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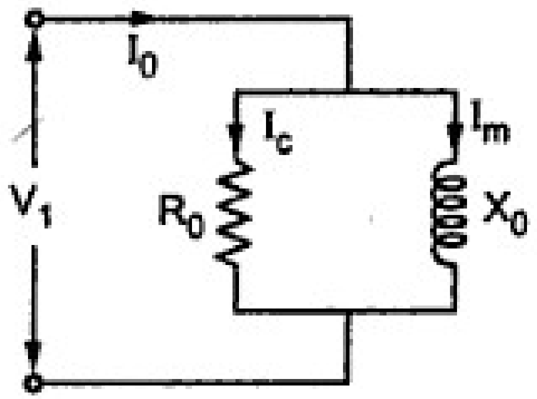
UNIT-IV

Equivalent circuit of a transformer

Equivalent circuit of a transformer

No load equivalent circuit:



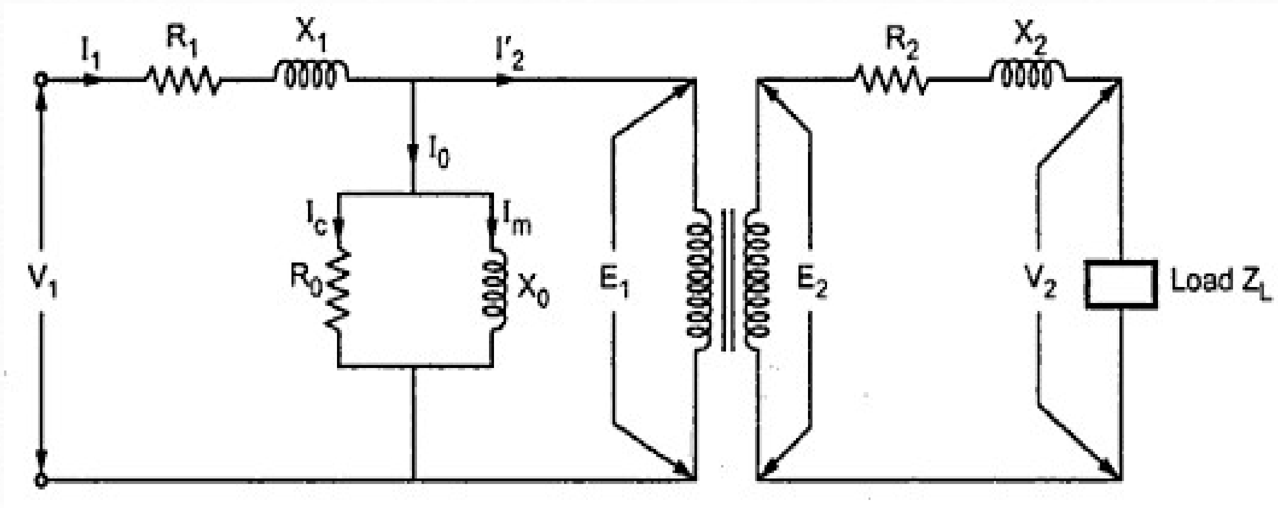
$$R_0 = \frac{V_1}{I_c}$$

$$X_0 = \frac{V_1}{I_m}$$

$$I_m = I_0 \sin \phi_0 = \text{Magnetising component}$$

$$I_c = I_0 \cos \phi_0 = \text{Active component}$$

Equivalent circuit parameters referred to primary and secondary sides respectively



Contd.,

- The effect of circuit parameters shouldn't be changed while transferring the parameters from one side to another side
- It can be proved that a resistance of R_2 in sec. is equivalent to R_2/k^2 will be denoted as R_2'(ie. Equivalent sec. resistance w.r.t primary) which would have caused the same loss as R_2 in secondary,

$$I_1^2 R_2' = I_2^2 R_2$$

$$R_2' = \left(\frac{I_2}{I_1}\right)^2 R_2$$

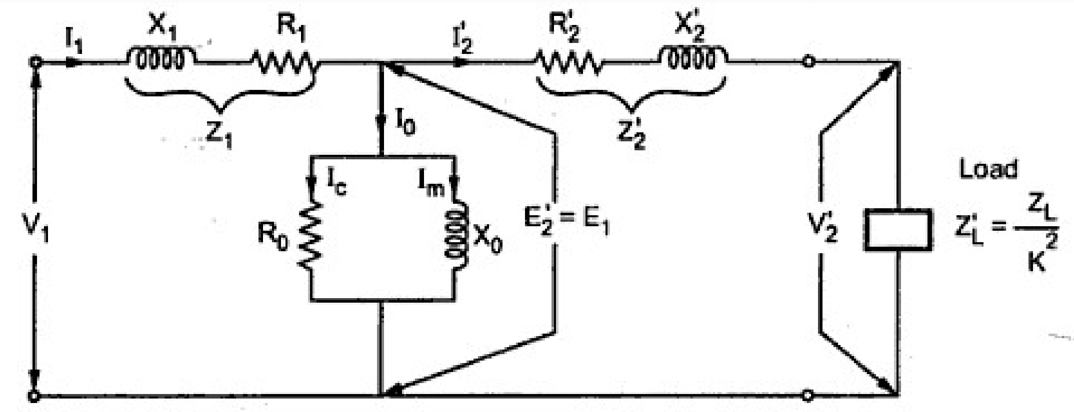
$$= \frac{R_2}{k^2}$$

Transferring secondary parameters to primary side

$$R'_2 = \frac{R_2}{K^2}, \quad X'_2 = \frac{X_2}{K^2}, \quad Z'_2 = \frac{Z_2}{K^2}$$

While $$E'_2 = \frac{E_2}{K}, \quad I'_2 = KI_2$$

where $$K = \frac{N_2}{N_1}$$



Exact equivalent circuit referred to primary

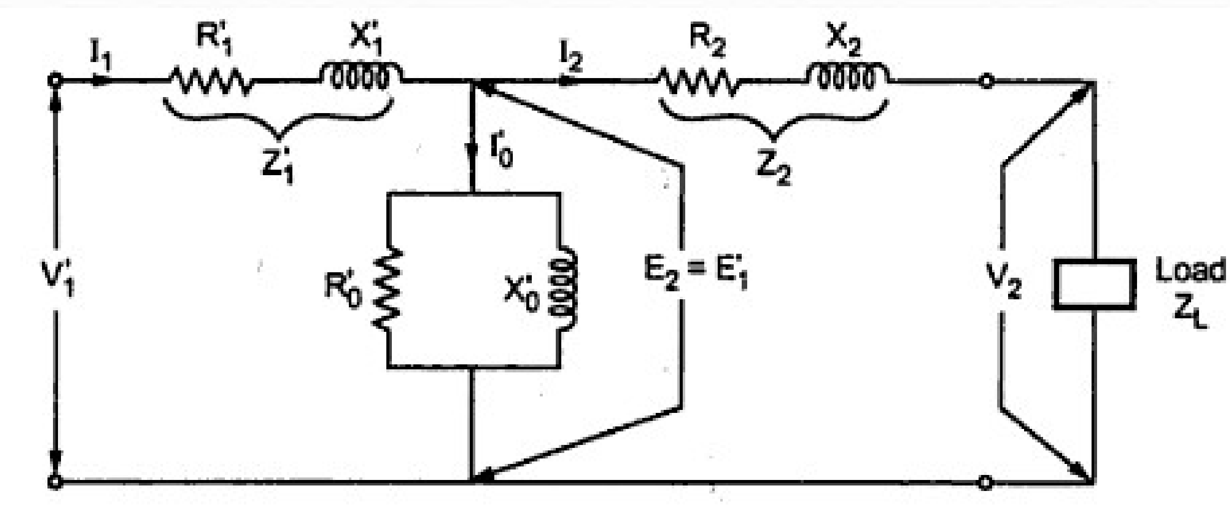
Equivalent circuit referred to secondary side

- Transferring primary side parameters to secondary side

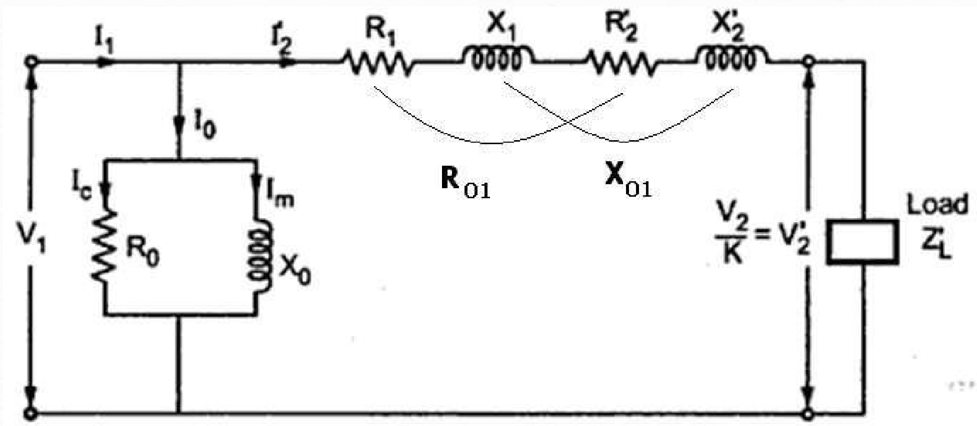
$$R_1' = K^2 R_1, \quad X_1' = K^2 X_1, \quad Z_1' = K^2 Z_1$$

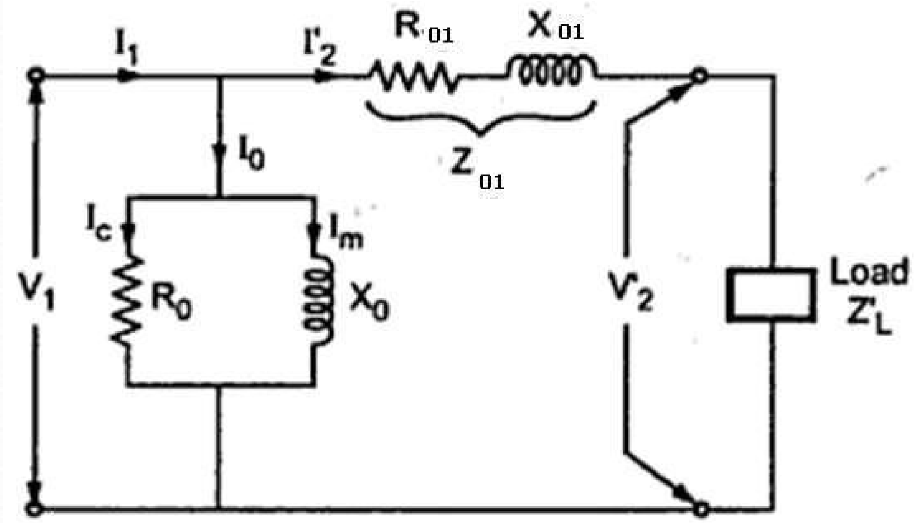
$$E_1' = K E_1, \quad I_1' = \frac{I_1}{K}, \quad I_0' = \frac{I_0}{K}$$

Similarly exciting circuit parameters are also transferred to secondary as R_o' and X_o'



Equivalent circuit w.r.t primary





where

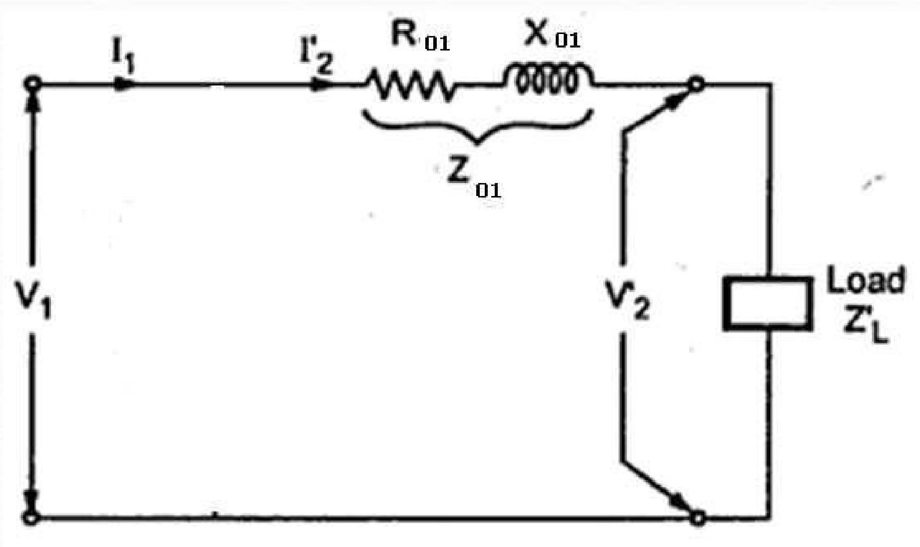
$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}$$

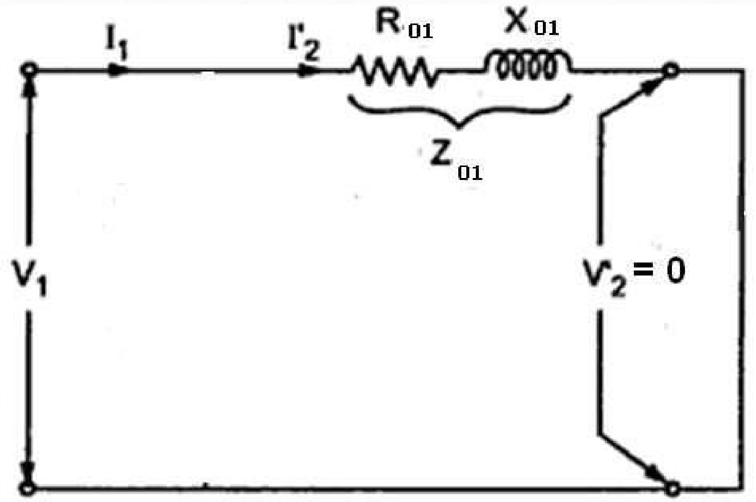
$$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2}$$

$$Z_{01} = R_{01} + j\,X_{01}$$

Approximate equivalent circuit

- Since the noload current is 1% of the full load current, the nolad circuit can be neglected





$$\text{Full load cu loss} = \mathrm{W}_{\mathrm{sc}} = I_{sc}^2 R_{01}$$

$$\mathrm{R}_{01} = \frac{\mathrm{W}_{\mathrm{sc}}}{I_{sc}^2}$$

$$Z_{01} = \frac{V_{sc}}{I_{sc}}$$

$$\therefore \mathrm{X}_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$$

Losses in a transformer

Core or Iron loss:

Hysteresis loss $W_h = \eta B^{1.6}_{max} f V$ watt;

eddy current loss $W_e = \eta B^2_{max} f^2 t^2$ watt

Copper loss:

Total Cu loss $= I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{01} + I_2^2 R_{02}$.

Transformer Voltage Regulation and Efficiency

The output voltage of a transformer varies with the load even if the input voltage remains constant. This is because a real transformer has series impedance within it. Full load Voltage Regulation is a quantity that compares the output voltage at no load with the output voltage at full load, defined by this equation:

$$\text{At noload } k = \frac{V_s}{V_p}$$

$$\text{Regulation up} = \frac{V_{S,nl} - V_{S,fl}}{V_{S,fl}} \times 100\%$$

$$\text{Regulation up} = \frac{(V_P / k) - V_{S,fl}}{V_{S,fl}}\, x\, 100\%$$

$$\text{Regulation down} = \frac{V_{S,nl} - V_{S,fl}}{V_{S,nl}} \times 100\%$$

$$\text{Regulation down} = \frac{(V_P / k) - V_{S,fl}}{V_{S,nl}}\, x\, 100\%$$

Ideal transformer, VR = 0%.

Voltage regulation of a transformer

$$\text{Voltage regulation} = \frac{\text{no - load voltage} - \text{full - load voltage}}{\text{no - load voltage}}$$

recall $\frac{V_s}{V_p} = \frac{N_s}{N_p}$

Secondary voltage on no-load $V_2 = V_1\left(\frac{N_2}{N_1}\right)$

V_2 is a secondary terminal voltage on full load

Substitute we have

$$\text{Voltage regulation} = \frac{V_1\left(\frac{N_2}{N_1}\right) - V_2}{V_1\left(\frac{N_2}{N_1}\right)}$$

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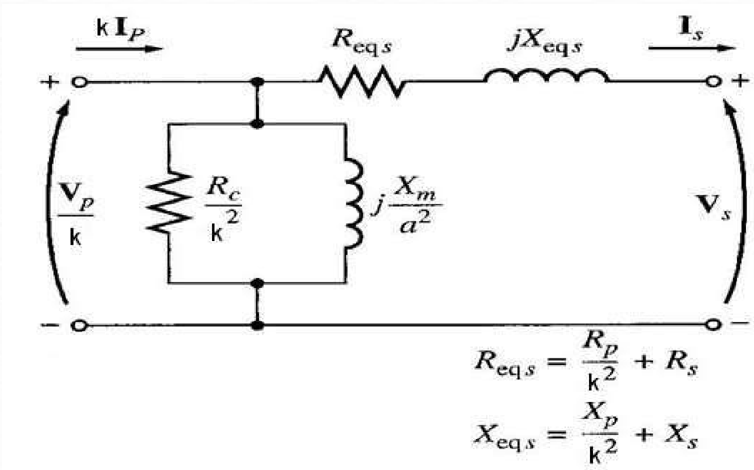
Transformer Phasor Diagram

To determine the voltage regulation of a transformer, it is necessary understand the voltage drops within it.



Transformer Phasor Diagram

Ignoring the excitation of the branch (since the current flow through the branch is considered to be small), more consideration is given to the series impedances (R_{eq} +jX_{eq}).

Voltage Regulation depends on magnitude of the series impedance and the phase angle of the current flowing through the transformer.

Phasor diagrams will determine the effects of these factors on the voltage regulation. A phasor diagram consist of current and voltage vectors.

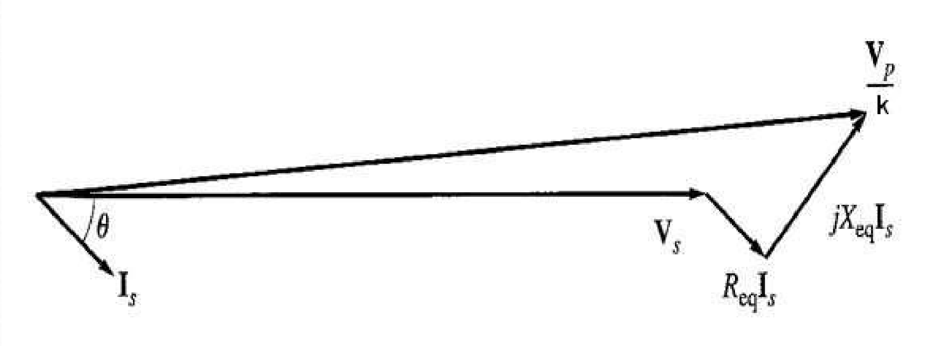
Assume that the reference phasor is the secondary voltage, V_S. Therefore the reference phasor will have 0 degrees in terms of angle.

Based upon the equivalent circuit, apply Kirchoff Voltage Law,

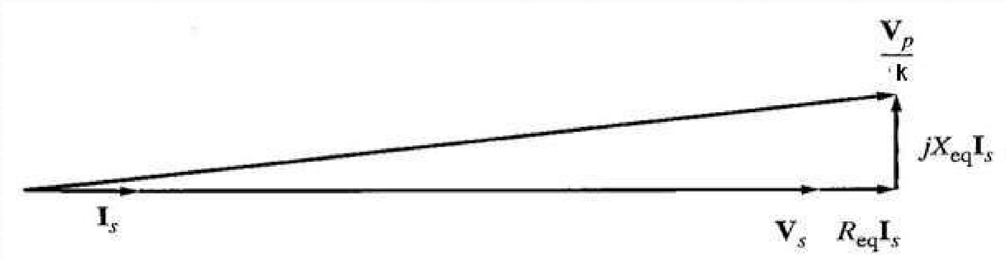
$$\frac{V_P}{k} = V_S + R_{eq} I_S + jX_{eq} I_S$$

Transformer Phasor Diagram

For lagging loads, V_P / a > V_S so the voltage regulation with lagging loads is > 0.



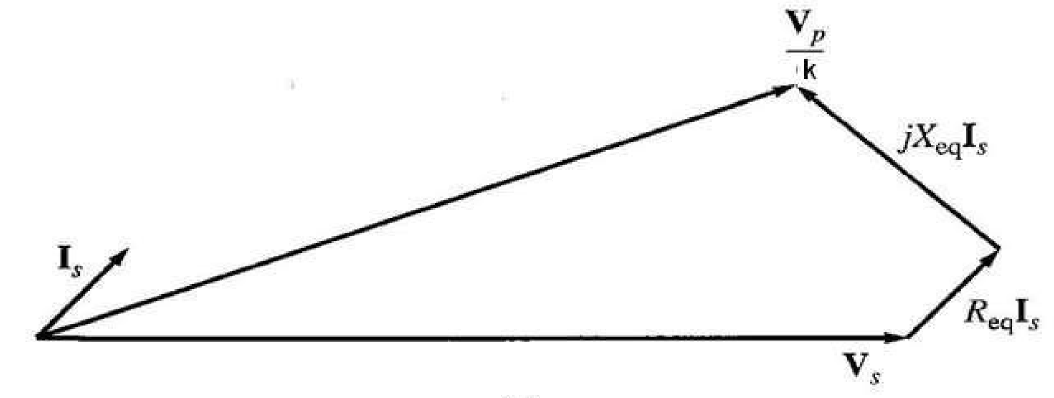
When the power factor is unity, V_S is lower than V_P so VR > 0.



(a)

Transformer Phasor Diagram

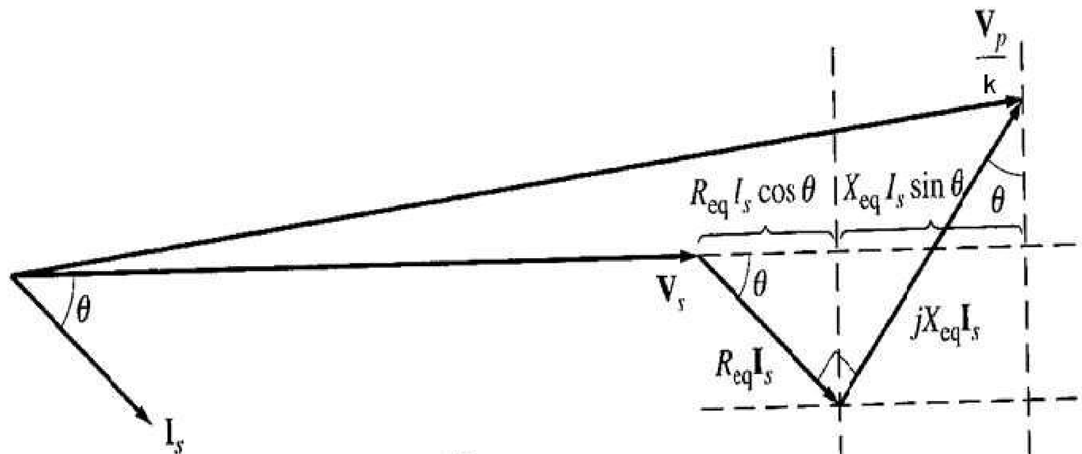
With a leading power factor, V_S is higher than the referred V_P so VR < 0



(b)

Transformer Phasor Diagram

For lagging loads, the vertical components of R_{eq} and X_{eq} will partially cancel each other. Due to that, the angle of V_P/a will be very small, hence we can assume that V_P/k is horizontal. Therefore the approximation will be as follows:



$$\frac{V_p}{k} \approx V_s + R_{eq} I_s \cos\theta + X_{eq} I_s \sin\theta$$

Formula: voltage regulation

In terms of secondary values

$$\%\ \text{regulation} = \frac{{}_0V_2 - V_2}{{}_0V_2} = \frac{I_2 R_{02}\cos\phi_2 \pm I_2 X_{02}\sin\phi_2}{{}_0V_2}$$

where '+' for lagging and '-'for leading

In terms of primary va lues

$$\%\ \text{regulation} = \frac{V_1 - V_2^{'}}{V_1} = \frac{I_1 R_{01}\cos\phi_1 \pm I_1 X_{01}\sin\phi_1}{V_1}$$

where '+' for lagging and '-'for leading

Transformer Efficiency

Transformer efficiency is defined as (applies to motors, generators and transformers):

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

$$\eta = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\%$$

Types of losses incurred in a transformer:

- Copper I^2R losses
- Hysteresis losses
- Eddy current losses

Therefore, for a transformer, efficiency may be calculated using the following:

$$\eta = \frac{V_S I_S \cos\theta}{P_{Cu} + P_{core} + V_S I_S \cos\theta} x100\%$$

Condition for maximum efficiency

$$\text{Cu loss} = I_1^2 R_{01} \quad \text{or} \quad I_2^2 R_{02} = W_{cu}$$

$$\text{Iron loss} = \text{Hysteresis loss} + \text{Eddy current loss} = W_h + W_e = W_i$$

Considering primary side,

$$\text{Primary input} = V_1 I_1 \cos\phi_1$$

$$\eta = \frac{V_1 I_1 \cos\phi_1 - \text{losses}}{V_1 I_1 \cos\phi_1} = \frac{V_1 I_1 \cos\phi_1 - I_1^2 R_{01} - W_i}{V_1 I_1 \cos\phi_1}$$

$$= 1 - \frac{I_1 R_{01}}{V_1 \cos\phi_1} - \frac{W_i}{V_1 I_1 \cos\phi_1}$$

Differentiating both sides with respect to I_1, we get

$$\frac{d\eta}{dI_1} = 0 - \frac{R_{01}}{V_1 \cos\phi_1} + \frac{W_i}{V_1 I_1^2 \cos\phi_1}$$

For η to be maximum, $\frac{d\eta}{dI_1} = 0$. Hence, the above equation becomes

$$\frac{R_{01}}{V_1 \cos\phi_1} = \frac{W_i}{V_1 I_1^2 \cos\phi_1} \quad \text{or} \quad W_i = I_1^2 R_{01} \quad \text{or} \quad I_2^2 R_{02}$$

or $\quad$ **Cu loss = Iron loss**

Contd.,

The output current corresponding to maximum efficiency is $I_2 = \sqrt{(W_i/R_{02})}$.

The load at which the two losses are equal $= \text{Full load} \times \sqrt{\left(\dfrac{\text{Iron loss}}{\text{F.L. Cu loss}}\right)}$

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All day efficiency

$$\text{ordinary commercial efficiency} = \frac{\text{out put in watts}}{\text{input in watts}}$$

$$\eta_{all\ \text{day}} = \frac{\text{output in kWh}}{\text{Input in kWh}}(\mathit{for}\ 24\ \text{hours})$$

- All day efficiency is always less than the commercial efficiency

APPLICATION AND USES

- The transformer used in television and photocopy machines
- The transmission and distribution of alternating power is possible by transformer
- Simple camera flash uses fly back transformer
- Signal and audio transformer are used couple in amplifier