inductance.

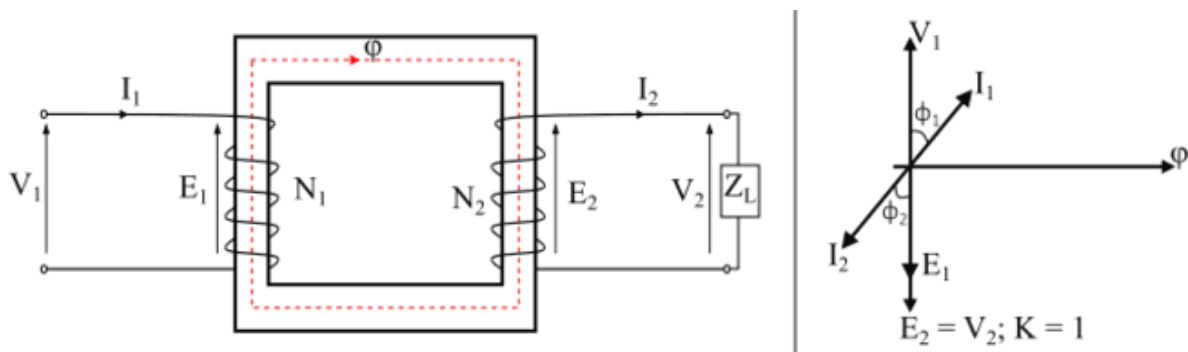


When an alternating voltage V_1 is applied to the primary winding, it draws a very small magnetising current I_m to establish the flux in the core, which lags behind the applied voltage by 90° . The magnetising current I_m produces an alternating flux ϕ_m which is proportional to and in phase with it. This alternating flux (ϕ_m) links the primary and secondary windings magnetically and induces EMF E_1 in the primary winding and EMF E_2 in the secondary winding.

The EMF induced in the primary winding E_1 is equal to and in opposition to the applied voltage V_1 (according to Lenz's law). The EMFs E_1 and E_2 lag behind the flux (ϕ_m) by 90° , although their magnitudes depend upon the number of turns in the primary and the secondary windings. From the phasor diagram of the ideal transformer on no-load, it is clear that the flux is common to both the windings, hence it can be taken as the reference phasor. Also, the EMFs E_1 and E_2 are in phase with each other, but E_1 is equal to V_1 and 180° out of phase with it.

Ideal Transformer On-Load

When a load is connected across the terminals of the secondary winding of the ideal transformer, the transformer is said to be loaded and a load current flows through the secondary winding and the load.



Consider an inductive load of impedance Z_L is connected across the secondary winding of the ideal transformer (see the figure). Then, the secondary EMF E_2 will cause a current I_2 to flow through the secondary winding and the load, which is given by,

$$I_2 = E_2 / Z_L = V_2 / Z_L$$

Since, for an ideal transformer, the EMF E_2 is equal to secondary terminal voltage V_2 .

Here, the load is inductive, therefore, the current I_2 will lag behind the E_2 or V_2 by an angle ϕ_2 . Also, the no-load current I_0 being neglected because the transformer is ideal one.

The current flowing in the secondary winding (I_2) sets up an mmf ($N_2 I_2$) which produces a flux ϕ_2 in opposite direction to the main flux (ϕ_m). As a result, the total flux in the core changes from its original value, however, the flux in the core should not change from its original value. Therefore, to maintain the flux in the core at its original value, the primary current must develop an MMF which can counter-balance the demagnetising effect of the secondary mmf $N_2 I_2$. Hence, the primary current I_1 must flow such that, $N_1 I_1 = N_2 I_2$

$$\Rightarrow I_1 = (N_2 / N_1) \times I_2 = K I_2$$

Therefore, the primary winding must draw enough current to neutralise the demagnetising effect of the secondary current so that the main flux in the core remains constant. Hence, when the secondary current (I_2) increases, the primary current (I_1) also increases in the same manner and keeps the mutual flux (ϕ_m) constant.

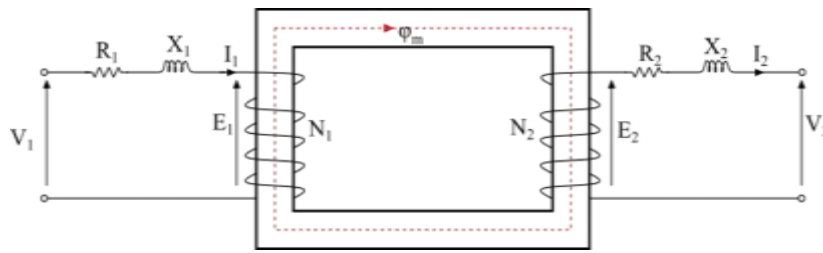
It is clear from the phasor diagram of the ideal transformer on-load that the secondary current I_2 lags behind the secondary terminal voltage V_2 by an angle of ϕ_2 .

PRACTICAL TRANSFORMER

A practical transformer is the one which has following properties—

- The primary and secondary windings have finite resistance.
 - There is a leakage flux, i.e., whole of the flux is not confined to the magnetic circuit.
 - The magnetic core has finite permeability, thus a considerable amount of mmf is required to establish flux in the core.
 - There are losses in the transformer due to winding resistances, hysteresis and eddy currents.
- Therefore, the efficiency of a practical transformer is less than 100%.

The figure shows a typical practical transformer, which possesses all the characteristics that are described above.



Transformer on No Load Condition

When the transformer is operating at no load, the secondary winding is open-circuited, which means there is no load on the secondary side of the transformer and, therefore, current in the secondary will be zero. While primary winding carries a small current I_0 called no-load current which is **2 to 10% of the rated current**.

This current is responsible for supplying the iron losses (hysteresis and eddy current losses) in the core and a very small amount of copper losses in the primary winding. The angle of lag depends upon the losses in the transformer. The power factor is very low and varies from **0.1 to 0.15**.

The no-load current consists of two components:

- **Reactive or magnetizing component I_m**
(It is in quadrature with the applied voltage V_1 . It produces flux in the core and does not consume any power).
- **Active or power component I_w** , also known as a working component
(It is in phase with the applied voltage V_1 . It supplies the iron losses and a small amount of primary copper loss).

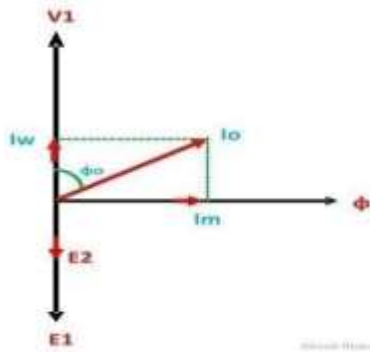
The following steps are given below to draw the phasor diagram:

1. The function of the magnetizing component is to produce the magnetizing flux, and thus, it will be in phase with the flux.
2. Induced EMF in the primary and the secondary winding lags the flux ϕ by 90 degrees.
3. The primary copper loss is neglected, and secondary current losses are zero as $I_2 = 0$.

Therefore, the current I_0 lags behind the voltage vector V_1 by an angle ϕ_0 called the no-load power factor angle and is shown in the phasor diagram above.

4. The applied voltage V_1 is drawn equal and opposite to the induced EMF E_1 because the difference between the two, at no load, is negligible.
5. Active component I_w is drawn in phase with the applied voltage V_1 .

6. Thephasorsumofmagnetizingcurrent I_m andtheworkingcurrent I_w givessthe no-load



current I_0 .

Fromthephasordiagramdrawnabove,thefollowingconclusionsaremade:

$$\text{Working component } I_w = I_0 \cos \phi_0$$

$$\text{No load current } I_0 = \sqrt{I_w^2 + I_m^2}$$

$$\text{Magnetizing component } I_m = I_0 \sin \phi_0$$

$$\text{Power factor } \cos \phi_0 = \frac{I_w}{I_0}$$

$$\text{No load power input } P_0 = V_1 I_0 \cos \phi_0$$

Thisisallabouttransformerinnoloadcondition.**TRANSFOR**

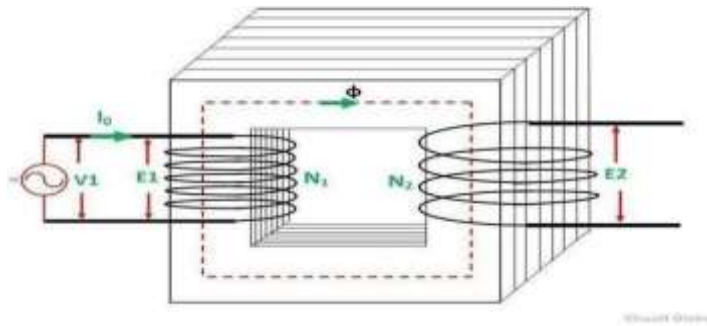
MERONLOAD CONDITION

When the transformer is on the loaded condition, the secondary of the transformer is connected to load. The load can be resistive, inductive or capacitive. The current I_2 flows through the secondary winding of the transformer. The magnitude of these secondary current depends on the terminal voltage V_2 and the load impedance. The phase angle between the secondary current and voltage depends on the nature of the load.

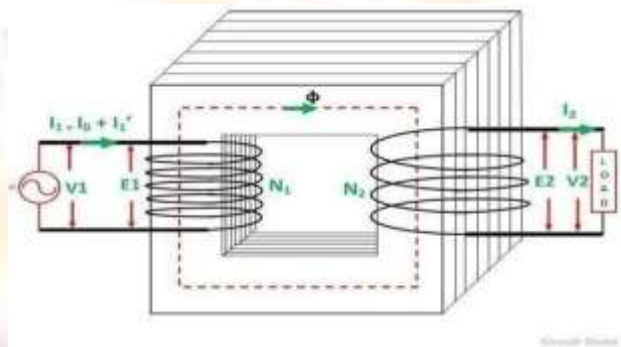
Operation of the Transformer on Load Condition

The Operation of the Transformer on Load Condition is explained below:

- When the secondary of the transformer is kept open, it draws the no-load current from the main supply. The no-load current induces the magnetomotive force $N_0 I_0$ and this force sets up the flux Φ in the core of the transformer. The circuit of the transformer at no load condition is shown in the figure below:



When the load is connected to the secondary of the transformer, I_2 current flows through their secondary winding. The secondary current induces the magnetomotive force $N_2 I_2$ on the secondary winding of the transformer. This force set up the flux ϕ_2 in the transformer core. The flux ϕ_2 opposes the flux ϕ , according to **Lenz's law**.



- As the flux ϕ_2 opposes the flux ϕ , the resultant flux of the transformer decreases and this flux reduces the induced EMF E_1 . Thus, the strength of the V_1 is more than E_1 and an additional primary current I_1' drawn from the main supply.

The additional current is used for restoring the original value of the flux in the core of the transformer so that $V_1 = E_1$. The primary current I_1' is in phase opposition with the secondary current I_2 . Thus, it is called the **primary counter-balancing current**.

- The additional current I_1' induces the magnetomotive force $N_1 I_1'$. And this force set up the flux ϕ_1 . The direction of the flux is the same as that of the ϕ and it cancels the flux ϕ_2 which induces because of the MMF $N_2 I_2$.

Now, $N_1 I_1' = N_2 I_2$

$$I_1' = \left(\frac{N_2}{N_1} \right) I_2 = K I_2$$

Therefore,

- The phase difference between V_1 and I_1 gives the power factor angle ϕ_1 of the primary side of the transformer.

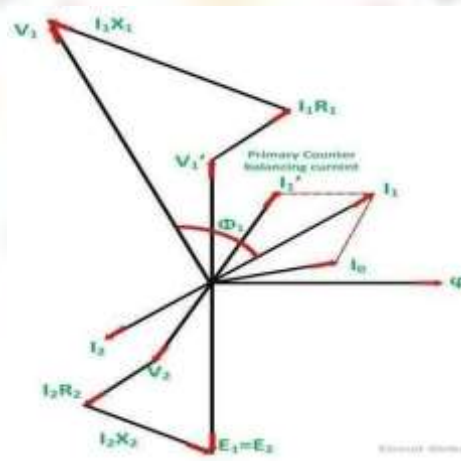
- The power factor of the secondary side depends upon the type of load connected to the transformer.

If the load is inductive as shown in the above phasor diagram, the power factor will be lagging, and if the load is capacitive, the power factor will be leading. The total primary current I_1 is the vector sum of the currents I_0 and I_1' . i.e

$$\bar{I}_1 = \bar{I}_0 + \bar{I}_1'$$

Phasor Diagram of Transformer on Inductive Load

The phasor diagram of the actual transformer when it is loaded inductively is shown below:



Phasor Diagram of the Transformer on Inductive Load Step 1

Draw the phasor diagram

- Take flux ϕ , as reference
- Induces emf E_1 and E_2 lag the flux by 90 degrees.
- The component of the applied voltage to the primary equal and opposite to induced emf in the primary winding. E_1 is represented by V_1' .
- Current I_0 lags the voltage V_1' by 90 degrees.
- The power factor of the load is lagging. Therefore current I_2 is drawn lagging E_2 by an angle ϕ_2 .
- The resistance and the leakage reactance of the windings result in a voltage drop, and hence secondary terminal voltage V_2 is the phasor difference of E_2 and voltage drop.

$V_2 = E_2 - \text{voltage drops}$

$I_2 R_2$ is in phase with I_2 and $I_2 X_2$ is in quadrature with I_2 .

- The total current flowing in the primary winding is the phasor sum of I_1' and I_0 .

- Primary applied voltage V_1 is the phasor sum of V_1' and the voltage drop in the primary winding.
- Current I_1' is drawn equal and opposite to the current I_2 $V_1 = V_1' + \text{voltage drop}$

$V_1' + \text{voltage drop}$

$I_1 R_1$ is in phase with I_1 and $I_1 X_1$ is in quadrature with I_1 .

The phasor difference between V_1 and I_1 gives the power factor angle ϕ_1 of the primary side of the transformer.

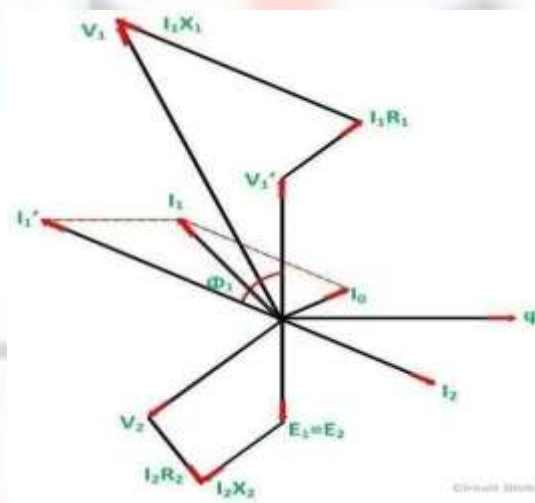
- The power factor of the secondary side depends upon the type of load connected to the transformer.
 - If the load is inductive as shown in the above phasor diagram, the power factor will be lagging, and if the load is capacitive, the power factor will be leading.
- Where $I_1 R_1$ is the resistive drop in the primary windings

$I_2 X_2$ is the reactive drop in the secondary winding Similarly

early

Phasor Diagram of Transformer on Capacitive Load

The Transformer on the Capacitive load (leading power factor load) is shown below in the phasor diagram.



Phasor Diagram of the Transformer on Capacitive Load Steps to

draw the phasor diagram at capacitive load

- Take flux ϕ as reference
- Induces emf E_1 and E_2 lag the flux by 90 degrees.
- The component of the applied voltage to the primary equal and opposite to induced emf in the primary winding. E_1 is represented by V_1' .
- Current I_0 lags the voltage V_1' by 90 degrees.
- The power factor of the load is leading. Therefore current I_2 is drawn leading E_2

- The resistance and the leakage reactance of the windings result in a voltage drop, and hence secondary terminal voltage V_2 is the phasor difference of E_2 and voltage drop.

$$V_2 = E_2 - \text{voltage drop}$$

$I_2 R_2$ is in phase with I_2 and $I_2 X_2$ is in quadrature with I_2 .

- Current I_1' is drawn equal and opposite to the current I_2
- The total current I_1 flowing in the primary winding is the phasor sum of I_1' and I_0 .
- Primary applied voltage V_1 is the phasor sum of V_1' and the voltage drop in the primary winding.

$$V_1 = V_1' + \text{voltage drop}$$

$I_1 R_1$ is in phase with I_1 and $I_1 X_1$ is in quadrature with I_1 .

- The phasor difference between V_1 and I_1 gives the power factor angle ϕ_1 of the primary side of the transformer.
- The power factor of the secondary side depends upon the type of load connected to the transformer.

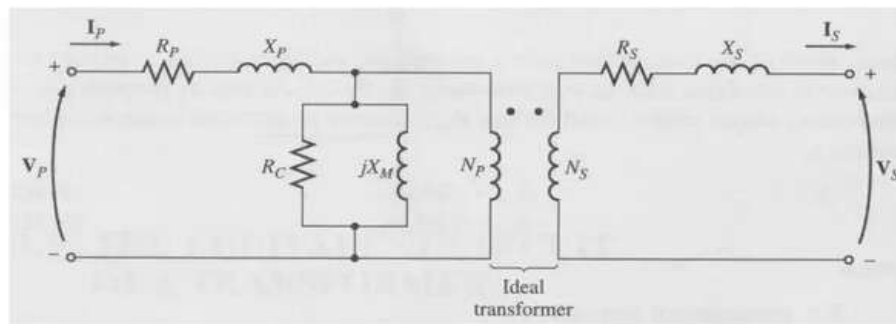
This is all about the phasor diagram on various loads.

The transformer's equivalent circuit:

To model a real transformer accurately, we need to account for the following losses:

1. COPPER LOSSES – RESISTIVE HEATING IN THE WINDINGS: $I^2 R$.
2. EDDY CURRENT LOSSES – RESISTIVE HEATING IN THE CORE: PROPORTIONAL TO THE SQUARE OF VOLTAGE APPLIED TO THE TRANSFORMER.
3. HYSTERESIS LOSSES – ENERGY NEEDED TO REARRANGE MAGNETIC DOMAINS IN THE CORE: NONLINEAR FUNCTION OF THE VOLTAGE APPLIED TO THE TRANSFORMER.
4. LEAKAGE FLUX – FLUX THAT ESCAPES FROM THE CORE AND FLUX THAT PASSES THROUGH ONE WINDING ONLY.

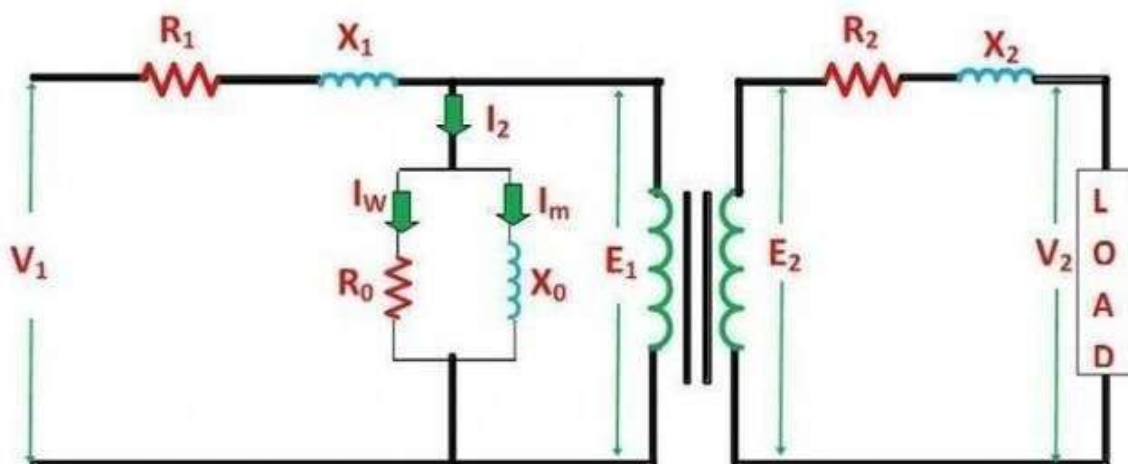
THE EXACT EQUIVALENT CIRCUIT OF A REAL TRANSFORMER



- ☐ Copper losses are modeled by the resistors R_P and R_S .
 - ☐ The leakage flux can be modeled by primary and secondary inductors.
 - ☐ The magnetization current can be modeled by a reactance X_M connected across the primary voltage source.
- e. The core-loss current can be modeled by a resistance R_C connected across the primary voltage source. Both magnetizing and core loss currents are nonlinear; therefore, X_M and R_C are just approximations.

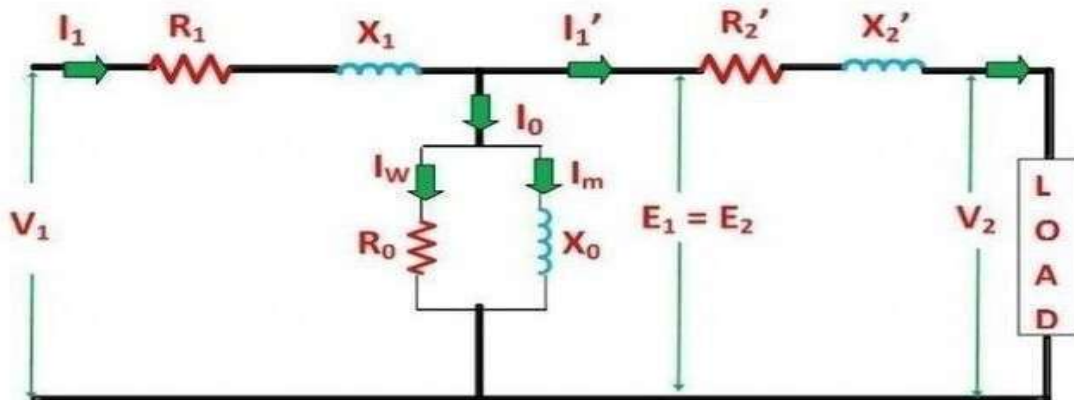
THE EXACT EQUIVALENT CIRCUIT OF A REAL TRANSFORMER:

The simplified equivalent circuit of a transformer is presented by considering all the properties of the transformer either on the primary or secondary side. The main equivalent circuit of the transformer is shown below in the diagram:



Equivalent Circuit of Transformer when all the quantities are referred to Primary side

In this method, to derive the equivalent circuit of transformer, all the features are to be considered as the primary side as presented in the figure below:



Equivalent Circuit of Transformer Referred to Primary Side (Reference: circuitglobe.com)

The following quantities are the values of resistance and reactance that can be calculated by the next equations. Secondary resistance based on the primary side is obtained as:

$$R_2' = R_2 / K^2$$

The equivalent resistance according to the primary consideration is obtained as:

$$R_{ep} = R_1 + R_2'$$

The secondary reactance based on the primary side is presented as:

$$X_2' = X_2 / K^2$$

The equivalent reactance depends upon the primary side is obtained as:

$$X_{ep} = X_1 + X_2'$$

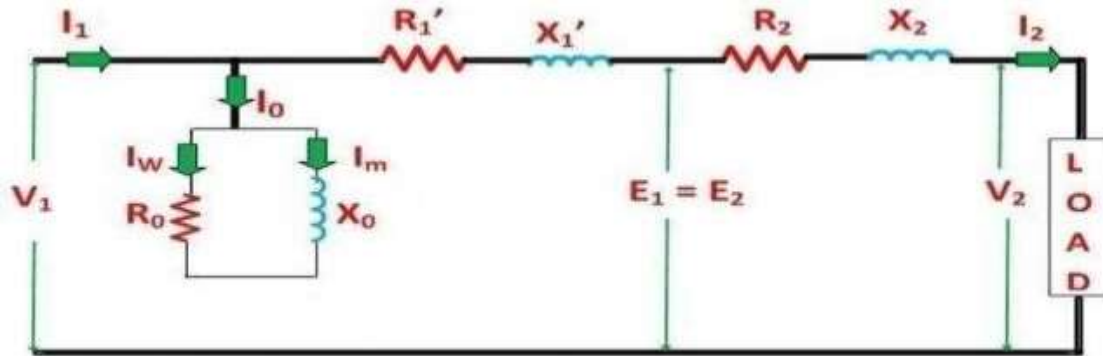
APPROXIMATE EQUIVALENT CIRCUIT OF TRANSFORMER

Due to the small value of I_0 in comparison with I_1 , it is less than 4 percent of the total load of the primary current and modifies the voltage reduction negligibly. As a result, it is a perfect approximation to reduce the excitation effect of the circuit in the approximate equivalent circuit of transformer method. The resistance and reactance of winding are arranged in a series configuration which can now be introduced as the equivalent reactance and resistance of transformer, based on any particular side. But in this method, it is the primary side or side 1 that determines the features of the circuit based on the next equation:

$$V_2' = KV_2$$

EQUIVALENT CIRCUIT OF TRANSFORMER WHEN ALL THE QUANTITIES ARE REFERRED TO SECONDARY SIDE

The equivalent circuit of transformer or the basic diagram is presented below when all the features are designed based on the secondary side.



Equivalent Circuit Referred to Secondary Side

The following properties are the values for resistance and reactance which can be obtained below. Basic resistance based on the secondary side is formulated as

$$R_1' = K^2 R_1$$

The equivalent value of resistance according to the secondary term is obtained as

$$R_{es} = R_2 + R_1'$$

The primary value of reactance based on the secondary side is presented as

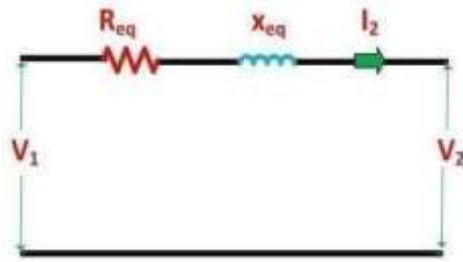
$$X_1' = K^2 X_1$$

And the equivalent value of reactance is obtained as

$$X_{es} = X_2 + X_1'$$

Because the no-load current or I_0 is commonly 2 to 4 percent of the full load value of rated current, the parallel configuration includes the R_0 resistance and X_0 reactance can be removed from the circuit without introducing any particular error in the performance of the transformer when the load is applied.

We can also apply further simplification in the equivalent circuit of transformer by removing the parallel terms in the circuit including R_0 and X_0 . This simplified diagram of the system is presented below:



SIMPLIFIED EQUIVALENT CIRCUIT OF TRANSFORMER

YPES OF LOSSES IN TRANSFORMERS:



An ideal **transformer** is very efficient, they don't have energy losses. It means power supplied to the transformer's input terminal must be equivalent to the power supplied to the transformer's output terminal. So the input power and output power in an ideal transformer are equal including zero **energy losses**.

In reality, both the input and output powers of the transformer will not equal because of electrical losses within the transformer. Because the transformer is a static device, it doesn't have any movable parts, so we cannot observe mechanical losses but electrical losses will occur like copper and iron. This article discusses an overview of different types of losses in a transformer.

There are different kinds of losses that will be occurred in the transformers such as copper, hysteresis, eddy, iron, stray & dielectric. The copper loss commonly occurs due to the resistance in the transformer winding whereas hysteresis losses will be occurred due to the magnetization change within the core.

1. Core Losses Or Iron Losses

Eddy current loss and hysteresis loss depend on the magnetic properties of the material used for the construction of the core. So, these losses are also known as core losses or iron losses.

Hysteresis loss in transformer: The reason is the reversal of magnetization in the transformer core. This loss depends on the volume and grade of the iron, frequency of magnetic reversals and value of flux density. We have the Steinmetz formula:

$$W_h = \eta B_{max}^{1.6} f V \text{ (watts)}$$

Where, η = Steinmetz hysteresis constant V = volume of the core in m^3

Eddy current loss in transformer: The AC current is supplied to the primary winding which sets up alternating magnetizing flux in the transformer. When this flux flows to a secondary winding, it produces induced emf in it. But some part of this flux also gets linked with other conducting parts such as steel core or iron body of the transformer, which will result in induced emf in those parts, causing small circulating current in them. This current is called as eddy current. Due to the current, some energy will be dissipated in the form of heat.

$$W_e = \eta B_{\max}^2 f^2 t^2 V \text{ (watts)}$$

Where, η = Steinmetz hysteresis constant
 V = volume of the core in m^3
 t = Thickness of core

Copper Loss

The ohmic resistance of the transformer windings creates copper loss. The copper loss for the primary winding is $I_1^2 R_1$ and for the secondary winding is $I_2^2 R_2$. Where, I_1 and I_2 are current in primary and secondary winding respectively, R_1 and R_2 are the resistances of primary and secondary winding respectively. We can see that $Cu \text{ loss}$ is proportional to square of the current, and current depends on the load. So that copper loss in transformer varies with the load.

Stray Loss

The reason for the types of loss is the occurrence of the leakage field. When compared with copper and iron losses, the percentage of stray losses are less, so these losses can be neglected.

Dielectric Loss

The oil of the transformer is the reason for this loss. Oil in transformer is an insulating material. When the oil in the transformer gets deteriorated then the transformer's efficiency will be affected.

VOLTAGE REGULATION OF TRANSFORMER:

Transformer's voltage regulation is the ratio of the difference between transformer no-load and full-load output voltage to its full-load output voltage expressed as a percentage (%).

In other words, transformer voltage regulation is the measure of supplying constant output voltage with different load currents.

In simple words, the change in magnitude of input and output voltage of the transformer is known as voltage regulation. i.e. the change in transformer secondary terminal voltage from no load to full load related to the no-load voltage is known as "voltage regulation".

Mathematically, the voltage regulation is expressed by the following formula.

$$\text{Voltage Regulation} = \frac{V_{\text{No Load}} - V_{\text{Full Load}}}{V_{\text{Full Load}}}$$

$$\% \text{ Voltage Regulation} = \frac{E_2 - V_2}{E_2} \times 100$$

Voltage regulation for primary winding of the transformer

$$\% \text{ Voltage Regulation} = \frac{E_1 - V_1}{E_1} \times 100$$

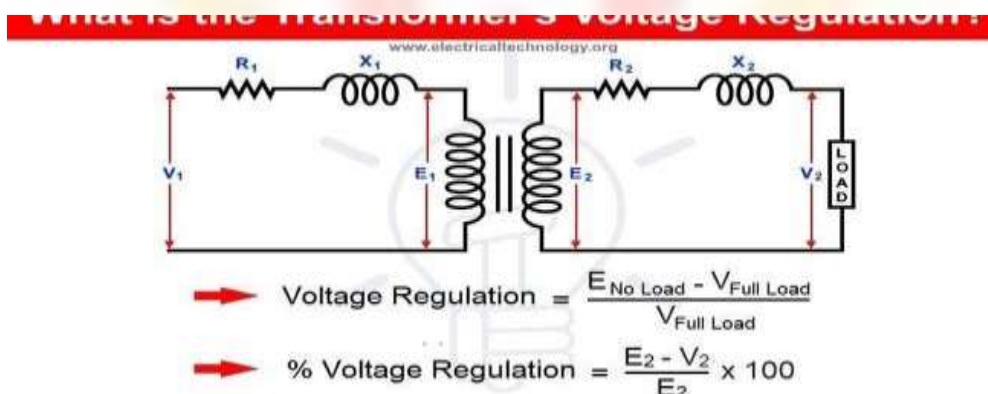
Where:

- ☐ E_1 = No load primary terminal voltage
- ☐ V_1 = Full load primary terminal voltage
- ☐ E_2 = No load secondary terminal voltage
- ☐ V_2 = Full load secondary terminal voltage

A [Transformer](#) will generally provide a higher output voltage at no load than when the transformer is fully loaded according to the [transformer nameplate](#) data rating capacity. Stated differently, under load, a transformer's output voltage drops slightly.

[Power transformer](#) should provide a constant output voltage (ideally as it is not possible in real). So it is the better option to have as much as a little variation in output voltage with different load currents. In this scenario, voltage regulation shows that how much a transformer can provide a constant secondary voltage with different loads connected to the transformer output.

The following basic transformer circuit and solved example will clear the concept of transformer's voltage regulation.



In first scenario, Suppose there is no load connected to the transformer's secondary, In this case of open circuit:

- ☐ No load current is flowing due to open circuit.
 - ☐ When no load current flows, there is no voltage drop and reactive drops across resistor and inductor respectively.
- Voltage drops across primary terminals are negligible.

In second scenario, the transformer is loaded i.e. there is a load connected to the secondary terminals of the transformer. In this case of loaded circuit:

- Load current is flowing due to completed circuit and load connected to these secondary terminals.
- Load current flows through the load, so there must be voltage drops across resistors and inductors.
- This way, the average value of voltage regulation is more than that of transformer at

no load. **TRANSFORMER EFFICIENCY**

Comparing system output with input will confirm transformer efficiency. The system is called better when its efficiency is high.

Measure Efficiency of Transformer

Like any other electrical machine, the efficiency of a transformer can be defined as the output power divided by the input power. That is efficiency = output/input.

In electrical devices, transformers are the most highly efficient ones. It is due to the fact that most of the transformers have full load efficiency between 95% to 98.5%.

As a transformer being highly efficient, the output value is equivalent to input value, and hence it is impractical to measure the efficiency of the transformer by using output/input.

Another method to find efficiency of a transformer is using, efficiency = (input - losses)/input = 1 - (losses/input).

Condition For Maximum Efficiency

You can see in the below formula: We have, $\text{Copper loss} = I_1^2 R_1$

$\text{Iron loss} = W_i$

$$\text{efficiency} = 1 - \frac{\text{losses}}{\text{input}} = 1 - \frac{I_1^2 R_1 + W_i}{V_1 I_1 \cos \Phi_1}$$

$$\eta = 1 - \frac{I_1 R_1}{V_1 \cos \Phi_1} - \frac{W_i}{V_1 I_1 \cos \Phi_1}$$

differentiating above equation with respect to I_1

$$\frac{d\eta}{dI_1} = 0 - \frac{R_1}{V_1 \cos \Phi_1} + \frac{W_i}{V_1 I_1^2 \cos \Phi_1}$$

$$\eta \text{ will be maximum at } \frac{d\eta}{dI_1} = 0$$

Hence efficiency η will be maximum at

$$\frac{R_1}{V_1 \cos \Phi_1} = \frac{W_i}{V_1 I_1^2 \cos \Phi_1}$$

$$\frac{I_1^2 R_1}{V_1 I_1^2 \cos \Phi_1} = \frac{W_i}{V_1 I_1^2 \cos \Phi_1}$$

$$I_1^2 R_1 = W_i$$

MBT

Hence, the efficiency of a transformer will be maximum when copper loss and iron losses are equal. So that, $\text{Copper loss} = \text{Iron loss}$.

Application of Transformer

The following are some of the most common uses for transformer:

- ☐ The transformer transmits electrical energy through wires over long distances.
- ☐ Transformers with multiple secondaries are used in radio and TV receivers, which require several different voltages.

Transformers are used as voltage regulators.

ROTATING MAGNETIC FIELD IN THREE-PHASE INDUCTION MOTOR:

When 3-phase supply is fed to the stator winding of the 3-phase induction motor, a rotating magnetic field (RMF) is produced. This magnetic field is such that it produces

donotremaininafixed

position on the stator but go on shifting their positions around the stator. For this reason, it is known as *rotating magnetic field (RMF)* or RMF.

Mathematically, it can be shown that the magnitude of this rotating magnetic field is constant and is equal to 1.5 times of the maximum flux (ϕ_m) due to current in any phase.

The speed of the rotating magnetic field is known as *asynchronous speed (NS)*. The value of synchronous speed depends upon the number of poles (P) on the stator and the supply frequency (f). Therefore,

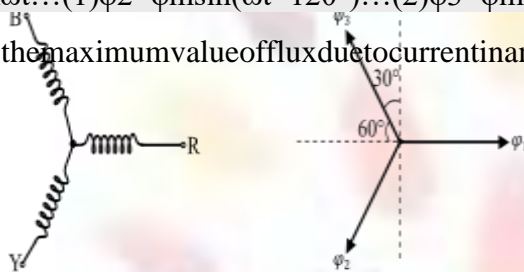
Synchronous speed, $N_s = 120f/PRPM$

Mathematical Analysis of Rotating Magnetic Field

Consider three identical coils which are displaced 120° apart from each other in space. Let these three coils are energised from a balanced 3-phase supply. Hence, each coil will produce an alternating flux along its own axis. Now, let the three instantaneous fluxes are given by,

$$\phi_1 = \phi_m \sin \omega t \dots (1) \quad \phi_2 = \phi_m \sin(\omega t - 120^\circ) \dots (2) \quad \phi_3 = \phi_m \sin(\omega t + 120^\circ) \dots (3)$$

Here, ϕ_m is the maximum value of flux due to current in any phase. The phasor diagram shows the



three fluxes.

To determine the magnitude of the resultant flux, resolve each flux into horizontal and vertical components and then find their phasor sum.

Thus, the resultant horizontal component of flux is given by, $\phi_h = \phi_1 - \phi_2 \cos 60^\circ - \phi_3 \cos 60^\circ = \phi_1 - (\phi_2 + \phi_3) \cos 60^\circ$

$$\Rightarrow \phi_h = \phi_1 - 1/2(\phi_2 + \phi_3)$$

$$\Rightarrow \phi_h = (\phi_m \sin \omega t) - 1/2[\phi_m \sin(\omega t - 120^\circ) + \phi_m \sin(\omega t + 120^\circ)]$$

$$\Rightarrow \phi_h = (\phi_m \sin \omega t) - \phi_m/2(\sin \omega t \cos 120^\circ - \cos \omega t \sin 120^\circ + \sin \omega t \cos 120^\circ + \cos \omega t \sin 120^\circ)$$

$$\Rightarrow \phi_h = \phi_m \sin \omega t - [\phi_m/2 \times (2 \sin \omega t) \times (-1/2)]$$

$$\Rightarrow \phi_h = 3/2 \phi_m \sin \omega t \dots (4)$$

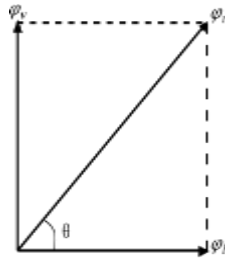
The resultant vertical component of the flux is given by, $\phi_v = 0 - \phi_2 \cos 30^\circ + \phi_3 \cos 30^\circ = (-\phi_2 + \phi_3) \cos 30^\circ$

$$\Rightarrow \phi_v = [-\phi_m \sin(\omega t - 120^\circ) + \phi_m \sin(\omega t + 120^\circ)] \cos 30^\circ$$

$$\Rightarrow \phi_v = \sqrt{3}/2 \phi_m [-(\sin \omega t \cos 120^\circ - \cos \omega t \sin 120^\circ) + (\sin \omega t \cos 120^\circ + \cos \omega t \sin 120^\circ)]$$

$$\Rightarrow \phi_v = \sqrt{3}/2 \phi_m (2 \cos \omega t \sin 120^\circ) = \sqrt{3}/2 \phi_m \times (2 \cos \omega t) \times \sqrt{3}/2$$

$$\Rightarrow \phi_v = 3/2 \phi_m \cos \omega t \dots (5)$$



Therefore, the resultant flux is given by,

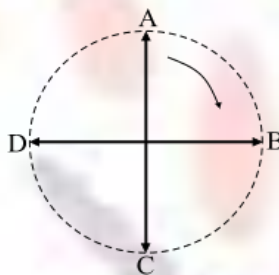
$$\phi_r = \sqrt{(\phi_h^2 + \phi_v^2)} = \sqrt{(3/2 \phi_m \sin \omega t)^2 + (3/2 \phi_m \cos \omega t)^2}$$

$$\Rightarrow \phi_r = 3/2 \phi_m (\sqrt{\sin^2 \omega t + \cos^2 \omega t}) = 3/2 \phi_m \dots (6)$$

Hence, from the eqn. (6) it is clear that the magnitude of the resultant rotating magnetic field is equal to 1.5 times of maximum value of the flux (ϕ_m) per phase. Also, the resultant flux (ϕ_r) is independent of time, i.e., it is constant flux.

$$\text{Again, } \tan \theta = \phi_v / \phi_h = (3/2 \phi_m \cos \omega t) / (3/2 \phi_m \sin \omega t) = \cot \omega t = \tan(90^\circ - \omega t)$$

$$\therefore \theta = (90^\circ - \omega t) \dots (7)$$



Eqn. (7) shows that the angle is the function of time. Hence,

- ☐ **Case 1** – At $\omega t = 0^\circ$; $\theta = 90^\circ$. It is corresponding to position A in the above figure.
- ☐ **Case 2** – At $\omega t = 90^\circ$; $\theta = 0^\circ$. It is corresponding to position B.
- ☐ **Case 3** – At $\omega t = 180^\circ$; $\theta = -90^\circ$. It is corresponding to position C.
- ☐ **Case 4** – At $\omega t = 270^\circ$; $\theta = -180^\circ$. It is corresponding to position D.

Hence, it can be seen that the resultant flux rotates in space in the clockwise direction with an angular velocity of ω radians per second. Therefore, for a machine of P poles,

$$\omega = 2\pi f; \text{ and } f = P N_s / 120;$$

The following conclusions can be drawn from the above discussion –

The 3-phase currents of a balanced 3-phase supply system produce a resultant flux of constant magnitude in the motor. The magnitude of the flux at every instant is $1.5 \phi_m$.

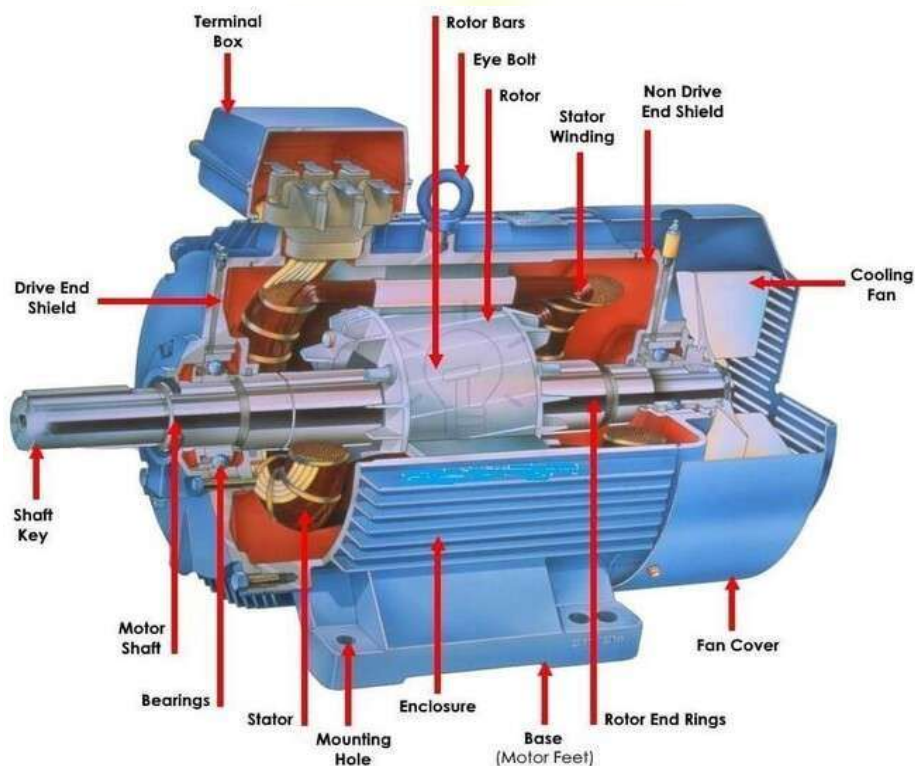
The resultant flux is rotating in nature and it rotates at an angular velocity same as that of the supply currents. The direction of rotation of the resultant flux depends upon the phase sequence of supply system.

CONSTRUCTION AND WORKING OF A THREE-PHASE INDUCTION MOTOR:

The three-phase induction motor is one of the A.C. motors, which is widely used for various purposes in industry. These motors never run at a Synchronous speed but a little less than the synchronous speed. The speed of these motors depends upon the supply frequency.

Therefore, these motors are not generally used for speed control. However, we prefer D.C. motors where large variations of speed are required. These motors are preferred in industry because they have low price, simple & rugged construction, can be manufactured with characteristics to suit the industrial requirement.

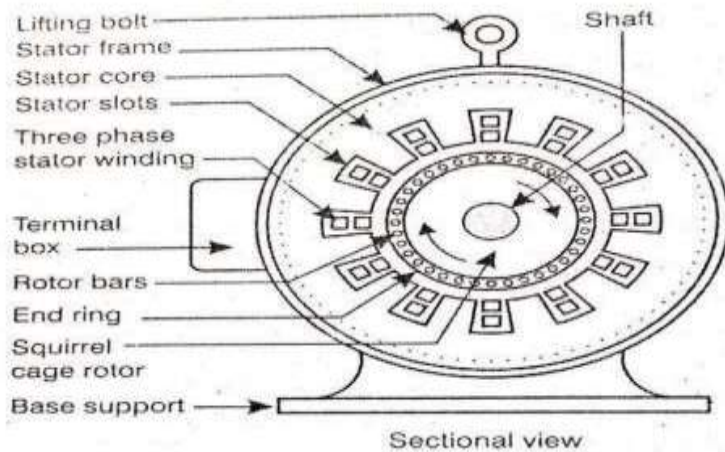
These motors differ from other types of motor, in that there is no electrical connection between the rotor & supply. The required voltage & current are induced by induction from the **stator winding** that is why, the name given is induction motor.



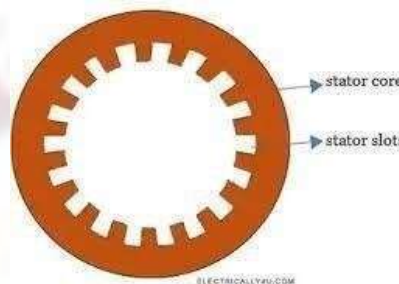
CONSTRUCTION OF THREE PHASE INDUCTION MOTOR:

It can be better understood if we see the construction of three phase induction motor which has two major parts :

1. Stationary part, known as Stator
2. Rotating part, known as Rotor.



Stator: It is the stationary part of the motor. It has three main parts:

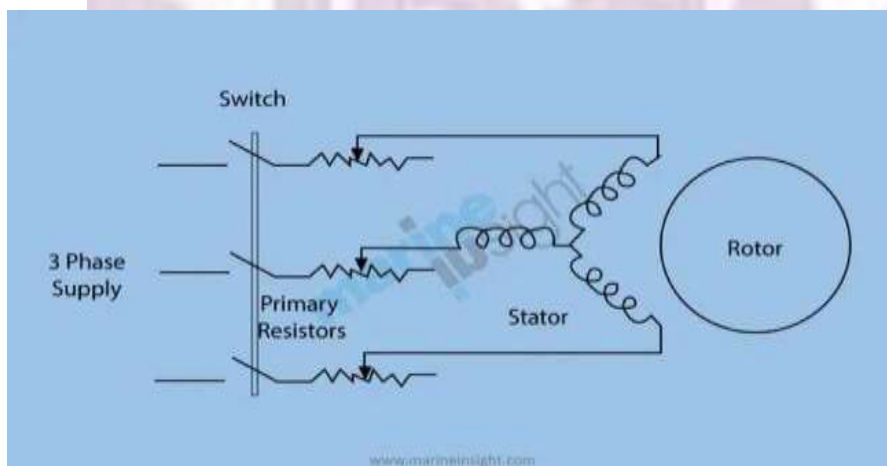


Frame or Yoke

It is the outer part of the three phase induction motor. Its main function of the frame is to support the stator core & stator winding. It acts as a covering, and it provides protection & mechanical strength to all the inner parts of the three phase induction motor.

Stator core

The main function of stator core is to carry the alternating flux. In order to reduce the eddy current loss, the stator core is laminated. The core is made up of thin silicon steel laminations. These are insulated from each other by varnish, the slots are cut on inner periphery of core stampings. The stator windings are placed in these slots.



Stator windings

Stator winding is made up of super enamelled copper wire. Three phase windings are placed in the stator core slots & six terminals are brought out. They may be star connected or may be delta connected. The windings are connected in star at starting.

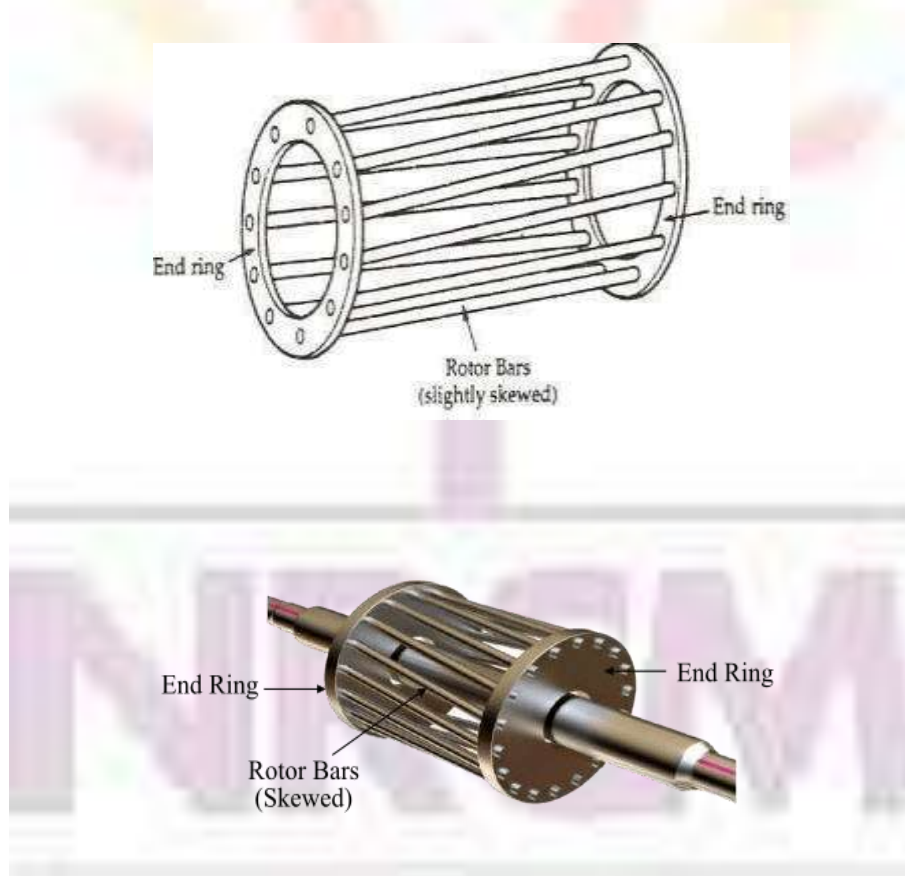
1. Rotor

It is a rotating part of the motor. It is mounted on the shaft. It consists of hollow laminated core having slots on its outer periphery. The windings placed in these slots (rotor winding) may be one of the following two types:

1. Squirrel cage rotor
2. Slip ring rotor or wound rotor or phase wound rotor.

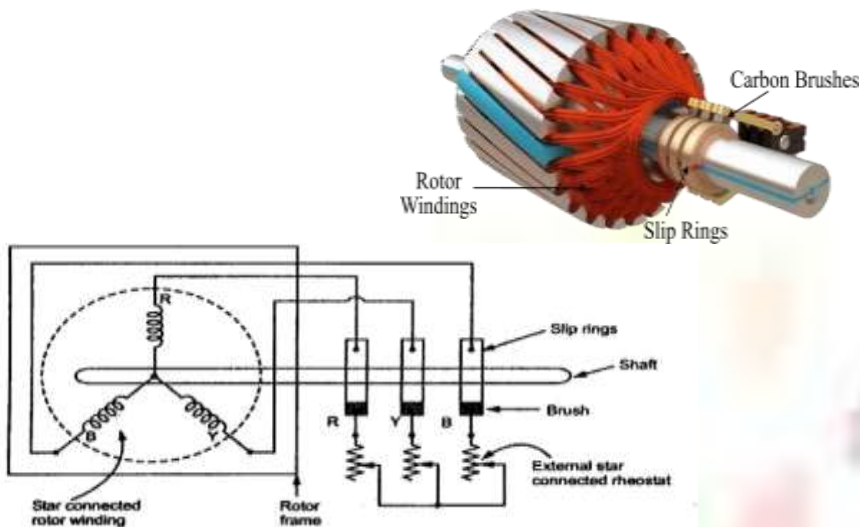
1. Squirrel cage rotor

The rotor consists of a cylindrical laminated core with parallel slots for carrying the rotor conductors. The **squirrel cage** rotor consists of aluminium, brass or copper bars. These aluminium, brass or copper bars are called rotor conductors & are placed in the slots on the periphery of the rotor. The rotor conductors are permanently shorted by the copper, or aluminium rings called the **end rings**. To provide mechanical strength, these rotor conductors are braced to the end ring & hence form a complete closed circuit resembling like a cage & hence got its name as squirrel cage induction motor.



2. Slip ring rotor or wound rotor or phase wound rotor

The wound rotor consists of a slotted armature. Insulated conductors are put in the slots & connected to form a three phase double layer distributed winding similar to the stator winding. The rotor windings are connected in star.



The open end of the star circuit is brought outside the rotor and connected to the insulated slip rings. The slip rings are mounted on the shaft with brushes placed on them. The brushes are connected to three phase variable resistors connected in star. The purpose of slip rings & brushes is to provide a means for connecting external resistors in the circuit.

PRINCIPLE AND WORKING OF 3 PHASE INDUCTION MOTOR

Induction motor works on the principle of electromagnetic induction.

When three phase supply is given to the stator winding, a rotating magnetic field of constant magnetic field is produced.

The speed of rotating magnetic field is synchronous speed, N_s r.p.m.

$$\rightarrow N_s = \frac{120f}{P} = \text{speed of rotating magnetic field}$$

- f = supply frequency

This rotating field produces an effect of rotating poles around a rotor. Let direction of this magnetic field is clockwise as shown.

Now at this instant rotor is stationary and stator flux R.M.F. is rotating. So it is obvious that there exists a relative motion between the R.M.F. and rotor conductors.

Now the R.M.F. gets cut by rotor conductors as R.M.F. sweeps over rotor conductors.

Whenever a conductor cuts the flux, e.m.f. gets induced in it. So e.m.f. gets induced in the rotor conductors called rotor induced e.m.f. this is electro-magnetic induction.

As rotor forms closed circuit, induced emf. circulates current through rotor called rotor current.

Any current carrying conductor produces its own flux. So rotor produces its flux called rotor flux. For assumed direction of rotor current, the direction of rotor flux is clockwise as shown.

This direction can be easily determined using right hand thumb rule. Now there are two fluxes, one R.M.F. and another rotor flux.

Both the fluxes interact with each. On left of rotor conductor, two fluxes are in same direction hence added up to get high flux area.

On right side of rotor conductor, two fluxes are in opposite direction hence they cancel each other to produce low flux area.

So rotor conductor experiences a force from left to right, due to interaction of the two fluxes. As all rotor conductors experience a force, overall rotor experiences a torque and starts rotating.

So interaction of the two fluxes is very essential for a motoring action. As seen from the figure, the direction of force is same as that of rotating magnetic field. Hence rotor starts rotating in the same direction as that of R.M.F.

ADVANTAGES OF INDUCTION MOTOR

The motor construction and the way electric power is supplied give the induction motor several benefits such as:

- They are robust and simple in construction with very few moving parts
- They can efficiently operate in a rugged and harsh environment such as in sea-going vessels
- The maintenance cost of 3-phase induction motor is less and unlike that of DC or synchronous motor, they do not have parts like brushes, commutators or slip rings etc.

3 Phase induction motor does not need any additional starting mechanism or arrangement as they can generate self-starting torque when three-phase AC supply is provided to them, unlike synchronous motors. However, the single-phase induction motor needs some auxiliary arrangement for the starting torque

- The final output of a three-phase motor is nearly 1.5 times the rating (output) of a single-phase motor of the same size.

DISADVANTAGES OF 3 PHASE INDUCTION MOTOR:

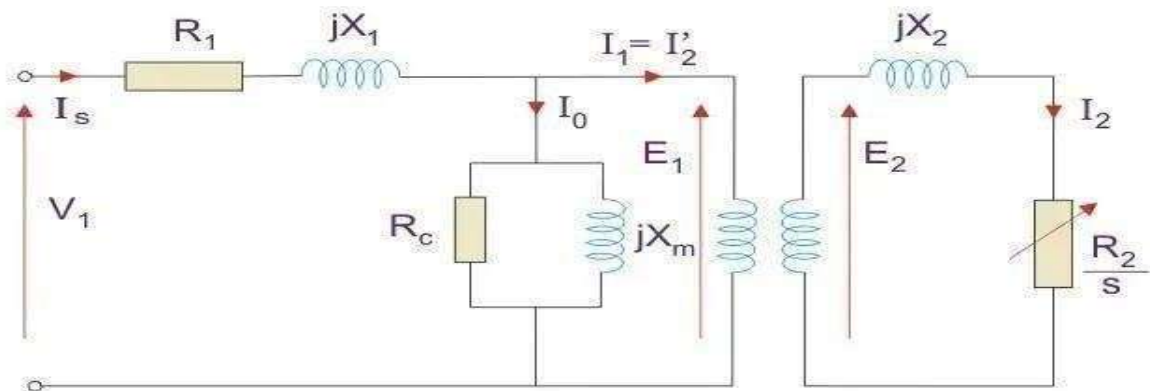
- During starting, it draws high initial starting current when attached to a heavy load. This causes a dip in voltage during the starting period of the machine. Soft starting methods are connected to the 3-phase electric motor to avoid this problem.
- Induction motor operates at lagging power factor which results in increased I^2R losses and efficiency reduction, especially at low load. To correct and improve the power factor, static capacitor banks can be used with this type of AC motor.
- Speed control of 3 phase induction motor is difficult as compared to DC motors. A variable frequency drive can be integrated with the induction motor for speed control.

EQUIVALENTCIRCUITOFANINDUCTIONMOTOR:

The equivalent circuit of any machine shows the various parameter of the machine such as its Ohmic losses and also other losses.

The losses are modeled just by inductor and resistor. The copper losses are occurred in the windings so the winding resistance is taken into account. Also, the winding has inductance for which there is a voltage drop due to inductive reactance and also a term called power factor comes into the picture. There are two types of equivalent circuits in case of a three-phase induction motor-

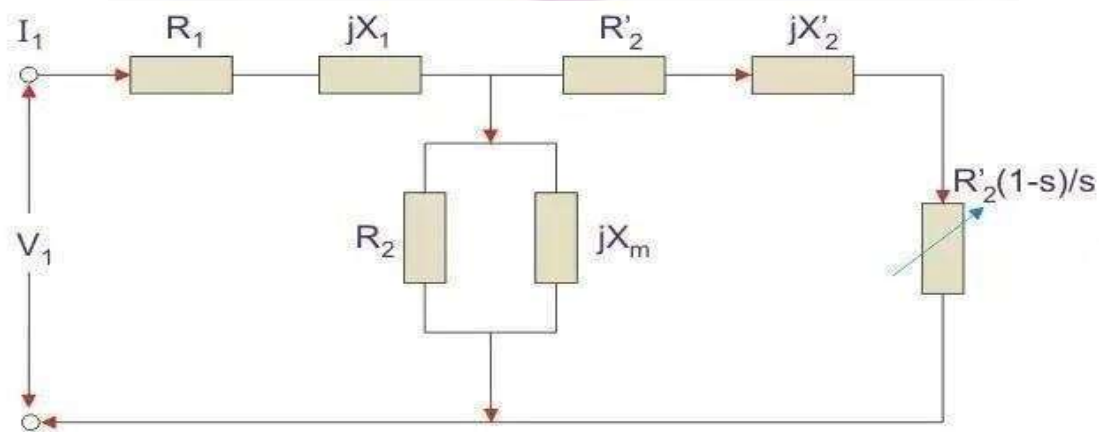
Exact Equivalent Circuit



Here, R_1 is the winding resistance of the stator. X_1 is the inductance of the stator winding. R_c is the core loss component.

X_m is the magnetizing reactance of the winding.

R_2/s is the power of the rotor, which includes output mechanical power and copper loss of rotor. If we draw the circuit with referred to the stator then the circuit will look like-



Here all the other parameters are same except-

R_2' is the rotor winding resistance with referred to stator winding. X_2' is the rotor winding inductance with referred to stator winding.

$R_2(1 - s) / s$ is the resistance which shows the power which is converted to mechanical power output or useful power. The power dissipated in that resistor is the useful power output or shaft power.

APPROXIMATE EQUIVALENT CIRCUIT

The approximate equivalent circuit is drawn just to simplify our calculation by deleting one node. The shunt branch is shifted towards the primary side. This has been done as the voltage drop between the stator resistance and inductance is less and there is not much difference between the supply voltage and the induced voltage. However, this is not appropriate due to following reasons-

1. The magnetic circuit of induction motor has a narrow gap so exciting current is larger compared to transformer so exact equivalent circuit should be used.
2. The rotor and stator inductance is larger in induction motor.
3. In induction motor, we used distributed windings.

This model can be used if approximate analysis has to be done for large motors. For smaller motors, we cannot use this.

Power Relation of Equivalent Circuit

1. Input power to stator- $3 V_1 I_1 \cos(\Theta)$. Where, V_1 is the stator voltage applied. I_1 is the current drawn by the stator winding. $\cos(\Theta)$ is the stator power factor.
2. Rotor input =
Power input - Stator copper and iron losses.
3. Rotor Copper loss = Slip \times power input to the rotor.
4. Developed Power = $(1 - s) \times$ Rotor input power.

TORQUE-SPEED & TORQUE-SLIP CHARACTERISTICS OF AN INDUCTION MOTOR

Torque-slip Characteristics

Torque-slip characteristics give the relation between torque and slip. The torque-slip characteristics show how the torque changes with a change in slip. The slip is defined as a ratio of synchronous speed and the actual speed of the rotor. The actual speed of the rotor varies with loading condition. Therefore, the slip changes with loading condition.

$$T = \frac{k s R_2 E_{20}^2}{R_2^2 + (s X_{20})^2}$$

In the previous article, we have derived the [torque equation of an induction motor](#).

From the above equation, if R_2 and X_{20} are kept constant, the torque depends on the slip. The torque-slip characteristics curve looks like a rectangular hyperbola. And this curve is divided into three regions;

- Low slip region
- Medium slip region
- High slip region

Low Slip Region

At synchronous speed, the slip of an induction motor is zero. Hence, the torque developed in the rotor is zero. Therefore, an [induction motor](#) always runs slightly less than the synchronous speed. And in this condition slip is very low.

$$T = \frac{k_1 s}{R_2}$$

When the slip is very low, $(sX_{20})^2$ is negligible compared to the R_2 . So, for low slip, if we consider rotor resistance R_2 as constant; $T \propto s$

Hence, at low slip conditions, the torque is directly proportional to the slip. This is the normal operating region for an induction motor. In the low-slip region, the torque-slip curve is a straight line.

Medium Slip Region

If the load increases, the speed of an induction motor decreases, and slip increases. As the slip increases, the term $(sX_{20})^2$ becomes high compared to Rotor resistance R_2 . And in this condition, we can neglect the rotor resistance R_2 .

$$T = \frac{k_3 R_2}{s X_{20}^2}$$
$$T \propto \frac{1}{s}$$

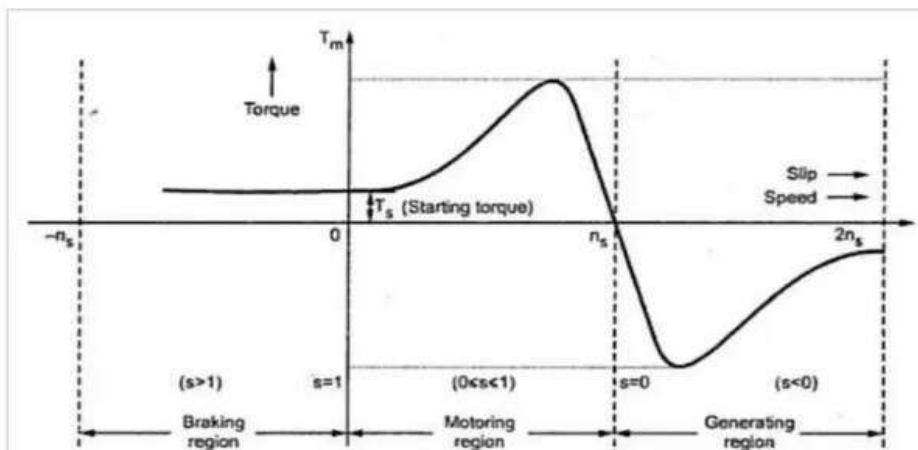
So, the torque is inversely proportional to the slip. During this region, the curve shape is a rectangular hyperbola and passes through the point of maximum torque. The maximum torque is achieved when $R_2 = sX_{20}$. This torque is known as pull-out torque or breakdown torque.

High Slip Region

If we increase the torque beyond the maximum torque point, the torque starts decreasing. This condition is when the load increases. During this condition, the motor speed decreases, and the overload protection must be activated to disconnect a motor from the supply. If the motor continuously runs in this region, the motor will damage due to overheating. This region in a torque-slip curve is a decreasing region after the maximum torque point.

Generally, the induction motor operates for the value of slip between zero to s_m . The slip s_m is a slip at the maximum torque point. The pull-out torque for an induction motor is 2 to 3 times of rated full-load torque for typical operation. Therefore, the motor can handle overload for short period.

without stalling. The torque-slip curve of the induction motor for constant rotor resistance is shown in the figure below.



Similar to the above

description, this curve is also divided into three parts;

- Motoring region
- Generating region
- Braking region

Motoring Region

In this mode of operation, the slip of an induction motor is between zero to one. When the stator is supplied by electric power, the rotor rotates below synchronous speed. And the torque of a motor varies from zero to full-load torque as slip varies from zero to one.

During this condition, the torque is directly proportional to the slip. Generally, the induction motor operates in this region. The slip is zero at synchronous speed and the slip is one at standstill condition.

Generating Region

In generating mode of operation, the induction motor runs above the synchronous speed and it behaves as an induction generator. The speed of a motor increases above synchronous speed with the help of external devices like a prime mover.

During generating region, the slip and torque both are negative. Hence, the machines receive mechanical energy and deliver electrical energy. During the generating region, the motor requires to supply reactive electric power.

Braking Region

In the braking region, the polarity of supply voltage is changed. Hence, the motor rotates in a reversed direction. This mode is used to stop the motor. This method of electrical braking is known as plugging. During braking mode, the slip is greater than one.

By this method, the motor stops within a short time. But the kinetic energy stored in the load is dissipated as heat. Therefore, during the braking, a very high amount of heat is generated. And also, if the stator is connected with the supply, it is also generated as heat. Hence, it is contained to disconnect the supply from the stator before entering the braking mode.

Losses in Induction Motor

There are two types of losses that occur in induction motors are- fixed and variable loss. The fixed losses are core loss and mechanical loss, and the variable losses are copper loss and stray loss.

In the energy conversion process, the electrical energy can not be fully converted into mechanical energy because some part of the electrical energy gets wasted in the form of heat energy. The energy wasted in the motor in the form of heat energy contributes to power loss.

The following losses occur in an induction motor.

1. Constant or Fixed loss
2. Variable Loss

Constant or Fixed Losses in Induction Motor

Constant losses are the losses that remain constant when the motor operates at its rated parameters. The constant losses can be divided into the following categories

1. Iron or core losses
2. Mechanical losses
3. Brush friction losses

Iron or Core Losses

Iron losses in an induction motor are the heat loss that gets dissipated in the core of the motor due to the alternating magnetic field created by the stator winding. These losses are also known as core losses or hysteresis losses.

Two types of iron losses occur in an induction motor: Hyst

- ☐ Hysteresis loss
- ☐ Eddy current loss

The magnetic core of an induction motor opposes changes in magnetic field direction and because of hysteresis lag the Hysteresis losses occur in the magnetic core. On the other hand, Eddy current losses occur because the alternating magnetic field induces a voltage in the core, and the induced voltage causes the setting up of circulating current (Eddy current), and this circulating current causes heat loss I^2R in the core.

The iron loss does not vary with load, therefore these are the fixed losses. The iron losses remain constant whether the motor is operating at no load or at full load. These losses happen because of magnetic flux and we know, the average magnetic flux remains constant, provided the voltage and frequency are maintained as per the motor's rated voltage & frequency. The iron losses can be reduced by the use of a high-quality laminated iron core.

Other factors like the shape and size of the core, the design of the rotor and stator slots, and operating temperature also affect the iron losses in the motor. A higher temperature leads to an increase in the core resistance, and consequently, it contributes to higher losses. Therefore, the motor should be adequately cooled to reduce the iron losses as well as the copper losses.

The iron loss can be mathematically calculated by the Steinmetz equation.

$$P_i = P_h + P_e$$

$$P_i = \eta B_m^{1.6} f V + K_e B_m^2 f^2 t^2 V^2$$

Where,

P_i = Iron or Core loss(Watt)

P_h = Hysteresis loss(watt)

P_e = Eddy current loss(Watt)

η = Steinmetz coefficient

B_m = Max. flux density(W_b/m^2)

f = frequency of magnetic reversal(H_z)

V = Volume of material(m^3)

t = Thickness of laminations(m)

Mechanical Losses

The mechanical losses in an induction motor occur because of two reasons. They are-

- ☐ Friction Loss
- ☐ Windage Loss

Friction and windage loss are the two types of mechanical losses that occur in an induction motor. The main reason for these losses is the rotation of the motor. The rotating parts like the rotor shaft, fans, and bearing cause mechanical losses in the rotating machine. All the rotating equipment has these types of losses.

The friction and windage losses depend on the speed of the motor, the quality of the bearings, and the design of the motor.

The friction loss occurs because of the rubbing of moving parts of the motor with each other, such as the friction between the rotor shaft and the bearings, friction between the balls and outer cage of the bearing, and friction between the inner and outer cage of the bearing. The motor takes extra power from the main to overcome this friction loss.

When the motor rotates, it has to cut the surrounding air for its rotation. The air exerts a force on the motor's components, like the rotor fans. The resistance offered by surrounding air causes windage loss. The windage loss depends on the speed of the motor and the density of the surrounding air.

Mechanical losses are constant losses and they do not vary with load. The mechanical losses can be minimized by the proper design of the rotor and the use of high-quality bearings.

Brush Friction Loss

Squirrel cage induction motors do not have carbon brushes, its rotor is internally short-circuited. Therefore, no brush friction loss occurs in this type of motor. However, the slip ring or wound rotor induction motors have slip rings and carbon brushes and the power loss on account of brush friction happens in these types of motors.

Carbon brushes collect the current from the slip ring. The slip ring is a rotating part and the carbon brushes are fixed parts, and there exists friction between the contact surfaces of the carbon brushes and the slip ring. Thus, the brush friction loss occurs in the slip ring induction motor.

Variable Losses in Induction Motor

The losses that vary with load are called variable losses. The major variable loss in an induction motor is **copper loss**. The following are the variable losses in an induction motor.

Copper Loss

The winding of the induction motor is made of copper that has finite resistance R . When current flows through the stator and rotor winding, the power loss (I^2R loss) in the form of heat occurs in the winding is called copper loss.

The copper loss occurs in the stator and rotor of an induction motor. The copper loss in the stator is called stator copper loss, and copper loss in the rotor is called the rotor copper loss. The copper losses depend on the magnitude of the current passing through the stator and rotor, and it is proportional to the square of the magnitude of the current. The copper loss in the induction motor can be calculated using the following formula.

$$P_c = P_s + P_r$$

$$P_c = I_s^2 R_s + I_r^2 R_r$$

Where,

P_c = Total copper loss

P_s = Stator copper loss

P_r = Rotor copper loss

I_s = Stator current

R_s = Stator winding Resistance

I_r = Rotor Current

R_r = Rotor Resistance

The total copper loss in a 3-phase induction motor is;

$$P_c = 3 I_s^2 R_s + 3 I_r^2 R_r$$

The current of the motor increases with an increase in load, therefore these losses are not constant and vary with load. The copper loss is also called a variable loss. The slip of the induction motor varies with load and losses also vary accordingly. Therefore, the copper loss in the rotor is also called slip power loss. The copper loss can be minimized by selecting a high-quality copper wire for stator and rotor winding.

The resistance of the winding also depends on the temperature. The resistance of the conductor increases with an increase in the temperature. Thus, the operating temperature also contributes to copper loss because copper loss is proportional to the resistance of the winding. Therefore, adequate cooling of the motor is a must for reducing the copper losses in the induction.

Stray Loss in induction motor

It is desired that all the generated flux in the stator must be 100% coupled to the rotor and there should be no leakage flux. However, practically the entire flux does not link to the rotor and some parts of the flux leak in the magnetic path. The leakage flux gets linked to other conductive parts of the motor such as the motor frame, bearings, and housing, and creates eddy currents that cause energy loss. Thus, the leakage flux contributes to stray loss in the induction motor. The stray loss is a variable loss, and it depends on the factors like design of the motor, core material, and the load on the motor.

The core of the motor should be of high-quality material in order to minimize the stray loss in the induction motor.

Power Stages In An Induction Motor

Power stages diagram showing the power losses in a different part of the machine. See Figure 1.

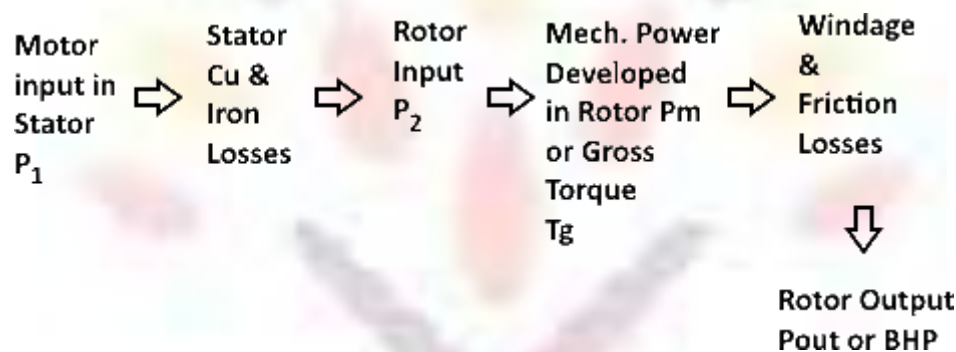


Figure 1: Power Stages and Losses Diagram of 3-Phase Induction Motor. The different stages of this diagram are discussed as follows:

1. Motor input = stator input = stator output + stator losses
2. Rotor input = stator output
3. Rotor gross output

Developed in rotor, $P_m = \text{Rotor input} - \text{rotor Cu. loss}$

Gross Torque developed, $T_g = \text{Rotor gross output} / 2\pi N$ --- (1)

If there were no Cu. Losses in the rotor, then the rotor output will be equal to rotor input and the rotor will run at synchronous speed:

$$T_g = \text{Rotor input} / 2\pi N_s \text{ --- (2)}$$

From equation (1) Rotor output = $T_g 2\pi N$ --- (3) From equation (2) Rotor input = $T_g \times 2\pi N_s$ --- (4)

As copper losses = Rotor input - Rotor output. So Rotor Cu. losses = $T_g 2\pi (N_s - N)$ --- (5) From equation (4) & (5)

RotorCu.Loss/RotorInput= $(N_s - N_r) / N_s$

) / N_s Or RotorCu.Loss = $S \times \text{Rotor input}$ --- (6)

And Rotor gross output = Rotor input - RotorCu.Loss --- (7) Rotor gross output = $(1 - S) \times \text{Rotor input}$

Rotor efficiency = Rotor gross output / Rotor Input

= $(1 - S) \times \text{Rotor Input} / \text{Rotor Input} = 1 - S$

Rotor efficiency = $1 - S = 1 - (N_s - N_r) / N_s = N_r / N_s$

= actual speed of rotor / Synchronous speed Note for 3-phases rotor Cu.Losses = $3I^2R$

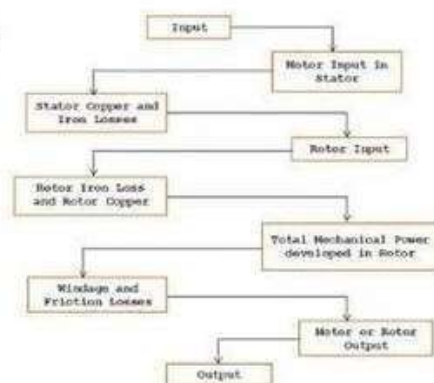
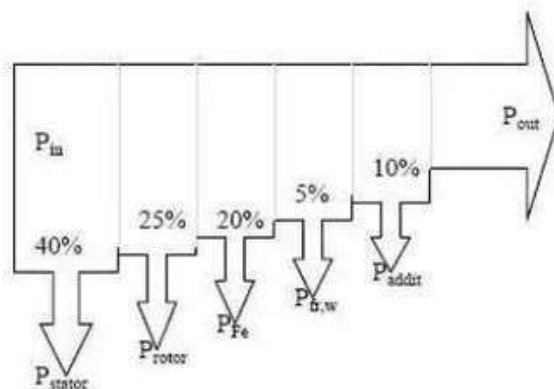
4. Motor output = Mechanical power developed in rotor, P_m windage and Friction losses.

If motor output is in B.H.P.

Motor output = B.H.P. $\times 746$ Watts

Efficiency of motor = output / Input = $((\text{B.H.P.} \times 746) / \text{Stator input}) \times 100$

Efficiency & Losses of an Induction Motor



Electrical 4 U



EFFICIENCY OF THREE PHASE INDUCTION MOTOR

Efficiency is defined as the ratio of the output to that of input,

$$\text{Efficiency, } \eta = \frac{\text{output}}{\text{input}}$$

Rotor efficiency of the three phase induction motor,

$$= \frac{\text{rotor output}}{\text{rotor input}}$$

= Gross mechanical power developed / rotor input

$$= \frac{P_m}{P_2}$$

Three phase induction motor efficiency,
$$= \frac{\text{power developed at shaft}}{\text{electrical input to the motor}}$$

Three phase induction motor efficiency

$$\eta = \frac{P_{out}}{P_{in}}$$

INDUCTION MOTOR SPEED CONTROL

From Stator Side

1. By Changing The Applied

Voltage: From the torque equation of induction motor,

$$T = \frac{k_1 s E_2^2 R_2}{\sqrt{(R_2^2 + (s X_2)^2)}} = \frac{3}{2\pi N_s} \frac{s E_a^2 R_a}{\sqrt{(R_2^2 + (s X_2)^2)}}$$

Rotor resistance R_2 is constant and if slip is small then $(s X_2)^2$ is so small that it can be neglected.

Therefore, $T = s E_a^2$ where E_a is rotor induced emf and $E_a \propto V$. Thus,

$T \propto V^2$, which means, if supplied voltage is decreased, the developed torque decreases. Hence, for providing the same load torque, the slip increases with decrease in voltage, and consequently, the speed decreases. This method is the easiest and cheapest, still rarely used, because

1. large change in supply voltage is required for relatively small change in speed.
2. large change in supply voltage will result in a large change in flux density, hence, this will disturb the magnetic condition of the motor.

2. By Changing The Applied Frequency

Synchronous speed of the rotating magnetic field of an induction motor is given by,

$$N_s = \frac{120 f}{P} \text{ (RPM)}$$

where, f = frequency of the supply and P = number of stator poles.

Hence, the synchronous speed changes with change in supply frequency. Actual speed of an induction motor is given as $N = N_s (1 - s)$. However, this method is not widely used. It may be used where, the induction motor is supplied by a dedicated generator (so that frequency can be easily varied by changing the speed of prime mover). Also, at lower frequency, the motor current may become too high due to decreased reactance. And if the frequency is increased beyond the rated value, the maximum torque developed falls while the speed rises.

3. Constant V/F Control Of Induction Motor

This is the most popular method for controlling the speed of an induction motor. As in above method, if the supply frequency is reduced keeping the rated supply voltage, the air gap flux will tend to saturate. This will cause excessive stator current and distortion of the stator flux wave. Therefore, the stator voltage should also be reduced in proportion to the frequency so as to maintain the air-gap flux constant. The magnitude of the stator flux is proportional to the ratio of the stator voltage and the frequency. Hence, if the ratio of voltage to frequency is kept constant, the flux remains constant. Also, by keeping V/F constant, the developed torque remains approximately constant. This method gives higher run-time efficiency. Therefore, majority of AC speed drives employ constant V/F method (or variable voltage, variable frequency method) for the speed control. Along with wider range of speed control, this method also offers 'soft start' capability.

4. Changing The Number Of Stator Poles

From the above equation of synchronous speed, it can be seen that synchronous speed (and hence, running speed) can be changed by changing the number of stator poles. This method is generally used for [squirrel cage induction motors](#), as squirrel cage rotor adapts itself for any number of stator poles. Change in stator poles is achieved by two or more independent stator windings wound for different number of poles in same slots.

For example, a stator is wound with two 3-phase windings, one for 4 poles and other for 6 poles. for supply frequency of 50 Hz

- i) synchronous speed when 4 pole winding is connected, $N_s = 120 * 50 / 4 = 1500 \text{ RPM}$
- ii) synchronous speed when 6

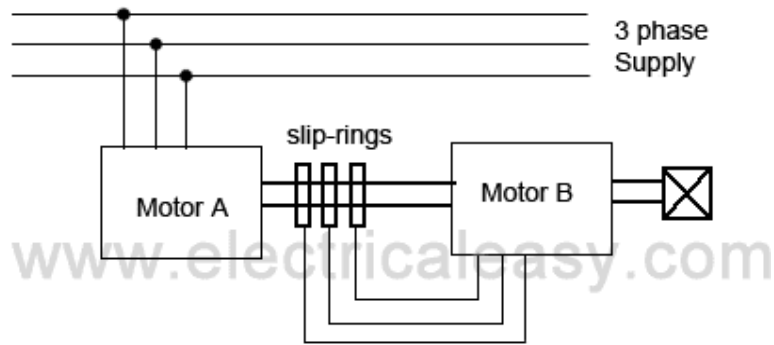
pole winding is connected, $N_s = 120 * 50 / 6 = 1000 \text{ RPM}$ **Speed Control From Rotor Side:**

1. Rotor Rheostat Control

This method is similar to that of [armature rheostat control of DC shunt motor](#). But this method is only applicable to [slip ring motors](#), as addition of external resistance in the rotor of squirrel cage motor is not possible.

2. Cascade Operation

In this method of speed control, two motors are used. Both are mounted on a same shaft so that both run at same speed. One motor is fed from a 3-phase supply and the other motor is fed from the induced EMF in first motor via slip-rings. The arrangement is as shown in following figure.



Motor A is called the main motor and motor B is called the auxiliary motor. Let, N_{s1} = frequency of motor A

N_{s2} = frequency of motor B

P_1 = number of poles stator of motor A P_2 = number of stator poles of motor

N = speed of the set and same for both motors f = frequency of the supply

Now, slip of motor A, $S_1 = (N_{s1} - N) / N_{s1}$.

frequency of the rotor induced emf in motor A, $f_1 = S_1 f$ Now, auxiliary motor B is supplied with the rotor induced emf

therefore, $N_{s2} = (120f_1) / P_2 = (120S_1f) / P_2$. now putting the value of $S_1 = (N_{s1} - N) / N_{s1}$

$$N_{s2} = \frac{120f(N_{s1} - N)}{P_2 N_{s1}}$$

At no load, speed of the auxiliary rotor is almost same as its synchronous speed.

i.e. $N = N_{s2}$.

from the above equations, it can be obtained that

$$N = \frac{120f}{P_1 + P_2}$$

With this method, four different speeds can be obtained

1. when only motor A works, corresponding speed = $N_{s1} = 120f / P_1$
2. when only motor B works, corresponding speed = $N_{s2} = 120f / P_2$
3. if cumulative cascading is done, speed of the set = $N = 120f / (P_1 + P_2)$
4. if differential cascading is done, speed of the set = $N = 120f / (P_1 - P_2)$

3. By Injecting EMF in Rotor Circuit

In this method, speed of an induction motor is controlled by injecting a voltage in rotor circuit. It is necessary that voltage (emf) being injected must have same frequency as of the slip frequency. However, there is no restriction to the phase of injected emf. If we inject emf which is in opposite phase with the rotor induced emf, rotor resistance will be increased. If we inject emf which is in phase with the rotor

induced emf, rotor resistance will decrease. Thus, by changing the phase of injected emf, speed can be controlled. The main advantage of this method is a wide range of speed control (above normal as well as below normal) can be achieved.

$$N_s = \frac{120 f}{P} \quad (\text{RPM})$$

where, f = frequency of the supply and P = number of stator poles.

Hence, the synchronous speed changes with change in supply frequency. Actual speed of an induction motor is given as $N = N_s (1 - s)$. However, this method is not widely used. It may be used where the induction motor is supplied by a dedicated generator (so that frequency can be easily varied by changing the speed of prime mover). Also, at lower frequency, the motor current may become too high due to decreased reactance. And if the frequency is increased beyond the rated value, the maximum torque developed falls while the speed rises.

STARTING METHODS OF THREE-PHASE INDUCTION MOTOR:

At starting the slip is unity and the motor impedance is quite small with rated applied voltage, thus the motor current is excessive due to the small motor impedance. This abnormal condition must be brought within normal limits using starting methods. The starters are used to start the motors in safe mode and to protect it from over withdrawn current. For small size motors, there is no need to use any starting method since smaller motors have higher per unit impedances and the starting period is rather short. Induction motors are practically a constant speed motor which account about 90 percent of the electrical drives used in industry.

The standard methods for starting of squirrel cage and wound rotor Induction motors are as follows:

1) Direct on-Line Starting: This method involves direct switching of three-phase stator on to the supply mains as in figure 1.

The motor draws low power factor starting current with (5-7) times of full load current (FLC). Due to this large current there is a considerable voltage drop in the power utility which causes undesirable dip in supply voltage thereby affecting the operation of other equipment connected to the power utility as well.

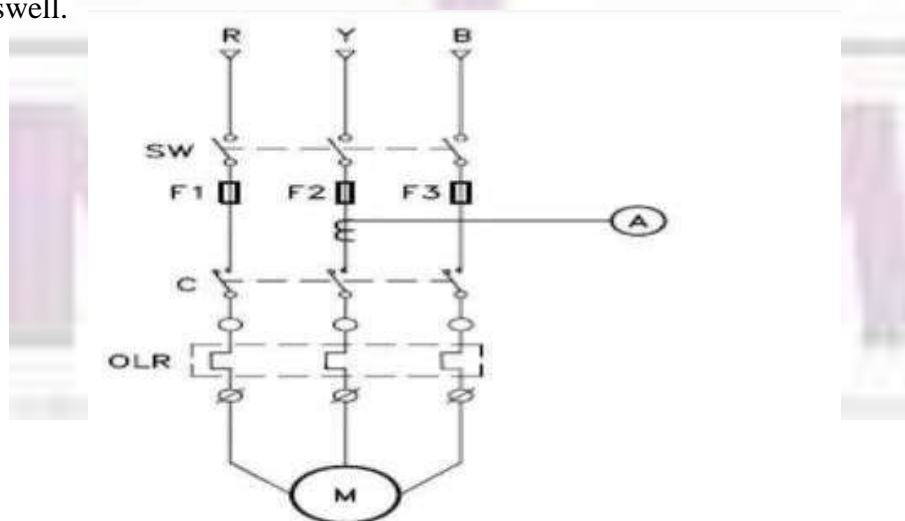


Figure 1 Power Circuit for DOL Starting

II. Autotransformer Starting: This method of starting is illustrated in figure 2, a fraction of 'x' (where x is always less than unity) of the supply voltage is applied to the stator terminals at the time of starting by means of an Autotransformer. This reduces the starting current of the motor. After the motor has accelerated close to its operating speed, the Autotransformer is disconnected, and full line voltage is applied to the stator windings by connecting it to the supply mains.

$$I_{AT} = x^2 I_{DOL}$$

I_{DOL} = Per phase starting current on direct switching to full voltage. I_{AT} = Per phase starting current by means of Autotransformer.

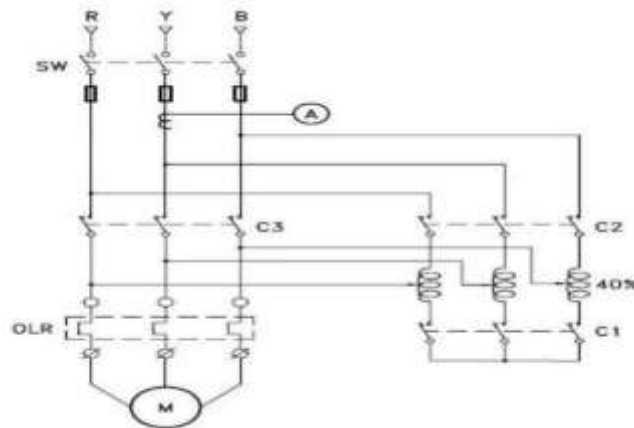


Figure 2 Power Circuit for Autotransformer Starting

III. Star-Delta Starting: This method may be adopted only for those motors which are designed to operate normally in delta. The six terminals from the stator winding must be available as shown in the figure 3 below. The terminals have been marked as a1, a2 for 'R' phase, b1, b2 for 'Y' phase and c1, c2 for 'B' phase. The terminals are connected to a TPDT (triple pole double throw) switch. During motor starting, the TPDT switch is in position 1, and hence the stator winding gets connected in star. Once the motor has accelerated to its steady state speed, the TPDT switch changes over to position 2. Now, the stator winding gets connected in delta. Hence, the motor has started in star connection thereby reducing the starting current and then switched to delta connection for its normal operation.

$$I_{Y-D} = I_{DOL}$$

I_{Y-D} = Starting line current with star-delta starter.

I_{DOL} = Starting line current with direct switching in delta.

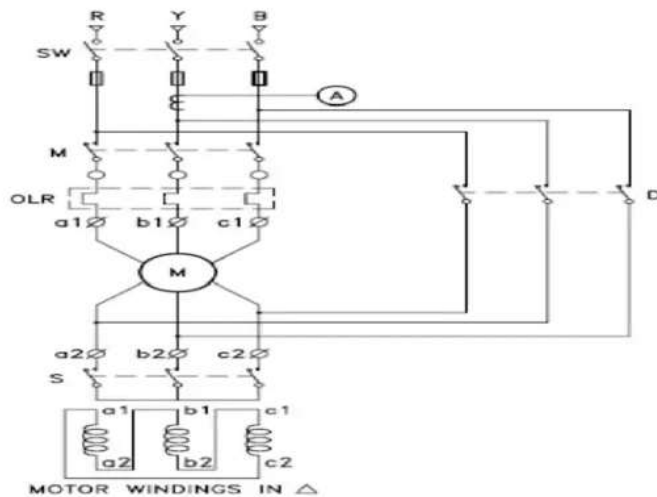


Figure 3 power Circuit for Star-Delta Starting

It is evident from the above that the starting torque available during DOL starting is also reduced to one-third in case of star-delta starting.

IV. The Stator Resistance (or Inductance) Starting:

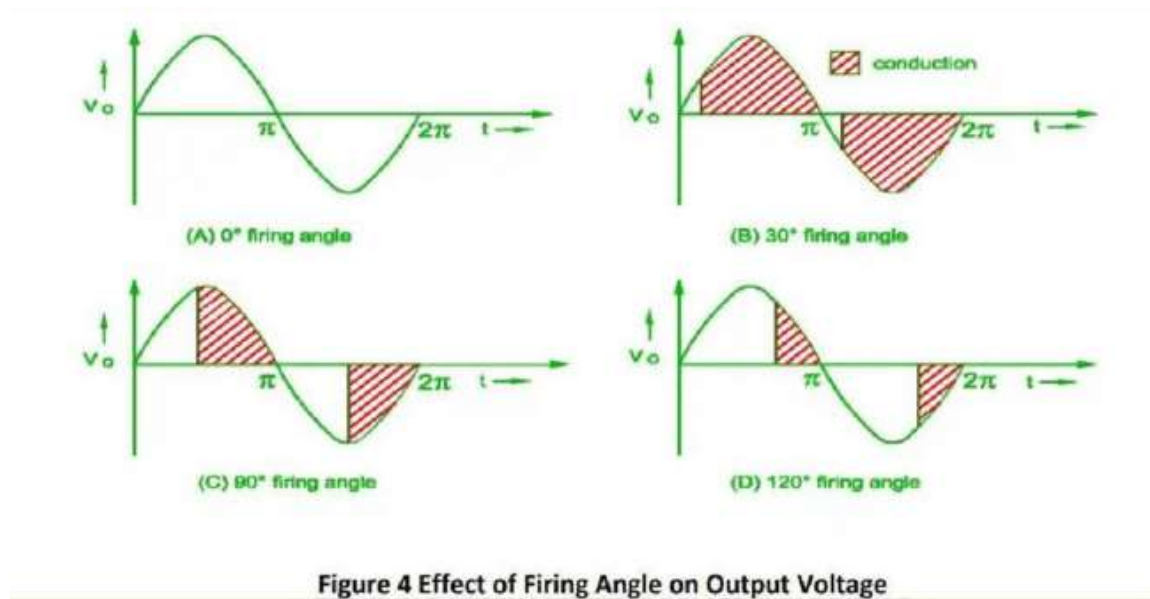
A suitable resistance (or inductance) is added in series with the stator and the motor is started with reduced current. As the motor speeds up, this resistance (or inductance) is gradually cut-off and finally the rated voltage is applied to the motor, and the motor achieves full speed.

V. Rotor-Rheostat Starting:

A suitable resistance is added (at starting) to the wound rotor windings via slip rings and then it is gradually cut, as the motor speeds up. Finally, when the rotor achieves full speed, the added resistance is fully cut-off.

VI. Electronic Soft Starting:

It is based upon the controlled rectifier or 'Thyristor'. By applying a firing pulse to the Thyristor, it switches from 'off' to 'on' until the current stops flowing through it, which occurs every half cycle of an AC supply. By controlling electronically, the Thyristor turn on point (firing angle), it is possible to regulate the energy passing through it. Figure 4 showing the effect of the firing angle of the Thyristor on the output voltage. By starting with a large delay angle and gradually reducing it, the motor terminal voltage is increased from a low value to full voltage, giving a smooth, stepless starting. Electronic soft starting has a simple, reliable and cost-effective piece of equipment.



APPLICATIONS OF THREE-PHASE INDUCTION MOTORS

- ☐ Lifts.
- ☐ Fans.
- ☐ Pumps.
- ☐ Large exhaust fans.
- ☐ Lathes.
- ☐ Crushers.
- ☐ Oil extracting.
- ☐ Mills.

CONSTRUCTION AND WORKING PRINCIPLE OF SINGLE-PHASE INDUCTION MOTOR:

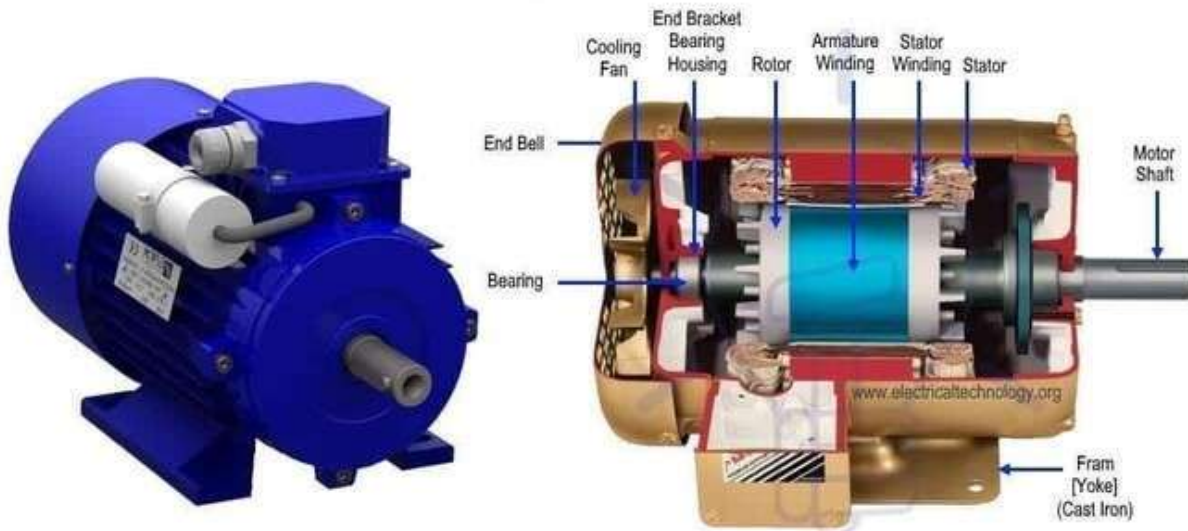
The single-phase motors are more preferred over a three-phase induction motor for domestic and commercial applications. Because for utility, only single-phase supply is available. So, in this type of application, the three-phase induction motor cannot be used.

CONSTRUCTION OF SINGLE-PHASE INDUCTION MOTOR

A single phase induction motor is similar to the three phase squirrel cage induction motor except there is single phase two windings (instead of one three phase winding in 3-phase motors) mounted on the stator and the cage winding rotor is placed inside the stator which freely rotates with the help of mounted bearings on the motor shaft.

The construction of a single-phase induction motor is similar to the construction of a three-phase induction motor.

Construction of Single-Phase Induction Motor



Similar to a three-phase induction motor, single-phase induction motor also has two main parts;

- Stator
- Rotor

Stator

In stator, the only difference is in the stator winding. The stator winding is single-phase winding instead of three-phase winding. The stator core is the same as the core of the three-phase induction motor.

In a single-phase induction motor, there are two windings are used in stator except in shaded-pole induction motor. Out of these two windings, one winding is the main winding and the second is an auxiliary winding.

The stator core is laminated to reduce the eddy current loss. The single-phase supply is given to the stator winding (main winding).

Rotor

Rotor of single-phase induction motor is the same as a rotor of squirrel cage induction motor. Instead of rotor winding, rotor bars are used and it is short-circuited at the end by end-rings. Hence, it makes a complete path in the rotor circuit. The rotor bars are braced to the end-rings to increase the mechanical strength of the motor.

The rotor slots are skewed at some angle to avoid magnetic coupling. And it also used to make a motor run smooth and quiet.



The following figure shows the stator and rotor of a 1-

phase induction motor. **WORKING OF SINGLE-PHASE INDUCTION MOTOR**

Single-phase AC supply is given to the stator winding (main winding). The alternating current flowing through the stator winding produces magnetic flux. This flux is known as the main flux.

Now we assume that the rotor is rotating and it is placed in a magnetic field produced by the stator winding. According to Faraday's law, the current starts flowing in the rotor circuit; it is a closed path. This current is known as rotor current.

Due to the rotor current, the flux is produced around the rotor winding. This flux is known as rotor flux.

There are two fluxes; **main flux which is produced by the stator** and **second is the rotor flux** which is produced by the rotor.

Interaction between main flux and rotor flux, the torque is produced in the rotor and it starts rotating.

The stator field is alternating in nature. The speed of the stator field is the same as synchronous speed. The synchronous speed of the motor depends on the number of poles and supply frequency.

It can be represented by two revolving fields. These fields are equal in magnitude and rotating in the opposite direction.

Let say Φ_m is a maximum field induced in the main winding. So, this field is divided into two equal parts and that is $\Phi_m/2$ and $\Phi_m/2$.

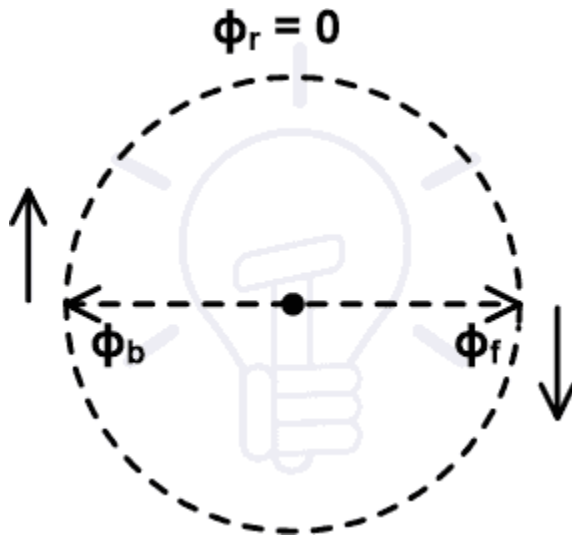
Out of these two fields, one field Φ is rotating in an anticlockwise direction and the second field Φ is rotating in a clockwise direction. Therefore, the resultant field is zero.

$$\Phi_r = \frac{\Phi_m}{2} - \frac{\Phi_m}{2}$$

$$\Phi_r = \Phi_f - \Phi_b$$

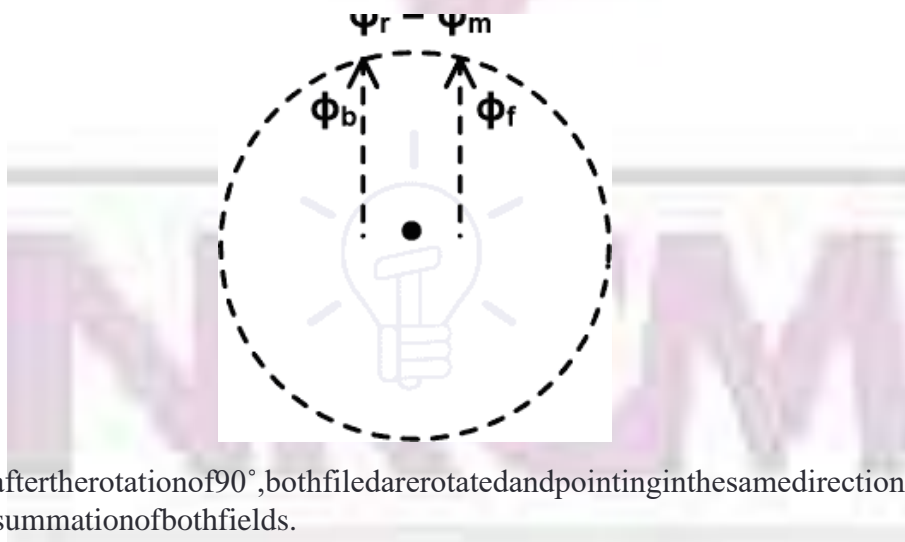
$$\Phi_r = 0$$

Now consider the resultant field at different instants.



When a motor starts, two fields are induced as shown in the above figure. These two fields are the same magnitude and opposite direction. So, resultant flux is zero.

In this condition, the stator field cannot cut by rotor field and resultant torque is zero. So, the rotor cannot rotate but it produces humming.



Now consider after the rotation of 90° , both fields are rotated and pointing in the same direction. Therefore, the resultant flux is a summation of both fields.

$$\Phi_r = \frac{\Phi_m}{2} + \frac{\Phi_m}{2}$$

$$\Phi_r = \Phi_f + \Phi_b$$

$$\Phi_r = 0$$

In this condition, the resultant field is equal to the maximum field induced by the stator. Now, both fields rotate separately and it is of alternating nature.

So, both fields cut by the rotor circuit and EMF induced in the rotor conductor. Due to this EMF, the current starts flowing in the rotor circuit and it induces a rotor flux.

Due to the interaction between stator flux and rotor flux motor continues to rotate. **This theory is known as Double Revolving Theory or double field revolving theory.**

Now, from the above explanation, we can conclude that the **single-phase induction motor is not self-starting.**

To make this motor self-starting motor, we need stator flux rotating in nature instead of alternating nature. This can be done by various methods.

Single-phase induction motor can be classified according to starting methods. Types of Single-phase Induction Motors

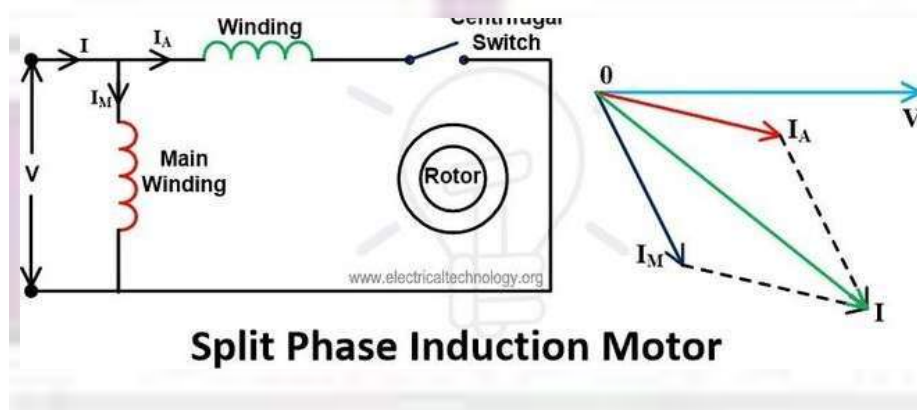
These single-phase induction motors are classified as;

- Split Phase Induction Motor
- Shaded Pole Induction Motor
- Capacitor Start Induction Motor
- Capacitor Start Capacitor Run Induction Motor
- Permanent Capacitor Induction Motor

Split Phase Induction Motor

In this type of motor, an extra winding is wound on the same core of the stator. So, there are two windings in the stator. One winding is known as the main winding or running winding and second winding is known as starting winding or auxiliary winding. A centrifugal switch is connected in series with the auxiliary winding. The auxiliary winding is highly resistive winding and the main winding is highly inductive winding. The auxiliary winding has few turns with a small diameter.

The aim of auxiliary winding is to create a phase difference between both fluxes produced by the main winding and rotor winding.



The connection diagram is as shown in the above figure. The current flowing through the main winding is I_M and current flowing through the auxiliary winding is I_A . Both windings are

parallel and supplied by voltage V .

The auxiliary winding is highly resistive in nature. So, the current I_A is almost in phase with supply voltage V .

The main winding is highly inductive in nature. So, the current I_M lags behind the supply voltage with a large angle.

The total stator flux is induced by the resultant current of these two windings. As shown in the phasor diagram, the resultant current is represented as (I) . It will create a phase difference between fluxes and resultant flux produces a rotating magnetic field. And the motor starts rotating.

Auxiliary winding only uses to start the motor. This winding is not useful in running condition. When the motor reaches 75 to 80 % of synchronous speed, the centrifugal switch opens. So, the auxiliary winding is out from the circuit. And motor runs on only main winding.

The phase difference created by this method is very small. Hence, the starting torque of this motor is poor. So, this motor is used in low starting torque applications like a fan, blower, grinder, pumps, etc.

Shaded Pole Induction Motor

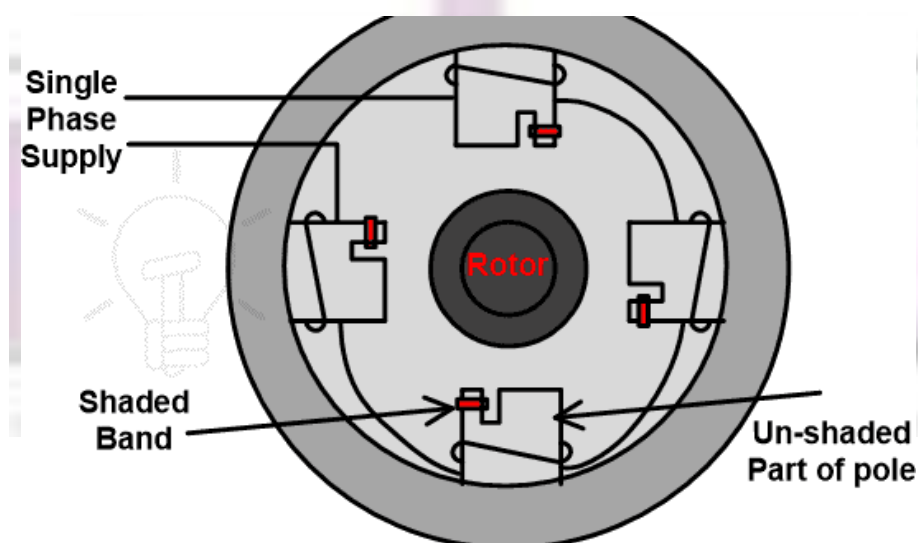
As compared to other types of single-phase induction motor, this motor has a different construction and working principle. This type of motor does not require auxiliary winding.

This motor has stator salient pole or projecting pole and the rotor is the same as squirrel cage induction motor. The stator poles are constructed specially to create a rotating magnetic field.

A pole of this motor is divided into two parts; shaded part and un-shaded part. It can be created by cutting pole into unequal distances.

A copper ring is placed in the small part of the pole. This ring is a highly inductive ring and it is known as a shaded ring or shaded band. The part at which shaded ring is placed is known as shaded part of the pole and the remaining part is an unshaded part.

The construction of this motor is as shown in the below figure.



Shaded Pole Induction Motor

When an alternating supply passing through the stator winding, an alternating flux induced in the stator coil. Due to this flux, some amount of flux will link with shaded ring and current will flow through shaded ring.

According to Len's law, the current passing through coil is opposite in nature, and flux produced due to this coil will oppose the main flux.

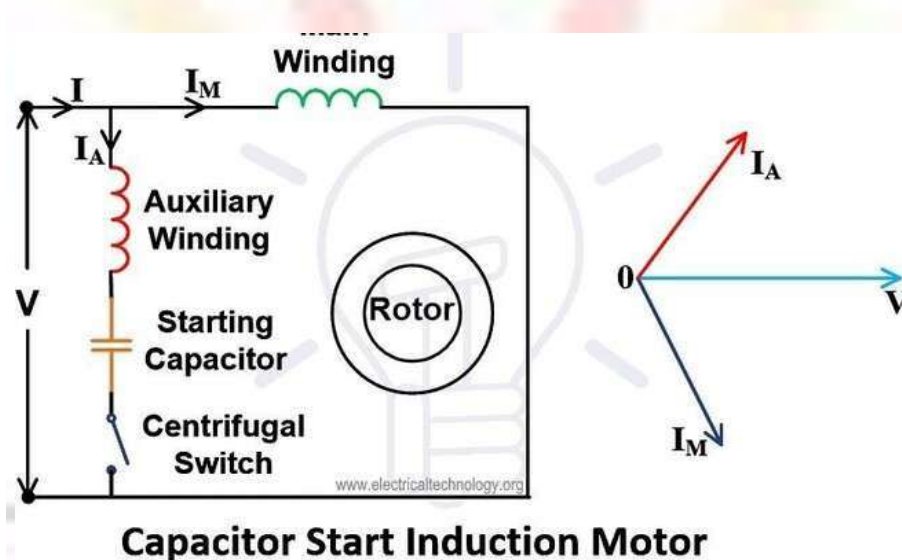
The shaded ring is a highly inductive coil. So, it will oppose the main flux when both fluxes are in the same direction and it will increase the main flux when both fluxes are in the opposite direction.

So, it will create a phase difference between the main flux (stator flux) and rotor flux. By this method, a phase difference is very less. Hence, the starting torque is very less. It is used in applications like toy motor, fan, blower, record player, etc.

Capacitor Start Induction Motor

This type of motor is an advanced version of the Split phase induction motor. The disadvantage of split-phase induction is low torque production. Because in this motor, the phase difference created is very less.

This disadvantage compensates in this motor with the help of a capacitor connected in series with auxiliary winding. The circuit diagram of this motor is as shown in the below figure.



The capacitor used in this motor is a dry-type capacitor. This is designed to use with alternating current. But this capacitor is not used for continuous operation.

In this method also, a centrifugal switch is used which disconnects the capacitor and auxiliary winding when the motor runs 75-80% of synchronous speed.

The current through auxiliary will lead the supply voltage by some angle. This angle is more than the angle increased in a split-phase induction motor.

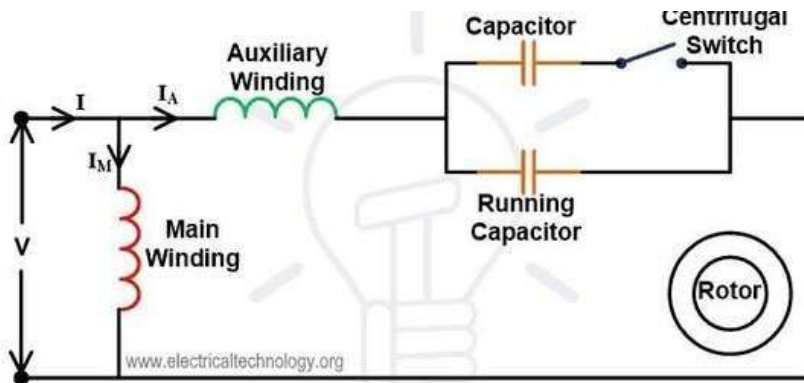
So, the starting torque of this motor is very high compared to the split-phase induction motor. The starting torque of this motor is 300% more than the full load torque.

Due to high starting torque, this motor is used in the applications where high starting torque is required like, a Lat h machine, compressor, drilling machines, etc.

Capacitor Start Capacitor Run Induction Motor

In this type of motor, two capacitors are connected in parallel with series in auxiliary winding. Out of these two capacitors, one capacitor is used only for starting (starting capacitor) and another capacitor is connected permanently with the motor (running capacitor).

The circuit diagram of this figure is as shown in the below figure



Capacitor Start Capacitor Run Induction Motor

The starting capacitor has high capacitance value and a running capacitor has low capacitance value. The starting capacitor is connected in series with a centrifugal switch that will open when the speed of the motor is 70% of synchronous speed.

During running conditions, both running winding and auxiliary winding connected with motor. The starting torque and efficiency of this motor are very high.

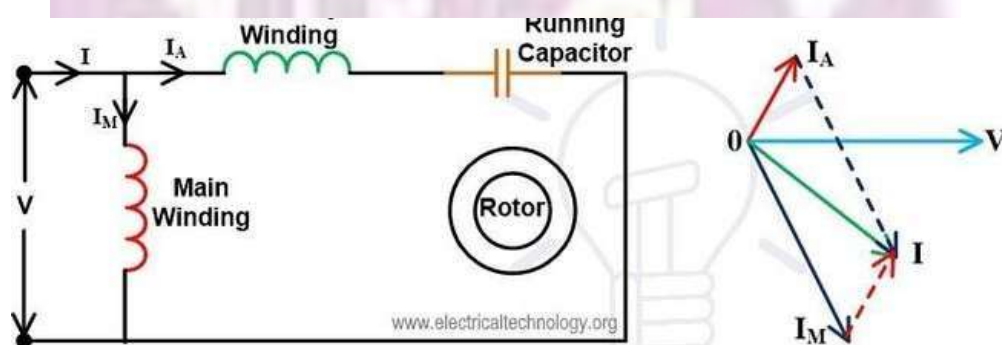
Therefore, this can be used in the application where high starting torque is required like a refrigerator, air conditioner, ceiling fan, compressor, etc.

Permanent Capacitor Induction Motor

The low-value capacitor is connected constantly with the auxiliary winding. Here, the capacitor has low capacitance.

The capacitor is used to increase the starting torque but it is low compared to the capacitor start induction motor.

The circuit diagram and phasor diagram of this motor is as shown in the below figure.



Permanent Capacitor Induction Motor

The power factor and efficiency of this motor are very high and also it has a high starting torque that is 80% of full load torque.

This type of motor is used in the application like an exhaust fan, blower, heater, etc. **APPLICATIONS**

OF SINGLE PHASE INDUCTION MOTORS

Single phase motors are not self starting and less efficient than [three phase induction motor](#) and available in 0.5 HP to 15 HP and still they are widely used for multiple purposes such as:

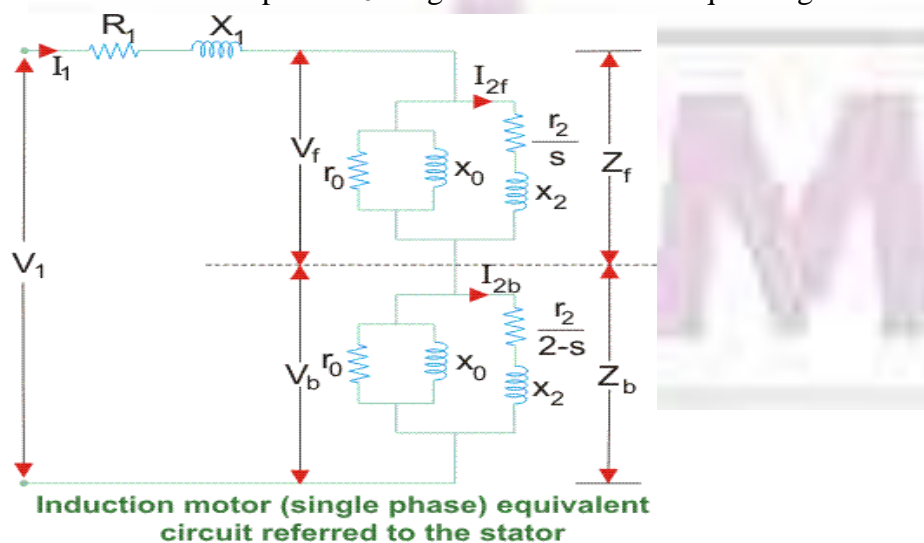
- Clocks
- Refrigerators, freezers and heaters
- Fans, table fans, ceiling fan, exhaust fans, air coolers and water coolers.
- Blowers
- Washing machines
- machine tools
- Dryers
- Typewriters, photostats and printers
- Water pumps and submersible
- Computers
- Grinders
- Drilling machines
- Other Home instrument, equipment and devices

etc. **EQUIVALENT CIRCUIT OF A SINGLE PHASE INDUCTION MOTOR**

There is a difference between single phase and three phase equivalent circuits. The [single phase induction motor circuit](#) is given by double revolving field theory which states that-

A stationary pulsating [magnetic field](#) might be resolved into two rotating fields, both having equal magnitude but opposite in direction. So the net torque induced is zero at standstill. Here, the forward rotation is called the rotation with slip s and the backward rotation is given with slip $2-s$. The equivalent circuit is-

In most of the cases the core loss component r_0 is neglected as this value is quite large and does not



affect much in the calculation.

Here, Z_f shows the forward impedance and Z_b shows the backward impedance.

Also, the sum of forward and backward slip is 2 so in case of backward slip, it is replaced by $(2-s)$. R_1 = Resistance of stator winding.

X_1 = Inductive reactance of the stator winding. X_m = Magnetising reactance. $R_2' = R_o$ rotor Reactance with referred to stator.

X_2' = Rotor inductive reactance with referred to stator.

DIFFERENCE BETWEEN SLIP RING & SQUIRREL CAGE INDUCTION MOTOR

The squirrel cage induction motor is the most popular type of AC motor. It is very commonly used in industries because it is very cheap, robust, efficient, and reliable. The slip ring motor has very little application in industries. Rarely 5%–10% slip ring motors are used in industries because it has several disadvantages like it required frequent maintenance, having high copper loss, etc.

One of the major difference between the slip ring and the squirrel cage motor is that the slip ring motor has an external resistance circuit for controlling the speed of the motor. Whereas in squirrel cage motor, it is not possible to add any external circuit because the bar of the motor is permanently slotted at the end of the ring. Some other differences between them are explained below in the comparison chart.

DC MACHINE

The DC machine can be classified into two types namely DC motors as well as DC generators. Most of the DC machines are equivalent to AC machines because they include AC currents as well as AC voltages in them. The output of the DC machine is DC output because they convert AC voltage to DC voltage. The conversion of this mechanism is known as the commutator, thus these machines are also named as commutating machines. DC machine is most frequently used for a motor. The main benefits of this machine include torque regulation as well as easy speed. The **applications of the DC machine** is limited to trains, mills, and mines. For example, underground subway cars, as well as trolleys, may utilize DC motors. In the past, automobiles were designed with DC dynamos for charging their batteries.

TYPE OF DC MOTORS

The excitation of the DC machine is classified into two types namely separate excitation, as well as self-excitation. In a separate excitation type of dc machine, the field coils are activated with a separate DC source. In the self-excitation type of dc machine, the flow of current throughout the field-winding is supplied with the machine. The principal kinds of DC machines are classified into four types which include the following.

- Permanent magnet type dc motor
- Separately excited dc motor

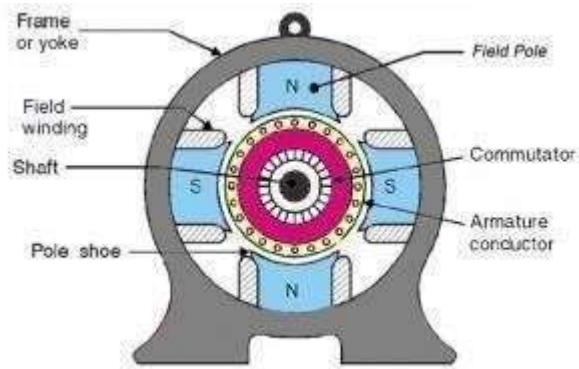
series wound motors

- Shunt wound DC motor
- Compound DC

motor. CONSTRUCTION OF DC MACHINE

LINE

The construction of the DC machine can be done using some of the essential parts like Yoke, Pole core & pole shoes, Pole coil & field coil, Armature core, Armature winding otherwise conductor, commutator, brushes & bearings. Some of the **parts of the DC machine** is discussed below.



Construction

Yoke

Another name of a yoke is the frame. The main function of the yoke in the machine is to offer mechanical support intended for poles and protects the entire machine from moisture, dust, etc. The materials used in the yoke are designed with cast iron, cast steel or otherwise rolled steel.

Pole and Pole Core

The pole of the DC machine is an electromagnet and the field winding is winding among pole. Whenever field winding is energized then the pole gives magnetic flux. The materials used for this are cast steel, cast iron or otherwise pole core. It can be built with the annealed steel laminations for reducing the power drop because of the eddy currents.

Pole Shoe

Pole shoe in the DC machine is an extensive part as well as to enlarge the region of the pole. Because of this region, flux can be spread out within the air-gap as well as extra flux can be passed through the air space toward armature. The materials used to build pole shoe is cast iron or otherwise cast steel, and also used annealed steel lamination to reduce the loss of power because of eddy currents.

Field Windings

In this, the windings are wound in the region of pole core & named as field coil. Whenever current is supplied through field winding then it electromagnetics the poles which generate required flux. The material used for field windings is copper.

Armature Core

Armature core includes a huge number of slots within its edge. The armature conductor is located in these slots. It provides the low-reluctance path toward the flux generated with field winding. The materials used in this core are permeability low-reluctance materials like iron or otherwise cast. The lamination is used to decrease the loss because of the eddy current.

Armature Winding

The armature winding can be formed by interconnecting the armature conductor. Whenever an armature winding is turned with the help of prime mover then the voltage, as well as magnetic flux, gets induced within it. This winding is allied to an exterior circuit. The materials used for this winding are conducting material like copper.

Commutator

The main function of the commutator in the DC machine is to collect the current from the armature conductor as well as supply the current to the load using brushes. And also provides uni-directional torque for DC-motor. The commutator can be built with a huge number of segments in the edge form of hard drawn copper. The Segments in the commutator are protected from the thin mica layer.

Brushes

Brushes in the DC machine gather the current from the commutator and supply it to the exterior load. Brushes wear with time to inspect frequently. The materials used in brushes are graphite or otherwise carbon which is in rectangular form.

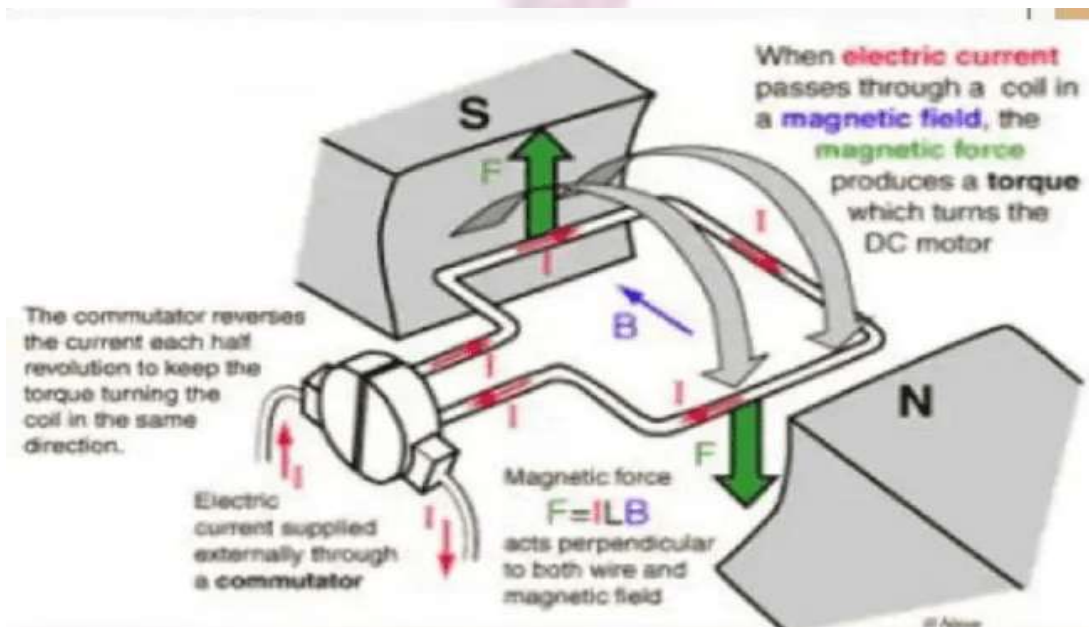
WORKING PRINCIPLE OF SEPARATELY EXCITED DC MOTOR

A same DC machine can be used as a motor or generator. Construction of a DC motor is same as that of a DC generator, however, the former converts electrical energy into mechanical energy. The principle of working of a DC motor is that "whenever a current carrying conductor is placed in a magnetic field, it experiences a mechanical force". The direction of this force is given by Fleming's left hand rule and its magnitude is given by $F = BIL$. When armature windings are connected to a DC supply, current flows in the winding. Magnetic field is provided by field winding excitation. In this case, current carrying armature conductors experience force due to the magnetic field, and this force will produce a torque to rotate the armature, thus rotating the machine shaft.

$$T = (ZP/2\pi A)\Phi I_a$$

When the armature of the motor is rotating, the conductors are also cutting the magnetic flux lines and hence according to the Faraday's law of electromagnetic induction, an emf induces in the armature conductors. The direction of this induced emf opposes the supplied armature current (I_a), hence it's called **Back emf** and given by the emf equation of DC generator;

$$E_b = K\Phi\omega_m, \text{ where } K = ZP/2\pi A$$



EMF EQUATION OF DC MACHINE

The **DC machine e.m.f.** can be defined as when the armature in the dc machine rotates, the voltage can be generated within the coils. In a generator, the e.m.f. of rotation can be called the generated e.m.f., and $E_g = E_r$. In the motor, the e.m.f. of rotation can be called the counter or back e.m.f., and $E_r = E_b$.

Let Φ is the useful flux for every pole within webers P is the total number of poles Z is the total number of conductors within the armature

n is the rotation speed for an armature in the revolution for each second

A is the no. of parallel lane throughout the armature among the opposite polarity brushes. Z/A is the no. of armature conductor within series for each parallel lane

As the flux for each pole is ' Φ ', every conductor slashes a flux ' $P\Phi$ ' within a single revolution.

The voltage produced for each conductor = flux slash for each revolution in WB / Time taken for a single revolution within seconds

As ' n ' revolutions are completed within a single second and 1 revolution will be completed within a $1/n$ second. Thus the time for a single armature revolution is $1/n$ sec.

The standard value of produced voltage for each conductor $P\Phi / 1/n = np\Phi$ volts

The voltage produced (E) can be decided with the no. of armature conductors within series I any single lane among the brushes thus, the whole voltage produced

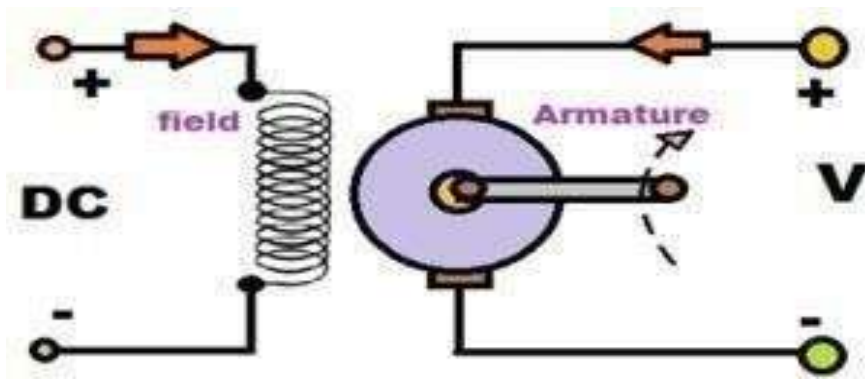
$E = \text{standard voltage for each conductor} \times \text{no. of conductors within series for each lane}$ $E = n.P.\Phi \times$

Z/A

The above equation is the e.m.f. equation of the DC machine.

SEPARATELY EXCITED DC MOTOR

In this section we will discuss about the separately excited dc motor. Like other DC motors, these motors also have both stator and rotor. Stator refers to the static part of motor, which consists of the field windings. And the rotor is the moving armature which contains armature windings or coils. Separately excited dc motor has field coils similar to that of shunt wound dc motor. The name suggests the construction of this type of motor. Usually, in other DC motors, the field coil and the armature coil both are energized from a single source. The field of them does not need any separate excitation. But, in separately excited DC motor, separate supply is provided for excitation of both field coil and armature coil. Figure below shows the separately excited dc motor.



Here, the field coil is energized from a separate DC voltage source and the armature coil is also energized from another source. Armature voltage source may be variable but, independent constant DC voltage is used for energizing the field coil. So, those coils are electrically isolated from each other, and this connection is the specialty of this type of DC motor.

OPERATING CHARACTERISTICS OF SEPARATELY EXCITED DC MOTOR

Both in shunt wound dc motor and separately excited dc motor field is supplied from constant voltage so that the field current is constant. Therefore these two motors have similar speed-armature current and torque – armature current characteristics. In this type of motor flux is assumed to be constant.

When the load increases, the output torque required to drive the load will increase. Hence, the motor speed will slow down. Consequently the internal generated voltage drops ($E_A = K\Phi\omega_m \downarrow$), increasing the armature current in motor $I_A = (V_s - E_A) / R_A$. As the armature current increases, the developed torque increases ($T_{dev} = K\Phi I_A \uparrow$) and finally the developed torque will be equal to the load torque at a lower mechanical speed of rotation ω_m .

Mechanical Load $\uparrow \omega_m \downarrow, I_A \uparrow, T_{dev} \uparrow$

1) TORQUE VS. ARMATURE CURRENT: Generally, the developed torque is directly proportional to armature current ($T_{dev} = K\Phi I_A$) and the relationship is in the form of a straight line, assuming the field flux Φ to be constant as the supply voltage is constant.

Since, heavy starting load needs high starting current, shunt motor should never be started on a heavy load.

2) SPEED VS. ARMATURE CURRENT

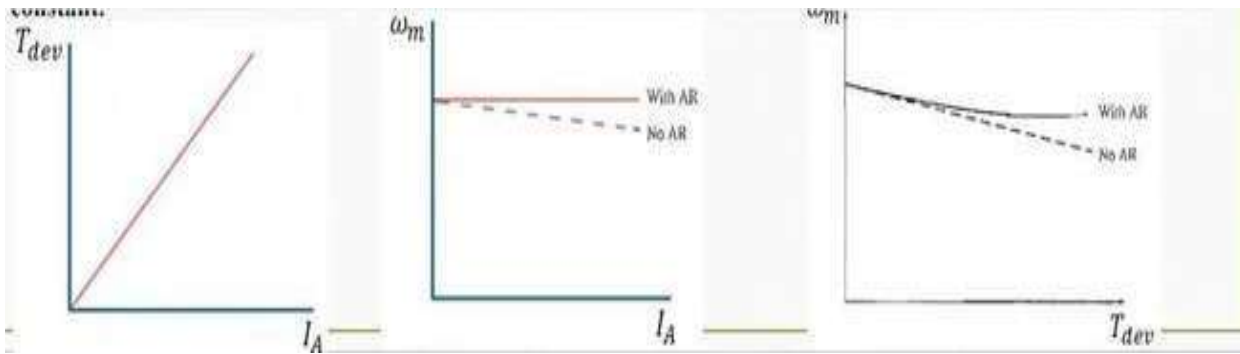
$$V_s = E_A + I_A R_A \text{ and } E_A = K\Phi\omega_m \Rightarrow V_s = K\Phi\omega_m + I_A R_A \Rightarrow \omega_m = (V_s - I_A R_A) / K\Phi$$

As flux Φ is assumed constant, the speed decreases with armature current increase. But practically, due to armature reaction, Φ decreases with increase in armature current, and hence the speed decreases slightly. Hence, a shunt motor can be assumed as a constant speed motor.

3) TORQUE VS. SPEED

$$\omega_m = (V_s - T_{dev} R_A / K\Phi) / K\Phi$$

As flux Φ is assumed constant, the speed decreases with developed torque increase. But practically, due to armature reaction, Φ decreases with increase in armature current, and hence the speed decreases slightly. Thus, at heavy loads, the motor speed is almost constant.

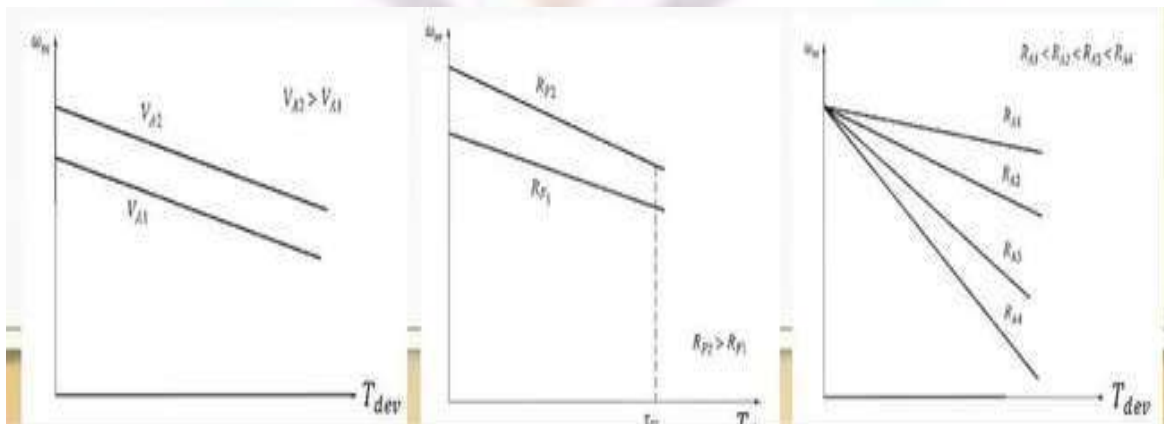


SPEED CONTROL OF SEPARATELY EXCITED DC MOTOR

Speed of this type of DC shunt motor is controlled by the following methods:

$$\omega_m = (V_s - I_A R_A) / K\Phi$$

1. **SUPPLY VOLTAGE CONTROL METHOD:** Adjusting the supply voltage applied to the armature without changing the voltage applied to the field. Hence, the flux is kept constant. This can be applied to separately excited motors only. Hence, at a certain load, since the flux is fixed, increasing the armature voltage, increases the motor speed.
2. **FLUX CONTROL METHOD:** Adjusting the field resistance $I_F = V_s / R_F$ (and thus the field flux). This can be applied to separately excited and shunt motors. Hence, for a constant supply voltage, at a certain load, increasing the flux decreases the motor speed.
3. **ARMATURE RESISTANCE CONTROL METHOD:** Inserting a resistor in series with the armature circuit. This can be applied to separately excited and shunt motors. Hence, for a constant supply voltage and fixed flux, at a certain load, increasing R_A decreases the motor speed.



ADVANTAGES

The advantages of this machine include the following.

- DC machines like DC motor have various advantages like starting torque is high, reversing, fast-starting & stopping, changeable speed through voltage input
- These are very easily controlled as well as cheaper when compared with AC
- Speed control is good
- Torque is high
- Operation is seamless
- Free from harmonics
- Installation and maintenance is easy

APPLICATIONS OF DC MOTORS

- Separately excited DC motors are often used as actuators in trains and automotive traction applications.
- For their constant speed characteristics, shunt DC motors are used in fixed speed applications such as fans.
- Since the series motors can give high torque per ampere (since their torque is directly proportional to the square of armature current), they can be used in applications that require high starting torque. Examples of these applications include starter motors in cars, and elevator motors.

SYNCHRONOUS GENERATOR – CONSTRUCTION AND WORKING PRINCIPLE

A *synchronous generator* is a synchronous machine which converts mechanical power into AC electric power through the process of electromagnetic induction.

Synchronous generators are also referred to as **alternators** or **AC generators**. The term "alternator" is used since it produces AC power. It is called synchronous generator because it must be driven at synchronous speed to produce AC power of the desired frequency.

A synchronous generator can be either *single-phase* or *poly-phase* (generally 3 phase).

CONSTRUCTION OF SYNCHRONOUS GENERATOR OR ALTERNATOR

An alternator consists of two main parts viz.

- **Stator** – The stator is the stationary part of the alternator. It carries the armature winding in which the voltage is generated. The output of the alternator is taken from the stator.
- **Rotor** – The rotor is the rotating part of the alternator. The rotor produces the main field flux.

Stator Construction of Alternator

The stator of the alternator includes several parts, viz. the frame, stator core, stator or armature windings, and cooling arrangement.

- The stator frame may be made up of cast iron for small-size machines and of welded steel for large-size machines.
- The stator core is assembled with high-grade silicon content steel laminations. These silicon steel laminations reduce the hysteresis and eddy losses in the stator core.
- The slots are cut on the inner periphery of the stator core. A 3-phase armature winding is put in these slots.
- The armature winding of the alternator is star connected. The winding of each phase is distributed over several slots. When current flows through the distributed armature winding, it produces an essential sinusoidal spaced distribution of EMF.

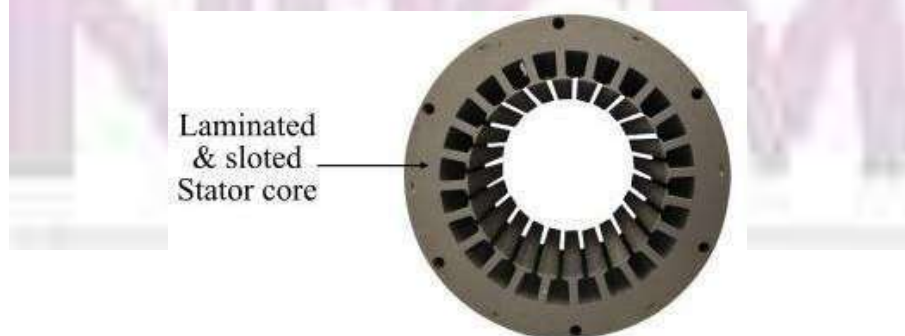


Fig. - Stator of Alternator

Rotor Construction of Alternator

The rotor of the alternator carries the field winding which is supplied with direct current through two slip rings by a separate DC source (also called exciter). The exciter is generally a small DC shunt generator mounted on the shaft of the alternator.

For the alternator, there are two types of rotor constructions are used viz. the salient-pole type and the cylindrical rotor type.

Salient Pole Rotor

The term *salient* means *projecting*. Hence, a *salient pole rotor* consists of poles projecting out from the surface of the rotor core. This whole arrangement is fixed to the shaft of the alternator as shown in the figure. The individual field pole windings are connected in series such that when the field winding is energised by the DC exciter, the adjacent poles have opposite polarities.

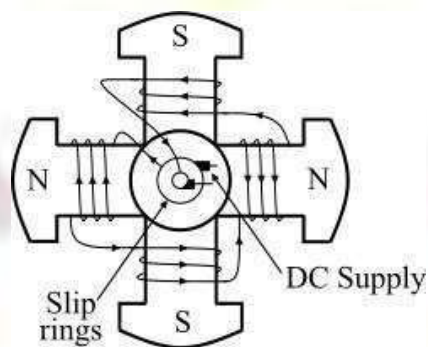
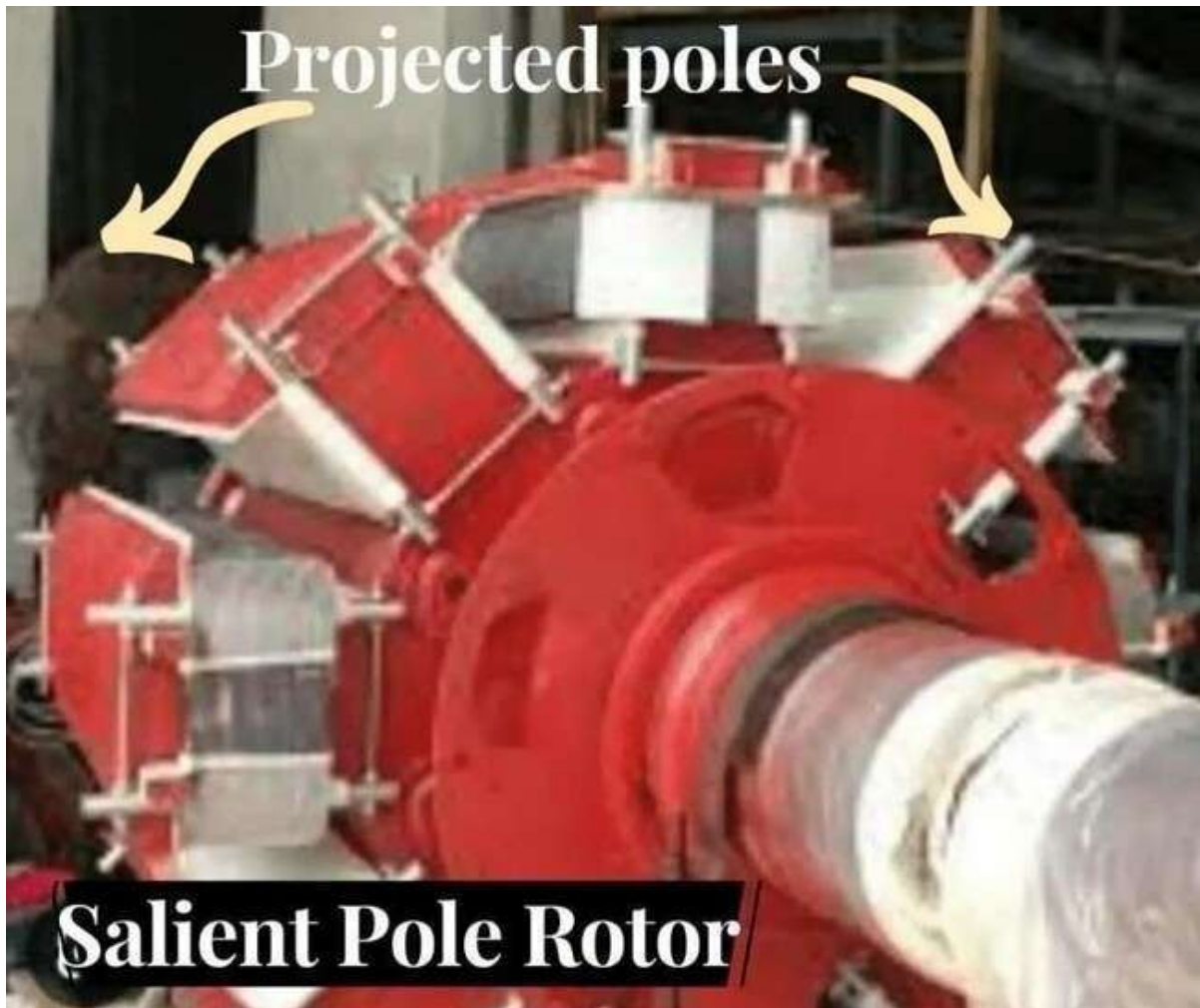


Fig. - Salient Pole Rotor



The salient pole type rotor is used in the low and medium speed (from 120 to 400 RPM) alternators such as those driven by the diesel engines or water turbines because of the following reasons –

- The construction of salient pole type rotor cannot be made strong enough to withstand the mechanical stresses to which they may be subjected at high speed.
- If the salient field pole type rotor is driven at high speed, then it would cause windage loss and would tend to produce noise.

Low speed rotors of the alternators possess a large diameter to provide the necessary space for the poles. As a result, the salient pole type rotors have large diameter and short axial length.

Cylindrical Rotor

The cylindrical rotors are made from solid forgings of high-grade nickel-chrome-molybdenum steel.

- The construction of the cylindrical rotor is such that there are no physical poles to be seen as in the salient pole rotor.
- In about two-thirds of the outer periphery of the cylindrical rotor, slots are cut at regular intervals and parallel to the rotor shaft.
- The field windings are placed in these slots and are excited by DC supply. The field winding is of *distributed type*.
- The unslotted portion of the rotor forms the pole faces.
- It is clear from the figure of the cylindrical rotor that the poles formed are non-salient, i.e., they do not project out from the rotor surface.

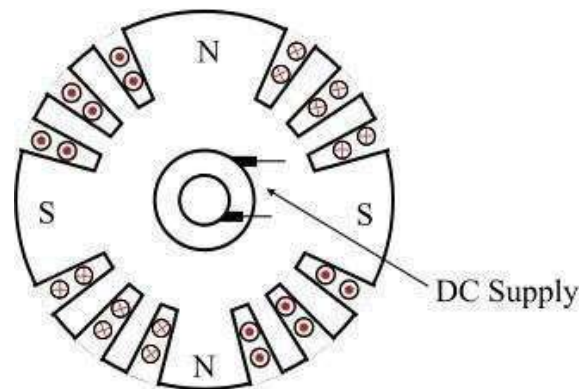


Fig. - Cylindrical Rotor



The cylindrical type rotor construction is used in the high-speed (1500 to 3000 RPM) alternators such as those driven by steam turbines because of the following reasons—

- The cylindrical type rotor construction provides a greater mechanical strength and permits more accurate dynamic balancing.
- It gives noiseless operation at high speeds because of the uniform air gap.
- The flux distribution around the periphery of the rotor is nearly as in a sine wave and hence a better EMF waveform is obtained.

A cylindrical rotor alternator has a comparatively small diameter and long axial length. The cylindrical rotor alternators are called **turbo-alternators** or **turbo-generators**. The alternator with cylindrical rotor has always a horizontal configuration in installation.

WORKING PRINCIPLE AND OPERATION OF ALTERNATOR

An alternator or synchronous generator works on the principle of electromagnetic induction, i.e., when the flux linking a conductor changes, an EMF is induced in the conductor. When the armature winding of an alternator is subjected to the rotating magnetic field, the voltage will be generated in the armature winding.

When the rotor field winding of the alternator is energised from the DC exciter, the alternate N and S poles are developed on the rotor. When the rotor is rotated in the anticlockwise direction by a prime mover, the armature conductors placed on the stator are cut by the magnetic field of the rotor.

poles. As a result, the EMF is induced in the armature conductors due to electromagnetic induction. This induced EMF is alternating one because the N and S poles of the rotor pass the armature conductors alternatively.

The direction of the generated EMF can be determined by the Fleming's right rule and the frequency of it is given by,

$$f = \frac{NsP}{120}$$

- Ns is the synchronous speed in RPM
- P is the number of rotor poles.
- The magnitude of the generated voltage depends upon the speed of rotation of the rotor and the DC field excitation current. For the balanced condition, the generated voltage in each phase of the winding is the same but differs in phase by 120° electrical.

E.M.F. EQUATION OF SYNCHRONOUS GENERATOR

The e.m.f. equation of this generator is shown below.

$$E_{ph} = 4.44 K_c K_d \Phi f T_{ph} \text{ Volts}$$

Where,

' P ' is poles

' Φ ' is Flux for each pole in Webers ' N ' is the speed in rpm (revolution per minute) ' f ' is the frequency in Hz

' T_{ph} ' is the number of turns connected in series per phase ' K_c ' is the span factor of the coil

' K_d '

is the distribution factor of the coil. **Applications of S**

ynchronous Generator

The applications of synchronous generator include the following.

- It is used in the systems where a stable speed is necessary.
- It is used to preserve the power factor (PF) of the system.
- It is used in power generation plants because of stable frequency.





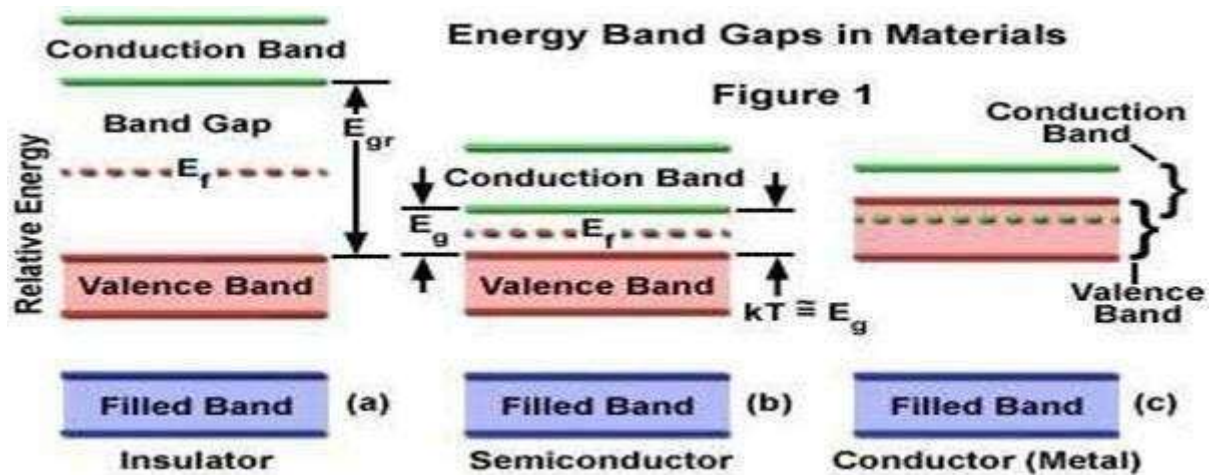
UNIT IV

PN JUNCTION DIODE

4.1 INTRODUCTION:

We start our study of nonlinear circuit elements. These elements (diodes and transistors) are made of semiconductors. A brief description of how semiconductor devices work is first given to understand their characteristics. You will see a rigorous analysis of semiconductors in the breadth courses.

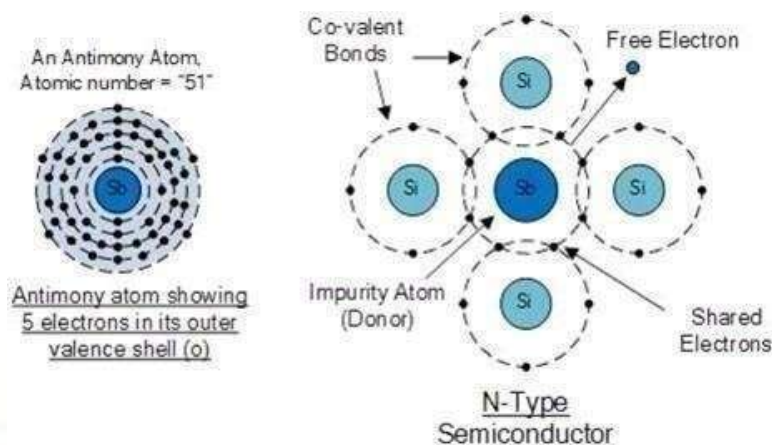
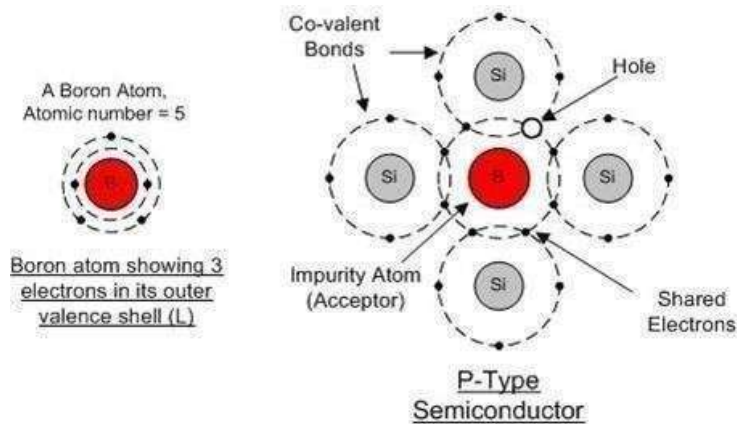
4.1.1 Energy Bands in Solids:



4.1.2 Semiconductors:

Semiconductor materials are mainly made of elements from group IVB of the periodic table like C (diamond), Si, Ge, SiC. These materials have 4 electrons in their outermost electronic shell. Each atom can form a "covalent" bond with four of its neighbors, sharing one electron with that atom. In this manner, each atom "sees" eight electrons in its outermost electronic shell (4 of its own, and one from each neighbor), completely filling that shell. It is also possible to form this type of covalent bond by combining elements from group IIIB (sharing three electrons) with an element from group VB (sharing five electrons). Examples of these semiconductors are GaAs or AlGaAs and are usually called "3-5" semiconductors. We focus mostly on Si semiconductors in this class. Figure below shows this covalent bond structure for Si. A pair of electrons and holes are shown. Note that Si forms a tetrahedron structure and an atom in the center of the tetrahedron shares electrons with atoms on each vertex. Figure below is a two-dimensional representation of such a structure. The left figure is for a pure Si semiconductor and an electron-hole pair is depicted. Both electrons and holes are called "mobile" carriers as they are responsible for carrying electric current.

If we add a small amount of an element from group VB, such as P, to the semiconductor, we create an n-type semiconductor and the impurity dopant is called an n-type dopant. Each of these new atoms also forms a covalent bond with four of its neighbors. However, as an n-type dopant has 5 valence electrons, the extra electron will be located in the "empty" energy band. As can be seen, there is no hole associated with this electron. In addition to electrons from the n-type dopant, there are electron-hole pairs in the solid from the base semiconductor (Si in the above figure) which are generated due to temperature effects. In an n-type semiconductor, the number of free electrons from the dopant is much larger than the number of electrons from electron-hole pairs. As such, an n-type semiconductor is considerably more conductive than the base semiconductor (in this respect, an n-type semiconductor is more like a "resistive" metal than a semiconductor).



In summary, in an n-type semiconductor there are two charge carriers: "holes" from the base semiconductor (called the "minority" carriers) and electrons from both the n-type dopant and electron-hole pairs (called the "majority" carrier).

Similarly, we can create a p-type semiconductor by adding an element from group IIIB, such as B, to the semiconductor. In this case, the p-type dopant generates holes. We will have two charge carriers: majority carriers are "holes" from the p-type dopant and electron-hole pairs, and minority carriers are electrons from the base semiconductor (from electron-hole pairs).

The charge carriers (electrons and holes) move in a semiconductor through two mechanisms: First, charge carriers would move from regions of higher concentration to lower concentration in order to achieve a uniform distribution throughout the semiconductor. This process is called "Diffusion" and is characterized by the diffusion coefficient, D . Second, charge carriers move under the influence of an electric field. This motion is called drift and is characterized by the mobility.

4.2 DIODE WORKING PRINCIPLE

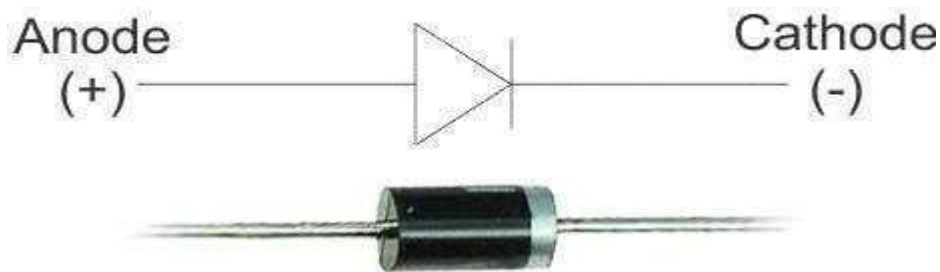
What is a Diode?

A diode is a device which only allows unidirectional flow of current if operated within a rated specified voltage level. A diode only blocks current in the reverse direction while the reverse voltage is within a limited range; otherwise, reverse barrier breaks and the voltage at which this breakdown occurs is called reverse breakdown voltage. The diode acts as a valve in the electronic and electric circuit. A P-N junction is the simplest form of the diode which behaves as an ideally short circuit when it is in forward biased and behaves as an ideally open circuit when it is in the reverse biased. Besides simple PN junction diodes, there are different types of diodes although the fundamental principle is more or less the same. So a particular arrangement of diodes can convert AC to pulsating DC, and hence, it is

sometimes also called as a rectifier. The name diode is derived from "di-ode" which means a device having two electrodes.

4.2.1 Symbol of Diode

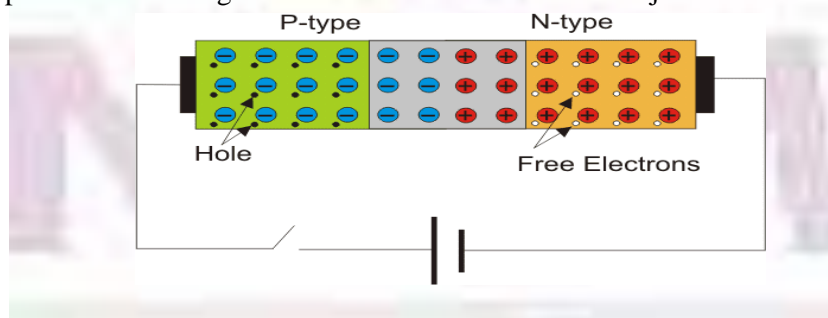
The symbol of a diode is shown below, the arrowhead points in the direction of conventional current flow.



A simple **PN junction diode** can be created by doping donor impurity in one portion and acceptor impurity in other portion of a silicon or germanium crystal block. These make a p n junction at the middle portion of the block beside which one portion is p-type (which is doped by trivalent or acceptor impurity) and other portion is n-type (which is doped by pentavalent or donor impurity). It can also be formed by joining a p-type (intrinsic semiconductor doped with a trivalent impurity) and n-type semiconductor (intrinsic semiconductor doped with a pentavalent impurity) together with a special fabrication technique such that a p-n junction is formed. Hence, it is a device with two elements, the p-type forms an anode and the n-type forms the cathode. These terminals are brought out to make the external connections.

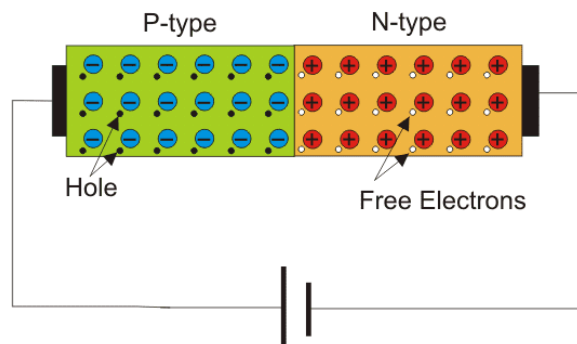
4.2.2 Working Principle of Diode

The n side will have a large number of electrons and very few holes (due to thermal excitation) whereas the p side will have a high concentration of holes and very few electrons. Due to this, a process called diffusion takes place. In this process free electrons from the n side will diffuse (spread) into the p side and combine with holes present there, leaving a positive immobile (not moveable) ion in the n side. Hence, few atoms on the p side are converted into negative ions. Similarly, few atoms on the n side will get converted to positive ions. Due to this large number of positive ions and negative ions will accumulate on the n-side and p-side respectively. This region so formed is called a depletion region. Due to the presence of these positive and negative ions a static electric field called as "barrier potential" is created across the p-n junction of the diode. It is called as "barrier potential" because it acts as a barrier and opposes the further migration of holes and electrons across the junction.

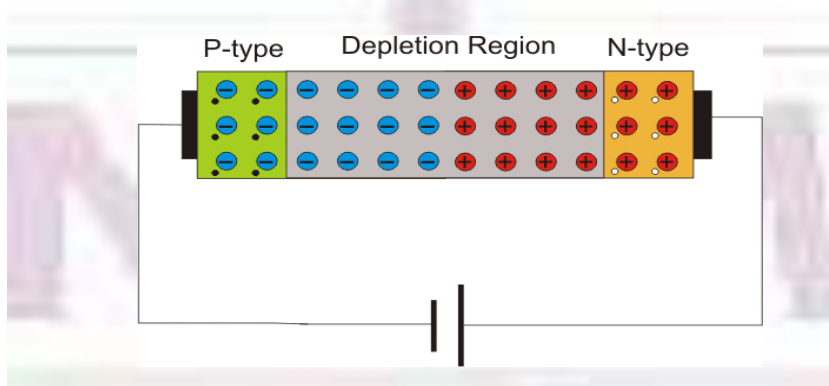


In a PN junction diode when the forward voltage is applied i.e. positive terminal of a source is connected to the p-type side, and the negative terminal of the source is connected to the n-type side, the diode is said to be in forward biased condition. We know that there is a barrier potential across the junction. This barrier potential is directed in the opposite of the forward applied voltage. So a diode can only allow current to flow in the forward direction when forward applied voltage is more than barrier potential of the junction. This voltage is called forward biased voltage. For silicon diode, it is 0.7 volts. For germanium diode, it is 0.3 volts. When forward applied voltage is more than this forward biased voltage, there will be forward current in the diode, and the diode will become short.

circuited. Hence, there will be no more voltage drop across the diode beyond this forward biased voltage, and forward current is only limited by the external resistance" > resistance connected in series with the diode. Thus, if forward applied voltage increases from zero, the diode will start conducting only after this voltage reaches just above the barrier potential or forward biased voltage of the junction. The time taken by this input voltage to reach that value or in other words the time taken by this input voltage to overcome the forward biased voltage is called recovery time.

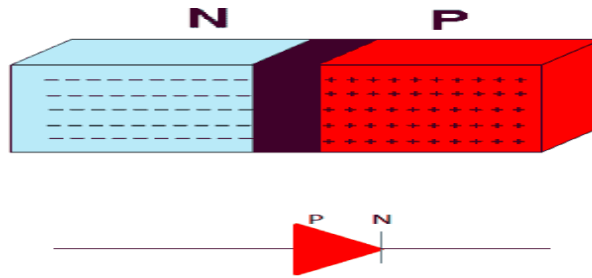


Now if the diode is reverse biased i.e. positive terminal of the source is connected to the n-type end, and the negative terminal of the source is connected to the p-type end of the diode, there will be no current through the diode except reverse saturation current. This is because at the reverse biased condition the depletion layer of the junction becomes wider with increasing reverse biased voltage. Although there is a tiny current flowing from n-type end to p-type end in the diode due to minority carriers. This tiny current is called reverse saturation current. Minority carriers are mainly thermally generated electrons and holes in p-type semiconductor and n-type semiconductor respectively. Now if reverse applied voltage across the diode is continually increased, then after certain applied voltage the depletion layer will destroy which will cause a huge reverse current to flow through the diode. If this current is not externally limited and it reaches beyond the safe value, the **diode** may be permanently destroyed. This is because, as the magnitude of the reverse voltage increases, the kinetic energy of the minority charge carriers also increase. These fast moving electrons collide with the other atoms in the device to knock-off some more electrons from them. The electrons so released further release much more electrons from the atoms by breaking the covalent bonds. This process is termed as carrier multiplication and leads to a considerable increase in the flow of current through the p-n junction. The associated phenomenon is called Avalanche Breakdown.



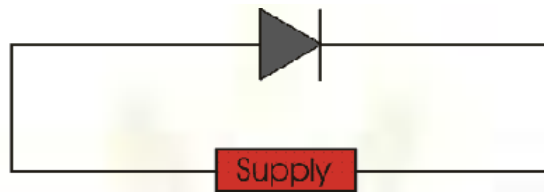
4.3 DIODE CHARACTERISTICS

Semiconductor materials (Si, Ge) are used to form variety of electronic devices. The most basic device is diode. Diode is a two terminal P-N junction device. P-N junction is formed by bringing a P type material in contact with N type material. When a P-type material is brought in contact with N-type material electrons and holes start recombining near the junction. This results in lack of charge carriers at the junction and thus the junction is called depletion region. Symbol of P-N junction is given as:



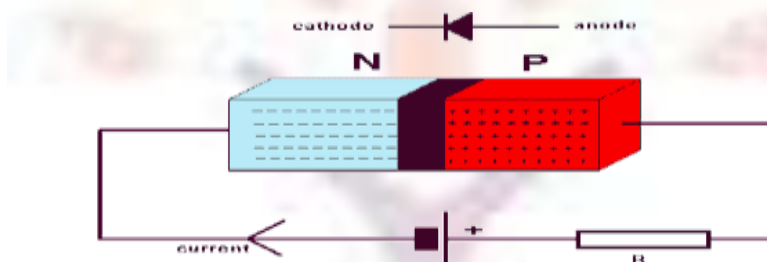
Biased i.e. when voltage is applied across the terminals of P-N junction, it is called diode.

Diode is unidirectional device that allows the flow of current in one direction only depending on the biasing.



4.3.1 Forward Biasing Characteristic of Diode

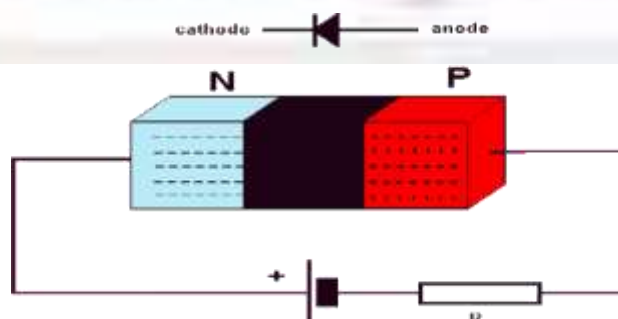
When, P terminal is more positive as compared to N terminal i.e. P- terminal connected to positive terminal of battery and N-terminal connected to negative terminal of battery, it is said to be forward biased.



Positive terminal of the battery repels majority carriers, holes, in P-region and negative terminal repels electrons in the N-region and push them towards the junction. This results in increase in concentration of charge carriers near junction, recombination takes place and width of depletion region decreases. As forward bias voltage is raised depletion region continues to reduce in width, and more and more carriers recombine. This results in exponential rise of current.

4.3.2 Reverse Biasing Characteristic of Diode

In reverse biasing P- terminal is connected to negative terminal of the battery and N- terminal to positive terminal of battery. Thus applied voltage makes N-side more positive than P-side.



Negative terminal of the battery attracts majority carriers, holes, in P-region and positive terminal attracts electrons in the N-region and pull them away from the junction. This results in decrease in concentration of charge carriers near junction and width of depletion region increases. A small amount of current flow due to minority carriers, called as reverse bias current or leakage current. As reverse bias voltage is raised depletion region continues to increase in width and no current flows. It can be concluded that diode acts only when forward biased. Operation of diode can be summarized in form of I-V diode characteristics graph.

For reverse bias diode, $V < 0$, $I_D = I_S$ Where, V = supply voltage I_D

= diode current I_S = reverse saturation current For forward bias, $V > 0$, $I_D = I_S(e^{V/NV_T} - 1)$

Where,

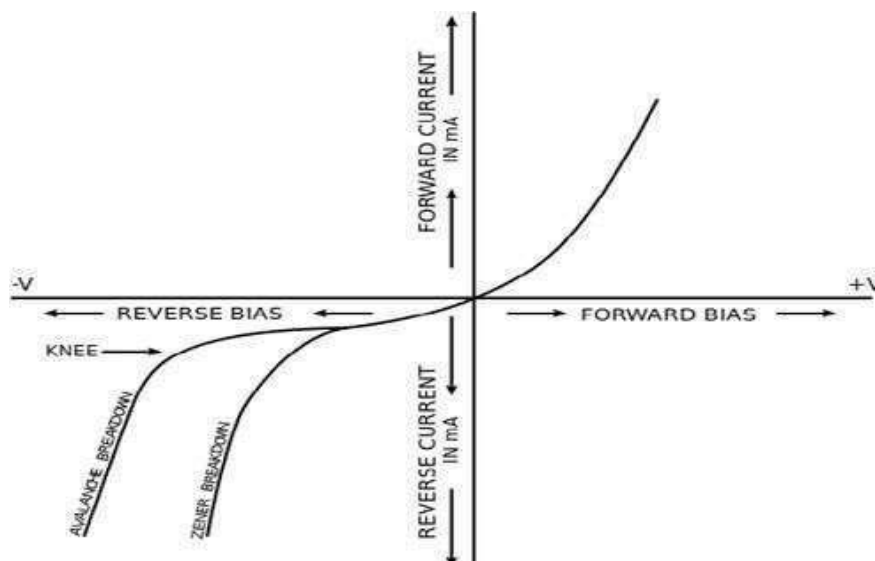
V_T = volt's equivalent of temperature $= KT/Q = T/11600$ $Q = e$

electronic charge $= 1.632 \times 10^{-19}$ C

K = Boltzmann's constant $= 1.38 \times 10^{-23}$

$N = 1$, for Ge

$= 2$, for Si



As reverse bias voltage is further raised, depletion region width increases and a point comes when junction breaks down. This results in large flow of current. Breakdown is the knee of diode characteristics curve. Junction breakdown takes place due to two phenomena

4.3.3 Avalanche Breakdown (for $V > 5V$)

Under very high reverse bias voltage kinetic energy of minority carriers become so large that they knock out electrons from covalent bonds, which in turn knock more electrons and this cycle continues until an undoes junction breakdowns.

4.3.4 Zener Effect (for $V < 5V$)

Under reverse bias voltage junction barrier tends to increase with increase in bias voltage. This results in very high static electric field at the junction. This static electric field breaks covalent bond and sets minority carriers free which contributes to reverse current. Current increases abruptly and junction breaks down.

P-N JUNCTION DIODE AND CHARACTERISTICS OF P-N JUNCTION

The volt-ampere characteristics of a diode explained by the following equations:

$$I = I_S (e^{V_D/(\eta V_T)} - 1)$$

Where

I = current flowing in the diode, I_0 =

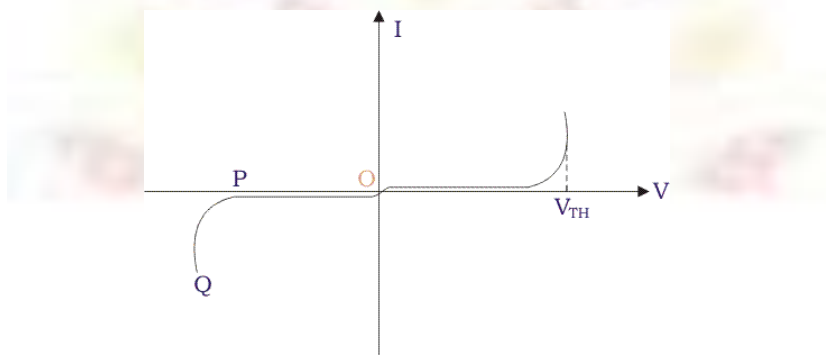
reverse saturation current

V_D = Voltage applied to the diode

V_T = volt-equivalent of temperature = $kT/q = T/11,600 = 26\text{mV}$ (@ room temp)

$\eta = 1$ (for Ge) and 2 (for Si)

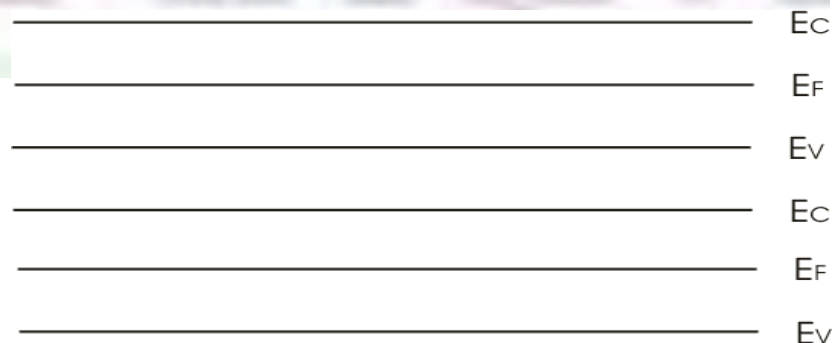
It is observed that **Ge** diodes have smaller cut-in-voltage when compared to **Si** diode. The reverse saturation current in **Ge** diode is larger in magnitude when compared to silicon diode.



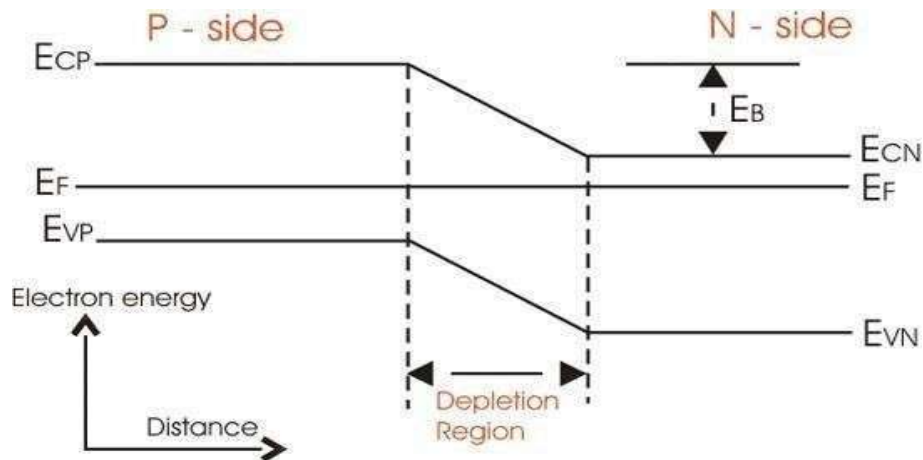
When V is positive, the junction is forward biased and when V is negative, the junction is reverse biased. When V is negative and less than V_{TH} , the current is very small. But when V exceeds V_{TH} , the current suddenly becomes very high. The voltage V_{TH} is known as threshold or cut-in voltage. For Silicon diode $V_{TH} = 0.6\text{ V}$. At a reverse voltage corresponding to the point P, there is an abrupt increment in reverse current. The PQ portion of the characteristics is known as breakdown region.

4.4 P-N Junction Band Diagram

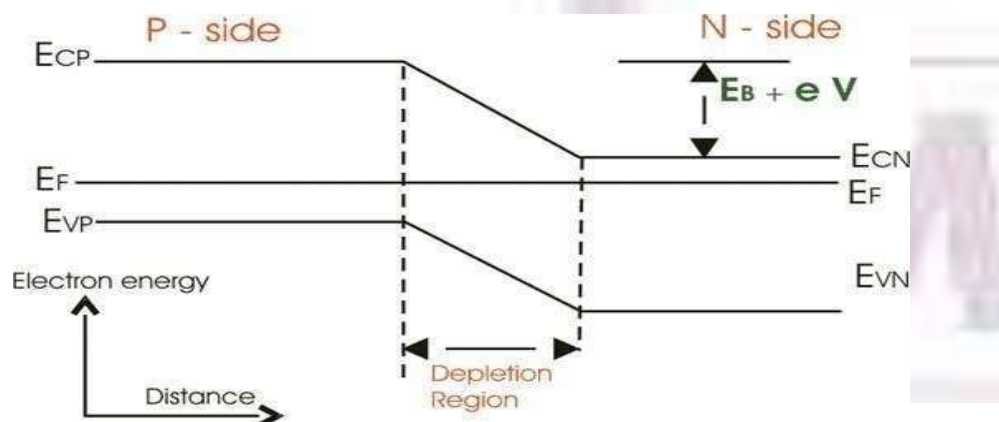
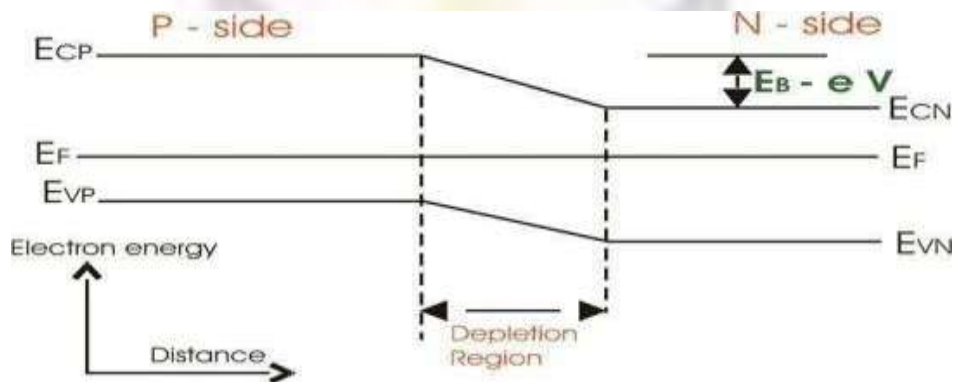
For an n-type semiconductor, the Fermi level E_F lies near the conduction band edge E_C but for a p-type semiconductor, E_F lies near the valence band edge E_V .



Now, when a p-n junction is built, the Fermi energy E_F attains a constant value. In this scenario the p-side conduction band edge. Similarly n-side valance band edge will be at higher level than E_{cn} , n- sides conduction band edge of p - side. This energy difference is known as barrier energy. The barrier energy is $E_B = E_{cp} - E_{cn} = E_{vp} - E_{vn}$



If we apply forward bias voltage V , across junction then the barrier energy decreases by an amount of eV and if V is reverse bias is applied the barrier energy increases by eV .



2.4. Diode Current Equation

The mathematical equation, which describes the forward and reverse characteristics of a semiconductor diode is called the **diode current equation**.

Let I = Forward (or reverse) diode current,

I_{RS} = Reverse saturation current,

V = External voltage (It is positive for forward bias and negative for reverse bias),

η = A constant

= 1 for germanium diodes, 2 for silicon diodes for relative low value of diode current (i.e., at or below the knee of the curve)

= 1 for germanium and silicon for higher levels of diode current (i.e., in the rapidly increasing section of the curve), and

V_T = Volt-equivalent of temperature. Its value is given by the relation, $\frac{T}{11600}$,

where T is the absolute temperature

= 26 mV at room temperature (300 K).

For a forward-biased diode, the current equation is given by the relation,

$$I = I_{RS} [e^{V/(\eta \times V_T)} - 1] \quad \dots(i)$$

Substituting the value of $V_T = 26 \text{ mV}$ or 0.026 V (at room temperature) in eqn. (i), we get

$$I = I_{RS} (e^{40V/\eta} - 1)$$

\therefore Diode current at or below the knee, for germanium,

$$I = I_{RS} (e^{40V} - 1) \quad (\because \eta = 1)$$

and, for silicon,

$$I = I_{RS} (e^{20V} - 1) \quad (\because \eta = 2)$$

When the value of applied voltage is greater than unity (i.e., for the diode current in the rapidly increasing section of curve), the equation of diode current for germanium or and silicon,

$$I = I_{RS} \cdot e^{20V} \quad (\because \eta = 2)$$

The current equation for a reverse biased diode may be obtained from eqn. (i) by changing the sign of the applied voltage (V). Thus the diode current for reverse bias,

$$I = I_{RS} [e^{-V/(\eta \times V_T)} - 1]$$

When $V \gg V_T$, then the term $e^{-V/(\eta \times V_T)} \ll 1$. Therefore $I = I_{RS}$. Thus the diode current under reverse bias is equal to the reverse saturation current as long as the external voltage is below its breakdown value.

4.5.1 APPLICATIONS OF DIODES

- Rectifying a voltage, such as turning AC into DC voltages
- Isolating signals from a supply
- Voltage Reference
- Controlling the size of a signal
- Mixing signals
- Detection signals
- Lighting
- Laser diodes

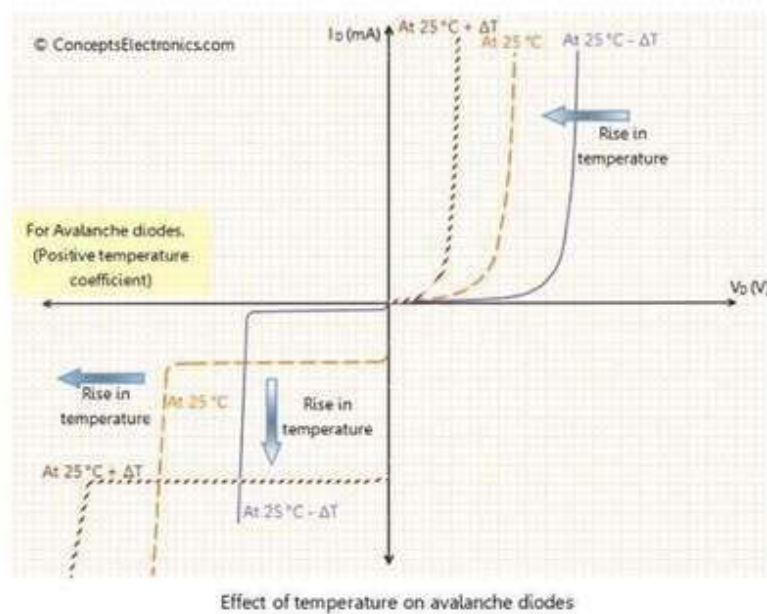


Effect of temperature on PN junction diode.

- PN junction diode parameters like cut in voltage, forward current in the forward bias and reverse current, reverse breakdown voltage and reverse saturation current in the reverse bias are dependent on temperature.
- The current that a PN junction diode can conduct at a given voltage is dependent upon the operating temperature. An increased temperature will result in a large number of broken covalent bonds increasing the large number of majority and minority carriers. Rise in temperature generates more electron-hole pair thus conductivity increases and thus increase in current. This amounts to a diode current larger than its previous diode current. The above phenomenon applies both to forward and reverse current.
- Mathematically diode current is given by

$$I = I_{RS} [e^{V/(\eta \times V_T)} - 1]$$

Hence from equation we conclude that the current should decrease with increase in temperature but exactly opposite occurs, there are two reasons:



Forward bias region :

- The effect of increased temperature and decrease in temperature on the forward characteristics curve of a PN junction diode is as shown in above figure. It may be noted that the forward characteristics of silicon diode shift to the left at rate of 2.5 mV per centigrade degree increase in temperature and shift towards right at rate of 25 mV per centigrade degree decrease in temperature.
- The cut-in voltage decreases as the temperature increases. The diode conducts at smaller voltage at large temperature.
- The cut-in voltage increases as the temperature decreases. The diode conducts at larger voltage at lower temperature.

Example:

At 25° C $V_D = 0.7V$

for 100° C Rise in temp. i.e., at 25+100=125° C

now we will find new V_D at 125° C then $100 \times 2.5mV = 0.25 V$

the new V_D will be reduce by 0.25 V

therefore new $V_D = 0.7 - 0.25 = 0.45 V$

At 25° C $V_D = 0.7V$

for 100° C decrease in temp. i.e., at 25-100= -75° C

now we will find new V_D at 75° C then $100 \times 2.5mV = 0.25 V$

the new V_D will be increase by 0.25 V

therefore new $V_D = 0.7 + 0.25 = 0.95 V$

Reverse bias region:

- The effect of increased temperature and decrease in temperature on the reverse characteristics curve of a PN junction diode is as shown in above figure. It may be noted that in the reverse bias region characteristics reverse current of silicon diode shift downwards with the increase in temperature and shift upward with decrease in temperature.
- In the reverse bias region the reverse current of diode doubles for every 10°C rise in temperature.

. Example:

- For 100°C Rise in temp. i.e., at $25+100=125^{\circ}\text{C}$

$25^{\circ}\text{C} \longrightarrow 10\text{nA}$

$35^{\circ}\text{C} \longrightarrow 20\text{nA}$

$45^{\circ}\text{C} \longrightarrow 40\text{nA}$

$55^{\circ}\text{C} \longrightarrow 80\text{nA}$

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$125^{\circ} \longrightarrow 10240\text{nA (OR)} 10.24\text{ }\mu\text{A}$

Therefore for 100°C Rise in temp. i.e., at $25+100=125^{\circ}\text{C}$ the reverse saturation current increases to greater than 10nA

- For 100°C decrease in temp. i.e., at $25-100 = -75^{\circ}\text{C}$ the reverse saturation current reduces to less than 10nA



Static and Dynamic Resistance of a Diode

Static forward resistances (R_F). A diode has a definite value of resistance when forward biased. It is given by the ratio of the D.C. voltage across the diode to D.C. current flowing through it.

$$\text{Mathematically, } R_F = \frac{V_F}{I_F}$$

R_F may be obtained graphically from the diode forward characteristics as shown in Fig. . From the operating point P, the static forward resistance,

$$R_F = \frac{0.8}{16} = 0.05 \, \Omega.$$

Dynamic or A.C. resistance. In practice we don't use static forward resistance, instead, we use the dynamic or A.C. resistance. The A.C. resistance of a diode, at a particular D.C. voltage, is equal to the reciprocal of the slope of the characteristic at that point ; i.e., the A.C. resistance,

$$r_{AC} = \frac{1}{\Delta I_F / \Delta V_F} = \frac{\Delta V_F}{\Delta I_F} = \frac{\text{Change in voltage}}{\text{Resulting change in current}}$$

Owing to the non-linear shape of the forward characteristic, the value of A.C. resistance of a diode is in the range of 1 to 25 Ω . Usually it is smaller than D.C. resistance of a diode.

Reverse resistance. When a diode is reverse biased, besides the forward resistance, it also possesses another resistance known as reverse resistance. It can be either D.C. or A.C. depending upon whether the reverse bias is direct or alternating voltage. Ideally, the reverse resistance of a diode is infinite. However, in actual practice, the reverse resistance is never infinite. It is due to the existence of leakage current in a reverse biased diode.

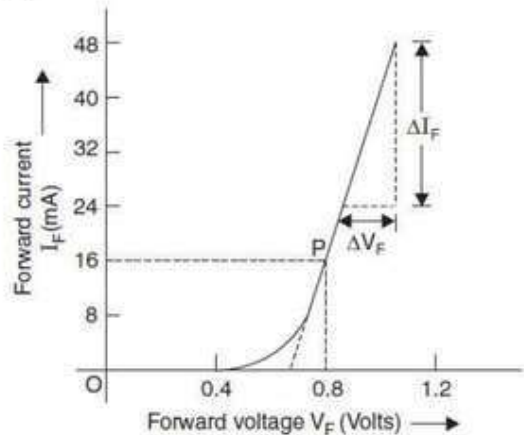
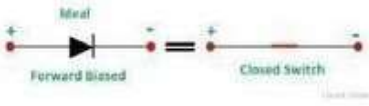
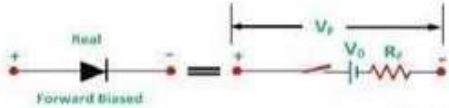
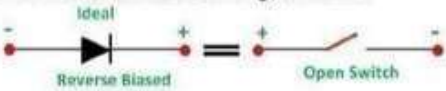
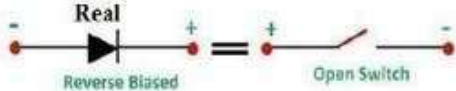
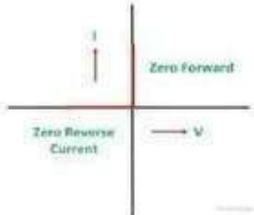
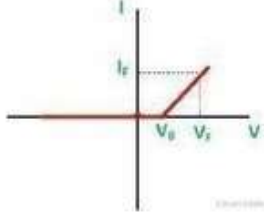


Fig. Static and dynamic forward resistances of a diode from the characteristic curve.



Ideal Diode and Practical Diode

Ideal Diode	Practical Diode
<p>1) A diode is said to be an Ideal Diode when it is forward biased and acts like a perfect conductor, with zero voltage across it. Similarly, when the diode is reverse biased, it acts as a perfect insulator with zero current through it.</p>	<p>1) A Practical diode contains barrier potential V_0 (0.7 V for silicon and 0.3 V for Germanium) and a forward resistance R_F of about 25 ohms. When a diode is forward biased and conducts a forward current I_F flows through it which causes a voltage drop $I_F R_F$ in the forward resistance. Hence, the forward voltage V_F applied across the Practical diode for conduction, has to overcome the following.</p> <ul style="list-style-type: none"> (i) Potential barrier (ii) Drop in forward resistance <p>i.e.,</p> $V_F = V_0 + I_F R_F$
<p>2) When the Ideal diode is forward biased it acts like a closed switch as shown in the figure below. An Ideal diode also acts like a switch</p>  <p>Diode resistance is zero i.e., $R_D = 0$ from ohms law $V_D = I_D R_D$ Therefore $V_D = 0$</p>	<p>2) The equivalent circuit of the Practical diode under forwarding bias condition is shown below. This circuit shows that a Practical diode still acts as a switch when forward biased, but the voltage required to operate this switch is V_F</p>  $V_F = V_0 + I_F R_F$
<p>3) Where as, if the diode is reverse biased, it acts like an open switch as shown in the figure below.</p>  <p>Diode resistance is infinity i.e., $R_D = \infty$ from ohms law $V_D = I_D R_D$ Therefore $I_D = 0$</p>	<p>3) For all the practical purposes, a diode is considered to be an open switch when reverse biased. It is because the value of reverse resistance is so high ($R_R > 100 \text{ M}\Omega$) that is considered to be an infinite for all practical purposes.</p> 
<p>4) The V-I characteristics of the Ideal diode are shown in the figure below</p> 	<p>4) The V-I characteristic of the Practical diode is shown below.</p> 

Equivalent Diode Circuits

An equivalent circuit is nothing but a combination of elements that best represents the actual terminal characteristics of the device. In simple language, it simply means the diode in the circuit can be replaced by other elements without severely affecting the behavior of circuit.

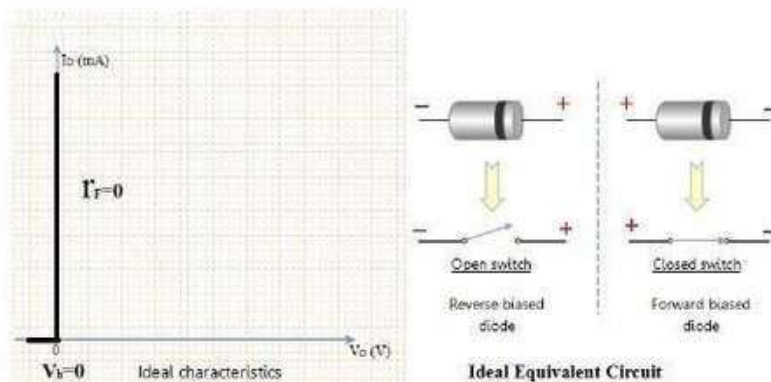
The diode can be modeled in three different ways depending on the accuracy required. Three models with increasing accuracy are listed below:

1. Ideal Diode Equivalent Circuit:
2. Constant voltage drop (or) Simplified Equivalent Circuit
3. Piece-Wise Linear Equivalent Circuit

1. Ideal Diode Equivalent Circuit:

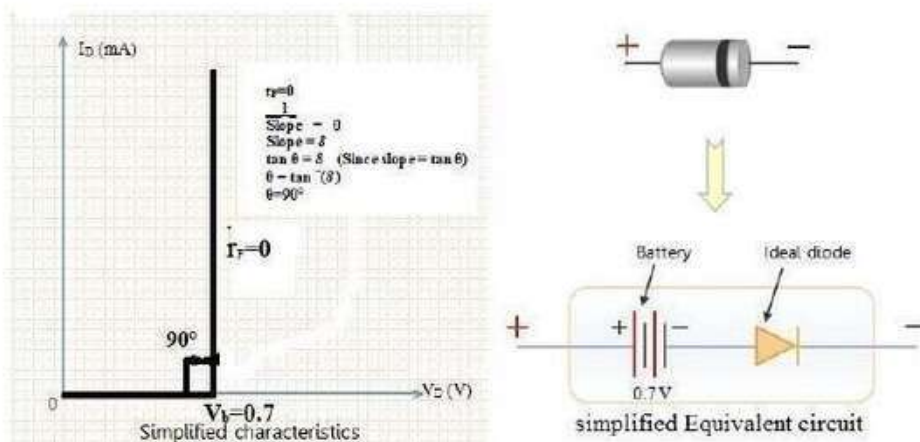
Figure indicates that the voltage drop across the diode is zero for any value of diode current. The ideal diode does not allow any current to flow in reverse biased condition. The current flowing through the diode is zero for any value of reverse biased voltage. Taking this into consideration, the ideal diode can be modeled as open or closed switch depending on the bias voltage.

- a) Ideal diode allows the flow of forward current for any value of forward bias voltage. Hence, Ideal diode can be modeled as closed switch under forward bias condition. This is shown in the figure.
- b) Ideal diode allows zero current to flow under reverse biased condition. Hence it can be modeled as open switch. This is indicated in the figure.



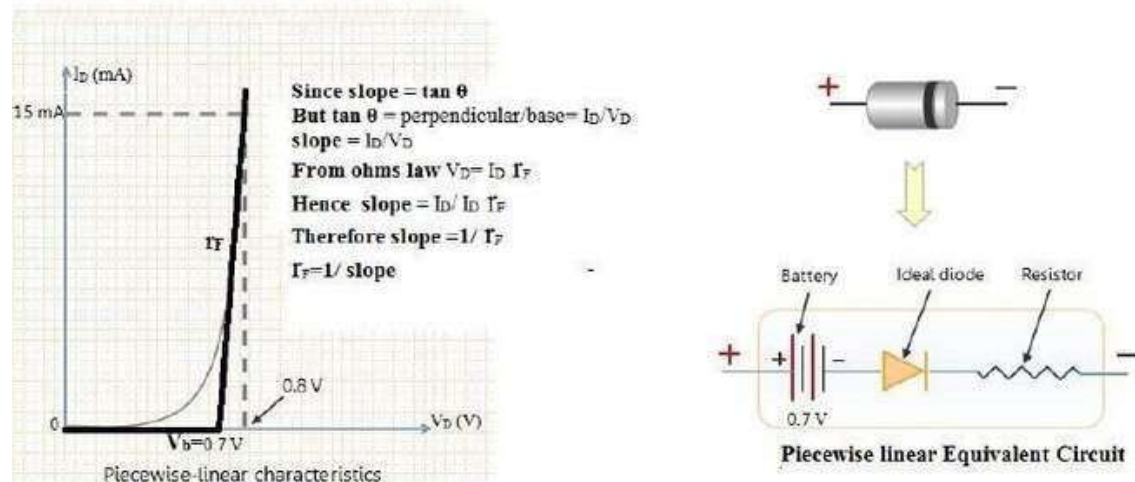
2. Constant Voltage Drop (or) Simplified Equivalent Circuit

The equivalent circuit in this case consists of a battery and an ideal diode. Consider the horizontal line from (0 to 0.7 V) in the curve. The horizontal line indicates that the current flowing through diode is zero for voltages between 0 and 0.7 V. To model this behavior, we put a battery of 0.7 V in the equivalent diode model. This does not mean that diodes are a source of voltage. When you measure the voltage across an isolated diode, the instrument will show zero value. The battery simply indicates that it opposes the flow of current in forward direction until 0.7 V. As the voltage becomes larger than 0.7 V, the current starts flowing in forward direction.



3. Piece-Wise Linear Equivalent Circuit

The piece-wise linear circuit, as the name suggests, is a model in which the characteristics of diode is approximated by "piece-wise linear" line segments. Now consider the straight line in the piece-wise linear characteristics. This straight line indicates constant slope. Slope in the V-I graph indicates resistance. So we add a resistor in the diode model. The value of resistance can be found from the graph. We can see from the graph that the diode current changes from 0 to 15 mA for a voltage change from 0.7 to 0.8 V. Thus the average value of resistance is $(0.8 \text{ V} - 0.7 \text{ V}) / (15 \text{ mA} - 0 \text{ mA}) = 6.67 \Omega$. Thus the value of resistance in the equivalent model is approximately 6.67Ω . The figure given below shows piece-wise linear characteristics of diode along with its model.



In the graph shown on left, the actual characteristics of diode is superimposed by piece-wise linear characteristics (shown in amber color). It is clear that the piece-wise linear characteristics do not exactly represent the characteristics of diode, especially near the knee of the curve. However it provides a good first approximation to the actual characteristics of the diode. Piece-wise linear characteristics can be obtained by replacing the diode in the circuit with a resistor, a battery and an ideal diode. This is shown in the right side of the above figure.

ZENER DIODE

A properly doped P-N junction crystal diode which has a sharp breakdown voltage is known as **Zener diode**.

The voltage-regulator diode is commonly called a '**Zener**' diode. It is a voltage limiting diode that has some applications in common with the older voltage-regulator gas tubes but serves a much wider field of application, because the devices cover a wide spectrum of voltages and power levels.

Performance/Operation

The electrical performance of a zener diode is based on the *avalanche characteristics* of the P-N junction. When a source of voltage is applied to a diode in the *reverse direction* (negative to anode), a reverse current I_R is observed (see Fig.). As the reverse potential is increased beyond the "Zener knee" avalanche breakdown becomes well developed at zener voltage V_Z . At voltage V_Z , the high counter resistance drops to a low value and the junction current increases rapidly. The current must of necessity be limited by an external resistance, since the voltage V_Z developed across the zener diode remains essentially constant. *Avalanche breakdown of the operating zener diode is not destructive as long as the rated power dissipation of the junction is not exceeded.*

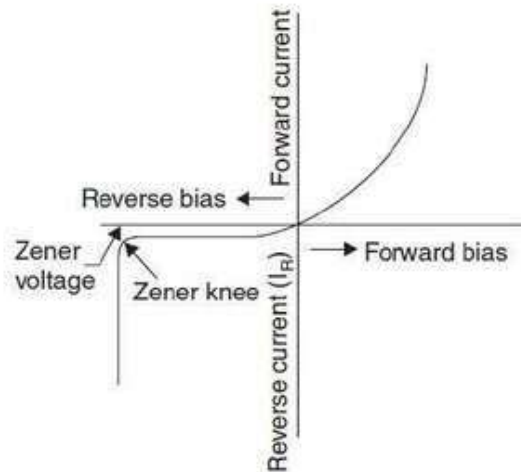


Fig.

Externally, the zener diode looks much like other silicon rectifying devices, and electrically it is capable of rectifying alternating current.

The following points about the *Zener diode* are worth noting :

- (i) It looks like an ordinary diode except that it is properly doped so as to have a sharp breakdown voltage.
 - (ii) It is always reverse connected i.e., it is *always reverse biased*.
 - (iii) It has sharp breakdown voltage, called Zener voltage V_Z .
 - (iv) When forward biased, its characteristics are just those of ordinary diode.
 - (v) It is not immediately burnt just because it has entered the breakdown region (The current is limited only by both external resistance and power dissipation of Zener diode).
- The location of Zener region can be controlled by varying the doping levels. An *increase in doping, producing an increase in the number of added impurities, will decrease the Zener potential.*
 - Zener diodes are available having Zener potentials of 1.8 to 200 V with power ratings from $\frac{1}{4}$ to 50 W. *Because of its higher temperature and current capability, silicon is usually preferred in the manufacture of Zener diodes.*



Equivalent Circuit of Zener Diode

The complete equivalent circuit of the Zener diode in the Zener region includes a small dynamic resistance and D.C. battery equal to the Zener potential, as shown in Fig. .

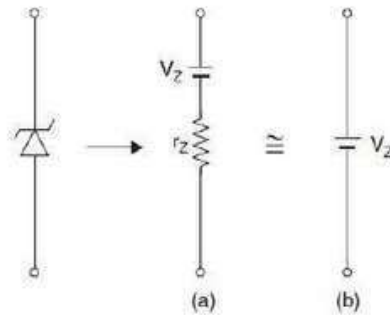


Fig. Zener equivalent circuit : (a) Complete ; (b) Approximately.

“ON” state. When reverse voltage across a Zener diode is equal to or more than breakdown voltage V_Z , the current increases very sharply. In this region curve is almost vertical ; it means that voltage across Zener diode is constant at V_Z even though the current through it changes. Therefore, in the breakdown region, an ideal Zener diode (this assumption is fairly reasonable as the impedance of Zener diode is *quite small in the breakdown region*) can be represented by a battery of voltage V_Z as shown in Fig. (b). Under such conditions, the Zener diode is said to be in the “ON” state.

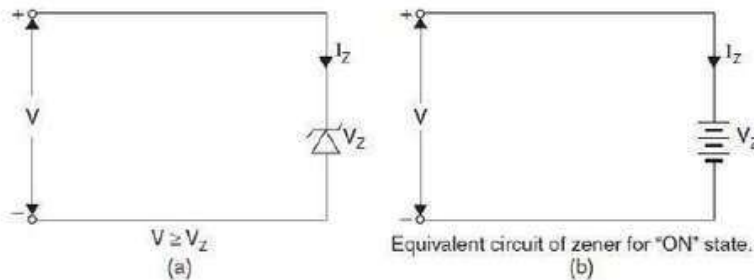
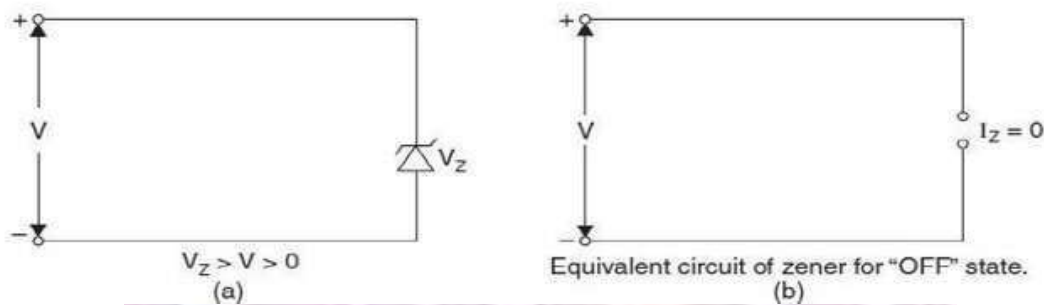


Fig.

“OFF” state. When the reverse voltage across the Zener diode is less than V_Z but greater than 0 V, the Zener diode is in the “OFF” stage. Under such conditions, the Zener diode can be represented by an open circuit as shown in Fig. (b).



3.3. Applications of Zener Diode

Zener diode serves in the following variety of applications :

1. Voltage reference or regulator element :

The primary use of a zener diode is as a *voltage reference or regulator element*. Fig. 22 shows the fundamental circuit for the Zener diode employed as a shunt regulator. In the circuit, diode element and load R_L draw current through the series resistance R_S . If E_{in} increases, the current through the Zener element will increase and thus maintain an essentially fixed voltage across R_L . This ability to maintain the desired voltage is determined by the temperature coefficient and the diode impedance of the zener device.

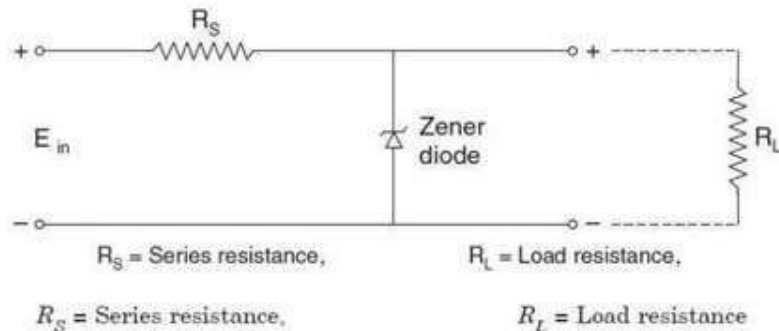


Fig. 22. Basic Zener-diode regulator circuit.

2. Shunt transistor regulator :

The Zener diode may also be used to control the reference voltage of a transistor regulated power supply. An example of this in a shunt transistor regulator is shown in Fig. 23, where Zener element is used to control the operating point of the transistor. The advantage of this circuit over that shown in Fig. 22 are *increased power handling capability and a regulating factor improved by utilizing the current gain of the transistor*.

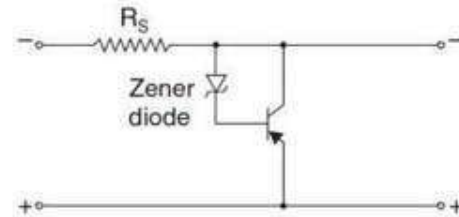


Fig. 23. Shunt transistor regulator.

3. Audio or r-f applications :

The Zener diode also finds use in audio or *r-f* (radio frequency) applications where a source of stable reference voltage is required, as in bias supplies. Frequently, *Zener diodes are connected in series package*, with, for example, one junction operating in the reverse within a single direction and possessing a positive temperature V_Z coefficient ; the remaining diodes are connected to operate in the forward direction and exhibit negative temperature V_Z coefficient characteristics. The net result



3.22. Zener Diode Regulator

The major application of zener diode in the electronic circuit is as a voltage regulator. It provides a constant voltage to the load from a source whose voltage may vary over sufficient range. The zener diode of zener voltage V_z is reverse connected across the load R_L across which constant voltage is desired. A resistor R is connected in series with the circuit which absorbs the output voltage fluctuation so as to maintain constant voltage (V_0) across the load.

Let a variable voltage V_{in} be applied across the load R_L . When the value of V_{in} is less than zener voltage V_z of the zener diode. No current flows through it and the same voltage appears across the load. When the input voltage V_{in} is more than V_z this will cause the zener diode to conduct a large current I_z .

In the above discussion it has been seen that when a zener diode of zener voltage V_z is connected in reverse direction parallel to the load. It maintains a constant voltage across the load equal to V_z and hence stabilises the output voltage.

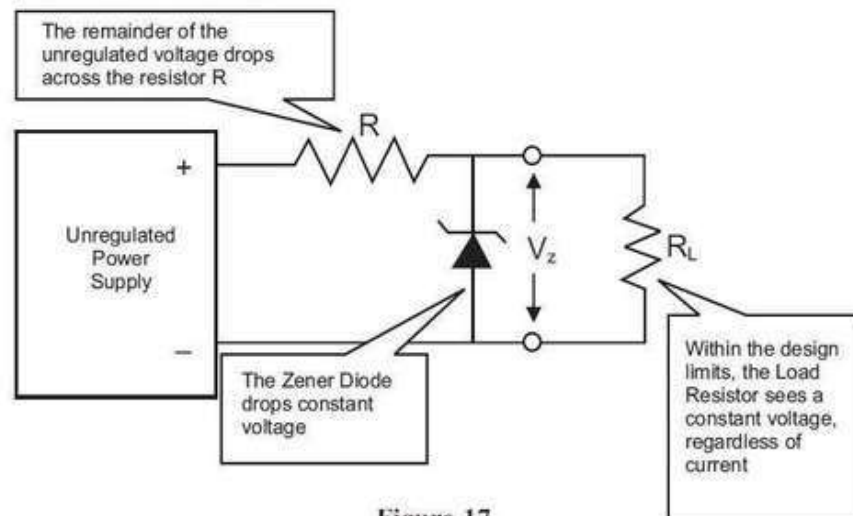


Figure-17

Example 4. Determine the current flowing through the Zener diode for the circuit shown in Fig. 24, if $R_L = 4000 \Omega$, input voltage is 50 volts, $R_S = 1800 \Omega$ and output voltage is 32 volts.

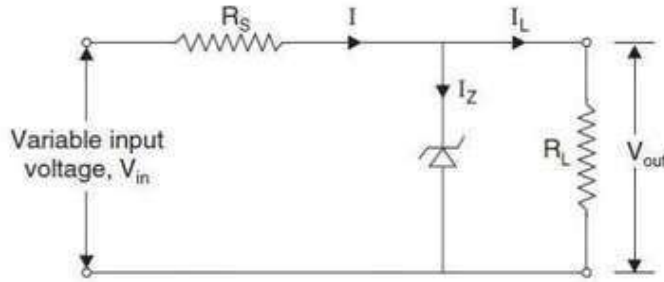


Fig. 24

Solution. Input voltage, $V_{in} = 50 \text{ V}$
 Output voltage, $V_{out} = 32 \text{ V}$
 Voltage drop in series resistor, $R_S = V_{in} - V_{out} = 50 - 32 = 18 \text{ V}$

Current through series resistance, $I = \frac{V_{in} - V_{out}}{R} = \frac{18}{1800} = .01 \text{ A or } 10 \text{ mA}$

Load current, $I_L = \frac{V_{out}}{R_L} = \frac{32}{4000} = 0.008 \text{ A or } 8 \text{ mA}$

Current through Zener diode, $I_Z = I - I_L = 10 - 8 = 2 \text{ mA. (Ans.)}$

Example 5. Determine the maximum and minimum values of Zener current if in the circuit shown in Fig. 24 the load resistance, $R_L = 4000 \Omega$, series resistance = 8000Ω , output voltage = 32 V and source voltage varies between 100 V and 128 V.

Solution. Refer to Fig. 23. Given : $R_L = 4000 \Omega$; $R = 8000 \Omega$; $V_{out} = 32 \text{ V}$;

Load current, $I_L = \frac{V_{out}}{R_L} = \frac{32}{4000} = 0.008 \text{ A or } 8 \text{ mA}$

The Zener current will be maximum when input voltage is maximum i.e., 128 V

Corresponding current through series resistance,

$$I = \frac{V_{in(max)} - V_{out}}{R_S} = \frac{128 - 32}{8000} = 0.012 \text{ A or } 12 \text{ mA}$$

Corresponding Zener current, $(I_Z)_{max} = I - I_L = 12 - 8 = 4 \text{ mA. (Ans.)}$

The Zener current will be minimum when input voltage is minimum i.e., 100 V.

Corresponding, current through series resistance,

$$I' = \frac{V_{in(min)} - V_{out}}{R_S} = \frac{100 - 32}{8000} = 0.0085 = 8.5 \text{ mA}$$

Corresponding Zener current, $(I_Z)_{min} = I' - I_L = 8.5 - 8 = 0.5 \text{ mA. (Ans.)}$

Example 6. In the simple Zener-diode based voltage regulator shown in Fig. 25, a 5.6 V, 0.25 W Zener diode is used. For reliable operation, the minimum I_Z should be 1 mA. The load R_L varies between 20Ω and 50Ω . Find the range of R_S for reliable and safe operation of the voltage regulator.

Solution. (i) Let $R_S = 20 \Omega$

$$I = \frac{5.6}{20} = 0.28 \text{ A}$$

$$R_S = \frac{10 - 5.6}{0.28 + 0.001} = 15.66 \Omega \approx 16 \Omega.$$

(ii) Let, $R_S = 50 \Omega$

$$I = \frac{5.6}{50} = 0.112 \text{ A}$$

$$R_S = \frac{10 - 5.6}{0.112 + 0.001} = 38.93 \Omega \approx 39 \Omega.$$

$\therefore R$ ranges from 16Ω to 39Ω . (Ans.)

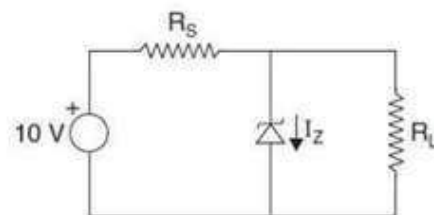


Fig. 25

Diode Junction capacitance:

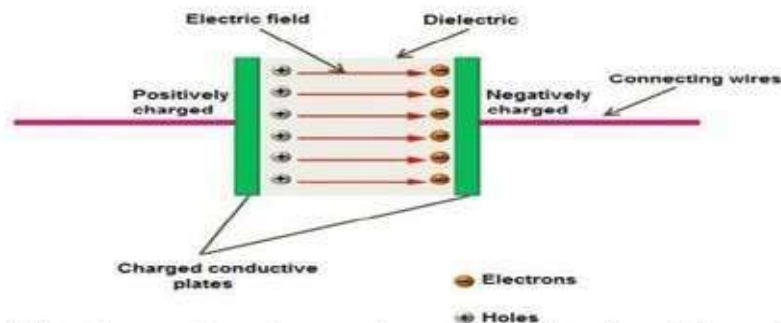
In a p-n junction diode, two types of capacitance take place. They are,

- Transition capacitance (C_T)
- Diffusion capacitance (C_D)

(a) Transition capacitance (C_T):

We know that capacitors store electric charge in the form of electric field. This charge storage is done by using two electrically conducting plates (placed close to each other) separated by an insulating material called dielectric.

The conducting plates or electrodes of the capacitor are good conductors of electricity. Therefore, they easily allow electric current through them. On the other hand, dielectric material or medium is poor conductor of electricity. Therefore, it does not allow electric current through it. However, it efficiently allows electric field.



When voltage is applied to the capacitor, charge carriers start flowing through the conducting wire. When these charge carriers reach the electrodes of the capacitor, they experience a strong opposition from the

dielectric or insulating material. As a result, a large number of charge carriers are trapped at the electrodes of the capacitor. These charge carriers cannot move between the plates. However, they exert an electric field between the plates. The charge carriers which are trapped near the dielectric material will store electric charge. The ability of the material to store electric charge is called capacitance.

In a basic capacitor, the capacitance is directly proportional to the size of electrodes or plates and inversely proportional to the distance between two plates.

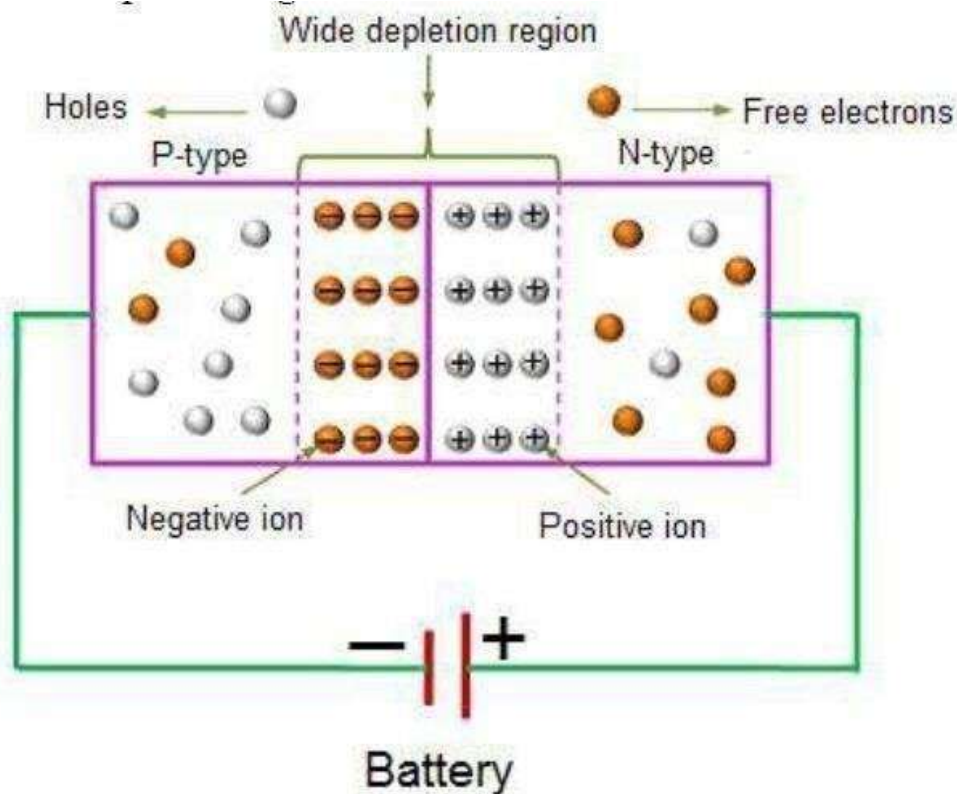
Just like the capacitors, a reverse biased p-n junction diode also stores electric charge at the depletion region. The depletion region is made of immobile positive and negative ions.

In a reverse biased p-n junction diode, the p-type and n-type regions have low resistance. Hence, p-type and n-type regions act like the electrodes or conducting plates of the capacitor. The depletion region of the p-n junction diode has high resistance. Hence, the depletion region acts like the dielectric or insulating material. Thus, p-n junction diode can be considered as a parallel plate capacitor.

In the depletion region, the electric charges (positive and negative ions) do not move from one place to another place. However, they exert an electric field or electric force. Therefore, charge is stored at the depletion region in the form of an electric field. The ability of a material to store electric charge is called capacitance. Thus, there exists a capacitance at the depletion region.

The capacitance at the depletion region changes with the change in applied voltage. When reverse bias voltage applied to the p-n junction diode is increased, a large number of holes (majority carriers) from the p-side and electrons (majority carriers) from the n-side are moved away from the p-n junction. As a result, the width of the depletion region increases whereas the size of p-type and n-type regions (plates) decreases.

We know that capacitance means the ability to store electric charge. The p-n junction diode with narrow depletion width and large p-type and n-type regions will store a large amount of electric charge whereas the p-n junction diode with wide depletion width and small p-type and n-type regions will store only a small amount of electric charge. Therefore, the capacitance of the reverse bias p-n junction diode decreases when voltage increases.



In a forward biased diode, the transition capacitance exist. However, the transition capacitance is very small compared to the diffusion capacitance. Hence, transition capacitance is neglected in forward biased diode. The amount of capacitance changed with increase in voltage is called transition capacitance. The transition capacitance is also known as depletion region capacitance, junction capacitance or barrier capacitance. Transition capacitance is denoted as C_T .

The change of capacitance at the depletion region can be defined as the change in electric charge per change in voltage.

$$C_T = dQ / dV$$

Where,

C_T = Transition capacitance

dQ = Change in electric charge

dV = Change in voltage

The transition capacitance can be mathematically written as,

$$C_T = \epsilon A / W$$

Where,

ϵ = Permittivity of the semiconductor

A = Area of plates or p-type and n-type regions

W = Width of depletion region

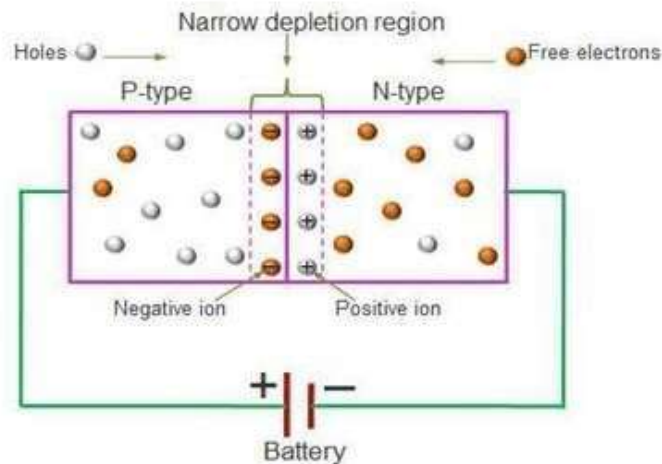
Diffusion capacitance (C_D):

Diffusion capacitance occurs in a forward biased p-n junction diode. Diffusion capacitance is also sometimes referred as storage capacitance. It is denoted as C_D .

In a forward biased diode, diffusion capacitance is much larger than the transition capacitance. Hence, diffusion capacitance is considered in forward biased diode.

The diffusion capacitance occurs due to stored charge of minority electrons and minority holes near the depletion region.

When forward bias voltage is applied to the p-n junction diode, electrons (majority carriers) in the n-region will move into the p-region and recombines with the holes. In the similar way, holes in the p-region will move into the n-region and recombines with electrons. As a result, the width of depletion region decreases.



The electrons (majority carriers) which cross the depletion region and enter into the p-region will become minority carriers of the p-region similarly; the holes (majority carriers) which cross the depletion region and enter into the n-region will become minority carriers of the n-region.

The accumulation of holes in the n-region and electrons in the p-region is separated by a very thin depletion region or depletion layer. This depletion region acts like dielectric or insulator of the capacitor and charge stored at both sides of the depletion layer acts like conducting plates of the capacitor.

Diffusion capacitance is directly proportional to the electric current or applied voltage. If large electric current flows through the diode, a large amount of charge is accumulated near the depletion layer. As a result, large diffusion capacitance occurs.

In the similar way, if small electric current flows through the diode, only a small amount of charge is accumulated near the depletion layer. As a result, small diffusion capacitance occurs.

When the width of depletion region decreases, the diffusion capacitance increases. The diffusion capacitance value will be in the range of nano farads (nF) to micro farads (μF).

The formula for diffusion capacitance is

$$C_D = dQ / dV$$

Where,

C_D = Diffusion capacitance

dQ = Change in number of minority carriers stored outside the depletion region

dV = Change in voltage applied across diode



4.5 HALF WAVE RECTIFIERS

Rectifiers are the circuits used to convert alternating current (AC) into direct current (DC). Half-Wave Rectifiers are designed using a diode (D) and a load resistor (R_L) as shown in Figure 1. In these rectifiers, only one-half of the input waveform is obtained at the output i.e. the output will comprise of either positive pulses or the negative pulses only. The polarity of the output voltage so obtained (across R_L) depends on the direction of the diode used in the circuit of half-wave rectifier. This is evident from the figure as Figure 1a shows the output waveform consisting of only positive pulses while the Figure 1b has only negative pulses in its output waveform.

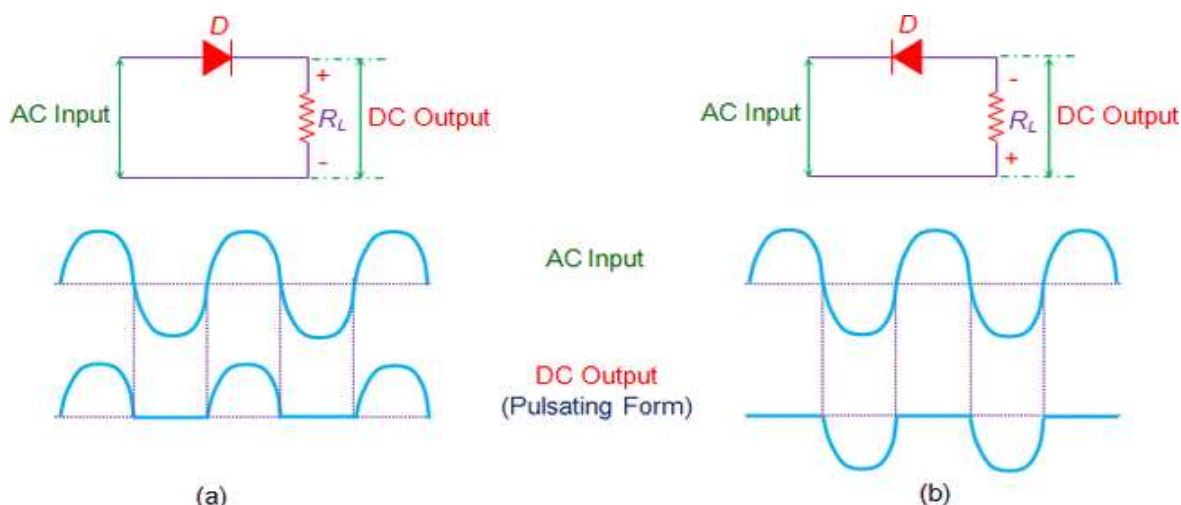


Figure 1 Half Wave Rectifier with Input and Output Waveforms

This is because, in Figure 1a the diode gets forward biased only during the positive pulse of the input which causes the current to flow across R_L , producing the output voltage.

Further for the same case, if the input pulse becomes negative, then the diode will be reverse biased and hence there will be no current flow and no output voltage. Similarly for the circuit shown in Figure 1b, the diode will be forward biased only when the input pulse is negative, and thus the output voltage will contain only the negative pulses. Further it is to be noted that the input to the half-wave rectifier can be supplied even via the transformer. This is advantageous as the transformer provides isolation from the power line as well as helps in obtaining the desired level of DC voltage. Next, one can connect a capacitor across the resistor in the circuit of half wave rectifier to obtain a smoother DC output (Figure 2). Here the capacitor charges through the diode D during the positive pulse of the input while it discharges through the load resistor R_L when the input pulse will be negative. Thus the output waveform of such a rectifier will have ripples in it as shown in the figure.

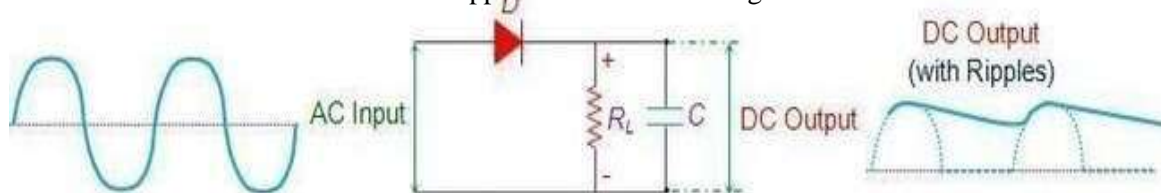


Figure 2 Half-Wave Rectifier with a RC Filter

Different parameters associated with the half wave rectifiers are

1. **Peak Inverse Voltage (PIV):** This is the maximum voltage which should be withstood by the diode under reverse biased condition and is equal to the peak of the input voltage, V_m .
2. **Average Voltage:** This is the DC content of the voltage across the load and is given by V_m/π . Similarly DC current is given as I_m/π , where I_m is the maximum value of the current.

3. **Ripple Factor (r):** It is the ratio of root mean square (rms) value of AC component to the DC component in the output and is given by

$$r = \sqrt{\left(\frac{V_{rms}}{V_{DC}}\right)^2 - 1}$$

Further, for half-wave rectifier, rms voltage is given as $V_m/2$ which results in the ripple factor of 1.21.

4. **Efficiency:** It is the ratio of DC output power to the AC input power and is equal to 40.6%.
5. **Transformer Utilization Factor:** It is the ratio of DC power delivered to the load to the AC rating of the transformer secondary and is equal to 0.287.
6. **Form Factor:** This is the ratio of rms value to the average value and is thus equal to 1.57 for half-wave rectifier.
7. **Peak Factor:** It is the ratio of peak value to the rms value and is equal to 2.

Half wave rectifiers are advantageous as they are cheap, simple and easy to construct. These are quite rarely used as they have high ripple content in their output. However they can be used in non-critical applications like those of charging the battery. They are also less preferred when compared to other rectifiers as they have low output power, low rectification efficiency and low transformer utilization factor. In addition, if AC input is fed via the transformer, then it might get saturated which in turn results in magnetizing current, hysteresis loss and/or result in the generation of harmonics. Lastly it is important to note that the explanation provided here applies only for the case where the diode is ideal. Although for a practical diode, the basic working remains the same, one will have to consider the voltage drop across the diode as well as its reverse saturation current into consideration during the analysis.

4.6 CENTRE TAP FULL WAVE RECTIFIERS

The circuits which convert the input alternating current (AC) into direct current (DC) are referred to as rectifiers. If such rectifiers rectify both the positive as well as negative pulses of the input waveform, then they are called Full-Wave Rectifiers. Figure 1 shows such a rectifier designed using a multiple winding transformer whose secondary winding is equally divided into two parts with a provision for the connection at its central point (and thus referred to as the centre-tapped transformer), two diodes (D_1 and D_2) and a load resistor (R_L). Here the AC input is fed to the primary winding of the transformer while an arrangement of diodes and the load resistor which yields the DC output, is made across its secondary terminals.

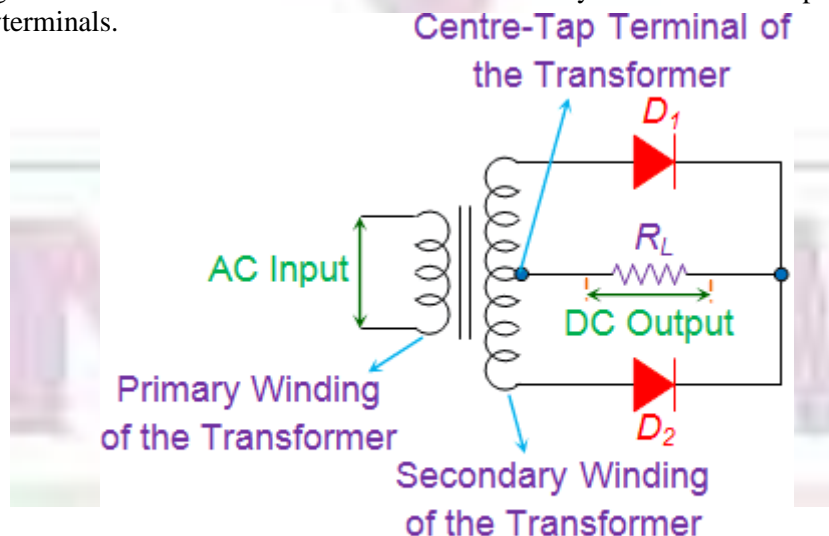


Figure 1 Full Wave Rectifier

The circuit can be analyzed by considering its working during the positive and the negative input pulses separately.

Figure 2a shows the case where the AC pulse is positive in nature i.e. the polarity at the top of the primary winding is positive while its bottom will be negative in polarity. This causes the top part of

the secondary winding to acquire a positive charge while the common centre-tap terminal of the transformer will become negative

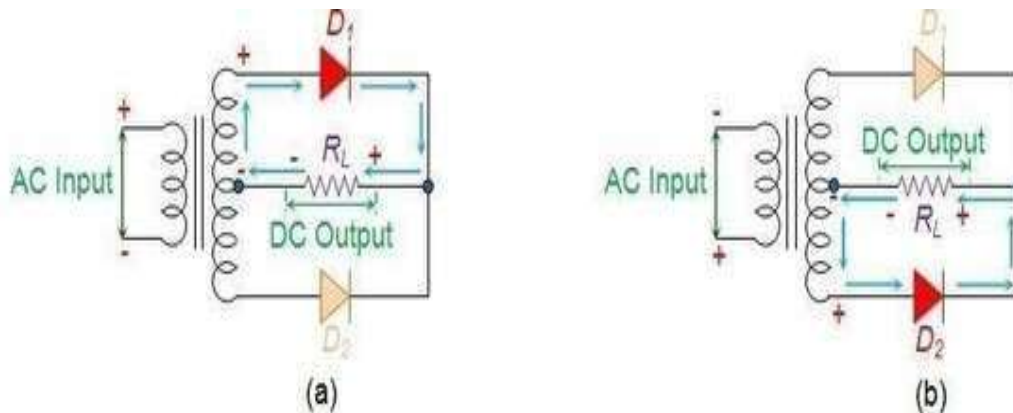


Figure 2 Conduction Path of Full Wave Rectifier for (a) Positive Input Pulse (b) Negative Input Pulse.

This causes the diode D_1 to be forward biased which in turn causes the flow of current through R_L along the direction shown in Figure 2a. However at the same time, diode D_2 will be reverse biased and hence acts like an open circuit. This causes the appearance of positive pulse across the R_L , which will be the DC output. Next, if the input pulse becomes negative in nature, then the top and the bottom of the primary winding will acquire the negative and the positive polarities respectively. This causes the bottom of the secondary winding to become positive while its centre-tapped terminal will become negative. Thus the diode D_2 gets forward biased while the D_1 will get reverse biased which allows the flow of current as shown in the Figure 2b. Here the most important thing to note is the fact that the direction in which the current flows via R_L will be identical in either case (both for positive as well as for negative input pulses). Thus we get the positive output pulse even for the case of negative input pulse (Figure 3), which indicates that both the half cycles of the input AC are rectified.

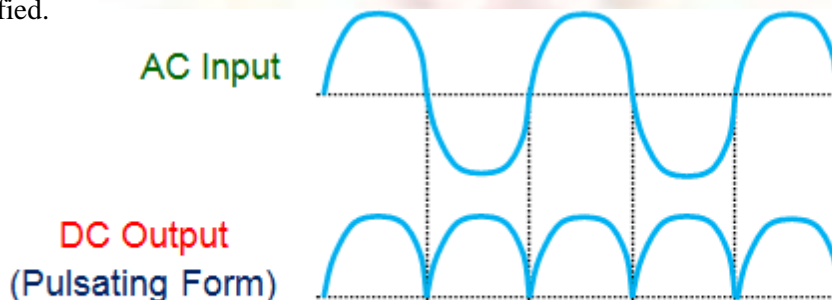


Figure 3 Input and Output Waveforms of Full Wave Rectifier

Such circuits are referred to as (i) **Centre-Tapped Full Wave Rectifiers** as they use a centre-tapped transformer, (ii) **Two-Diode Full-Wave Rectifiers** because of the use of two diodes and/or (iii) **Bi-Phase Circuits** due to the fact that in these circuits, the output voltage will be the phasor addition of the voltages developed across the load resistor due to two individual diodes, where each of them conducts only for a particular half-cycle. However as evident from Figure 3, the output of the rectifier is not pure DC but pulsating in nature, where the frequency of the output waveform is seen to be double of that at the input. In order to smoothen this, one can connect a capacitor across the load resistor as shown by the Figure 4. This causes the capacitor to charge via the diode D_1 as long as the input positive pulse increases in its magnitude. By the time the input pulse reaches the positive maxima, the capacitor would have charged to the same magnitude. Next, as long as the input positive pulse keeps

decreasing, the capacitor tries to hold the charge acquired (being an energy-storage element).

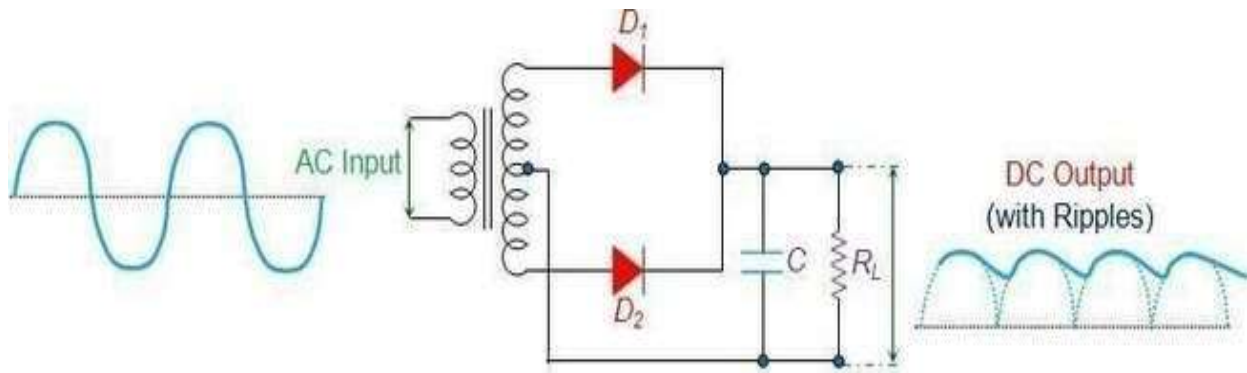


Figure 4 Full-Wave Rectifier with a RC Filter

However there will be voltage-loss as some amount of charge gets lost through the path provided by the load resistor (nothing but discharging phenomenon). Further, as the input pulse starts to go low to reach the negative maxima, the capacitor again starts to charge via the path provided by the diode D_2 and acquires an almost equal voltage but with opposite polarity. Next, as the input voltage starts to move towards 0V, the capacitor slightly discharges via R_L . This charge-discharge cycle of the capacitor causes the ripples to appear in the output waveform of the full-wave rectifier with RC filter as shown in Figure 4.

Different parameters and their values for the centre-tapped full-wave rectifiers are

1. **Peak Inverse Voltage (PIV):** This is the maximum voltage which occurs across the diodes when they are reverse biased. Here it will be equal to twice the peak of the input voltage, $2V_m$.
2. **Average Voltage:** It is the DC voltage available across the load and is equal to $2V_m/\pi$. The corresponding DC current will be $2I_m/\pi$, where I_m is the maximum value of the current.
3. **Ripple Factor (r):** This is the ratio of the root mean square (rms) value of AC component to the DC component at the output. It is given by and will be equal to 0.482 as the rms voltage for a full-wave rectifier is given as

$$r = \sqrt{\left(\frac{V_{rms}}{V_{DC}}\right)^2 - 1}$$

$$\frac{V_m}{\sqrt{2}}$$

4. **Efficiency:** This is the ratio of DC output power to the AC input power and is equal to 81.2%.
5. **Transformer Utilization Factor (TUF):** This factor is expressed as the ratio of DC power delivered to the load to the AC rating of the transformer secondary. For the full-wave rectifier this will be 0.693.
6. **Form Factor:** This is the ratio of rms value to the average value and is equal to 1.11.
7. **Peak Factor:** It is the ratio of peak value to the rms value and is equal to $\sqrt{2}$ for the full-wave rectifiers.

Further it is to be noted that the two-diode full-wave rectifier shown in Figure 1 is costly and bulky in size as it uses the complex centre-tapped transformer in its design. Thus one may resort to another type of full-wave rectifier called Full-Wave Bridge Rectifier (identical to Bridge Rectifier) which might or might not involve the transformer (even if used, will not be as complicated as a centre-tap one). It also offers higher TUF and higher PIV which makes it ideal for high power applications. However it is to be noted that the full wave bridge rectifier uses four diodes instead of two, which in turn increases the magnitude of voltage drop across the diodes, increasing the heating loss. **Full wave rectifiers** are used in general power supplies, to charge a battery and to provide power to the devices like motors, LEDs, etc. However due to the ripple content in the output waveform, they are not

preferred for audio applications. Further these are advantageous when compared to half-wave rectifiers as they have higher DC output power, higher transformer utilization factor and lower ripple content, which can be made more smoother by using π -filters. All these merits mask-up its demerit of being costly in comparison to the half-wave rectifiers due to the use of increased circuit elements. At last, it is to be noted that the explanation provided here considers the diodes to be ideal in nature. So, in case of practical diodes, one will have to consider the voltage drop across the diode, its reverse saturation current and other diode characteristics into account and reanalyze the circuit. Nevertheless the basic working remains the same.

4.10 FULL WAVE BRIDGE RECTIFIERS

Bridge Rectifiers are the circuits which convert alternating current (AC) into direct current (DC) using the diodes arranged in the bridge circuit configuration. They usually comprise of four or more number of diodes which cause the output generated to be of the same polarity irrespective of the polarity at the input. Figure 1 shows such a bridge rectifier composed of four diodes D_1 , D_2 , D_3 and D_4 in which the input is supplied across two terminals A and B in the figure while the output is collected across the load resistor R_L connected between the terminals C and D.

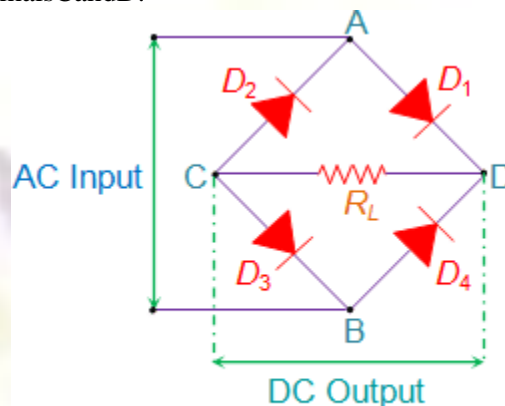


Figure 1 Bridge Rectifier

Now consider the case where in the positive pulse appears at the AC input, i.e. the terminal A is positive while the terminal B is negative. This causes the diodes D_1 and D_3 to get forward biased and at the same time, the diodes D_2 and D_4 will be reverse biased.

As a result, the current flows along the short-circuited path created by the diodes D_1 and D_3 (considering the diodes to be ideal), as shown by Figure 2a. Thus the voltage developed across the load resistor R_L will be positive towards the end connected to terminal D and negative at the end connected to the terminal C.

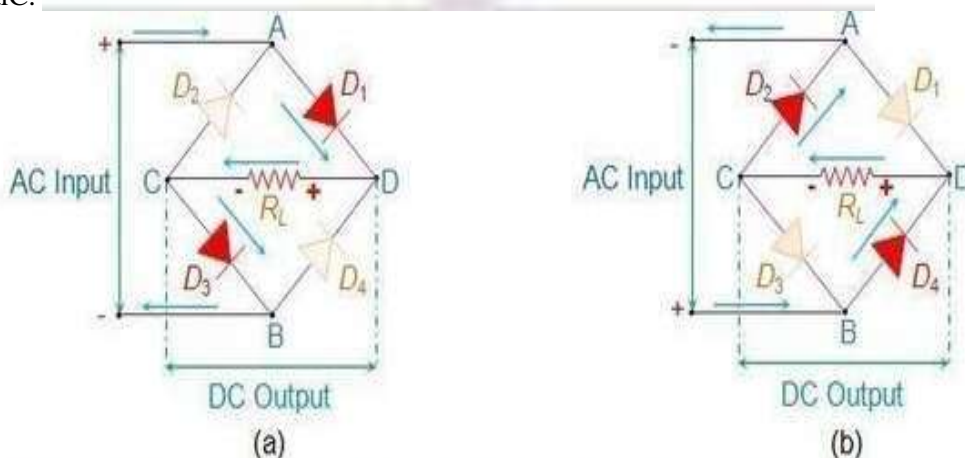


Figure 2 Current Path Through the Bridge Rectifier for (a) Positive half-cycle (b) Negative Half-Cycle

Next if the negative pulse appears at the AC input, then the terminals A and B are negative and positive respectively. This forward biases the diodes D_2 and D_4 , while reverse biasing D_1 and D_3 which causes the current to flow in the direction shown by Figure 2b. At this instant, one has to note that the polarity of the voltage developed across R_L is identical to that produced when the incoming AC pulse was positive in nature. This means that for both positive and negative pulse, the output of the bridge rectifier will be

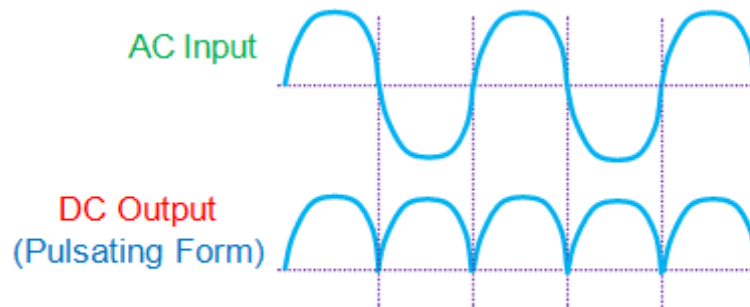


Figure 3 Input-Output Waveforms of a Bridge Rectifier

identical in polarity as shown by the waveforms in Figure 3.

However it is to be noted that the bridge rectifier's DC will be pulsating in nature. In order to obtain pure form of DC, one has to use a capacitor in conjunction with the bridge circuit (Figure 4).

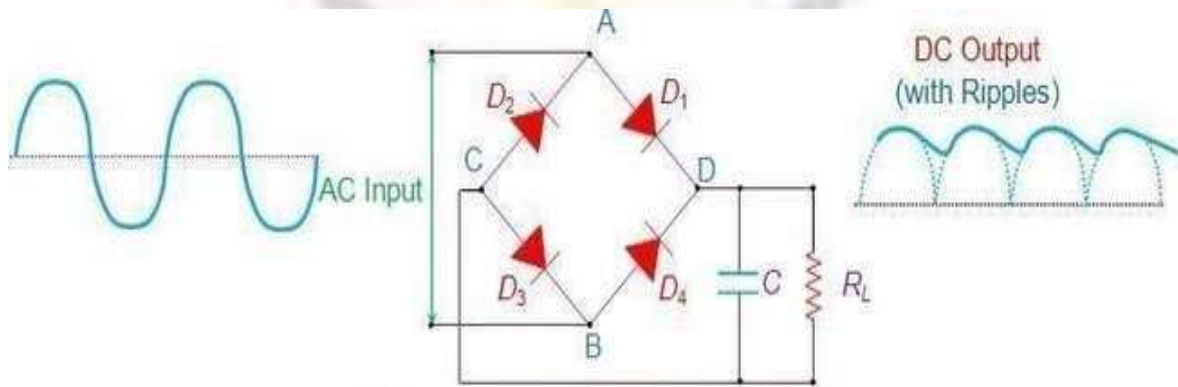


Figure 4 Bridge Rectifier with a RC Filter

In this design, the positive pulse at the input causes the capacitor to charge through the diodes D_1 and D_3 . However as the negative pulse arrives at the input, the charging action of the capacitor ceases and it starts to discharge via R_L . This results in the generation of DC output which will have ripples in it as shown in the figure. This ripple factor is defined as the ratio of AC component to the DC component in the output voltage. In addition, the mathematical expression for the ripple voltage is given by the equation

$$V_r = \frac{I_L}{fC}$$

Where, V_r represents the ripple voltage.

I_L represents the load current.

f represents the frequency of the ripple

which will be twice the input frequency. C is the Capacitance.

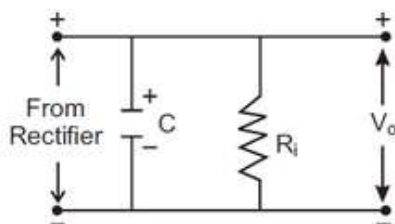
Further, the **bridge rectifiers** can be majorly of two types, viz., Single-Phase Rectifiers and Three-Phase Rectifiers. In addition, each of these can be either Uncontrolled or Half-Controlled or Full-Controlled. Bridge rectifiers for a particular application are selected by considering the load current requirements. These bridge rectifiers are quite advantageous as they can be constructed with or without a transformer and are suitable for high voltage applications. However here two diodes will be conducting for every half-cycle and thus the voltage drop across the diodes will be higher. Lastly one has to note that apart from converting AC to DC, **bridge rectifiers** are also used to detect the amplitude of modulated radio signals and to supply polarized voltage for welding applications.

3.23.1. Filters Circuit

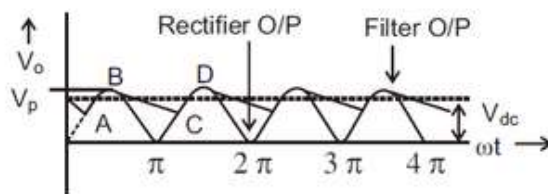
An electronic circuit or device which blocks the a.c. components but allows the d.c. components of the rectifiers to pass to the load is called a filter circuit.

Types of filter circuit:-

- (i) Shunt Capacitor Filter
- (ii) Series Inductor Filter
- (iii) Choke Input (LC) Filter
- (iv) Capacitor Input (pi) Filter
- (i) Shunt Capacitor Filter**



a) Capacitor Filter



b) Waveform

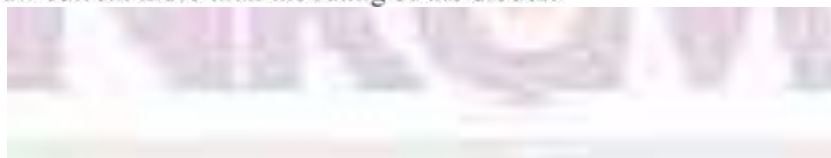
Figure-18

Working

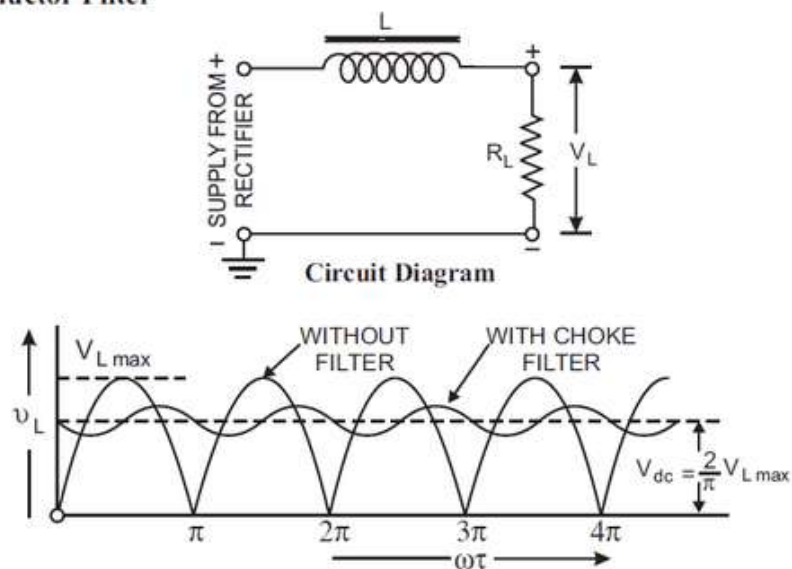
The working of a shunt capacitor filter can be explained with the help of a wave diagram shown in fig.18 the dotted pulsating wave shows the output of a full wave rectifier. When the rectifier voltage is increasing the capacitor is charged to $+V_m$. at point b the rectifier voltage tries to fall but the charged capacitor immediately tries to send the current back to rectifier. In the process the rectifier diodes are reverse biased and stop giving supply to the load. Thus the capacitor discharges (B to C) through the load. The capacitor continues to discharge until the source voltage becomes more than the capacitor voltage. The diode again starts conducting and the capacitor is again charged to peak value $+V_m$ (point). During this time the rectifier supplies the charging current I and the load current.

From above it is clear that capacitor not only remove the a.c. component but also improves the output voltage. The smoothness and magnitude of output voltage depends upon the time constant CR . The longer the time period the steadier is the output voltage. This can be achieved by using a large value of capacitor.

However the maximum value of the capacitance that can be employed is limited by the current that can be safely handled by the diode. The diodes employed in the rectifier circuit can deliver maximum current as per their rating. Therefore the size of the capacitor has to be limited so that it may not draw current more than the rating of the diodes.



(ii) Series Inductor Filter



Output Voltage Waveforms Full-Wave Rectifier with Series Inductor Filter
Figure-19

A series inductor filter is shown. In this case an inductor is just connected in series with load. The inductor has the inherent property to oppose the change of current. This property of inductor utilised here to suppress the a.c. component (ripples) from the output of the rectifier.

The reactance ($X=2\pi fL$) of the inductor is large for high frequencies and offers more opposition to them but it allows the d.c. component of the rectifier output. Hence an inductance blocks the a.c. components but allows the d.c. components to reach the load. Thus it smooths out the rectifier output as shown fig.-19.

(iii) Choke Input LC Filter

A choke input LC filter is shown fig.-20. In this case an inductor is connected in series and a capacitor is parallel with the load.

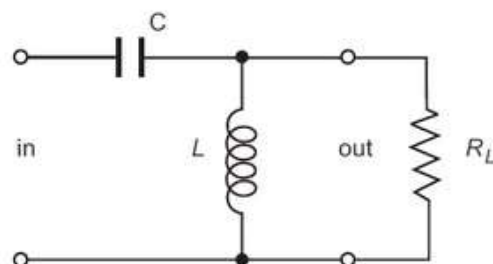


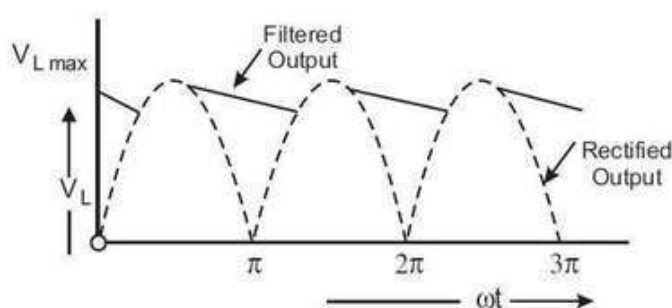
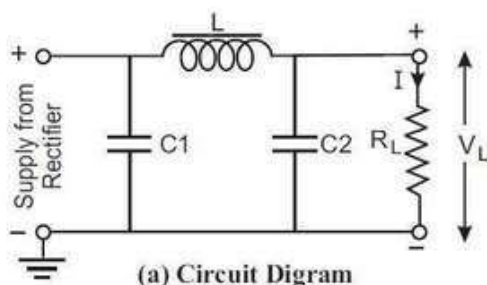
Figure-20

The output of a full wave rectifier contains a.c. components of a fundamental frequencies 100 Hz. The inductor offers a high opposition to the a.c. component and blocks it but allows the d.c. component to pass through the low reactance of the capacitor. Hence almost pure d.c. reaches at

the load. Although the output of this filter is almost d.c. but still it contains small a.c. component. To improve it further one or more sets of LC filter may be applied further.

(iv) Capacitor Input (PIE) Filter

A capacitor input filter is shown fig.-21(a). In this case an additional capacitor C , is connected in the beginning across the output terminals of the rectifier. Since its shape is like the Greek letter (PIE) it is named as pie rectifier.



(b) Rectified and Filtered Output Voltage Waveform Full-Wave Rectifier
with Capacitor Input Filter
Figure-21

The filter action of three components $C1$, L and $C2$ is described below:

- (I) **Action of $C1$:** It provides an easy path to the a.c. components and by pass it and blocks d.c. components which continues its journey through the inductor choke. It also increases the magnitude of V_{av} because of its charging and discharging action.
- (II) **Action of L :** It provides an easy path to d.c. component but blocks the a.c. components because of its high reactance.
- (III) **Action of $C2$:** Any a.c. component which the inductor has failed to block is by passed by this capacitor and only pure d.c. appears across the load.



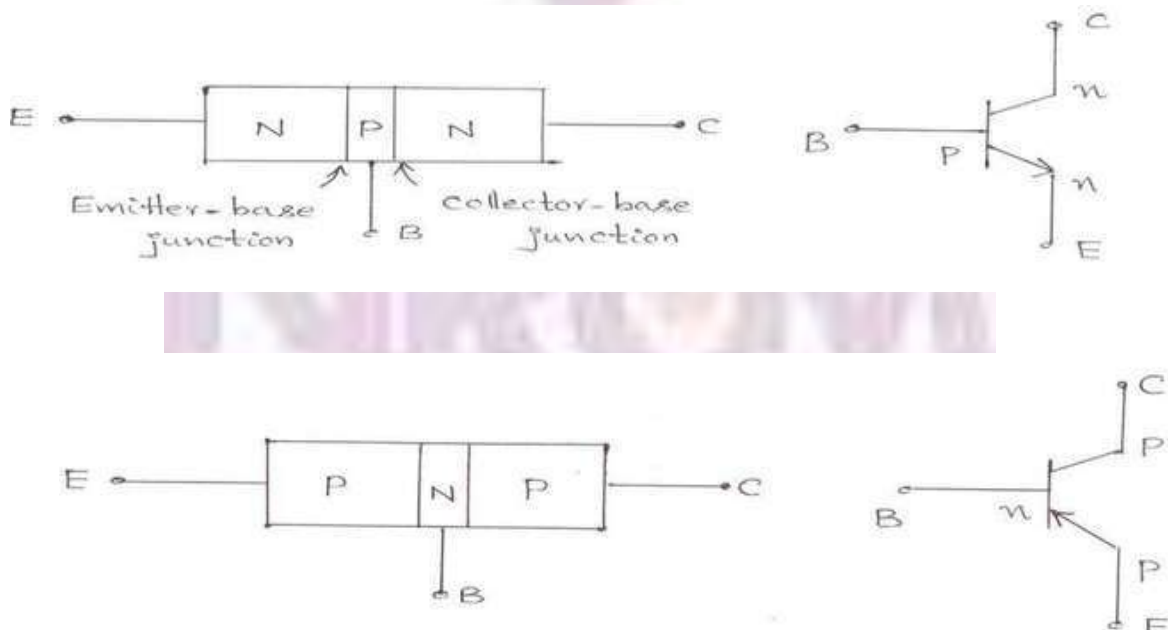
UNIT V

BIPOLAR JUNCTION TRANSISTOR AND JFET

5.1 INTRODUCTION

- The transistor was invented in 1947 by John Bardeen, Walter Brattain and William Shockley at Bell Laboratory in America. A transistor is a semiconductor device, commonly used as an amplifier or an electrically controlled switch.
- There are two types of transistors:
 - 1) Unipolar Junction Transistor
 - 2) Bipolar Junction Transistor
- In Unipolar transistor, the current conduction is only due to one type of carriers i.e., majority charge carriers. The current conduction in bipolar transistor is because of both the types of charge carriers i.e., holes and electrons. Hence it is called as Bipolar Junction Transistor and it is referred to as BJT.
- BJT is a semiconductor device in which one type of semiconductor material is sandwiched between two opposite types of semiconductor i.e., an n-type semiconductor is sandwiched between two p-type semiconductors or a p-type semiconductor is sandwiched between two n-type semiconductors.
- Hence the BJTs are of two types. They are:
 - 1) n-p-n Transistor
 - 2) p-n-p Transistor

The two types of BJTs are shown in the figure below.



- The arrow head represents the conventional current direction from p to n. Transistor has three terminals.
1)Emitter 2)Base 3)Collector

➤ Transistor has two p-n junctions. They are:

- 1) Emitter-Base Junction
- 2) Collector-Base Junction

Emitter: Emitter is heavily doped because it is to emit the charge carriers.

Base: The charge carriers emitted by the emitter should reach collector passing through the base. Hence base should be very thin and to avoid recombination, and to provide more collector current base is lightly doped.

Collector: Collector has to collect the most of charge carriers emitted by the emitter. Hence the area of cross-section of collector is more compared to emitter and it is moderately doped.

Transistor can be operated in three regions.

- 1) Active region.
- 2) Saturation region.
- 3) Cut-Off region.

Active Region: For the transistor to operate in active region base-emitter junction is forward biased and collector-base junction is reverse biased.

Saturation Region: Transistor to be operated in saturation region if both the junctions i.e., collector-base junction and base-emitter junction are forward biased.

Cut-Off Region: For the transistor to operate in cut-off region both the junctions i.e., base-emitter junction and collector-base junction are reverse biased.

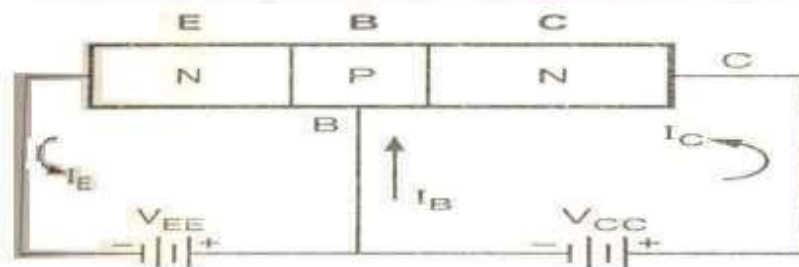
Transistor can be used as

- 1) Amplifier
- 2) Switch

For the transistor to act as an amplifier, it should be operated in active region. For the transistor to act as a switch, it should be operated in saturation region for ON state, and cut-off region for OFF state.

5.2 TRANSISTOR OPERATION:

Working of an N-P-N Transistor:



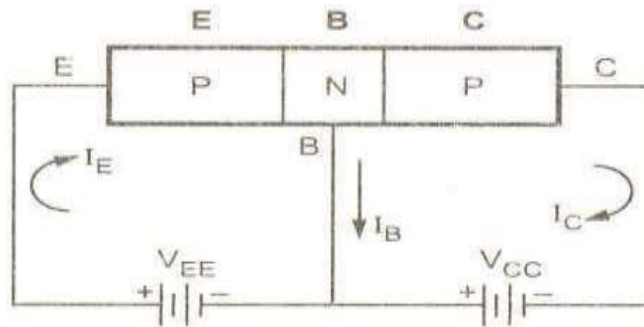
The n-p-n transistor with base-emitter junction forward biased and collector-base junction reverse biased is as shown in figure.

As the base-emitter junction is forward biased, the majority carriers emitted by the n-type emitter i.e., electrons have a tendency to flow towards the base, which constitutes the emitter current I_E . As the base is p-type, there is a chance of recombination of electrons emitted by the emitter with the holes in the p-type base. But as the base is very thin and lightly doped

only few electrons emitted by the p-type emitter less than 5% combines with the holes in the p-type base, the remaining more than 95% electrons emitted by the n-type emitter cross over into the collector region constitute the collector current.

The current distributions are as shown in fig $I_E = I_B + I_C$

Working of a P-N-P Transistor:



The p-n-p transistor with base to emitter junction is forward biased and collector to base junction reverse biased is as shown in figure. As the base to emitter junction is forward biased, the majority carriers emitted by the p-type emitter i.e., holes have a tendency to flow towards the base which constitutes the emitter current I_E . As the base is n-type, there is a chance of recombination of holes emitted by the emitter with the electrons in the n-type base. But as the base is very thin and lightly doped, only a few electrons (less than 5%) combine with the holes emitted by the p-type emitter, the remaining 95% charge carriers cross over into the collector region to constitute the collector current. The current distributions are shown in figure.

$$I_E = I_B + I_C$$

5.1 TRANSISTOR CIRCUIT CONFIGURATIONS:

Following are the three types of transistor circuit configurations:

- 1) Common-Base (CB)
- 2) Common-Emitter (CE)
- 3) Common-Collector (CC)

Here the term 'Common' is used to denote the transistor lead which is common to the input and output circuits. The common terminal is generally grounded. It should be remembered that regardless of the circuit configuration, the emitter is always forward-biased while the collector is always reverse-biased.

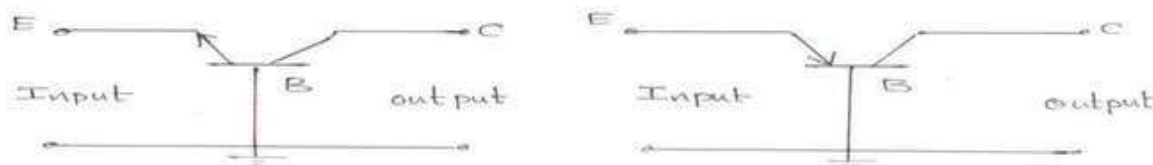


Fig. Common-Base Configuration

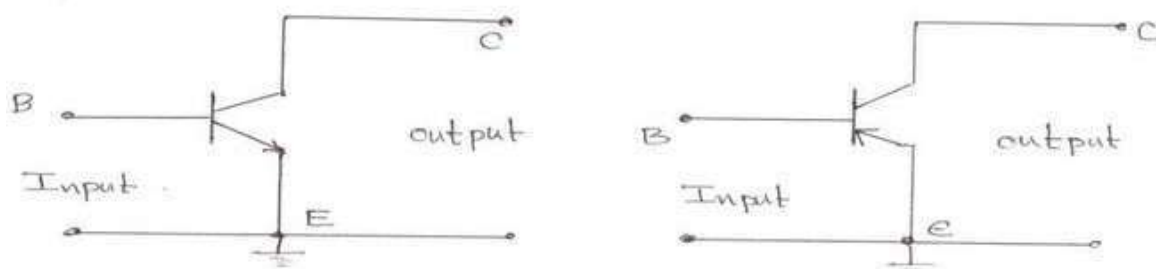


Fig.Common–EmitterConfiguration

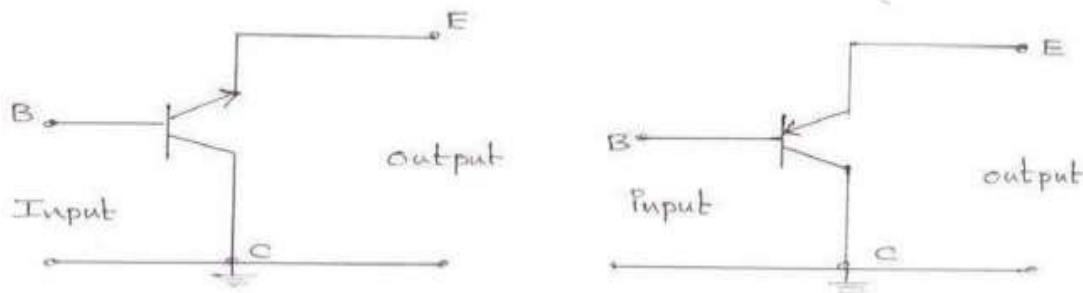


Fig.Common–CollectorConfiguration

5.1.1 Common–Base(CB)Configurations:

In this configuration, the input signal is applied between emitter and base while the output is taken from collector and base. As base is common to input and output circuits, hence the name common-base configuration. Figures show the common-base P-N-P transistor circuit.

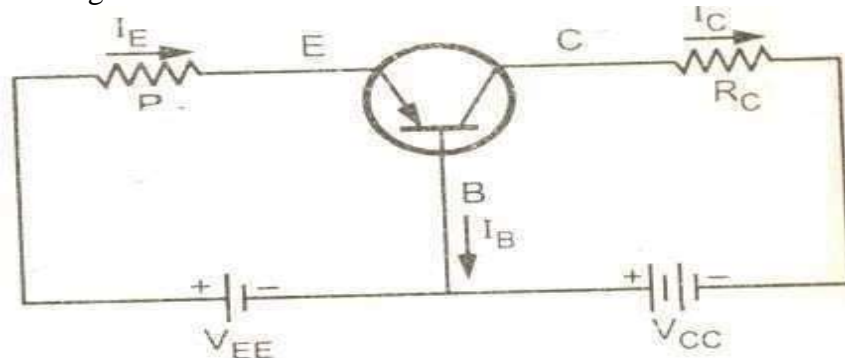


Fig.Common– BasePNPtransistoramplifier.

Current Amplification Factor(α):

When no signal is applied, then the ratio of the collector current to the emitter current is called α_{dc} of a transistor.

$$\alpha_{dc} = \frac{\Delta I_C}{\Delta I_E}$$

(Negative sign signifies that I_E flows into transistor while I_C flows out of it). ‘ α ’ of a transistor is a measure of the quality of a transistor. Higher is the value of ‘ α ’, better is the transistor in the sense that collector current approaches the emitter current. By considering only magnitudes of the currents, $I_C = \alpha I_E$ and hence $I_B = I_E - I_C$. Therefore,

$$I_B = I_E - \alpha I_E = I_E(1 - \alpha)$$

For all practical purposes, $\alpha_{dc} = \alpha_{ac} = \alpha$ and practical values in commercial transistors range from 0.9 to 0.99.

Total Collector Current:

The total collector current consists of the following two parts

- i) I_E current due to majority carriers
- ii) I_{CBO} current due to minority carriers

$$\text{Total collector current } I_C = \alpha I_E + I_{CBO}$$

The collector current can also be expressed as $I_C = \alpha(I_B + I_C) + I_{CBO}$ (Q $I_E = I_B + I_C$)

$$\begin{aligned}\Rightarrow I_C(1 - \alpha) &= \alpha I_B + I_{CBO} \\ \Rightarrow I_C &= \left(\frac{\alpha}{1 - \alpha}\right) I_B + \left(\frac{1}{1 - \alpha}\right) I_{CBO}\end{aligned}$$

5.1.2 COMMON-EMITTER (CE) CONFIGURATION:

In this configuration, the input signal is applied between base and emitter and the output is taken from collector and emitter. As emitter is common to input and output circuits, hence the name common emitter configuration.

Figure shows the Common-Emitter P-N-P transistor circuit.

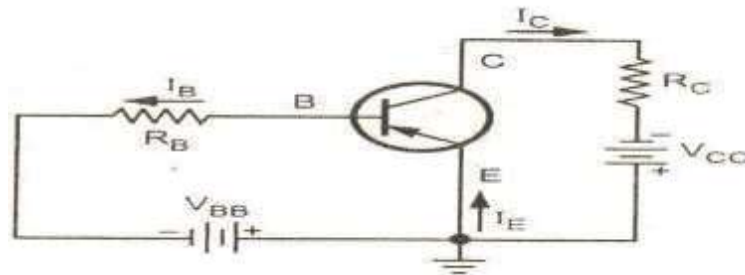


Fig. Common-Emitter PNP transistor amplifier.

Current Amplification Factor (β):

When no signal is applied, then the ratio of collector current to the base current is called ddc beta (β_{dc}) of a transistor.

$$\beta_{dc} = \beta = \frac{I_C}{I_B} \quad \dots\dots\dots(1)$$

When signal is applied, the ratio of change in collector current to the change in base current is defined as base current amplification factor. Thus,

$$\beta_{dc} = \beta = \frac{\Delta I_C}{\Delta I_B} \quad \dots\dots\dots(2)$$

From equation (1), $I_C = \beta I_B$

Almost in all transistors, the base current is less than 5% of the emitter current. Due to this fact, ' β ' ranges from 20 to 500. Hence this configuration is frequently used when appreciable current gain as well as voltage gain is required.

Total Collector Current:

$$\text{The Total collector current } I_C = \beta I_B + I_{CEO} \quad \dots\dots\dots(3)$$

Where I_{CEO} is the leakage current.

$$\text{But, we have, } I_C = \left(\frac{\alpha}{1-\alpha} \right) I_B + \left(\frac{1}{1-\alpha} \right) I_{CBO} \dots\dots\dots(4)$$

Comparing equations (3) and (4), we get

$$\beta = \frac{\alpha}{1-\alpha} \text{ and } I_{CEO} = \frac{1}{1-\alpha} I_{CBO} \dots\dots\dots(5)$$

Relation between α and β :

$$\text{We know that } \alpha = \frac{I_C}{I_E} \text{ and } \beta = \frac{I_C}{I_B}$$

$$I_E = I_B + I_C \quad (\text{or}) \quad I_B = I_E - I_C$$

$$\text{Now} \quad \beta = \frac{I_C}{I_E - I_C} = \frac{\frac{I_C}{I_E}}{1 - \frac{I_C}{I_E}} = \frac{\alpha}{1-\alpha} \dots\dots\dots(6)$$

$$\Rightarrow \beta(1-\alpha) = \alpha \quad (\text{or}) \quad \beta = \alpha(1+\beta)$$

$$\Rightarrow \alpha = \frac{\beta}{1+\beta} \dots\dots\dots(7)$$

$$\text{It can be seen that } 1-\alpha = \frac{1}{1+\beta} \dots\dots\dots(8)$$

5.1.3 COMMON-COLLECTOR(CC)CONFIGURATION:

In this configuration, the input signal is applied between base and collector and the output is taken from the emitter. As collector is common to input and output circuits, hence the name common collector configuration. Figure shows the common collector PNP transistor circuit.

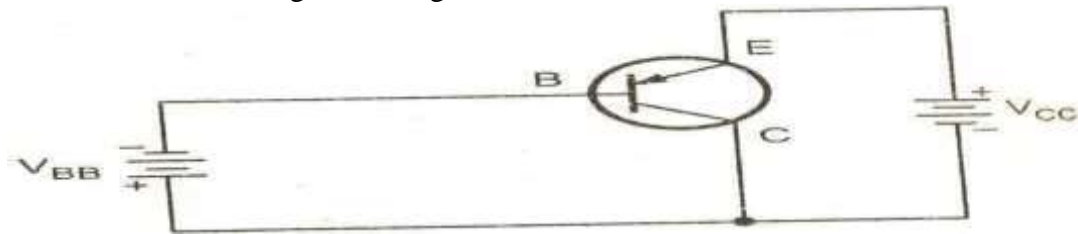


Fig. Common Collector PNP transistor amplifier.

Current Amplification Factor (γ):

When no signal is applied, then the ratio of emitter current to the base current is called as dc gamma (γ_{dc}) of the transistor.

$$\gamma_{dc} = \gamma = \frac{I_E}{I_B} \dots\dots\dots(1)$$

5.4 CHARACTERISTICS OF COMMON-BASE CIRCUIT:

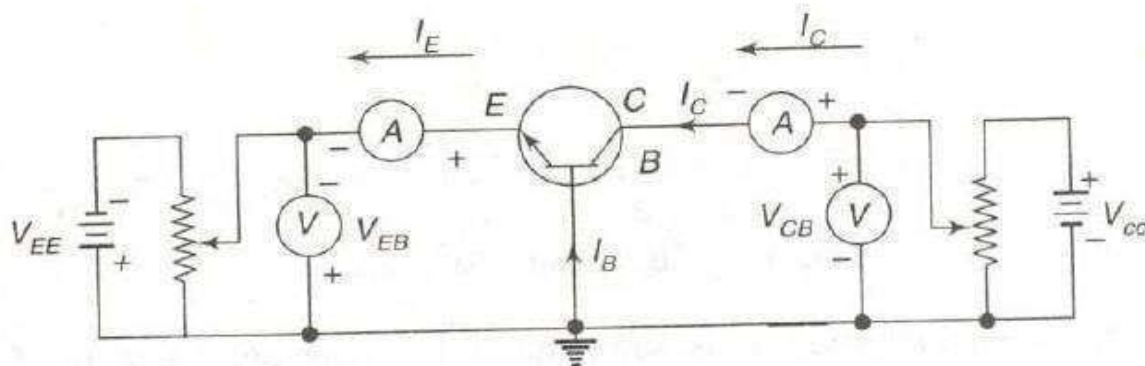


Fig. Circuit to determine CB static characteristics.

Input Characteristics:

To determine the input characteristics, the collector-base voltage V_{CB} is kept constant at zero volts and the emitter current I_E is increased from zero in suitable equal steps by increasing V_{EB} . This is repeated for higher fixed values of V_{CB} . A curve is drawn between emitter current I_E and emitter-base voltage V_{EB} at constant collector-base voltage V_{CB} .

The input characteristics thus obtained are shown in figure below.

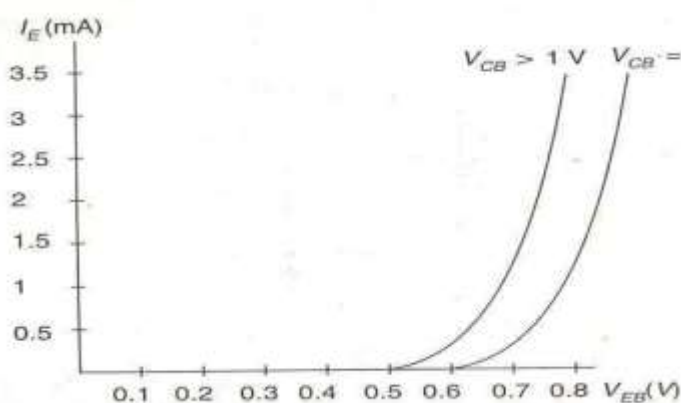
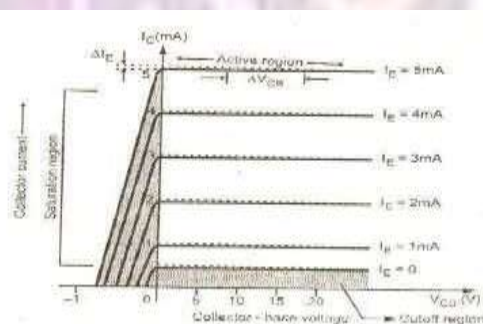


Fig. CB Input Characteristics.

Output Characteristics:

To determine the output characteristics, the emitter current I_E is kept constant at a suitable value by adjusting the emitter-base voltage V_{EB} . Then V_{CB} is increased in suitable equal steps and the collector current I_C is noted for each value of I_E . Now the curves of I_C versus V_{CB} are plotted for constant values of I_E and the output characteristics thus obtained are shown in figure below.



From the characteristics, it is seen that for a constant value of I_E , I_C is independent of V_{CB} and the curves are parallel to the axis of V_{CB} . Further, I_C flows even when V_{CB} is equal to zero. As the emitter-base junction is forward biased, the majority carriers, i.e., electrons, from the emitter are injected into the base region. Due to the action of the internal potential barrier at the reverse biased collector-base junction, they flow to the collector region and give rise to I_C even when V_{CB} is equal to zero.

It is the slope of CB output characteristics I_C versus V_{CB} .

5.5 CHARACTERISTICS OF COMMON-EMITTER CIRCUIT:

The circuit diagram for determining the static characteristic curves of the an N-P-N transistor in the common emitter configuration is shown in figure below.

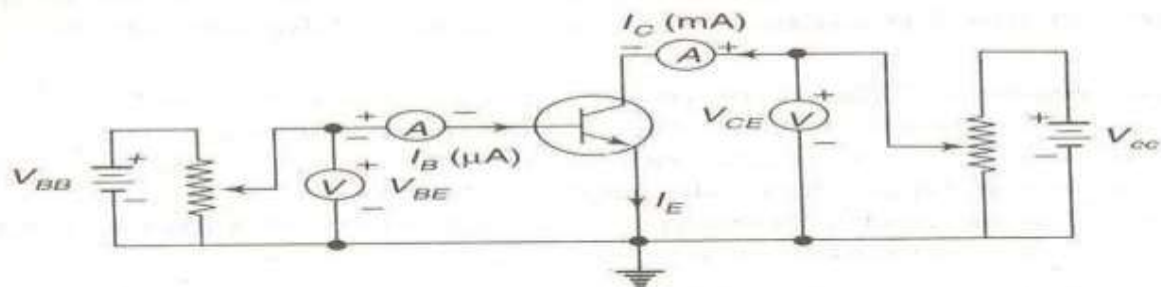


Fig. Circuit to determine CE Static characteristics.

Input Characteristics:

To determine the input characteristics, the collector to emitter voltage is kept constant at zero volts and base current is increased from zero in equal steps by increasing V_{BE} in the circuit. The value of V_{BE} is noted for each setting of I_B . This procedure is repeated for higher fixed values of V_{CE} , and the curves of I_B versus V_{BE} are drawn.

The input characteristics thus obtained are shown in figure below.

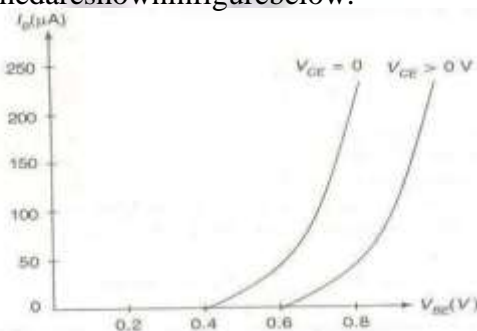


Fig. CE Input Characteristics.

When $V_{CE}=0$, the emitter-base junction is forward biased and the junction behaves as a forward biased diode. When V_{CE} is increased, the width of the depletion region at the reverse biased collector-

base junction will increase. Hence the effective width of the base will decrease. This effect causes a decrease in the base current I_B . Hence, to get the same value of I_B as that for $V_{CE}=0$, V_{BE} should be increased. Therefore, the curves shift to the right as V_{CE} increases.

Output Characteristics:

To determine the output characteristics, the base current I_B is kept constant at a suitable value by adjusting base-emitter voltage, V_{BE} . The magnitude of collector-emitter voltage V_{CE} is increased in suitable equal steps from zero and the collector current I_C is noted for each setting of V_{CE} . Now the curves of I_C versus V_{CE} are plotted for different constant values of I_B . The output characteristics thus obtained are shown in figure below.

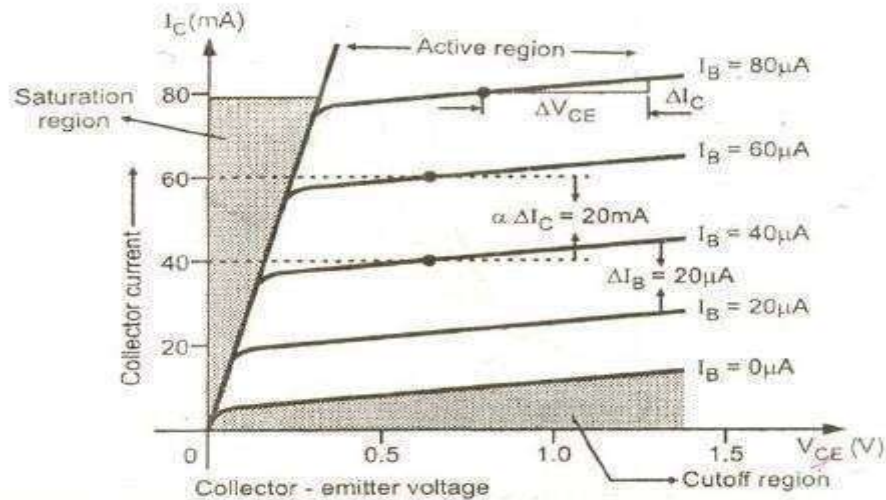


Fig. CE Output characteristics

The output characteristics of common emitter configuration consist of three regions: Active, Saturation and Cut-off regions.

Active Region:

The region where the curves are approximately horizontal is the "Active" region of the CE configuration. In the active region, the collector junction is reverse biased. As V_{CE} is increased, reverse bias increases. This causes the depletion region to spread more in the base than in the collector, reducing the chances of recombination in the base. This increases the value of α_{dc} . This Early effect causes collector current to rise more sharply with increasing V_{CE} in the active region of output characteristics of CE transistor.

Saturation Region:

If V_{CE} is reduced to a small value such as 0.2V, then collector-base junction becomes forward biased, since the emitter-base junction is already forward biased by 0.7V. The input junction in CE configuration is base to emitter junction, which is always forward biased to operate transistor in active region. Thus input characteristics of CE configuration are similar to forward characteristics of p-n junction diode. When both the junctions are forward biased, the transistor operates in the saturation region, which is indicated on the output characteristics. The saturation value of V_{CE} , designated $V_{CE(Sat)}$, usually ranges between 0.1V to 0.3V.

Cut-Off Region:

When the input base current is made equal to zero, the collector current is the reverse leakage current I_{CEO} . Accordingly, in order to cut off the transistor, it is not enough to reduce $I_B=0$. Instead, it is necessary to reverse bias the emitter junction slightly. We shall define cut off as the condition where the collector current is equal to the reverse saturation current I_{CO} and the emitter current is zero.

5.5 Characteristics of common collector circuit:

The circuit diagram for determining the static characteristics of an N-P-N transistor in the common collector configuration is shown in fig. below.

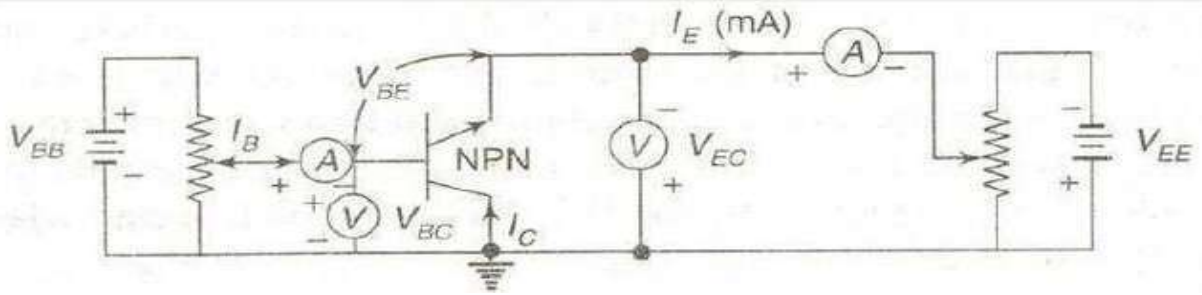


Fig. Circuit to determine CC static characteristics.

Input Characteristics:

To determine the input characteristic, V_{EC} is kept at a suitable fixed value. The base-collector voltage V_{BC} is increased in equal steps and the corresponding increase in I_B is noted. This is repeated for different fixed values of V_{EC} . Plots of V_{BC} versus I_B for different values of V_{EC} shown in figure are the input characteristics.

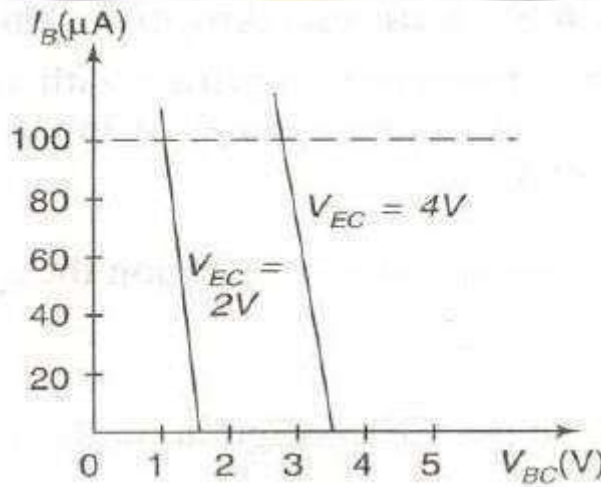


Fig. CC Input Characteristics.

Output Characteristics:

The output characteristics shown in figure below are the same as those of the common emitter configuration.

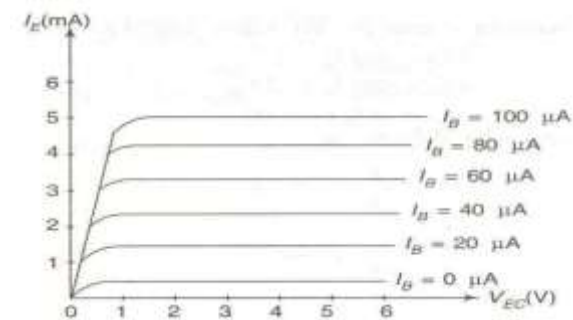
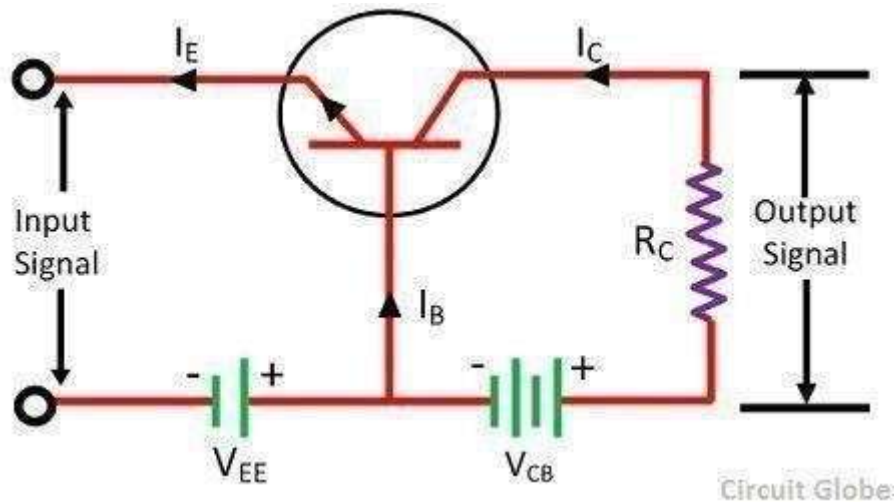


Fig. CC Output characteristics.

Transistor as an Amplifier:

The transistor raises the strength of a weak signal and hence acts as an amplifier. The transistor amplifier circuit is shown in the figure below. The transistor has three terminals namely emitter, base and collector. The emitter and base of the transistor are connected in forward bias and the collector-base region is in reverse bias. The forward bias means the P-region of the transistor is connected to the positive terminal of the supply and the negative region is connected to the N-terminal and in reverse bias just opposite of it has occurred.



The input signal or weak signal is applied across the emitter-base and the output is obtained to the load resistor R_C which is connected in the collector circuit. The DC voltage V_{EE} is applied to the input circuit along with the input signal to achieve the amplification. The DC voltage V_{EE} keeps the emitter-base junction under the forward biased condition regardless of the polarity of the input signal and is known as a bias voltage.

In the collector circuit, a load resistor R_C of high value is connected. When collector current flows through such a high resistance, it produces a large voltage drop across it. Thus, a weak signal (0.1 V) applied to the input circuit appears in the amplified form (10 V) in the collector circuit.



FIELD EFFECT TRANSISTOR

INTRODUCTION

1. The Field effect transistor is abbreviated as FET, it is another semiconductor device like a BJT which can be used as an amplifier or switch.
2. The Field effect transistor is a voltage operated device. Whereas Bipolar junction transistor is a current controlled device. Unlike BJT a FET requires virtually no input current.
3. This gives it an extremely high input resistance, which is its most important advantage over a bipolar transistor.
4. FET is also a three terminal device, labeled as source, drain and gate.
5. The source can be viewed as BJT's emitter, the drain as collector, and the gate as the counterpart of the base.
6. The material that connects the source to drain is referred to as the channel.
7. FET operation depends only on the flow of majority carriers, therefore they are called unipolar devices. BJT operation depends on both minority and majority carriers.
8. As FET has conduction through only majority carriers it is less noisy than BJT.
9. FETs are much easier to fabricate and are particularly suitable for ICs because they occupy less space than BJTs.
10. FET amplifiers have low gain bandwidth product due to the junction capacitive effects and produce more signal distortion except for small signal operation.
11. The performance of FET is relatively unaffected by ambient temperature changes. As it has a negative temperature coefficient at high current levels, it prevents the FET from thermal breakdown. The BJT has a positive temperature coefficient at high current levels which leads to thermal breakdown.

CLASSIFICATION OF FET:

There are two major categories of field effect transistors:

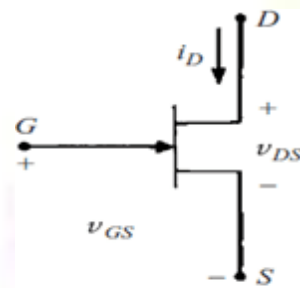
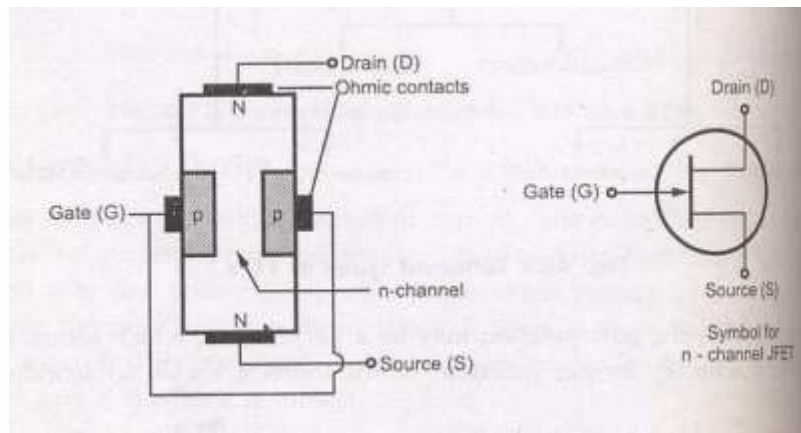
1. Junction Field Effect Transistors
2. MOSFETs

1. Junction Field Effect Transistors

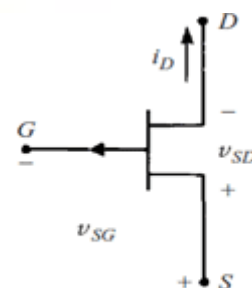
- Junction Field Effect Transistors are further subdivided into P-channel and N-channel devices.
- When the channel is of N-type the JFET is referred to as an N-channel JFET, when the channel is of P-type the JFET is referred to as a P-channel JFET.

CONSTRUCTION OF N-CHANNEL JFET

A piece of N-type material, referred to as a channel, has two smaller pieces of P-type material attached to its sides, forming PN junctions. The channel ends are designated as the drain and source. And the two pieces of P-type material are connected together and their terminal is called the gate. Since this channel is in the N-type bar, the FET is known as N-channel JFET.



N-channel FET



P-channel FET

These schematic symbols for the P-channel and N-channel JFETs are shown in the figure

OPERATION OF N-CHANNEL JFET:-

The overall operation of the JFET is based on varying the width of the channel to control the drain current.

A piece of N type material referred to as the channel, has two smaller pieces of P type material attached to its sides, forming PN Junctions. The channel's ends are designated the drain and the source. And the two pieces of P type material are connected together and their terminal is called the gate. With the gate terminal not connected and the potential applied positive at the drain and negative at the source, a drain current I_D flows.

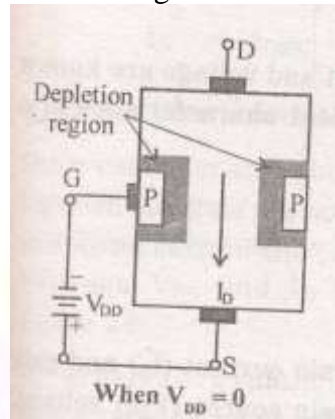
When the gate is biased negative with respect to the source the PN junctions are reverse biased and depletion regions are formed. The channel is more lightly doped than the P type gate blocks, so the depletion regions penetrate deeply into the channel. Since the depletion region is a region depleted of charge carriers it behaves as an insulator. The result is that the channel is narrowed. Its resistance is increased and I_D is reduced. When the negative gate bias voltage is further increased, the depletion regions meet at the center and I_D is cut off completely.

There are two ways to control the channel width

1. By varying the value of V_{GS}
2. And by Varying the value of V_{DS} holding V_{GS} constant

By varying the value of V_{GS} :-

We can vary the width of the channel and in turn vary the amount of drain current. This can be done by varying the value of V_{GS} . This point is illustrated in the fig below. Here we are dealing with N channel FET. So channel is of N type and gate is of P type that constitutes a PN junction. This PN junction is always reverse biased in JFET operation. The reverse bias is applied by a battery voltage V_{GS} connected between the gate and the source terminal. The positive terminal of the battery is connected to the source and negative terminal to gate.



- 1) When a PN junction is reverse biased the electrons and holes diffuse across junction by leaving immobile ions on the N and P sides, the region containing these immobile ions is known as depletion regions.
- 2) If both P and N regions are heavily doped then the depletion region extends symmetrically on both sides.
- 3) But in N channel FET P region is heavily doped than N type thus depletion region extends more in N region than P region.
- 4) So when no V_{DS} is applied the depletion region is symmetrical and the conductivity becomes zero. Since there are no mobile carriers in the junction.
- 5) As the reverse bias voltage is increased the thickness of the depletion region also increases. i.e. the effective channel width decreases.
- 6) By varying the value of V_{GS} we can vary the width of the channel.

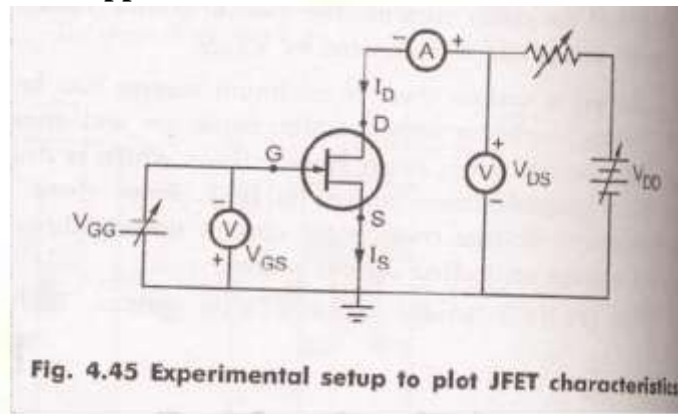
2 Varying the value of V_{DS} holding V_{GS} constant:-

- 1) When no voltage is applied to the gate i.e. $V_{GS} = 0$, V_{DS} is applied between source and drain the electrons will flow from source to drain through the channel constituting drain current I_D .
- 2) With $V_{GS} = 0$ for $I_D = 0$ the channel between the gate junctions is entirely open. In response to a small applied voltage V_{DS} , the entire bar acts as a simple semiconductor resistor and the current I_D increases linearly with V_{DS} .
- 3) The channel resistances are represented as R_D and R_S as shown in the fig.



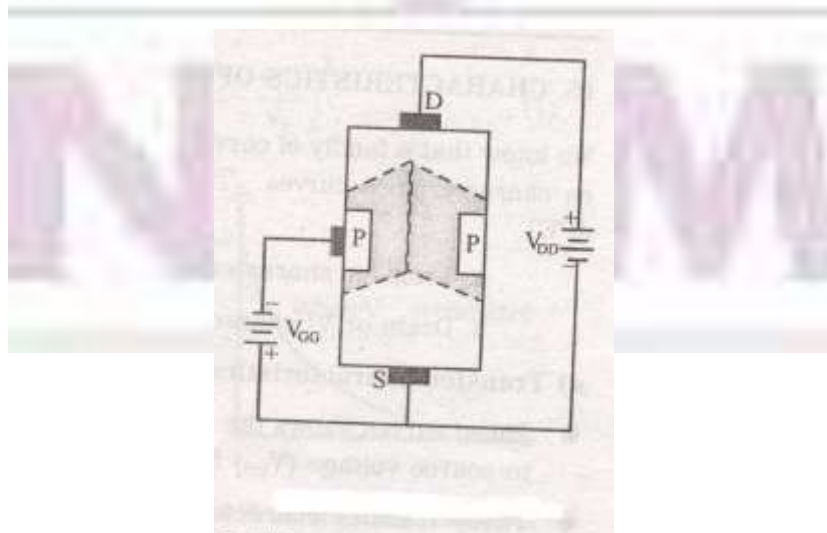
- 4) This increasing drain current I_D produces a voltage drop across r_D which reverse biases the gate to source junction, ($R_D > R_S$). Thus the depletion region is formed which is not symmetrical.
- 5) The depletion region i.e. developed penetrates deeper into the channel near drain and less towards source because $V_{R_D} \gg V_{R_S}$. So reverse bias is higher near drain than at source.
- 6) As a result growing depletion region reduces the effective width of the channel. Eventually a voltage V_{DS} is reached at which the channel is pinched off. This is the voltage where the current I_D begins to level off and approach a constant value.
- 7) So, by varying the value of V_{DS} we can vary the width of the channel holding V_{GS} Constant.

When both V_{GS} and V_{DS} is applied:-



It is of course in principle not possible for the channel to close Completely and there by reduce the current I_D to Zero for, if such indeed, could be the case the gate voltage V_{GS} is applied in the direction to provide additional reverse bias

- 1) When voltage is applied between the drain and source with a battery V_{DD} , the electrons flow from source to drain through the narrow channel existing between the depletion regions. This constitutes the drain current I_D , its conventional direction is from drain to source.
- 2) The value of drain current is maximum when no external voltage is applied between gate and source and is designated by I_{DSS} .



- 3) When V_{GS} is increased beyond Zero the depletion regions are widened. This reduces the effective width of the channel and therefore controls the flow of drain current through the channel.
- 4) When V_{GS} is further increased a stage is reached at which the depletion regions touch each other that means the entire channel is closed with depletion region. This reduces the drain current to Zero.

CHARACTERISTICS OF N-CHANNEL JFET:-

The family of curves that shows the relation between current and voltage are known as characteristic curves.

There are two important characteristics of a JFET.

- 1) Drain or V_I Characteristics
- 2) Transfer characteristics

1. Drain Characteristics:

Drain characteristics show the relation between the drain to source voltage V_{DS} and drain current I_D . In order to explain typical drain characteristics let us consider the curve with $V_{GS} = 0V$.

- 1) When V_{DS} is applied and it is increasing the drain current I_D also increases linearly up to a knee point.
- 2) This shows that FET behaves like an ordinary resistor. This region is called as ohmic region.
- 3) I_D increases with increase in drain to source voltage. Here the drain current increases slowly as compared to ohmic region.

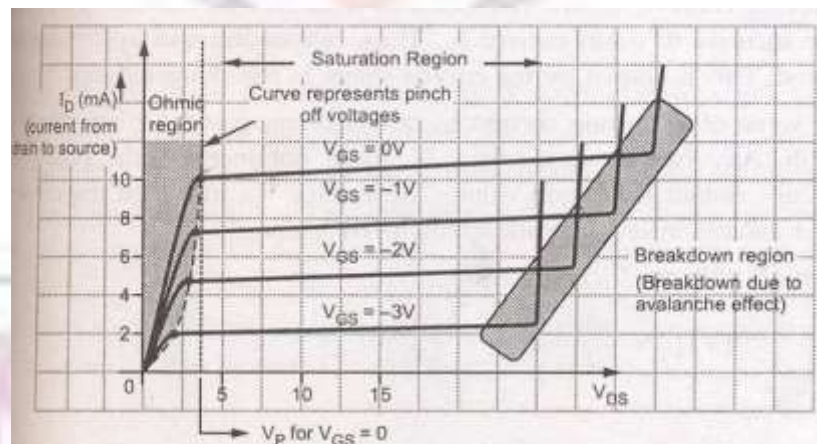


Fig. 4.44 Drain V-I characteristics of n-channel JFET

- 4) It is because of the fact that there is an increase in V_{DS} . This in turn increases the reverse bias voltage across the gate-source junction. As a result of this depletion region grows in size thereby reducing the effective width of the channel.
- 5) All the drain to source voltage corresponding to point the channel width is reduced to a minimum value and is known as pinch off.
- 6) The drain to source voltage at which channel pinch off occurs is called pinch off voltage (V_P).

PINCHOFFRegion:

- 1) This is the region shown by the curve as saturation region.
- 2) It is also called as saturation region or constant current region. Because of the channel is occupied with depletion region, the depletion region is more towards the drain and less towards the source, so the channel is limited, with this only limited number of carriers are only allowed to cross this channel from source drain causing a current that is constant in this region. To use FET as an amplifier it is operated in this saturation region.
- 3) In this drain current remains constant at its maximum value I_{DSS} .
- 4) The drain current in the pinch off region depends upon the gate to source voltage and is given by the relation

$$I_D = I_{DSS} [1 - V_{GS}/V_P]^2$$

This is known as Shockley's relation.

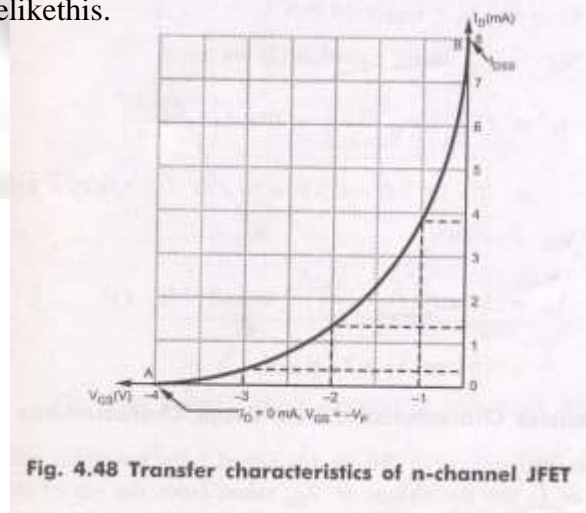
BREAKDOWNREGION:

- 1) The region is shown by the curve. In this region, the drain current increases rapidly as the drain to source voltage is increased.
- 2) It is because of the gate to source junction due to avalanche effect.
- 3) The avalanche break down occurs at progressively lower value of V_{DS} because the reverse bias gate voltage adds to the drain voltage thereby increasing effective voltage across the gate junction.
This causes
 1. The maximum saturation drain current is smaller
 2. The ohmic region portion decreased.
- 4) It is important to note that the maximum voltage V_{DS} which can be applied to FET is the lowest voltage which causes available breakdown.

2. TRANSFER CHARACTERISTICS:

These curves show the relationship between drain current I_D and gate to source voltage V_{GS} for different values of V_{DS}

- 1) First adjust the drain to source voltage to some suitable value, then increase the gate to source voltage in small suitable value.
- 2) Plot the graph between gate to source voltage along the horizontal axis and current I_D on the vertical axis. We shall obtain a curve like this.



- 3) As we know that if V_{GS} is more negative curves drain current to reduce. where V_{GS} is made sufficiently negative, I_D is reduced to zero. This is caused by the widening of the depletion region to a point where it is completely close to the channel. The value of V_{GS} at the cutoff point is designed as V_{GSoff}
- 4) The upper end of the curve as shown by the drain current value is equal to I_{DSS} that is when $V_{GS}=0$ the drain current is maximum.
- 5) While the lower end is indicated by a voltage equal to V_{GSoff}
- 6) If V_{GS} continuously increases, the channel width is reduced, then $I_D=0$
- 7) It may be noted that the curve is part of the parabola; it may be expressed as
$$I_D = I_{DSS} [1 - V_{GS}/V_{GSoff}]^2$$

DIFFERENCE BETWEEN V_P AND V_{GSoff} :

- 1) V_P is the value of V_{GS} that causes the JFET to become a constant current component. It is measured at $V_{GS}=0V$ and has a constant drain current of $I_D=I_{DSS}$.
Where V_{GSoff} is the value of V_{GS} that reduces I_D to approximately zero.

Why the gate to source junction of a JFET be always reverse biased?

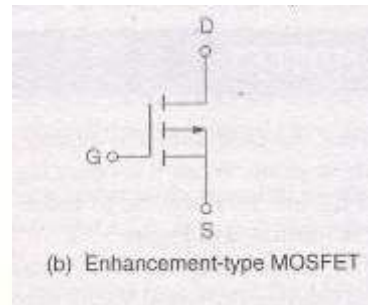
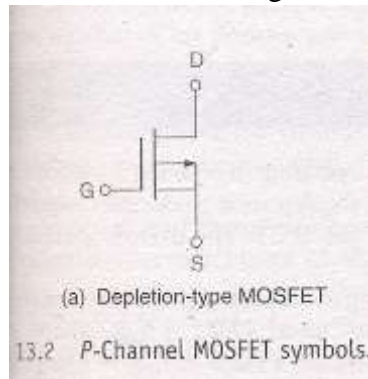
The gate to source junction of a JFET is never allowed to become forward biased because the gate material is not designed to handle any significant amount of current. If the junction is allowed to become forward biased, current is generated through the gate material. This current may destroy the component.

There is one more important characteristic of JFET reverse biasing i.e. JFET 's have extremely high characteristic gate input impedance. This impedance is typically in the high mega ohm range. With the advantage of extremely high input impedance it draws no current from the source. The high input impedance of the JFET has led to its extensive use in integrated circuits. The low current requirements of the component makes it perfect for use in ICs. Where thousands of transistors must be etched on to a single piece of silicon. The low current draw helps the IC to remain relatively cool, thus allowing more components to be placed in a smaller physical area.



MOSFET:

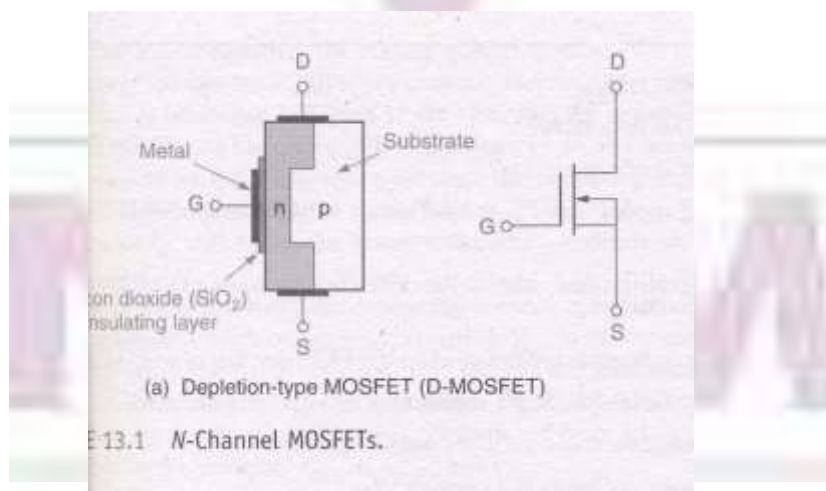
- We now turn our attention to the Insulated Gate FET or Metal Oxide Semiconductor FET which is having the greater commercial importance than the junction FET.
- Most MOSFETs however are triodes, with the substrate internally connected to the source. The circuit symbols used by several manufacturers are indicated in the Fig below.



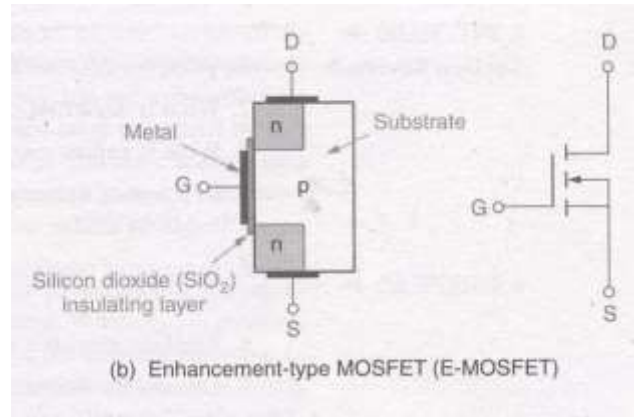
(a) Depletion type MOSFET (b) Enhancement type MOSFET

Both of them are P-channel

- Here are two basic types of MOSFETs
 - (1) Depletion type
 - (2) Enhancement type MOSFET.
- D-MOSFETs can be operated in both the depletion mode and the enhancement mode.
- E-MOSFETs are restricted to operate in enhancement mode. The primary difference between them is their physical construction.
- The construction difference between the two is shown in the fig given below.



As we can see the DMOSFET has a physical channel between the source and drain terminals (Shaded area)



The E MOSFET on the other hand has no such channel physically. It depends on the gate voltage to form a channel between the source and the drain terminals.

Both MOSFETs have an insulating layer between the gate and the rest of the component. This insulating layer is made up of SiO_2 a glass like insulating material. The gate material is made up of metal conductor. Thus going from gate to substrate, we can have metal oxide semiconductor which is where the term MOSFET comes from.

Since the gate is insulated from the rest of the component, the MOSFET is sometimes referred to as an insulated gate FET or IGFET.

The foundation of the MOSFET is called the substrate. This material is represented in the schematic symbol by the center line that is connected to the source.

In the symbol for the MOSFET, the arrow is placed on the substrate. As with JFET an arrow pointing in represents an n-channel device, while an arrow pointing out represents a p-channel device.

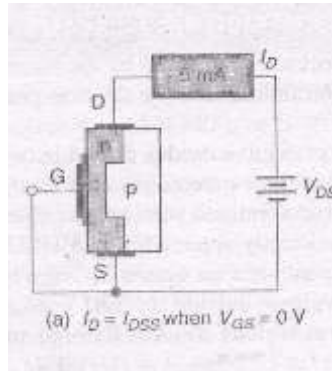
CONSTRUCTION OF AN N-CHANNEL MOSFET:

The N-channel MOSFET consists of a lightly doped p type substance into which two heavily doped n+ regions are diffused as shown in the Fig. These n+ sections, which will act as source and drain.

A thin layer of insulation silicon dioxide (SiO_2) is grown over the surface of the structure, and holes are cut into oxide layer, allowing contact with the source and drain. Then the gate metal area is overlaid on the oxide, covering the entire channel region. Metal contacts are made to drain and source and the contact to the metal over the channel area is the gate terminal. The metal area of the gate, in conjunction with the insulating dielectric oxide layer and the semiconductor channel, forms a parallel plate capacitor. The insulating layer of SiO_2 is the reason why this device is called the insulated gate field effect transistor. This layer results in an extremely high input resistance (10^{10} to 10^{15} ohms) for MOSFET.

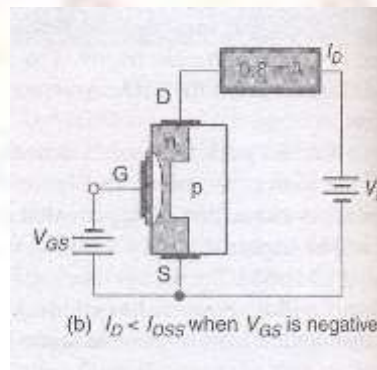
DEPLETION MOSFET

The basic structure of D-MOSFET is shown in the fig. An N-channel is diffused between source and drain with the device an appreciable drain current I_{DSS} flows for zero gate to source voltage, $V_{GS}=0$.



Depletion mode operation:-

- 1) The above fig shows the D-MOSFET operating conditions with gate and source terminals shorted together ($V_{GS}=0V$)
- 2) At this stage $I_D = I_{DSS}$ where $V_{GS}=0V$, with this voltage V_{DS} , an appreciable drain current I_{DSS} flows.
- 3) If the gate to source voltage is made negative i.e. V_{GS} is negative. Positive charges are induced in the channel through the SiO_2 of the gate capacitor.
- 4) Since the current in a FET is due to majority carriers (electrons for an N-type material), the induced positive charges make the channel less conductive and the drain current drops as V_{GS} is made more negative.
- 5) The redistribution of charge in the channel causes an effective depletion of majority carriers, which accounts for the designation depletion MOSFET.
- 6) That means biasing voltage V_{GS} depletes the channel of free carriers. This effectively reduces the width of the channel, increasing its resistance.
- 7) Note that negative V_{GS} has the same effect on the MOSFET as it has on the JFET.

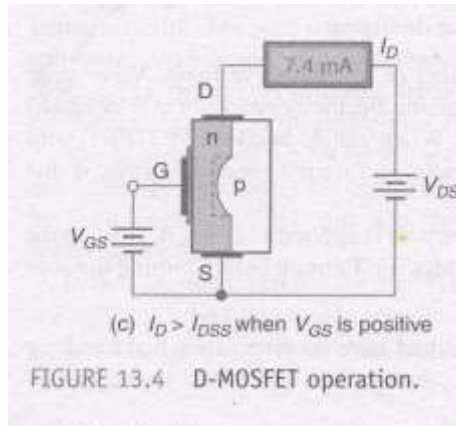


- 8) As shown in the fig above, the depletion layer generated by V_{GS} (represented by the white space between the insulating material and the channel) cuts into the channel, reducing its width. As a result, $I_D < I_{DSS}$. The actual value of I_D depends on the value of I_{DSS} , $V_{GS\text{OFF}}$ and V_{GS} .

Enhancement mode operation of the D-MOSFET:-

- 1) This operating mode is a result of applying a positive gate to source voltage V_{GS} to the device.
- 2) When V_{GS} is positive, the channel is effectively widened. This reduces the resistance of the channel, allowing I_D to exceed the value of I_{DSS} .
- 3) When V_{GS} is given positive, the majority carriers in the p-type are holes. The holes in the P-type substrate are repelled by the +ve gate voltage.
- 4) At the same time, the conduction band electrons (minority carriers) in the P-type material are attracted towards the channel by the +ve gate voltage.
- 5) With the buildup of electrons near the channel, the area to the right of the physical channel effectively becomes an N-type material.

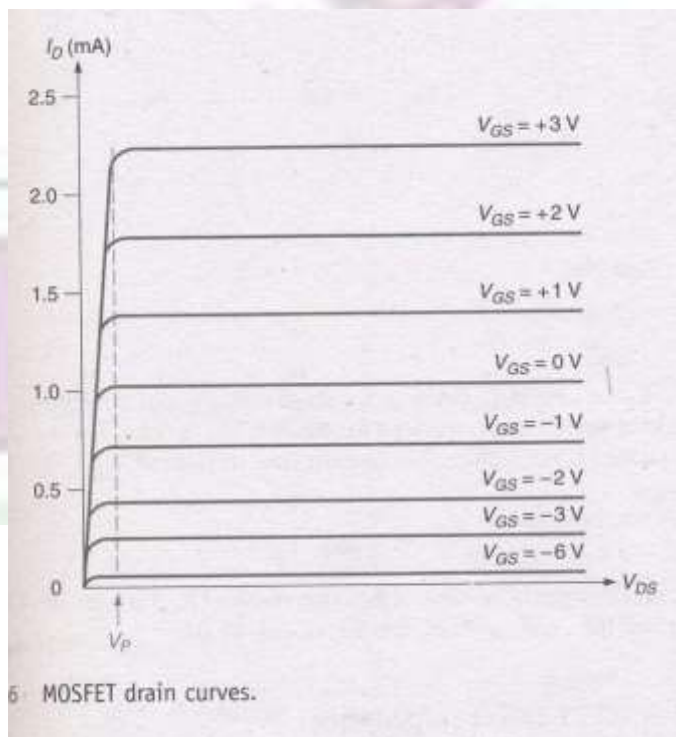
- 6) The extended n-type channel now allows more current, $I_D > I_{DSS}$



Characteristics of Depletion MOSFET:-

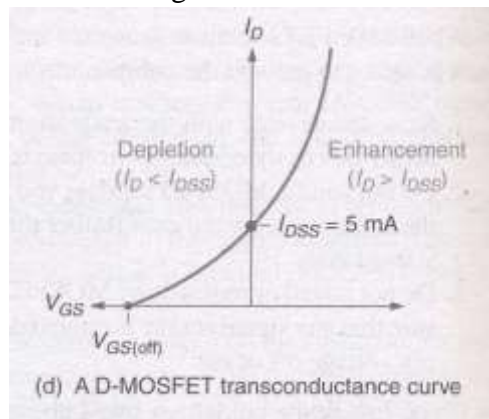
The fig. shows the drain characteristics for the N-channel depletion type MOSFET

- 1) The curves are plotted for both V_{GS} positive and V_{GS} negative voltages
- 2) When $V_{GS}=0$ and negative the MOSFET operates in depletion mode when V_{GS} is positive, the MOSFET operates in the enhancement mode.
- 3) The difference between JFET and DMOSFET is that JFET does not operate for positive values of V_{GS} .
- 4) When $V_{DS}=0$, there is no conduction takes place between source to drain, if $V_{GS} < 0$ and $V_{DS} > 0$ then I_D increases linearly.
- 5) But as $V_{GS}=0$ induces positive charges holes in the channel, and controls the channel width. Thus the conduction between source to drain is maintained as constant, i.e. I_D is constant.
- 6) If $V_{GS} > 0$ the gate induces more electrons in channel side, it is added with the free electrons generated by source. again the potential applied to gate determines the channel width and maintains constant current flow through it as shown in Fig



TRANSFER CHARACTERISTICS:-

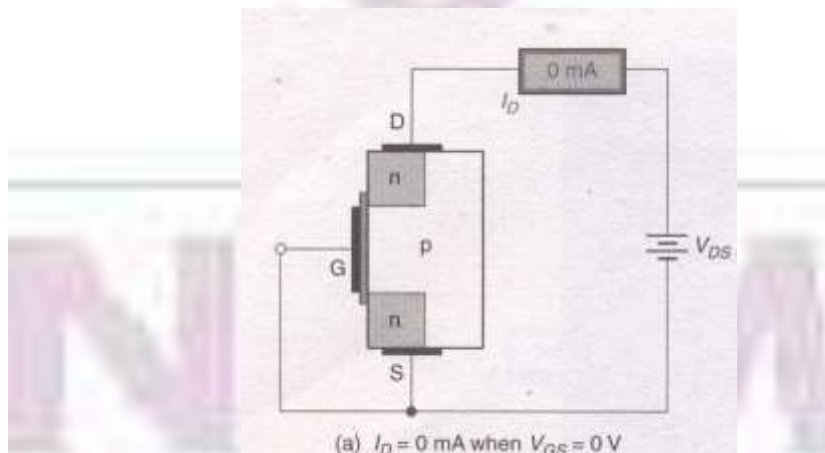
The combination of 3 operating states i.e. $V_{GS}=0V$, $V_{GS}<0V$, $V_{GS}>0V$ is represented by the D MOSFET transconductance curves shown in Fig.



- 1) Here in this curve it may be noted that the region AB of the characteristic is similar to that of JFET.
- 2) This curve extends for the positive values of V_{GS}
- 3) Note that $I_D = I_{DSS}$ for $V_{GS} = 0V$ when V_{GS} is negative, $I_D < I_{DSS}$ when $V_{GS} = V_{GS(off)}$, I_D is reduced to approximately 0mA. Where V_{GS} is positive $I_D > I_{DSS}$. So obviously I_{DSS} is not the maximum possible value of I_D for a MOSFET.
- 4) The curve is similar to JFET so that the D MOSFET has the same transconductance equation.

E-MOSFETS

The E MOSFET is capable of operating only in the enhancement mode. The gate potential must be positive w.r.t. to source.

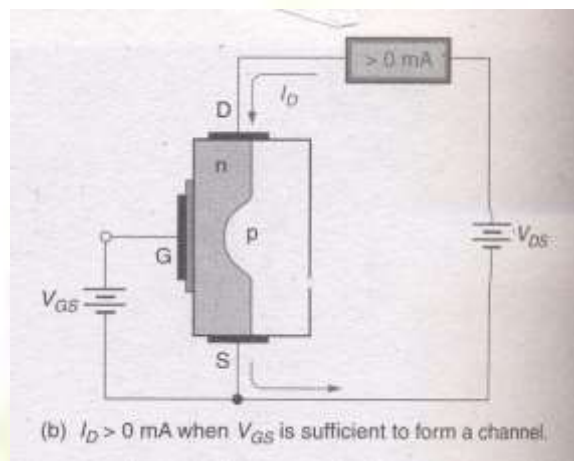


- 1) when the value of $V_{GS} = 0V$, there is no channel connecting the source and drain materials.
- 2) As a result, there can be no significant amount of drain current.
- 3) When $V_{GS} = 0$, the V_{DD} supply tries to force free electrons from source to drain but the presence of p-region does not permit the electron to pass through it. Thus there is no drain current at $V_{GS} = 0$.

- 4) If V_{GS} is positive, it induces a negative charge in the p-type substrate just adjacent to the SiO_2

layer.

- 5) As the holes are repelled by the positive gate voltage, the minority carrier electrons attracted toward this voltage. This forms an effective N-type bridge between source and drain providing a path for drain current.
- 6) This +ve gate voltage forms a channel between the source and drain.
- 7) This produces a thin layer of N-type channel in the P-type substrate. This layer of free electrons is called N-type inversion layer.

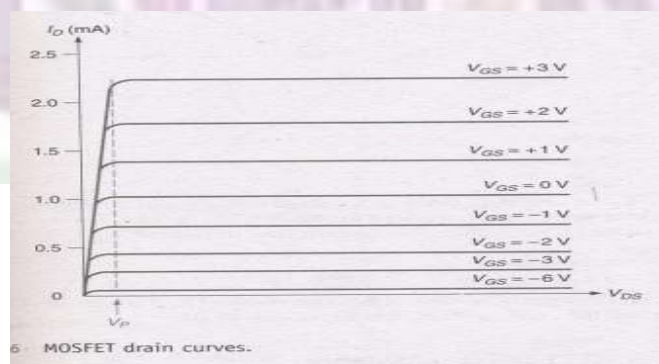


- 8) The minimum V_{GS} which produces this inversion layer is called threshold voltage and is designated by $V_{GS(th)}$. This is the point at which the device turns on is called the threshold voltage $V_{GS(th)}$
- 9) When the voltage V_{GS} is $< V_{GS(th)}$ no current flows from drain to source.
- 10) However when the voltage $V_{GS} > V_{GS(th)}$ the inversion layer connects the drain to source and we get significant values of current.

CHARACTERISTICS OF MOSFET:

1. DRAIN CHARACTERISTICS:

The voltage drain characteristics of an N-channel enhancement mode MOSFET are given in.



2. TRANSFER CHARACTERISTICS:

- 1) The current I_{DSS} at $V_{GS} \leq 0$ is very small being of the order of a few nanoamps.
- 2) As V_{GS} is made +ve, the current I_D increases slowly at first, and then much more rapidly with an increase in V_{GS} .
- 3) The standard transconductance formula will not work for the E-MOSFET.
- 4) To determine the value of I_D at a given value of V_{GS} we must use the following relation

$$I_D = K[V_{GS} - V_{GS(Th)}]^2$$

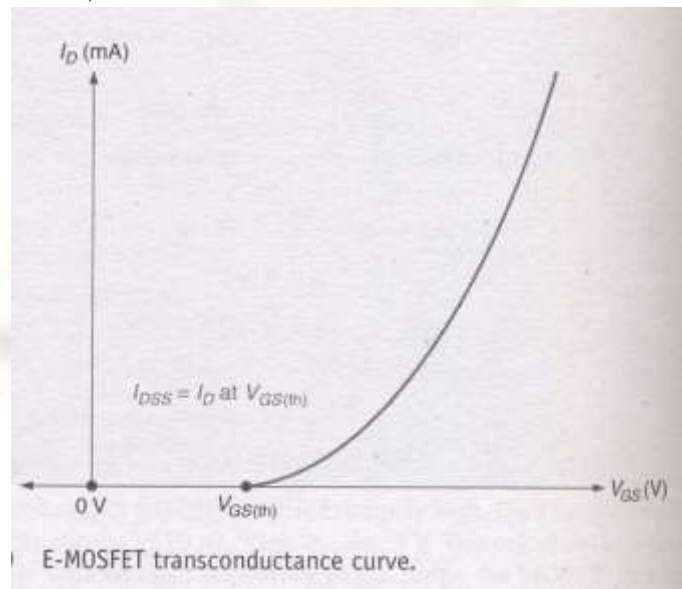
Where K is constant for the MOSFET. found as

$$K = \frac{I_{D(on)}}{[V_{GS(on)} - V_{GS(Th)}]^2}$$

From the data specifications sheets, the 2N7000 has the following ratings.

$I_{D(on)} = 75 \text{ mA (minimum)}$. And

$V_{GS(th)} = 0.8 \text{ (minimum)}$



APPLICATION OF MOSFET

One of the primary contributions to electronics made by MOSFETs can be found in the area of digital (computer electronics). The signals in digital circuits are made up of rapidly switching dc levels. This signal is called as a rectangular wave, made up of two dc levels (or logic levels). These logic levels are 0V and +5V.

A group of circuits with similar circuitry and operating characteristics is referred to as a logic family. All the circuits in a given logic family respond to the same logic levels, have similar speed and power-handling capabilities, and can be directly connected together. One such logic family is complementary MOS (or CMOS) logic. This logic family is made up entirely of MOSFETs.

COMPARISON OF MOSFET WITH JFET

- a. In enhancement and depletion types of MOSFET, the transverse electric field induced across an insulating layer deposited on the semiconductor material controls the conductivity of the channel.
- b. In the JFET the transverse electric field across the reverse-biased PN junction controls the conductivity of the channel.

- c. The gate leakage current in a MOSFET is of the order of 10^{-12} A. Hence the input resistance of a MOSFET is very high in the order of 10^{10} to $10^{15} \Omega$. The gate leakage current of a JFET is of the order of 10^{-9} A., and its input resistance is of the order of $10^8 \Omega$.

- d. The output characteristics of the JFET are flatter than those of the MOSFET, and hence the drain resistance of a JFET (0.1 to $1\text{M}\Omega$) is much higher than that of a MOSFET (1 to $50\text{k}\Omega$).
- e. JFETs are operated only in the depletion mode. The depletion type MOSFET may be operated in both depletion and enhancement mode.
- f. Comparing to JFET, MOSFETs are easier to fabricate.
- g. Special digital CMOS circuits are available which involve near zero power dissipation and very low voltage and current requirements. This makes them suitable for portable systems.

